

Goldbach's conjectures says, "An even number which is above 6, can be represented as sum of 2 odd primes".

For example, $20=3+17$, $20=7+13$, there are 2 ways. Or 4 ways if commutativity of addition is counted.

Now consider a related problem,

Define $f(n)$, which represents the product of primes less than or equal to n , that is $f(n) = \prod_{p \leq n} p$.

For an even number $s \geq 8$, consider $h = g(s) = f(\lfloor \sqrt{s} \rfloor) + s$, obviously h is even, and h is greater than s .

Then according to the conjecture, h (quite probably) can be represented as $p_1 + p_2$.

If h can indeed be represented as this form, and, $p_1 \leq s - 3$, then, it's easy to show that $p_3 = s - p_1$ is a prime.

(p , p_1 , p_2 , p_3 are all primes)

For a concrete example,

Let's say $s = 60$, then $h = 2 \times 3 \times 5 \times 7 + 60 = 270$.

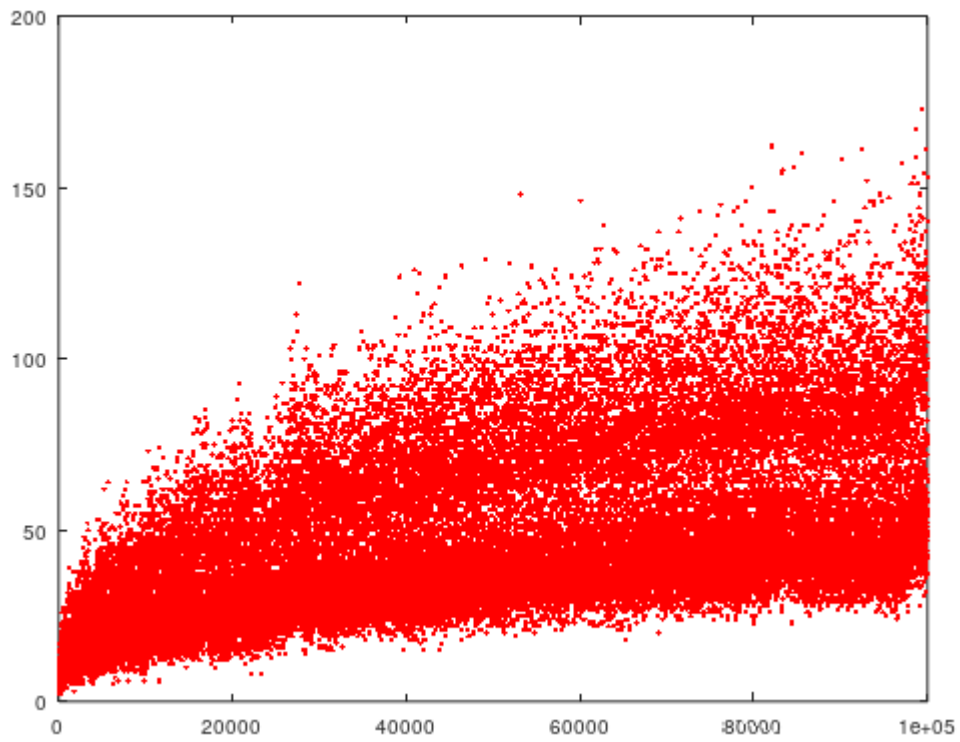
Now we list the ways h be represented as sum of primes, $7 + 263$, $13 + 257$, $19 + 251$, $29 + 241$ and so on.

Then we have $60 - 7 = 53$, $60 - 13 = 47$, $60 - 19 = 41$, $60 - 29 = 31$, and so on. We know 53, 47, 41, 31 must be primes.

Or we can say, there are "solutions" in common when consider how to represent s and h as sum of primes.

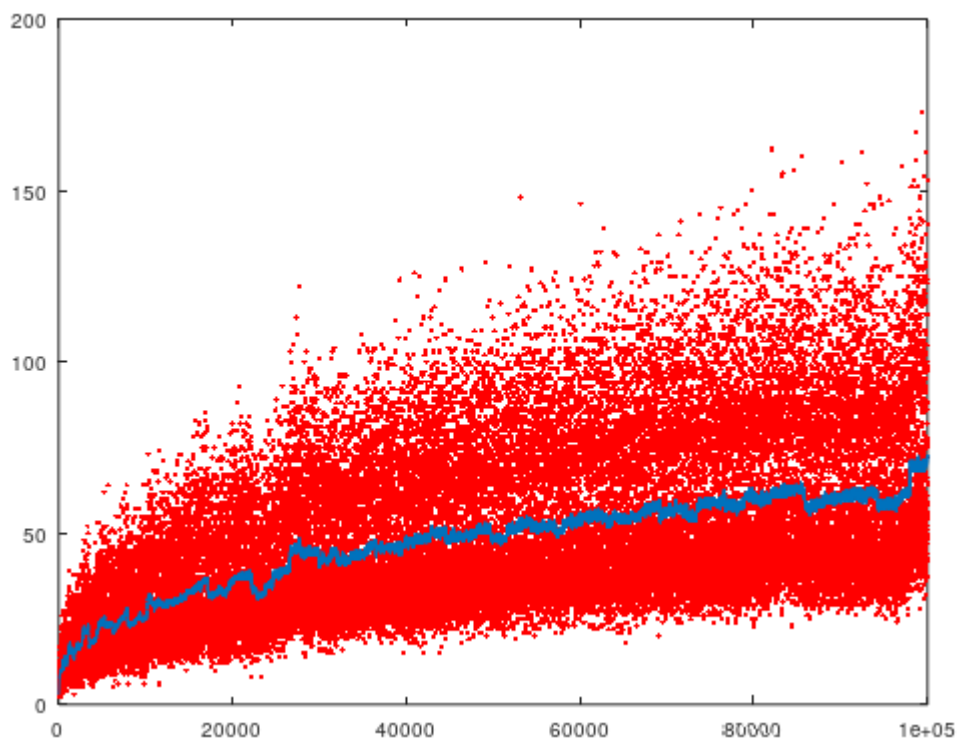
The Groovy script is to count the number of these common "solutions" for each s .

Plot the graph,

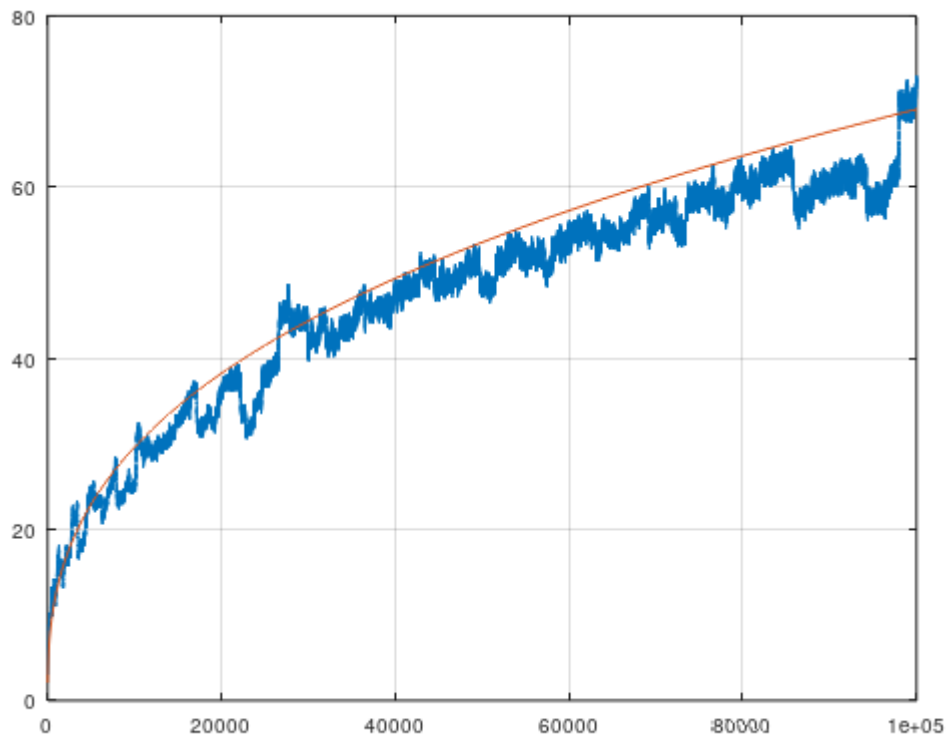


Looks like chaos, but there seems a trend to go upward.

Plot the moving average, every 50 points,



This “moving average” can be fitted using a simple function quite well.



The function (red line) is $y = x^{\frac{1}{e}}$.