HCM (Hard c-means)

optimization problem

$$\underset{u,v}{\text{minimize}} \sum_{i=1}^{C} \sum_{k=1}^{N} u_{i,k} \|x_k - v_i\|_2^2 \quad \text{subject to} \quad \sum_{i=1}^{C} u_{i,k} = 1 \quad \text{and} \quad u_{i,k} \in \{0, 1\}$$

$$u_{i,k} = \begin{cases} 1 & (i = \arg\min_{1 \le j \le C} \{\|x_k - v_i\|_2^2\}) \\ 0 & (\text{otherwise}) \end{cases},$$

$$v_i = \frac{\sum_{k=1}^{N} u_{i,k} x_k}{\sum_{k=1}^{N} u_{i,k}}.$$

BFCM (Bezdek-type fuzzy c-means with variables controlling cluster sizes)

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} \sum_{i=1}^{C} \sum_{k=1}^{N} (\pi_i)^{1-m} (u_{i,k})^m \|x_k - v_i\|_2^2$$

$$d_{i,k} = \|x_k - v_i\|_2^2 = \left(\sqrt{\sum_{\ell=1}^{M} (x_{k,\ell} - v_{i,\ell})^2} \right)^2,$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C} \frac{\pi_j}{\pi_i} \left(\frac{d_{j,k}}{d_{i,k}} \right)^{\frac{1.0}{1.0-m}}}, \quad v_i = \frac{\sum_{k=1}^{N} (u_{i,k})^m x_k}{\sum_{k=1}^{N} (u_{i,k})^m},$$

$$\pi_i = \frac{1.0}{\sum_{j=1}^{C} \left(\sum_{k=1}^{N} \frac{(u_{j,k})^m d_{j,k}}{(u_{i,k})^m d_{i,k}} \right)^{\frac{1.0}{m}}}.$$

KLFCM(KL-divergence based Fuzzy *c*-means with variables controlling cluster sizes)

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} \sum_{i=1}^{C}\sum_{k=1}^{N} u_{i,k}\|x_k - v_i\|_2^2 + \lambda^{-1}\sum_{i=1}^{C}\sum_{k=1}^{N} u_{i,k}\log\left(\frac{u_{i,k}}{\pi_i}\right)$$

$$d_{i,k} = \|x_k - v_i\|_2^2,$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C}\frac{\pi_j}{\pi_i}\exp\left(-\lambda(d_{j,k} - d_{i,k})\right)}, \quad v_i = \frac{\sum_{k=1}^{N} u_{i,k}x_k}{\sum_{k=1}^{N} u_{i,k}},$$

$$\pi_i = \frac{\sum_{k=1}^{N} u_{i,k}}{N}.$$

QFCM (q-divergence based Fuzzy *c*-means with variables controlling cluster sizes)

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} \sum_{i=1}^{C}\sum_{k=1}^{N}(\pi_i)^{1-m}(u_{i,k})^m\|x_k - v_i\|_2^2 + \frac{\lambda^{-1}}{m-1}\sum_{i=1}^{C}\sum_{k=1}^{N}(\pi_i)^{1-m}(u_{i,k})^m$$

$$d_{i,k} = \|x_k - v_i\|_2^2,$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C}\frac{\pi_j}{\pi_i}\left(\frac{1.0-\lambda(1.0-m)d_{j,k}}{1.0-\lambda(1.0-m)d_{i,k}}\right)^{\frac{1.0}{1.0-m}}}, \quad v_i = \frac{\sum_{k=1}^{N}(u_{i,k})^m x_k}{\sum_{k=1}^{N}(u_{i,k})^m},$$

$$\pi_i = \frac{1.0}{\sum_{j=1}^{C}\left(\sum_{k=1}^{N}\frac{(u_{j,k})^m(1.0-\lambda(1.0-m)d_{j,k})}{(u_i,k)^m(1.0-\lambda(1.0-m)d_{i,k})}\right)^{\frac{1.0}{m}}}.$$

ref : `https://www.jstage.jst.go.jp/article/fss/30/0/30_452/_pdf`

FCCM

optimization problem

$$\underset{u,w,\pi}{\text{minimize}} \sum_{i=1}^{C}\sum_{k=1}^{N}\sum_{\ell=1}^{M} -u_{i,k}w_{i,\ell}x_{k,\ell} + \lambda_1^{-1}\sum_{i=1}^{C}\sum_{k=1}^{N} u_{i,k}\log\left(\frac{u_{i,k}}{\pi_i}\right) + \lambda_2^{-1}\sum_{i=1}^{C}\sum_{\ell=1}^{M} w_{i,\ell}\log(w_{i,\ell})$$

$$d_{i,k} = -\sum_{\ell=1}^{M} w_{i,\ell}x_{k,\ell}, \tag{1}$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C}\frac{\pi_j}{\pi_i}\exp\left(-\lambda(d_{j,k}-d_{i,k})\right)}, \tag{2}$$

$$w_{i,\ell} = \frac{1.0}{\sum_{r=1}^{M}\exp\left(\lambda_2\sum_{k=1}^{N}(u_{i,k}x_{k,r}-u_{i,k}x_{k,\ell})\right)}, \quad \pi_i = \frac{1}{N}\sum_{k=1}^{N} u_{i,k}. \tag{3}$$

ref : `http://ieeexplore.ieee.org/document/944403/`

FCCMb

optimization problem

$$\underset{u,w,\pi}{\text{minimize}} - \sum_{i=1}^{C} \sum_{k=1}^{N} \sum_{\ell=1}^{M} (\pi_i)^{1-m_1} (u_{i,k})^{m_1} (w_{i,\ell})^{m_2} x_{k,\ell}$$

$$d_{i,k} = - \sum_{\ell=1}^{M} (w_{i,\ell})^{m_2} x_{k,\ell},$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C} \frac{\pi_j}{\pi_i} \left(\frac{d_{j,k}}{d_{i,k}} \right)^{\frac{1.0}{1.0-m_1}}}, \quad w_{i,\ell} = \frac{1.0}{\sum_{r=1}^{M} \left(\sum_{k=1}^{N} \frac{(u_{i,k})^{m_1} x_{k,r}}{(u_{i,k})^{m_1} x_{k,\ell}} \right)^{\frac{1.0}{1.0-m_2}}},$$

$$\pi_i = \frac{1.0}{\sum_{j=1}^{C} \left(\sum_{k=1}^{N} \frac{(u_{j,k})^{m_1} d_{j,k}}{(u_{i,k})^{m_1} d_{i,k}} \right)^{\frac{1.0}{m_1}}}.$$

ref : `https://www.fujipress.jp/jaciii/jc/jacii001900060852/`

BFCCM

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} - \sum_{i=1}^{C} \sum_{k=1}^{N} \sum_{\ell=1}^{M} (\pi_i)^{1-m} (u_{i,k})^m \log\left(w_{i,\ell}\right) x_{k,\ell}$$

$$d_{i,k} = -\sum_{\ell=1}^{M} \log(w_{i,\ell}) x_{k,\ell},$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C} \frac{\pi_j}{\pi_i} \left(\frac{d_{j,k}}{d_{i,k}}\right)^{\frac{1.0}{1.0-m}}}, \quad w_{i,\ell} = \frac{\sum_{k=1}^{N} (u_{i,k})^m x_{k,\ell}}{\sum_{r=1}^{M} \sum_{k=1}^{N} (u_{i,k})^m x_{k,r}},$$

$$\pi_i = \frac{1.0}{\sum_{j=1}^{C} \left(\sum_{k=1}^{N} \frac{(u_{j,k})^m d_{j,k}}{(u_{i,k})^m d_{i,k}}\right)^{\frac{1.0}{m}}}.$$

KLFCCM

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} - \sum_{i=1}^{C} \sum_{k=1}^{N} \sum_{\ell=1}^{M} u_{i,k} \log\left(w_{i,\ell}\right) x_{k,\ell} + \lambda^{-1} \sum_{i=1}^{C} \sum_{k=1}^{N} u_{i,k} \log\left(\frac{u_{i,k}}{\pi_i}\right)$$

$$d_{i,k} = -\sum_{\ell=1}^{M} \log(w_{i,\ell}) x_{k,\ell},$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C} \frac{\pi_j}{\pi_i} \exp\left(-\lambda(d_{j,k} - d_{i,k})\right)}, \quad w_{i,\ell} = \frac{\sum_{k=1}^{N} u_{i,k} x_{k,\ell}}{\sum_{r=1}^{M} \sum_{k=1}^{N} u_{i,k} x_{k,r}},$$

$$\pi_i = \frac{1}{N} \sum_{k=1}^{N} u_{i,k}.$$

ref : http://ieeexplore.ieee.org/abstract/document/6891747/

QFCCM

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} - \sum_{i=1}^{C} \sum_{k=1}^{N} \sum_{\ell=1}^{M} (\pi_i)^{1-m} (u_{i,k})^m \log(w_{i,\ell}) x_{k,\ell} + \frac{\lambda^{-1}}{m-1} \sum_{i=1}^{C} \sum_{k=1}^{N} (\pi_i)^{1-m} (u_{i,k})^m$$

$$d_{i,k} = -\sum_{\ell=1}^{M} \log(w_{i,\ell}) x_{k,\ell},$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C} \frac{\pi_j}{\pi_i} \left(\frac{1.0-\lambda(1.0-m)d_{j,k}}{1.0-\lambda(1.0-m)d_{i,k}} \right)^{\frac{1.0}{1.0-m}}}, \quad w_{i,\ell} = \frac{\sum_{k=1}^{N} (u_{i,k})^m x_{k,\ell}}{\sum_{r=1}^{M} \sum_{k=1}^{N} (u_{i,k})^m x_{k,r}},$$

$$\pi_i = \frac{1.0}{\sum_{j=1}^{C} \left(\sum_{k=1}^{N} \frac{(u_{j,k})^m (1.0-\lambda(1.0-m)d_{j,k})}{(u_i,k)^m (1.0-\lambda(1.0-m)d_{i,k})} \right)^{\frac{1.0}{m}}}.$$

ref : `http://ieeexplore.ieee.org/document/7337853/`

BFCCMM

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} \ -\frac{1}{t}\sum_{i=1}^{C}\sum_{k=1}^{N}\sum_{\ell=1}^{M}(u_{i,k})^m\left((w_{i,\ell})^t-1\right)x_{k,\ell}$$

$$d_{i,k} = -\frac{1.0}{t}\sum_{\ell=1}^{M}\left((w_{i,\ell})^t-1.0\right)x_{k,\ell},$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C}\frac{\pi_j}{\pi_i}\left(\frac{d_{j,k}}{d_{i,k}}\right)^{\frac{1.0}{1.0-m}}}, \quad w_{i,\ell} = \frac{1.0}{\sum_{r=1}^{M}\left(\sum_{k=1}^{N}\frac{(u_{i,k})^m x_{k,r}}{(u_{i,k})^m x_{k,\ell}}\right)^{\frac{1.0}{1.0-t}}},$$

$$\pi_i = \frac{1.0}{\sum_{j=1}^{C}\left(\sum_{k=1}^{N}\frac{(u_{j,k})^m d_{j,k}}{(u_{i,k})^m d_{i,k}}\right)^{\frac{1.0}{m}}}.$$

KLFCCMM

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} -\frac{1}{t}\sum_{i=1}^{C}\sum_{k=1}^{N}\sum_{\ell=1}^{M} u_{i,k}((w_{i,\ell})^t - 1)x_{k,\ell} + \lambda^{-1}\sum_{i=1}^{C}\sum_{k=1}^{N} u_{i,k}\log\left(\frac{u_{i,k}}{\pi_i}\right)$$

$$d_{i,k} = -\frac{1.0}{t}\sum_{\ell=1}^{M}\left((w_{i,\ell})^t - 1.0\right)x_{k,\ell},$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C}\frac{\pi_j}{\pi_i}\exp\left(-\lambda(d_{j,k} - d_{i,k})\right)}, \quad w_{i,\ell} = \frac{1.0}{\sum_{r=1}^{M}\left(\sum_{k=1}^{N}\frac{u_{i,k}x_{k,r}}{u_{i,k}x_{k,\ell}}\right)^{\frac{1.0}{1.0-t}}},$$

$$\pi_i = \frac{1}{N}\sum_{k=1}^{N} u_{i,k}.$$

ref : `http://ieeexplore.ieee.org/document/6982814/`

QFCCMM

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} -\frac{1}{t}\sum_{i=1}^{C}\sum_{k=1}^{N}\sum_{\ell=1}^{M}(\pi_i)^{1-m}(u_{i,k})^m((w_{i,\ell})^t-1)x_{k,\ell} + \frac{\lambda^{-1}}{m-1}\sum_{i=1}^{C}\sum_{k=1}^{N}(\pi_i)^{1-m}(u_{i,k})^m$$

$$d_{i,k} = -\frac{1.0}{t}\sum_{\ell=1}^{M}\left((w_{i,\ell})^t - 1.0\right)x_{k,\ell},$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C}\frac{\pi_j}{\pi_i}\left(\frac{1.0-\lambda(1.0-m)d_{j,k}}{1.0-\lambda(1.0-m)d_{i,k}}\right)^{\frac{1.0}{1.0-m}}}, \quad w_{i,\ell} = \frac{1.0}{\sum_{r=1}^{M}\left(\sum_{k=1}^{N}\frac{(u_{i,k})^m x_{k,r}}{(u_{i,k})^m x_{k,\ell}}\right)^{\frac{1.0}{1.0-t}}},$$

$$\pi_i = \frac{1.0}{\sum_{j=1}^{C}\left(\sum_{k=1}^{N}\frac{(u_{j,k})^m(1.0-\lambda(1.0-m)d_{j,k})}{(u_i,k)^m(1.0-\lambda(1.0-m)d_{i,k})}\right)^{\frac{1.0}{m}}}.$$

ref : http://ieeexplore.ieee.org/document/7337853/(式注意)

BFCS

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} \sum_{i=1}^{C} \sum_{k=1}^{N} (\pi_i)^{1-m} (u_{i,k})^m (1 - x_k^\top v_i)$$

$$d_{i,k} = 1.0 - x_k^\top v_i,$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C} \frac{\pi_j}{\pi_i} \left(\frac{d_{j,k}}{d_{i,k}}\right)^{\frac{1.0}{1.0-m}}}, \quad v_i = \frac{\sum_{k=1}^{N} (u_{i,k})^m x_k}{\| \sum_{k=1}^{N} (u_{i,k})^m x_k \|_2},$$

$$\pi_i = \frac{1.0}{\sum_{j=1}^{C} \left(\sum_{k=1}^{N} \frac{(u_{j,k})^m d_{j,k}}{(u_{i,k})^m d_{i,k}} \right)^{\frac{1.0}{m}}}.$$

ref : `http://ieeexplore.ieee.org/document/6891670/`

KLFCS

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} \sum_{i=1}^{C} \sum_{k=1}^{N} u_{i,k}(1 - x_k^\top v_i) + \lambda^{-1} \sum_{i=1}^{C} \sum_{k=1}^{N} u_{i,k} \log\left(\frac{u_{i,k}}{\pi_i}\right)$$

$$d_{i,k} = 1.0 - x_k^\top v_i,$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C} \frac{\pi_j}{\pi_i} \exp\left(-\lambda(d_{j,k} - d_{i,k})\right)}, \quad v_i = \frac{\sum_{k=1}^{N} u_{i,k} x_k}{\|\sum_{k=1}^{N} u_{i,k} x_k\|_2},$$

$$\pi_i = \frac{\sum_{k=1}^{N} u_{i,k}}{N}.$$

QFCS

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} \sum_{i=1}^{C}\sum_{k=1}^{N}(\pi_i)^{1-m}(u_{i,k})^m(1-x_k^\top v_i)+\frac{\lambda^{-1}}{m-1}\sum_{i=1}^{C}\sum_{k=1}^{N}(\pi_i)^{1-m}(u_{i,k})^m$$

$$d_{i,k} = 1.0 - x_k^\top v_i,$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C}\frac{\pi_j}{\pi_i}\left(\frac{1.0-\lambda(1.0-m)d_{j,k}}{1.0-\lambda(1.0-m)d_{i,k}}\right)^{\frac{1.0}{1.0-m}}}, \quad v_i = \frac{\sum_{k=1}^{N}(u_{i,k})^m x_k}{\|\sum_{k=1}^{N}(u_{i,k})^m x_k\|_2},$$

$$\pi_i = \frac{1.0}{\sum_{j=1}^{C}\left(\sum_{k=1}^{N}\frac{(u_{j,k})^m(1.0-\lambda(1.0-m)d_{j,k})}{(u_i,k)^m(1.0-\lambda(1.0-m)d_{i,k})}\right)^{\frac{1.0}{m}}}.$$

BPCS

optimization problem

$$\underset{u,v}{\text{minimize}} \sum_{k=1}^{N} (u_{1,k})^m (1 - x_k^\top v_1) - \alpha \sum_{k=1}^{N} (1 - u_{1,k})^m$$

$$d_{1,k} = 1.0 - x_k^\top v_1,$$

$$u_{1,k} = \frac{1.0}{1.0 + \left(\frac{m d_{1,k}}{\alpha}\right)^{\frac{1.0}{m-1.0}}}, \quad v_1 = \frac{\sum_{k=1}^{N} (u_{1,k})^m x_k}{\| \sum_{k=1}^{N} (u_{1,k})^m x_k \|_2},$$

ref : `https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=531779`

EPCS

optimization problem

$$\underset{u,v}{\text{minimize}} \sum_{k=1}^{N} u_{1,k}(1 - x_k^\top v_1) + \lambda^{-1} \sum_{k=1}^{N} u_{1,k} \log(u_{1,k}) - \alpha \sum_{k=1}^{N} u_{1,k}$$

$$d_{1,k} = 1.0 - x_k^\top v_1,$$

$$u_{1,k} = \exp(\lambda(\alpha - d_{1,k}) - 1.0), \quad v_1 = \frac{\sum_{k=1}^{N} u_{1,k} x_k}{\| \sum_{k=1}^{N} u_{1,k} x_k \|_2},$$

refhttps://link.springer.com/chapter/10.1007/978-3-319-23240-9_10

TPCS

optimization problem

$$\underset{u,v}{\text{minimize}} \sum_{k=1}^{N}(u_{1,k})^m(1 - x_k^\top v_1) + \frac{\lambda^{-1}}{m-1}\sum_{k=1}^{N}\left((u_{1,k})^m - u_{1,k}\right) - \alpha\sum_{k=1} u_{1,k}$$

$$d_{1,k} = 1.0 - x_k^\top v_1,$$

$$u_{1,k} = \left(\frac{1.0 - \lambda(1.0 - m)d_{1,k}}{m^{-1.0}\,(1.0 - \alpha\lambda(1.0 - m))}\right)^{\frac{1.0}{1.0-m}}, \quad v_1 = \frac{\sum_{k=1}^{N}(u_{1,k})^m x_k}{\|\sum_{k=1}^{N}(u_{1,k})^m x_k\|_2},$$

BPCCM

optimization problem

$$\underset{u,v}{\text{minimize}} - \sum_{k=1}^{N} \sum_{\ell=1}^{M} (u_{1,k})^m \left(\log(w_{1,\ell}) x_{k,\ell} - \log(\Gamma(x_{k,\ell} + 1))\right) - \alpha \sum_{k=1}^{N} (1 - u_{1,k})^m$$

$$d_{1,k} = -\left(\log(w_{1,\ell}) x_{k,\ell} - \log(\Gamma(x_{k,\ell} + 1.0))\right),$$

$$u_{1,k} = \frac{1.0}{1.0 + \left(\frac{m d_{1,k}}{\alpha}\right)^{\frac{1.0}{m-1.0}}}, \quad w_{1,\ell} = \frac{\sum_{k=1}^{N} (u_{1,k})^m x_{k,\ell}}{\sum_{r=1}^{M} \sum_{k=1}^{N} (u_{1,k})^m x_{k,r}},$$

EPCCM

optimization problem

$$\underset{u,v}{\text{minimize}}$$
$$-\sum_{k=1}^{N} u_{1,k}\left(\log(w_{1,\ell})x_{k,\ell} - \log(\Gamma(x_{k,\ell}+1))\right) + \lambda^{-1}\sum_{k=1}^{N} u_{1,k}\log(u_{1,k}) - \alpha\sum_{k=1}^{N} u_{1,k}$$

$$d_{1,k} = -\left(\log(w_{1,\ell})x_{k,\ell} - \log(\Gamma(x_{k,\ell}+1.0))\right),$$
$$u_{1,k} = \exp(\lambda(\alpha - d_{1,k}) - 1.0), \quad w_{1,\ell} = \frac{\sum_{k=1}^{N} u_{1,k}x_{k,\ell}}{\sum_{r=1}^{M}\sum_{k=1}^{N} u_{1,k}x_{k,r}},$$

TPCCM

optimization problem

$$\underset{u,v}{\text{minimize}} - \sum_{k=1}^{N} (u_{1,k})^m \left(\log(w_{1,\ell}) x_{k,\ell} - \log(\Gamma(x_{k,\ell}+1))\right) + \frac{\lambda^{-1}}{m-1} \sum_{k=1}^{N} \left((u_{1,k})^m - u_{1,k}\right) - \alpha \sum_{k=1} u_{1,k}$$

$$d_{1,k} = -\left(\log(w_{1,\ell}) x_{k,\ell} - \log(\Gamma(x_{k,\ell} + 1.0))\right),$$

$$u_{1,k} = \left(\frac{1.0 - \lambda(1.0-m) d_{1,k}}{m^{-1.0}\left(1.0 - \alpha\lambda(1.0-m)\right)}\right)^{\frac{1.0}{1.0-m}}, \quad w_{1,\ell} = \frac{\sum_{k=1}^{N} (u_{1,k})^m x_{k,\ell}}{\sum_{r=1}^{M} \sum_{k=1}^{N} (u_{1,k})^m x_{k,r}},$$

KLFCCMP

optimization problem

$$\underset{u,\alpha,\pi}{\text{minimize}}\ J_{KLFCCMP}$$

$$J_{KLFCCMP} = -\sum_{i=1}^{C}\sum_{k=1}^{N} u_{i,k}\left(\log\Gamma(s_i) - \log\Gamma(s_i + \textstyle\sum_{\ell=1}^{M} x_{k,\ell}) + \sum_{\ell:(x_{k,l})\geq 1} log(\alpha_{i,\ell})\right) + \lambda^{-1}\sum_{i=1}^{C}\sum_{k=1}^{N} u_{i,k}\log(\frac{u_{i,k}}{\pi_i})\ ,\ s_i = \sum_{\ell=1}^{M}\alpha_{i,\ell}$$

$$d_{i,k} = -\log\Gamma(s_i) + \log\Gamma(s_i + \sum_{\ell=1}^{M} x_{k,\ell}) - \sum_{\ell:(x_{k,\ell})\geq 1} log(\alpha_{i,\ell})$$

$$u_{i,k} = \frac{1.0}{\sum_{j=1}^{C}\frac{\pi_j}{\pi_i}\exp(-\lambda(d_{j,k} - d_{i,k}))},\quad \pi_i = \frac{\sum_{k=1}^{N} u_{i,k}}{N}$$

$$\alpha_{i,\ell} = \frac{\sum_{k=1}^{N} u_{i,k}}{\sum_{k=1}^{N} u_{i,k}\psi(s_i + x_{k,\ell}) - \sum_{k=1}^{N} u_{i,k}\psi(s_i)}.$$

ref : https://link.springer.com/chapter/10.1007/978-3-319-67422-3_9

OCS-BFCS

optimization problem

$$\underset{u,v,\pi}{\text{minimize}} \sum_{i=1}^{C} \sum_{k=1}^{N} (\pi_i)^{1-m} (u_{i,k})^m (1 - x_k^\top v_i)$$

$$\begin{aligned}
d_{i,k} &= 1.0 - x_k^\top v_i, \\
x_{k,\ell} &= \frac{\left(\sum_{i=1}^{C} (\pi_i)^{1.0-m} (u_{i,k})^m v_{i,\ell}\right) \sqrt{1.0 - \sum_{r=1}^{M} (x_{k,\ell})^{2.0} (1.0 - y_{k,\ell})}}{\sqrt{\sum_{r=1}^{M} \left(\sum_{i=1}^{C} (\pi_i)^{1.0-m} (u_{i,k})^m v_{i,r} (1.0 - y_{k,r})\right)^{2.0}}}, \\
u_{i,k} &= \frac{1.0}{\sum_{j=1}^{C} \frac{\pi_j}{\pi_i} \left(\frac{d_{j,k}}{d_{i,k}}\right)^{\frac{1.0}{1.0-m}}}, \quad v_i = \frac{\sum_{k=1}^{N} (u_{i,k})^m x_k}{\| \sum_{k=1}^{N} (u_{i,k})^m x_k \|_2}, \\
\pi_i &= \frac{1.0}{\sum_{j=1}^{C} \left(\sum_{k=1}^{N} \frac{(u_{j,k})^m d_{j,k}}{(u_{i,k})^m d_{i,k}}\right)^{\frac{1.0}{m}}}.
\end{aligned}$$

ref : `https://ieeexplore.ieee.org/document/956035/`

TFIDF

TF-IDF

- $\mathrm{tf}_{k,\ell}$: k 番目の文書の ℓ 番目の単語の頻出回数
- N : 全文書数
- Df_ℓ : ℓ 番目の単語が出現する文書数

$$\mathrm{TF}_{k,\ell} = \begin{cases} 1 + \log(\mathrm{tf}_{k,\ell}) & \mathrm{tf}_{k,\ell} > 0 \text{ のとき} \\ 0 & \text{それ以外のとき} \end{cases}$$

$$\mathrm{IDF}_\ell = \log\left(\frac{\mathrm{N}}{\mathrm{Df}_\ell}\right)$$

$$\mathrm{TF.IDF}_{k,\ell} = \mathrm{TF}_{k,\ell} \times \mathrm{IDF}_\ell$$

LBM(Bernoulli) (Latent Block Model)

optimization problem

$$\underset{u,w,\pi,\rho,\alpha}{\text{maximize}} \sum_{i=1}^{C^{row}} \sum_{j=1}^{C^{col}} \sum_{k=1}^{N} \sum_{\ell=1}^{M} u_{i,k} w_{j,\ell} \log\left(\text{prob}(x_{k,\ell}; \alpha_{i,j})\right)$$

$$+ \sum_{i=1}^{C^{row}} \sum_{k=1}^{N} u_{i,k} \log\left(\frac{\pi_i}{u_{i,k}}\right) + \sum_{j=1}^{C^{col}} \sum_{\ell=1}^{M} w_{j,\ell} \log\left(\frac{\rho_j}{w_{i,\ell}}\right)$$

$$\alpha_{i,j} \in \mathbb{R}^{C^{row} \times C^{col}}, \text{ subject to } \sum_{j=1}^{C^{col}} \rho_j = 1 \text{ and } \sum_{j=1}^{C^{col}} w_{j,\ell} = 1$$

$$\text{Bernoulli : prob}(x_{k,\ell}; \alpha_{i,j}) = (\alpha_{i,j})^{x_{k,\ell}} (1 - (\alpha_{i,j})^{(1-x_{k,\ell})})$$

$$u_{i,k} = \frac{\pi_i \exp\left(\sum_{j=1}^{C^{col}} \sum_{\ell=1}^{M} w_{j,\ell} x_{k,\ell} \log(\frac{\alpha_{i,j}}{1-\alpha_{i,j}}) + \sum_{j=1}^{C^{col}} \sum_{\ell=1}^{M} w_{j,\ell} \log(1-\alpha_{i,j})\right)}{\sum_{i'=1}^{C^{row}} \pi_{i'} \exp\left(\sum_{j=1}^{C^{col}} \sum_{\ell=1}^{M} w_{j,\ell} x_{k,\ell} \log(\frac{\alpha_{i',j}}{1-\alpha_{i',j}}) + \sum_{j=1}^{C^{col}} \sum_{\ell=1}^{M} w_{j,\ell} \log(1-\alpha_{i',j})\right)},$$

$$w_{j,\ell} = \frac{\rho_j \exp\left(\sum_{i=1}^{C^{row}} \sum_{k=1}^{N} u_{i,k} x_{k,\ell} \log(\frac{\alpha_{i,j}}{1-\alpha_{i,j}}) + \sum_{i=1}^{C^{row}} \sum_{k=1}^{N} u_{i,k} \log(1-\alpha_{i,j})\right)}{\sum_{j'=1}^{C^{col}} \rho_{j'} \exp\left(\sum_{i=1}^{C^{row}} \sum_{k=1}^{N} u_{i,k} x_{k,\ell} \log(\frac{\alpha_{i,j'}}{1-\alpha_{i,j'}}) + \sum_{i=1}^{C^{row}} \sum_{k=1}^{N} u_{i,k} \log(1-\alpha_{i,j'})\right)},$$

$$\pi_i = \frac{1}{N} \sum_{k=1}^{N} u_{i,k} \, , \rho = \frac{1}{M} \sum_{\ell=1}^{M} w_{j,\ell} \, , \alpha_{i,j} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} u_{i,k} w_{j,\ell} x_{k,\ell}}{\sum_{k=1}^{N} \sum_{\ell=1}^{M} u_{i,k} w_{j,\ell}}$$

Algorithm 1 LBM(Bernoulli)

initialize $u, w, \pi_i = \frac{1}{N}\sum_{k=1}^{N} u_{i,k}$, $\rho_j = \frac{1}{M}\sum_{\ell=1}^{M} w_{j,\ell}$

repeat

    $s^u_{i,\ell} \in \mathbb{R}^{C^{row}\times M} = \sum_{k=1}^{N} u_{i,k} x_{k,\ell}$

    repeat

$$w_{j,\ell} = \frac{\rho_j \exp\Big(\sum_{i=1}^{C^{row}} s^u_{i,\ell} \log(\frac{\alpha_{i,j}}{1-\alpha_{i,j}}) + \sum_{i=1}^{C^{row}} \sum_{k=1}^{N} u_{i,k} \log(1-\alpha_{i,j})\Big)}{\sum_{j'=1}^{C^{col}} \rho_{j'} \exp\Big(\sum_{i=1}^{C^{row}} s^u_{i,\ell} \log(\frac{\alpha_{i,j'}}{1-\alpha_{i,j'}}) + \sum_{i=1}^{C^{row}} \sum_{k=1}^{N} u_{i,k} \log(1-\alpha_{i,j'})\Big)}$$

$$\rho_j = \frac{1}{M}\sum_{\ell=1}^{M} w_{j,\ell}, \, , \alpha_{i,j} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M} u_{i,k} w_{j,\ell} x_{k,\ell}}{\sum_{k=1}^{N}\sum_{\ell=1}^{M} u_{i,k} w_{j,\ell}}$$

    until convergence $\{w, \rho, \alpha\}$

    $s^w_{j,k} \in \mathbb{R}^{C^{col}\times N} = \sum_{\ell=1}^{M} w_{j,\ell} x_{k,\ell}$

    repeat

$$u_{i,k} = \frac{\pi_i \exp\Big(\sum_{j=1}^{C^{col}} s^w_{j,k} \log(\frac{\alpha_{i,j}}{1-\alpha_{i,j}}) + \sum_{j=1}^{C^{col}} \sum_{\ell=1}^{M} w_{j,\ell} \log(1-\alpha_{i,j})\Big)}{\sum_{i'=1}^{C^{row}} \pi_{i'} \exp\Big(\sum_{j=1}^{C^{col}} s^w_{j,k} \log(\frac{\alpha_{i',j}}{1-\alpha_{i',j}}) + \sum_{j=1}^{C^{col}} \sum_{\ell=1}^{M} w_{j,\ell} \log(1-\alpha_{i',j})\Big)}$$

$$\pi_i = \frac{1}{N}\sum_{k=1}^{N} u_{i,k} \, , \alpha_{i,j} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M} u_{i,k} w_{j,\ell} x_{k,\ell}}{\sum_{k=1}^{N}\sum_{\ell=1}^{M} u_{i,k} w_{j,\ell}}$$

    until convergence $\{u, \pi, \alpha\}$

until convergence $\{s^u \, , s^w\}$

FuzzyLBM(Binomial) (Latent Block Model)

optimization problem

$$\underset{u,w,\pi,\rho,\alpha}{\text{maximize}} \sum_{i=1}^{C^{row}} \sum_{j=1}^{C^{col}} \sum_{k=1}^{N} \sum_{\ell=1}^{M} u_{i,k} w_{j,\ell} \log\left(\text{prob}(x_{k,\ell};\alpha_{i,j})\right) + \lambda_1^{-1} \sum_{i=1}^{C^{row}} \sum_{k=1}^{N} u_{i,k} \log\left(\frac{\pi_i}{u_{i,k}}\right) + \lambda_2^{-1} \sum_{j=1}^{C^{col}} \sum_{\ell=1}^{M} w_{j,\ell} \log\left(\frac{\rho_j}{w_{j,\ell}}\right)$$

$$\text{Binomial : prob}(x_{k,\ell};\alpha_{i,j}) = \binom{n}{x_{k,\ell}} (\alpha_{i,j})^{x_{k,\ell}} (1-(\alpha_{i,j})^{(n-x_{k,\ell})})$$

$$u_{i,k} = \frac{\pi_i \exp\left(\lambda_1 \sum_{j=1}^{C^{col}} \sum_{\ell=1}^{M} w_{j,\ell} x_{k,\ell} \log\left(\frac{\alpha_{i,j}}{1-\alpha_{i,j}}\right) + n\lambda_1 \sum_{j=1}^{C^{col}} \sum_{\ell=1}^{M} w_{j,\ell} \log(1-\alpha_{i,j})\right)}{\sum_{i'=1}^{C^{row}} \pi_{i'} \exp\left(\lambda_1 \sum_{j=1}^{C^{col}} \sum_{\ell=1}^{M} w_{j,\ell} x_{k,\ell} \log\left(\frac{\alpha_{i',j}}{1-\alpha_{i',j}}\right) + n\lambda_1 \sum_{j=1}^{C^{col}} \sum_{\ell=1}^{M} w_{j,\ell} \log(1-\alpha_{i',j})\right)},$$

$$w_{j,\ell} = \frac{\rho_j \exp\left(\lambda_2 \sum_{i=1}^{C^{row}} \sum_{k=1}^{N} u_{i,k} x_{k,\ell} \log\left(\frac{\alpha_{i,j}}{1-\alpha_{i,j}}\right) + n\lambda_2 \sum_{i=1}^{C^{row}} \sum_{k=1}^{N} u_{i,k} \log(1-\alpha_{i,j})\right)}{\sum_{j'=1}^{C^{col}} \rho_{j'} \exp\left(\lambda_2 \sum_{i=1}^{C^{row}} \sum_{k=1}^{N} u_{i,k} x_{k,\ell} \log\left(\frac{\alpha_{i,j'}}{1-\alpha_{i,j'}}\right) + n\lambda_2 \sum_{i=1}^{C^{row}} \sum_{k=1}^{N} u_{i,k} \log(1-\alpha_{i,j'})\right)},$$

$$\pi_i = \frac{1}{N} \sum_{k=1}^{N} u_{i,k}\ ,\ \rho = \frac{1}{M} \sum_{\ell=1}^{M} w_{j,\ell}\ ,\ \alpha_{i,j} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} u_{i,k} w_{j,\ell} x_{k,\ell}}{n \sum_{k=1}^{N} \sum_{\ell=1}^{M} u_{i,k} w_{j,\ell}}$$

Algorithm 2 FuzzyLBM(Binomial)

initialize $u, w, \pi_i = \frac{1}{N}\sum_{k=1}^{N} u_{i,k}$, $\rho_j = \frac{1}{M}\sum_{\ell=1}^{M} w_{j,\ell}$

repeat

$\quad s^u_{i,\ell} \in \mathbb{R}^{C^{row}\times M} = \sum_{k=1}^{N} u_{i,k}x_{k,\ell}$

$\quad$ **repeat**

$$w_{j,\ell} = \frac{\rho_j \exp\Big(\lambda_2 \sum_{i=1}^{C^{row}} s^u_{i,\ell}\log(\frac{\alpha_{i,j}}{1-\alpha_{i,j}}) + n\lambda_2 \sum_{i=1}^{C^{row}}\sum_{k=1}^{N} u_{i,k}\log(1-\alpha_{i,j})\Big)}{\sum_{j'=1}^{C^{col}} \rho_{j'} \exp\Big(\lambda_2 \sum_{i=1}^{C^{row}} s^u_{i,\ell}\log(\frac{\alpha_{i,j'}}{1-\alpha_{i,j'}}) + n\lambda_2 \sum_{i=1}^{C^{row}}\sum_{k=1}^{N} u_{i,k}\log(1-\alpha_{i,j'})\Big)}$$

$$\rho_j = \frac{1}{M}\sum_{\ell=1}^{M} w_{j,\ell}\ ,\ \alpha_{i,j} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M} u_{i,k}w_{j,\ell}x_{k,\ell}}{n\sum_{k=1}^{N}\sum_{\ell=1}^{M} u_{i,k}w_{j,\ell}}$$

$\quad$ **until** convergence $\{w, \rho, \alpha\}$

$\quad s^w_{j,k} \in \mathbb{R}^{C^{col}\times N} = \sum_{\ell=1}^{M} w_{j,\ell}x_{k,\ell}$

$\quad$ **repeat**

$$u_{i,k} = \frac{\pi_i \exp\Big(\lambda_1 \sum_{j=1}^{C^{col}} s^w_{j,k}\log(\frac{\alpha_{i,j}}{1-\alpha_{i,j}}) + n\lambda_1 \sum_{j=1}^{C^{col}}\sum_{\ell=1}^{M} w_{j,\ell}\log(1-\alpha_{i,j})\Big)}{\sum_{i'=1}^{C^{row}} \pi_{i'} \exp\Big(\lambda_1 \sum_{j=1}^{C^{col}} s^w_{j,k}\log(\frac{\alpha_{i',j}}{1-\alpha_{i',j}}) + n\lambda_1 \sum_{j=1}^{C^{col}}\sum_{\ell=1}^{M} w_{j,\ell}\log(1-\alpha_{i',j})\Big)}$$

$$\pi_i = \frac{1}{N}\sum_{k=1}^{N} u_{i,k}\ ,\ \alpha_{i,j} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M} u_{i,k}w_{j,\ell}x_{k,\ell}}{n\sum_{k=1}^{N}\sum_{\ell=1}^{M} u_{i,k}w_{j,\ell}}$$

$\quad$ **until** convergence $\{u, \pi, \alpha\}$

until convergence $\{s^u\ ,\ s^w\}$