Lesson 13

random stuff

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Maths Olympiad Club

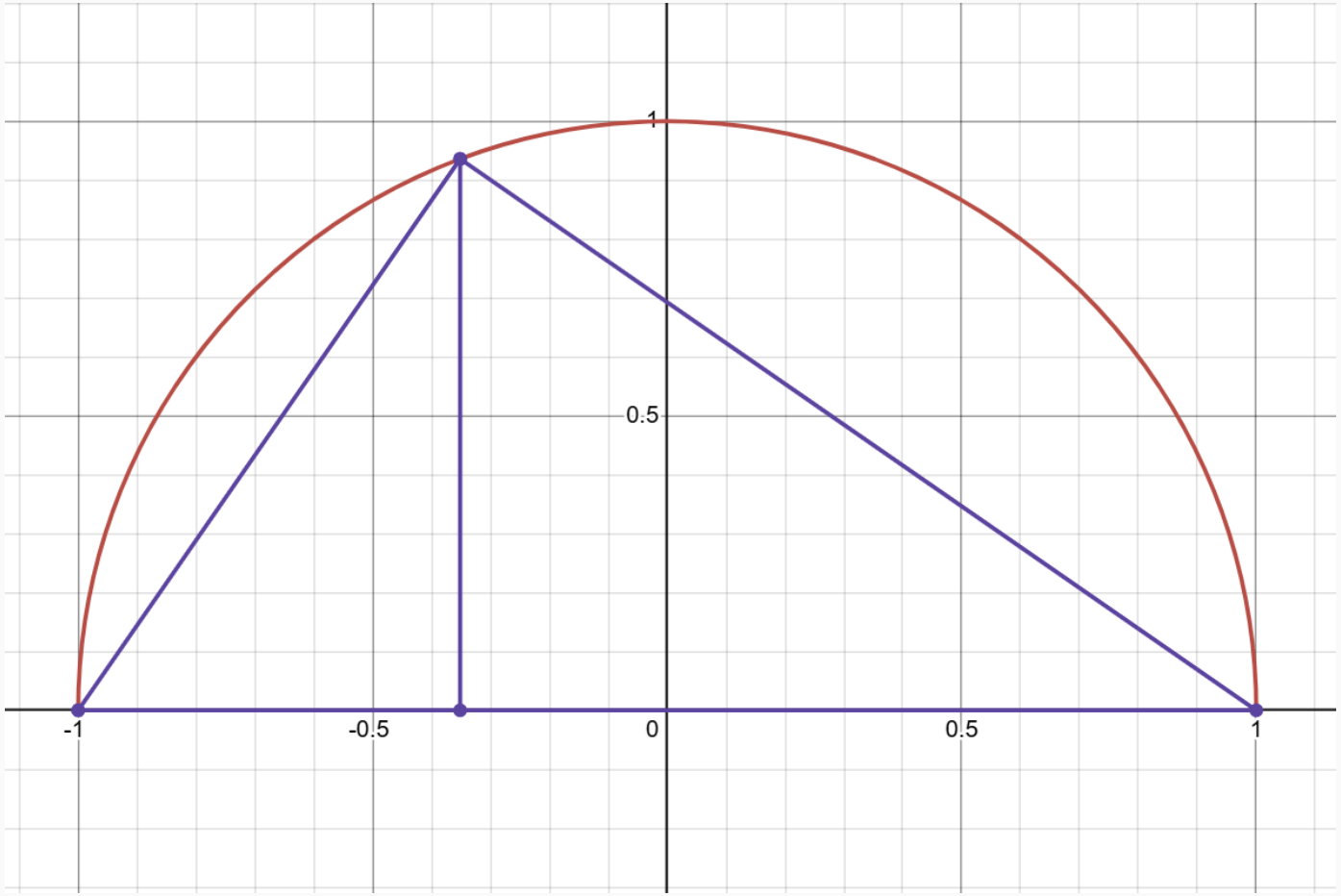
1. **lets have a look at desmos**

2. A whole new world

3. There's still nothing here

go to desmos

desmos semicircle thing



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remember this?

remember that: $\frac{a+b+c+\dots}{n} \geq \sqrt[n]{a \times b \times c \times \cdots}$

lets try this

For a, b, $b \in R^+$, show that

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} \geq 3$$

a new question

remember that: $\frac{a+b+c+\dots}{n} \geq \sqrt[n]{a \times b \times c \times \cdots}$

lets try this

Find the maximum of $2 - a - \frac{1}{2a}$ for all positive a.

a new question

lets try this

Find the maximum of $2 - a - \frac{1}{2a}$ for all positive a.

We can rewrite the given expression as $2 - \left(a + \frac{1}{2a}\right)$. To maximize the whole expression, we must minimize $a + \frac{1}{2a}$. Since a is positive, so is $\frac{1}{2a}$. This means AM - GM will hold for a and $\frac{1}{2a}$.

By AM - GM, the arithmetic mean of a and $\frac{1}{2a}$ is at least their geometric mean, or $\frac{\sqrt{2}}{2}$. This means the sum of a and $\frac{1}{2a}$ is at least $\sqrt{2}$. We can prove that we can achieve this minimum for $a + \frac{1}{2a}$ by plugging in $a = \frac{\sqrt{2}}{2}$ by solving $a + \frac{1}{2a} = \sqrt{2}$ for a.

Plugging in $a = \frac{\sqrt{2}}{2}$ into our original expression that we wished to maximize, we get that $2 - a - \frac{1}{2a} = \boxed{2 - \sqrt{2}}$, which is our answer.

one more question

remember that: $\frac{a+b+c+\dots}{n} \geq \sqrt[n]{a \times b \times c \times \cdots}$

lets try this

Find **all** real solutions to $2^x + x^2 = 2 - \frac{1}{2^x}$

one more question

lets try this

Find **all** real solutions to $2^x + x^2 = 2 - \frac{1}{2^x}$

rearrange to get $2^x - \frac{1}{2^x} = 2 - x^2$

using AM $\geq$ GM we can see that

$$2^x - \frac{1}{2^x} \geq 2$$

$$\implies 2 - x^2 \geq 2$$

$\iff x^2 \leq 0$ which can only be possible if $x = 0$.

Therefore this is our only answer.

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