

Learning image transformations via convolutional neural networks: a review

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Abstract

Recent methodological innovations in deep learning and associated advancements in computational hardware have significantly impacted the various core subfields of quantitative medical image analysis. The generalizability, computational efficiency, and open-source availability of deep learning algorithms, particularly those utilizing convolutional neural networks, have produced paradigm shifts within the field. This impact is evident from topical prevalence in the literature, conference and workshop themes, and winning methodologies in relevant competitions. In this work, we review the various state-of-the-art, approaches to learning and prediction and/or optimizing image transformations using convolutional neural networks. Although of primary importance within the quantitative imaging domain, image registration algorithmic development, in the context of these deep learning strategies, has received comparatively less attention than its counterparts (e.g., image segmentation). Nevertheless, significant progress has been made in this particular subfield which has been presented in various research venues. We contextualize these contributions within the broader scope of deep learning advancements and, in so doing, attempt to facilitate the leveraging and further development of such techniques within the medical imaging research community.

Key words: deep learning, diffeomorphisms, image registration, spatial normalization

Introduction

Determining the spatial correspondence between imaging domains is frequently a critical component in quantitative image analysis workflows. The evolution of image registration theoretical and technological development has led to increasingly high quality transformational mappings that have significantly improved performance in related processing tasks (e.g., image segmentation via joint label fusion [1]) and imaging-based statistical analyses involving template-based analysis (e.g., sparse canonical correlation analysis [2]). Several reviews [3–8] have charted this chronology and provided insight into related issues such as algorithmic classification, available implementations, evaluation strategies, and speculation concerning future directions of the field. While prescient in many respects, speculation vis-à-vis the resurgence of deep learning was understandably limited due to its recent explosion in popularity and research focus.

The foundational concepts that form the basis for contemporary deep learning research dates back decades (e.g., [9]). Since this early seminal work, major developmental milestones include the *Neocognitron*, an early neural network for character recognition [10], and convolutional neural networks (“CNNs” or “ConvNets”) utilized in speech [11] and visual signal processing [12], largely inspired by the visual cell types of the feline visual cortex [13]. The major elements of CNNs are localized connections, convolutions, and subsampling (or “pooling”) [14]. Furthermore, it is the deep, or “hidden,” layering that characterizes modern CNNs and is the reason for the extreme performance gains seen with modern architectures. The training of such architectures is made computationally tractable with gradient-based optimization using backpropagation (first performed in [12]) and the advent of GPU-based hardware [14]. An illustration of a bare-bones CNN configuration is provided in Figure 1 which illustrates the core components of convolution and max pooling. Architectural novelty derives from innovative arrangements of these core (and other) network components and the connections between them.

A key event in the widespread adoption of CNNs was the 2012 ImageNet Large Scale Visual Recognition Challenge for object classification [15]. The winning entry, a CNN-based architecture colloquially known as *AlexNet* [16], reduced the error rate by almost half over other entries. The following years’ competitions were dominated by CNN variants such as VGG [17], GoogLeNet [18], and ResNet

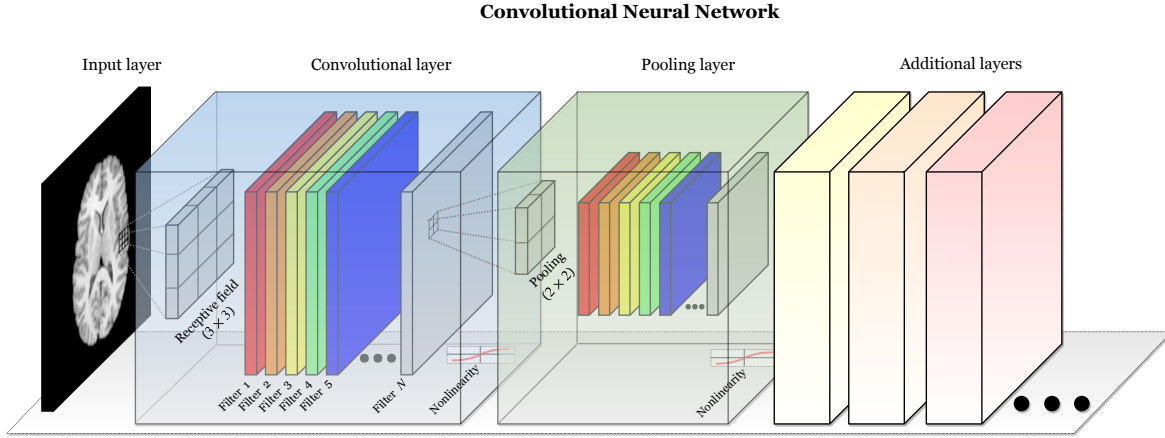


Figure 1: The basic elements of the prop convolutional neural network. The convolutional layer comprises several filters which are optimized in terms of their responses to various features found in the input layer. Pooling is used to extract salient features and reduce computational complexity and passed on to subsequent layers.

[19] with performance ultimately exceeding human performance in 2015 [20]. Additional competition outlets including conference-based venues (e.g., NeurIPS) and community-based platforms, such as Kaggle¹, continue to highlight the salience of CNNs as paradigmatic solutions to computational problems. This is in addition to the sheer number of formal research reports discussed in the same conferences and published in dedicated journals. Notable reviews by key figures in the field include those of Yann LeCun [14] and Jürgen Schmidhuber [22].

Early work specific to medical imaging date back to the 1990s with classification tasks providing the majority of use cases (e.g., lung nodule classification [23, 24] and breast tissue differentiation [25, 26]). Despite the early adoption by certain research groups, widespread uptake did not occur until much later. Although several deep learning overviews specific to medical imaging have been recently presented in the research literature:

- editorial [27],
- specific to generative adversarial networks (GANs) [28],
- specific to MRI [29],
- applications [30], and

¹Following the 2017 ImageNet challenge, in which the vast majority of teams surpassed the 5% classification error rate threshold, the ImageNet organizers ceded management to the Kaggle community which maintains a running performance assessment in ostensible perpetuity [21].

- general reviews [31–34];

discussion of adoption within the community is limited. However, one can informally gauge this evolution from utilization of alternative machine learning techniques to predominately CNN-based approaches from the various competitions held simultaneously with medical imaging conferences. For example, the annual Multimodal Brain Tumor Segmentation (BraTS) Challenge has taken place under the auspices of the International Conference on Medical Image Computing and Computer Assisted Intervention (MICCAI) since 2012 wherein large sets of training data are provided to the competitors who attempt to perform a voxelwise labeling of the constituent components of tumors from multimodal MR image data. The winning entries from the first two years employed random forest classifiers for segmentation [35]. Although variations of the traditional random forest scheme continued to be well represented in the 2014 Challenge, convolutional neural networks made an appearance [36]. By 2018, CNN-based pipelines were, by far, the most common [37] with specific preference being that of the U-net architecture [38] which, as we describe below, features prominently in image registration.

- Early work in medical image registration with the GPU focused on interfacing with the hardware directly [39, 40]. One of the review papers listed this as well.

Spatial transformer networks

In 2015 Jaderberg and his fellow co-authors described a powerful new module, known as the spatial transformer network (STN) [41], which figures prominently in many of the image registration approaches that we review below. Although conceptually relatively straightforward, the obvious influence reflected in recent work will most likely continue in future research which is why we review this important network component.

Generally, STNs enhance CNNs by permitting a flexibility which allows for an explicit spatial invariance that goes beyond the implicitly limited translational invariance associated with the architecture’s pooling layers. In many image-based tasks (e.g., localization or segmentation), designing an algorithm that can account for possible pose or geometric variation of the object(s) of interest within the image is crucial for maximizing performance. The STN is a fully differentiable layer which can be

inserted anywhere in the CNN to learn the parameters of the transformation of the input feature map (not necessarily an image) which renders the output in such a way to optimize the network based on the specified loss function. The added flexibility and the fact that there is no manual supervision or special handling required makes this module an essential addition for any CNN-based toolkit.

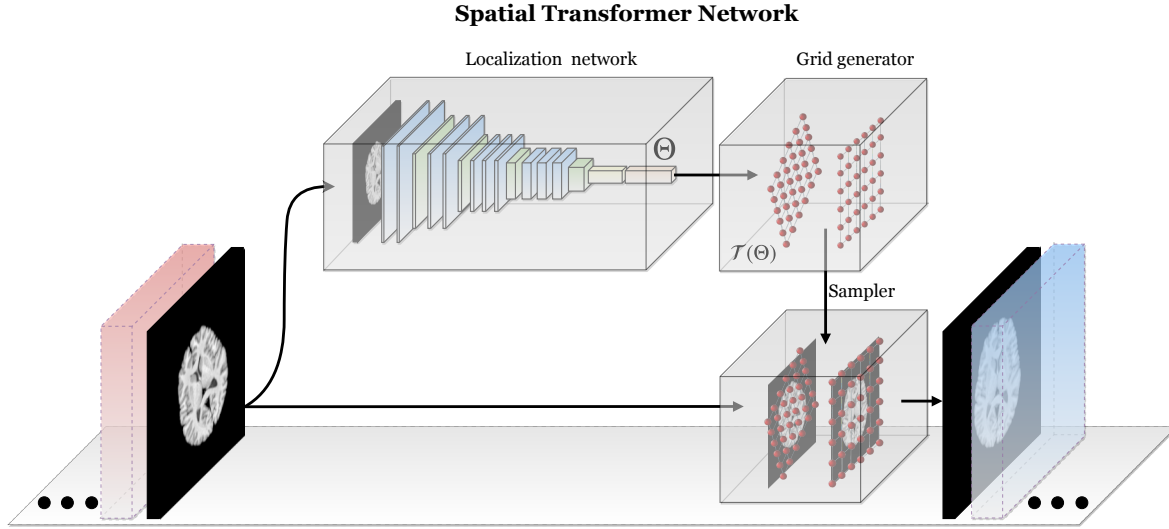


Figure 2: Diagrammatic illustration of the spatial transformer network. The STN can be placed anywhere within a CNN to provide spatial invariance for the input feature map. Core components include the localization network used to learn/predict the parameters which transform the input feature map. The transformed output feature map is generated with the grid generator and sampler.

An STN comprises three principal components: 1) a localization network, 2) a grid generator, and 3) a sampler (see Figure 3). The localization network uses the input feature map to learn/regress the transformation parameters which optimize a specified loss function. In many examples provided, this amounts to transforming the input feature map to a quasi-canonical configuration to facilitate, for example, classification. The actual architecture of the localization network is fairly flexible and any conventional architecture, such as a fully connected network (FCN), is suitable as long as the output maps to the continuous estimate of the transformation parameters. These transformation parameters are then applied to the output of the grid generator which are simply the regular coordinates of the input image (or some normalized version thereof). The sampler, or interpolator, is used to map the transformed input feature map to the coordinates of the output feature map.

Inverse compositional transformer networks

[42] Inspired by the IC-LK algorithm, we advocate an improved extension to the STN framework that (a) propagates warp parameters, rather than image intensities

Diffeomorphic transformer networks

Although discussion of transform generalizability was included in the original STN paper [41], discussion was limited to affine, attention (scaling + translation), and thin-plate spline transforms which all fill the requirements of differentiability. This was later extended to encompass a diffeomorphic transformer network (DTN) [43] based on continuous piecewise affine-based (CPAB) transformations [44]. **This section needs to be expanded.**

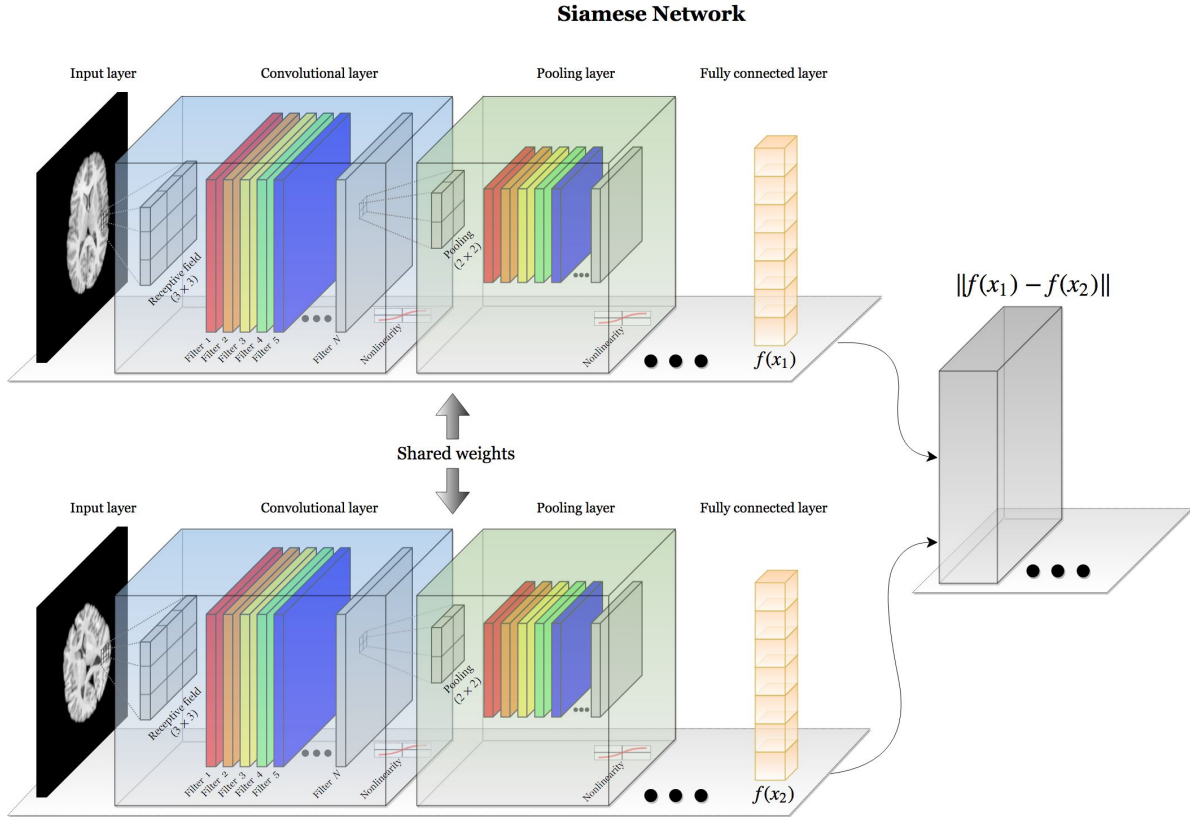


Figure 3: Diagrammatic illustration of the spatial transformer network.

Quicksilver

- Quicksilver uses a patch-based approach to predict the “momentum” of the LDDMM framework in determining the correspondence relationship between fixed and moving image patch pairs.
 - The large deformation diffeomorphic metric mappings (LDDMM) framework for image matching stems from the theoretical foundations underlying diffeomorphic *flows* ([45–47]), or transformations which are differentiable with differentiable inverses. The collection of *paths* between two images which describe the possible mappings between images is described by the functional (equation here) XX.
 - Computationally efficient algorithms for solving such diffeomorphic flows have been subsequently proposed (e.g., [48–53]).
 - The Euler-Lagrange equation for the functional XX can be written as a system of equations which incorporates a “momentum” term, $m_t = \text{Jac}(\phi_{t,1}^v)(I \circ \phi_{t,0} - J \circ \phi_{t,1})$. Several algorithms (e.g., [48, 52]) take advantage of the fact that the initial momentum, m_0 , completely encodes the optimum path describing the transform between images I and J .
 - Following [54], the initial momentum is calculated from the equation $m(x, 0) = \alpha(x, 0) \nabla I_0$ where α is a time-varying scalar field function and ∇I_0 is the spatial derivative of the image I .
 - In contrast to scalar formulations of the momenta (e.g., [52]), a vector formulation is proposed in [55], to avoid potential confounding effects of noise in the image gradient (i.e., better behaved numerically).
 - The prediction network is used to estimate the vector momentum
- Network architecture
 - Encoder/decoder structure for performing voxelwise image regression.
 - There are two separate encoders—one for the fixed image and one for the moving image.
 -
 - $15 \times 15 \times 15$ patches selected at stride = 14.

- Thus, Quicksilver is a supervised approach wherein it uses multiple image pairs and the corresponding ground-truth estimations of the momentum scalar maps to perform prediction of momentum on unseen image pairs (but patch-based).
- Written in pytorch? (torch (facebook) in python) and available at
- Misc. notes
 - Initial momentum is not generally smooth so smoothing is applied after prediction
 - Training data employed regular LDDMM shooting results using PyCA (page 385)
 - They also use dropout layers to induce a probabilistic network
 - They also train a correction network (Section 2.3). The prediction → correction networks is diagrammed in Figure 3.
 - The PyCA LDDMM, prediction, and prediction + correction are included in the evaluation.
 - Preprocessing:
 - * Skull-stripping (FreeSurfer and AutoSeg)
 - * Initial affine registration — NiftyReg

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