```
function [thetalist, success, theta_mat, err_ang, err_lin] = IKinBodyIterates(Blist, M,↙
T, thetalist0, max_iter, eomg, ev, filename)
% Takes Blist: The joint screw axes in the end-effector frame when the
%              manipulator is at the home position, in the format of a
%              matrix with the screw axes as the columns,
%       M: The home configuration of the end-effector,
%       T: The desired end-effector configuration Tsd,
%       thetalist0: An initial guess of joint angles that are close to
%                   satisfying Tsd,
%       max_iter: the maximum number iteration trying find the solution
%       eomg: A small positive tolerance on the end-effector orientation
%             error. The returned joint angles must give an end-effector
%             orientation error less than eomg,
%       ev: A small positive tolerance on the end-effector linear position
%           error. The returned joint angles must give an end-effector
%           position error less than ev.
%       filename: the csv filename to save the configuration
% Returns thetalist: Joint angles that achieve T within the specified
%                    tolerances,
%         success: A logical value where TRUE means that the function found
%                  a solution and FALSE means that it ran through the set
%                  number of maximum iterations without finding a solution
%                  within the tolerances eomg and ev.
%           theta_mat: the matrix of the all thetalist on all iterations
%           err_ang: list of angular errors
%           err_lin: list of linear errors
% Uses an iterative Newton-Raphson root-finding method.
% The maximum number of iterations before the algorithm is terminated has
% been hardcoded in as a variable called maxiterations. It is set to 20 at
% the start of the function, but can be changed if needed.
% Example Inputs:
%
% clear; clc;
% Blist = [[0; 0; -1; 2; 0; 0], [0; 0; 0; 0; 1; 0], [0; 0; 1; 0; 0; 0.1]];
% M = [[-1, 0, 0, 0]; [0, 1, 0, 6]; [0, 0, -1, 2]; [0, 0, 0, 1]];
% T = [[0, 1, 0, -5]; [1, 0, 0, 4]; [0, 0, -1, 1.6858]; [0, 0, 0, 1]];
% thetalist0 = [1.5; 2.5; 3];
% eomg = 0.01;
% ev = 0.001;
% max_iter = 30;
% filename = 'thetalist';
% [thetalist, success, theta_mat, err_ang, err_lin] = IKinBodyIterates(Blist, M, T,↙
thetalist0, max_iter, eomg, ev, filename)
%
% Outputs:
% --------------- Iteration 1 ---------------
% Joint Vector:
%     1.5000    2.5000    3.0000
%
% SE(3) end-effector config:
%    -0.0707    0.9975         0   -4.4887
```

```
%     0.9975    0.0707         0    4.3183
%          0         0   -1.0000    1.7000
%          0         0         0    1.0000
%
%           error twist V_b: (0.000, 0.000, 0.071, -0.300, -0.522, 0.014)
% angular error ||omega_b||: 7.079633e-02
%  linear error     ||v_b||: 6.025602e-01
%
% --------------- Iteration 2 ---------------
% Joint Vector:
%     1.5824    2.9748    3.1531
%
% SE(3) end-effector config:
%    -0.0001    1.0000         0   -4.9744
%     1.0000    0.0001         0    3.9423
%          0         0   -1.0000    1.6847
%          0         0         0    1.0000
%
%           error twist V_b: (0.000, 0.000, 0.000, 0.058, -0.026, -0.001)
% angular error ||omega_b||: 1.107873e-04
%  linear error     ||v_b||: 6.311800e-02
%
% --------------- Iteration 3 ---------------
% Joint Vector:
%     1.5707    2.9997    3.1415
%
% SE(3) end-effector config:
%     0.0000    1.0000         0   -4.9997
%     1.0000   -0.0000         0    4.0003
%          0         0   -1.0000    1.6858
%          0         0         0    1.0000
%
%           error twist V_b: (0.000, 0.000, -0.000, -0.000, -0.000, 0.000)
% angular error ||omega_b||: 4.608708e-06
%  linear error     ||v_b||: 4.444047e-04
%
% thetalist =
%     1.5707
%     2.9997
%     3.1415
% success =
%    1
% theta_mat =
%     1.5000    2.5000    3.0000
%     1.5824    2.9748    3.1531
%     1.5707    2.9997    3.1415
% err_ang =
%     0.0708
%     0.0001
%     0.0000
% err_lin =
```

```
%     0.6026
%     0.0631
%     0.0004


    theta_curr = thetalist0;
    theta_mat  = [];
    err_ang    = [];
    err_lin    = [];

    for i = 1 : max_iter
        fprintf('--------------- Iteration %d ---------------\n', i)

        theta_mat(i, :) = theta_curr';

        fprintf('Joint Vector:\n')
        disp(theta_curr')

%         calculate the transformation Tbd from body frame to desired frame
        Tsb = FKinBody(M, Blist, theta_curr);
        Tbd = TransInv(Tsb)*T;

        fprintf('SE(3) end-effector config:\n')
        disp(Tsb)

%         calculate the error twist vector
        Vb_skrw = MatrixLog6(Tbd);
        Vb_vec  = se3ToVec(Vb_skrw);

        wb = Vb_vec(1:3, :);
        vb = Vb_vec(4:6, :);

        fprintf('          error twist V_b: ')

        fprintf('(')
        for j = 1:6
            if j == 6
                fprintf('%.3f', Vb_vec(j))
            else
                fprintf('%.3f, ', Vb_vec(j))
            end
        end
        fprintf(')\n')

%         calculate the error
        err_ang(i, 1) = norm(wb);
        err_lin(i, 1) = norm(vb);

        fprintf('angular error ||omega_b||: %d\n', norm(wb))
        fprintf(' linear error     ||v_b||: %d\n\n', norm(vb))
```

```
%         return the result if converges
        if norm(wb) < eomg && norm(vb) < ev
            thetalist = theta_curr;
            success   = true;
            writematrix(theta_mat, sprintf('%s.csv', filename));
            return
        end

%         calculate the psuedo jacobian and the new theta
        J   = JacobianBody(Blist, theta_curr);
        J_p = pinv(J);
        dtheta = J_p*Vb_vec;

        theta_curr = theta_curr + dtheta;
        theta_curr = atan2(sin(theta_curr), cos(theta_curr));
    end

%     Fail if not concerge after all iterations
    thetalist = NaN;
    success = false;
end
```

```
% MECH_ENG 449 Robotics Manipulation
% Allen Liu
% Homework 2

close all
clear variables
clc

%% Configuration constants
W1 = .109;  % m
W2 = .082;  % m
L1 = .425;  % m
L2 = .392;  % m
H1 = .089;  % m
H2 = .095;  % m

M = [-1 0 0 L1+L2;
      0 0 1 W1+W2;
      0 1 0 H1-H2;
      0 0 0 1];

Tsd = [ 0.7071  0    0.7071     -0.3;
        0.7071  0   -0.7071     -0.5;
             0  1         0      0.5;
             0  0         0        1];

B1 = [  0   1   0   W1+W2    0      L1+L2]';
B2 = [  0   0   1   H2      -L1-L2  0]';
B3 = [  0   0   1   H2      -L2     0]';
B4 = [  0   0   1   H2       0      0]';
B5 = [  0  -1   0  -W2       0      0]';
B6 = [  0   0   1   0        0      0]';

B_mat = [B1 B2 B3 B4 B5 B6];

%% Init variables
start_x = zeros(2, 1);
start_y = zeros(2, 1);
start_z = zeros(2, 1);

%% Short Iteration
fprintf('---------------------------------- Short Iteration↙
----------------------------------\n');

theta0_vec = [-9 36 -18 -112 0 88]';

T_init = FKinBody(M, B_mat, theta0_vec);
start_x(1) = T_init(1, 4);
start_y(1) = T_init(2, 4);
start_z(1) = T_init(3, 4);
```

```
display(theta0_vec)

[theta_vec, succ, tmat_short, ea_short, el_short] = IKinBodyIterates(B_mat, M, Tsd,↙
theta0_vec, 1000, 0.001, 0.0001, 'short_iterates');

if succ
    display(theta_vec)
end

%% Long Iteration

fprintf('---------------------------------- Long Iteration↙
----------------------------------\n');
theta0_vec = [0 0 0 0 0 0]';
display(theta0_vec)

T_init = FKinBody(M, B_mat, theta0_vec);
start_x(2) = T_init(1, 4);
start_y(2) = T_init(2, 4);
start_z(2) = T_init(3, 4);

[theta_vec, succ, tmat_long, ea_long, el_long] = IKinBodyIterates(B_mat, M, Tsd,↙
theta0_vec, 1000, 0.001, 0.0001, 'long_iterates');

if succ
    display(theta_vec)
end

%% Prepare for plotting

short_iter = readmatrix('short_iterates.csv');
long_iter  = readmatrix('long_iterates.csv');

n = size(long_iter, 1);

x1 = zeros(n, 1);
y1 = zeros(n, 1);
z1 = zeros(n, 1);


for i = 1:n
    thetalist = long_iter(i, :)';
    T = FKinBody(M, B_mat, thetalist);
    pos = T(1:3, 4);

    x1(i) = pos(1);
    y1(i) = pos(2);
    z1(i) = pos(3);
end
```

```
n = size(short_iter, 1);
x2 = zeros(n, 1);
y2 = zeros(n, 1);
z2 = zeros(n, 1);

for i = 1:n
    thetalist = short_iter(i, :)';
    T = FKinBody(M, B_mat, thetalist);
    pos = T(1:3, 4);

    x2(i) = pos(1);
    y2(i) = pos(2);
    z2(i) = pos(3);
end

end_x = Tsd(1, 4);
end_y = Tsd(2, 4);
end_z = Tsd(3, 4);

%% 3d-plot of e-e positions
figure

plot3(x1, y1, z1,'b-', LineWidth=1.5)
hold on
plot3(x2, y2, z2, 'r-', LineWidth=1.5)

plot3(start_x, start_y, start_z, 'ro', MarkerSize=10, LineWidth=3)
plot3(end_x, end_y, end_z, 'kx', MarkerSize=10, LineWidth=3)

hold off
xlabel('$x$ [m]', Interpreter='latex')
ylabel('$y$ [m]', Interpreter='latex')
zlabel('$z$ [m]', Interpreter='latex')
title('\textbf{Trajectory of the End-Effector position}', Interpreter='latex')
legend('Long Iteration', 'Short Iteration', 'Start', 'End')
grid minor

%% Angular error
figure
hold on
plot(ea_short, LineWidth=2)
plot(ea_long, LineWidth=2)
hold off

title('\textbf{Angular Error} $\epsilon_{\omega}$', Interpreter='latex')
xlabel('Number of iterations', Interpreter='latex')
ylabel('Error $\epsilon$', Interpreter='latex')
legend('Short Iteration', 'Long Iteration', Interpreter='latex')
grid minor

%% Linear error
```

```
figure
hold on
plot(el_short, LineWidth=2)
plot(el_long, LineWidth=2)
hold off

title('\textbf{Linear Error} $\epsilon_v$', Interpreter='latex')
xlabel('Number of iterations', Interpreter='latex')
ylabel('Error $\epsilon$', Interpreter='latex')
legend('Short Iteration', 'Long Iteration', Interpreter='latex')
grid minor
```

```
---------------------------------- Short Iteration ----------------------------------

theta0_vec =

    -9
    36
   -18
  -112
     0
    88

--------------- Iteration 1 ---------------
Joint Vector:
    -9    36   -18  -112     0    88

SE(3) end-effector config:
    0.8748   -0.2546    0.4121   -0.0863
    0.3957   -0.1152   -0.9111   -0.2487
    0.2794    0.9602         0    0.7128
         0         0         0    1.0000

          error twist V_b: (0.051, 0.358, -0.280, -0.348, -0.167, 0.082)
angular error ||omega_b||: 4.575661e-01
 linear error     ||v_b||: 3.949115e-01

--------------- Iteration 2 ---------------
Joint Vector:
   -2.4559   -1.5461    1.7629    0.3790   -0.1069   -0.5927

SE(3) end-effector config:
    0.7150   -0.0422    0.6979   -0.1369
    0.6992    0.0425   -0.7137   -0.3582
    0.0004    0.9982    0.0598    0.3558
         0         0         0    1.0000

          error twist V_b: (0.060, 0.011, -0.000, -0.216, 0.145, -0.010)
angular error ||omega_b||: 6.091898e-02
 linear error     ||v_b||: 2.598339e-01

--------------- Iteration 3 ---------------
Joint Vector:
   -2.4536   -1.2214    0.9700    0.3353   -0.0807   -0.0840

SE(3) end-effector config:
    0.7190   -0.0044    0.6950   -0.2733
    0.6950    0.0051   -0.7190   -0.4716
   -0.0003    1.0000    0.0068    0.4917
         0         0         0    1.0000

          error twist V_b: (0.007, 0.017, 0.000, -0.039, 0.008, 0.002)
angular error ||omega_b||: 1.824818e-02
```

```
 linear error     ||v_b||: 3.983725e-02

-------------- Iteration 4 ---------------
Joint Vector:
   -2.4444   -1.1251    0.8116    0.3059   -0.0878    0.0076

SE(3) end-effector config:
    0.7073    0.0005    0.7069   -0.2989
    0.7069   -0.0005   -0.7073   -0.4991
   -0.0000    1.0000   -0.0007    0.4983
         0         0         0    1.0000

          error twist V_b: (-0.001, 0.000, 0.000, -0.001, 0.002, -0.000)
angular error ||omega_b||: 7.398803e-04
 linear error     ||v_b||: 2.182614e-03

-------------- Iteration 5 ---------------
Joint Vector:
   -2.4444   -1.1168    0.7949    0.3220   -0.0882   -0.0000

SE(3) end-effector config:
    0.7071   -0.0000    0.7071   -0.3000
    0.7071    0.0000   -0.7071   -0.5000
   -0.0000    1.0000    0.0000    0.5000
         0         0         0    1.0000

          error twist V_b: (0.000, 0.000, 0.000, -0.000, 0.000, -0.000)
angular error ||omega_b||: 3.528689e-06
 linear error     ||v_b||: 2.410856e-05


theta_vec =

   -2.4444
   -1.1168
    0.7949
    0.3220
   -0.0882
   -0.0000

---------------------------------- Long Iteration ----------------------------------

theta0_vec =

     0
     0
     0
     0
     0
     0
```

```
-------------- Iteration 1 ---------------
Joint Vector:
     0     0     0     0     0     0

SE(3) end-effector config:
   -1.0000         0         0    0.8170
         0         0    1.0000    0.1910
         0    1.0000         0   -0.0060
         0         0         0    1.0000

          error twist V_b: (0.000, -2.356, 0.000, -0.269, 0.506, -1.653)
angular error ||omega_b||: 2.356140e+00
 linear error     ||v_b||: 1.749618e+00

-------------- Iteration 2 ---------------
Joint Vector:
   -2.0234   -0.8808    0.5450    1.8602    0.3327   -1.5243

SE(3) end-effector config:
    0.4509    0.2923    0.8434   -0.0715
    0.8926   -0.1450   -0.4270   -0.5734
   -0.0025    0.9453   -0.3263    0.5148
         0         0         0    1.0000

          error twist V_b: (-0.329, -0.314, 0.055, -0.075, -0.053, -0.224)
angular error ||omega_b||: 4.584319e-01
 linear error     ||v_b||: 2.424753e-01

-------------- Iteration 3 ---------------
Joint Vector:
   -2.4616   -0.9278    0.6999    2.0109   -0.0018   -1.8565

SE(3) end-effector config:
    0.7758    0.0560    0.6285   -0.3028
    0.6267    0.0475   -0.7778   -0.4905
   -0.0734    0.9973    0.0018    0.5378
         0         0         0    1.0000

          error twist V_b: (-0.002, 0.106, 0.074, -0.003, -0.038, 0.009)
angular error ||omega_b||: 1.288508e-01
 linear error     ||v_b||: 3.911863e-02

-------------- Iteration 4 ---------------
Joint Vector:
   -2.4450   -0.1825    2.6933    2.1767    0.0225    1.5956

SE(3) end-effector config:
    0.7667   -0.0144    0.6419   -0.0280
    0.6420    0.0173   -0.7665   -0.2724
   -0.0001    0.9997    0.0225   -0.0609
         0         0         0    1.0000
```

```
          error twist V_b: (0.022, 0.088, 0.001, -0.355, 0.561, -0.009)
angular error ||omega_b||: 9.107370e-02
 linear error     ||v_b||: 6.636733e-01

-------------- Iteration 5 ---------------
Joint Vector:
   -2.5567   -0.8981    1.5668    1.3888    0.0050   -2.0562

SE(3) end-effector config:
    0.8325    0.0014    0.5541   -0.3016
    0.5541   -0.0044   -0.8325   -0.4288
    0.0013    1.0000   -0.0044    0.2225
         0         0         0    1.0000

          error twist V_b: (-0.004, 0.198, -0.002, -0.044, 0.278, 0.056)
angular error ||omega_b||: 1.981935e-01
 linear error     ||v_b||: 2.865519e-01

-------------- Iteration 6 ---------------
Joint Vector:
   -2.4401   -0.5683   -2.1865    0.7770    0.0391    1.9780

SE(3) end-effector config:
    0.7538   -0.0236    0.6567    0.0612
    0.6571    0.0271   -0.7533   -0.1982
   -0.0000    0.9994    0.0359    0.5061
         0         0         0    1.0000

          error twist V_b: (0.036, 0.068, 0.001, -0.470, -0.006, -0.026)
angular error ||omega_b||: 7.725728e-02
 linear error     ||v_b||: 4.708463e-01

-------------- Iteration 7 ---------------
Joint Vector:
   -2.6659    2.0593    2.3053   -1.0416    0.1226    2.9642

SE(3) end-effector config:
    0.8274   -0.0147    0.5614    0.3768
    0.5616    0.0167   -0.8272   -0.0201
    0.0027    0.9998    0.0221    0.1775
         0         0         0    1.0000

          error twist V_b: (0.022, 0.189, -0.001, -0.828, 0.324, -0.058)
angular error ||omega_b||: 1.903799e-01
 linear error     ||v_b||: 8.914120e-01

-------------- Iteration 8 ---------------
Joint Vector:
   -2.7835    2.0239   -1.5581    2.2332    0.4205   -2.6976
```

```
SE(3) end-effector config:
    0.7408    0.0919    0.6654   -0.0229
    0.6708   -0.1528   -0.7257   -0.2049
    0.0350    0.9840   -0.1748   -0.3977
         0         0         0    1.0000

          error twist V_b: (-0.175, 0.046, -0.039, -0.387, 0.905, -0.057)
angular error ||omega_b||: 1.851943e-01
 linear error     ||v_b||: 9.855996e-01

--------------- Iteration 9 ---------------
Joint Vector:
   -0.7726    1.5331    2.2916   -2.6164   -1.4299   -1.0245

SE(3) end-effector config:
   -0.9497   -0.2732   -0.1533   -0.2063
    0.2075   -0.9152    0.3456    0.3694
   -0.2347    0.2964    0.9258   -0.0460
         0         0         0    1.0000

          error twist V_b: (0.956, -1.934, -1.228, -0.697, 0.817, 0.108)
angular error ||omega_b||: 2.482548e+00
 linear error     ||v_b||: 1.079807e+00

--------------- Iteration 10 ---------------
Joint Vector:
    0.4633   -0.1150   -1.2003   -2.4281    0.8305    2.6999

SE(3) end-effector config:
    0.0651   -0.5294   -0.8459    0.3001
   -0.7135   -0.6173    0.3314    0.3336
   -0.6976    0.5820   -0.4180    0.5611
         0         0         0    1.0000

          error twist V_b: (-1.199, 2.284, -0.283, -0.305, 0.442, 1.227)
angular error ||omega_b||: 2.595300e+00
 linear error     ||v_b||: 1.339520e+00

--------------- Iteration 11 ---------------
Joint Vector:
    2.7046    0.0286   -2.8193   -1.9660    1.5606   -2.2459

SE(3) end-effector config:
    0.9708    0.2357   -0.0445   -0.0152
    0.2363   -0.9716    0.0095   -0.1141
   -0.0410   -0.0197   -0.9990    0.1255
         0         0         0    1.0000

          error twist V_b: (-1.558, 0.458, -0.403, -0.386, 0.469, -0.160)
angular error ||omega_b||: 1.673139e+00
 linear error     ||v_b||: 6.287374e-01
```

```
-------------- Iteration 12 ---------------
Joint Vector:
    1.3960    2.7766    2.0358   -1.8135    0.0156    2.3248

SE(3) end-effector config:
   -0.0893   -0.1312   -0.9873   -0.2529
   -0.5670   -0.8083    0.1587   -0.3337
   -0.8188    0.5740   -0.0022    0.4212
         0         0         0    1.0000

          error twist V_b: (-1.182, 2.270, 0.380, -0.018, 0.134, 0.170)
angular error ||omega_b||: 2.587085e+00
 linear error     ||v_b||: 2.166622e-01

-------------- Iteration 13 ---------------
Joint Vector:
    0.9345   -0.0021   -0.5358   -2.4541    2.8877    2.4052

SE(3) end-effector config:
    0.5116    0.5832    0.6310    0.4251
    0.3793    0.5056   -0.7749    0.6252
   -0.7710    0.6358    0.0374    0.3877
         0         0         0    1.0000

          error twist V_b: (-0.010, 0.112, 0.882, -1.255, -0.468, 0.357)
angular error ||omega_b||: 8.893505e-01
 linear error     ||v_b||: 1.386243e+00

-------------- Iteration 14 ---------------
Joint Vector:
    0.8941    3.1010    2.9718   -3.0279    3.0173    3.0506

SE(3) end-effector config:
    0.7177    0.0048    0.6963   -0.0595
    0.6963   -0.0120   -0.7176   -0.0299
    0.0050    0.9999   -0.0120    0.2472
         0         0         0    1.0000

          error twist V_b: (-0.012, 0.015, -0.005, -0.501, 0.255, 0.165)
angular error ||omega_b||: 1.988532e-02
 linear error     ||v_b||: 5.854349e-01

-------------- Iteration 15 ---------------
Joint Vector:
    2.9593   -2.3959    1.7689   -0.9314    0.9778   -1.5461

SE(3) end-effector config:
   -0.9866   -0.1192   -0.1114   -0.1274
    0.1611   -0.8210   -0.5477   -0.1339
   -0.0261   -0.5583    0.8292    0.6742
```

```
         0         0         0    1.0000

          error twist V_b: (1.138, -1.181, -2.183, -0.437, 0.348, -0.175)
angular error ||omega_b||: 2.730631e+00
 linear error     ||v_b||: 5.851866e-01

--------------- Iteration 16 ---------------
Joint Vector:
   -0.7165   -3.0231    2.8741   -1.0422    2.1769    0.1864

SE(3) end-effector config:
    0.5570   -0.8179   -0.1445    0.1004
    0.5859    0.5102   -0.6296   -0.0048
    0.5887    0.2661    0.7633    0.2248
         0         0         0    1.0000

          error twist V_b: (1.184, 0.367, -0.561, -0.513, 0.348, 0.368)
angular error ||omega_b||: 1.360609e+00
 linear error     ||v_b||: 7.208779e-01

--------------- Iteration 17 ---------------
Joint Vector:
   -1.4429   -1.7944   -0.5886   -1.4828    2.9542    2.8414

SE(3) end-effector config:
   -0.0618   -0.1076   -0.9923   -0.0296
   -0.9143    0.4049    0.0130    0.4531
    0.4004    0.9080   -0.1234    0.8341
         0         0         0    1.0000

          error twist V_b: (0.433, 2.349, -0.345, 0.058, -0.398, 1.208)
angular error ||omega_b||: 2.413564e+00
 linear error     ||v_b||: 1.273322e+00

--------------- Iteration 18 ---------------
Joint Vector:
    1.1450    0.0883    0.6344   -1.4966    3.1318   -0.4296

SE(3) end-effector config:
    0.3807   -0.1432    0.9135    0.2994
    0.8609   -0.3058   -0.4067    0.7255
    0.3376    0.9413    0.0068   -0.2751
         0         0         0    1.0000

          error twist V_b: (-0.056, -0.364, -0.342, -1.210, 1.003, 0.187)
angular error ||omega_b||: 5.025482e-01
 linear error     ||v_b||: 1.582182e+00

--------------- Iteration 19 ---------------
Joint Vector:
    0.8984    0.4738    1.3190    0.5884   -3.0202    2.3857
```

```
SE(3) end-effector config:
    0.5515   -0.0700    0.8312    0.1239
    0.8342    0.0454   -0.5497    0.1999
    0.0008    0.9965    0.0835   -0.4116
         0         0         0    1.0000

          error twist V_b: (0.083, -0.201, -0.009, -0.809, 0.908, 0.153)
angular error ||omega_b||: 2.177695e-01
 linear error     ||v_b||: 1.225277e+00

--------------- Iteration 20 ---------------
Joint Vector:
    0.2563   -0.6846   -0.4491    0.2202   -2.7584   -0.8867

SE(3) end-effector config:
    0.9998    0.0143    0.0142    0.5252
    0.0178   -0.2958   -0.9551    0.1717
   -0.0094    0.9551   -0.2960    0.6306
         0         0         0    1.0000

          error twist V_b: (-0.289, 0.763, -0.106, -1.052, -0.068, 0.318)
angular error ||omega_b||: 8.230099e-01
 linear error     ||v_b||: 1.101067e+00

--------------- Iteration 21 ---------------
Joint Vector:
    0.2470   -2.9260    1.5947   -0.6935    2.8127   -2.0265

SE(3) end-effector config:
    0.9944   -0.0488    0.0940   -0.2485
    0.1041    0.2868   -0.9523   -0.0303
    0.0195    0.9567    0.2903    0.6262
         0         0         0    1.0000

          error twist V_b: (0.290, 0.679, 0.082, -0.250, -0.188, 0.388)
angular error ||omega_b||: 7.426343e-01
 linear error     ||v_b||: 4.980760e-01

--------------- Iteration 22 ---------------
Joint Vector:
    1.8140    2.6828    2.2184    3.0066    2.6844    1.3910

SE(3) end-effector config:
   -0.3112    0.3671    0.8766    0.0630
    0.9261    0.3241    0.1930   -0.4009
   -0.2132    0.8719   -0.4408    0.2548
         0         0         0    1.0000

          error twist V_b: (-0.299, -1.034, 0.445, -0.241, 0.137, -0.380)
angular error ||omega_b||: 1.164658e+00
```

```
 linear error     ||v_b||: 4.701491e-01

-------------- Iteration 23 ---------------
Joint Vector:
    1.1198   -3.0559    1.4281   -2.2562   -3.0826    2.2250

SE(3) end-effector config:
    0.3966    0.0330    0.9174   -0.2451
    0.9013    0.1756   -0.3960   -0.4440
   -0.1741    0.9839    0.0399    0.5900
         0         0         0    1.0000

          error twist V_b: (0.072, -0.373, 0.166, -0.070, -0.096, -0.017)
angular error ||omega_b||: 4.148192e-01
 linear error     ||v_b||: 1.199673e-01

-------------- Iteration 24 ---------------
Joint Vector:
    1.0982   -2.6680    1.5790   -0.5425    2.9319   -1.6325

SE(3) end-effector config:
    0.4666   -0.1840    0.8651   -0.0724
    0.8845    0.0966   -0.4565   -0.0784
    0.0004    0.9782    0.2078    0.6530
         0         0         0    1.0000

          error twist V_b: (0.208, -0.299, -0.032, -0.478, -0.157, 0.051)
angular error ||omega_b||: 3.653000e-01
 linear error     ||v_b||: 5.055024e-01

-------------- Iteration 25 ---------------
Joint Vector:
    1.8201    2.0082    3.0829    0.9642    3.0769   -0.3038

SE(3) end-effector config:
   -0.3053   -0.0373    0.9515   -0.0242
    0.9492    0.0681    0.3073   -0.0149
   -0.0762    0.9970    0.0146   -0.0231
         0         0         0    1.0000

          error twist V_b: (0.055, -1.097, 0.060, -0.582, 0.504, -0.151)
angular error ||omega_b||: 1.099644e+00
 linear error     ||v_b||: 7.850997e-01

-------------- Iteration 26 ---------------
Joint Vector:
   -1.4306    0.1094   -2.7629    0.0055    1.1356   -2.5848

SE(3) end-effector config:
   -0.7713    0.5582    0.3058    0.1500
   -0.0435   -0.5256    0.8496   -0.0351
```

```
    0.6350    0.6421    0.4296    0.3453
         0         0         0    1.0000

          error twist V_b: (-0.513, -2.029, -0.932, -0.275, 0.383, -0.691)
angular error ||omega_b||: 2.290666e+00
 linear error     ||v_b||: 8.364298e-01

-------------- Iteration 27 ---------------
Joint Vector:
   -1.7980   -1.1991   -2.1934   -0.0386   -0.5340   -3.0289

SE(3) end-effector config:
    0.6846   -0.0127    0.7288    0.2228
    0.7160    0.1989   -0.6692    0.1666
   -0.1364    0.9799    0.1453    0.4906
         0         0         0    1.0000

          error twist V_b: (0.148, -0.032, 0.135, -0.842, -0.055, 0.087)
angular error ||omega_b||: 2.032438e-01
 linear error     ||v_b||: 8.480546e-01

-------------- Iteration 28 ---------------
Joint Vector:
   -0.4238    1.1661    2.8426   -0.1961   -1.8666    2.3627

SE(3) end-effector config:
    0.0299    0.8259    0.5631    0.0662
    0.7334    0.3646   -0.5737    0.0635
   -0.6791    0.4301   -0.5948    0.0229
         0         0         0    1.0000

          error twist V_b: (-0.586, -0.312, 0.968, -0.851, 0.134, -0.088)
angular error ||omega_b||: 1.173596e+00
 linear error     ||v_b||: 8.663674e-01

-------------- Iteration 29 ---------------
Joint Vector:
   -1.6844    2.3914    1.4661    2.6226   -2.8858    2.0034

SE(3) end-effector config:
    0.1304    0.3352   -0.9331    0.1027
    0.2067    0.9112    0.3563    0.6381
    0.9697   -0.2393    0.0495   -0.0326
         0         0         0    1.0000

          error twist V_b: (2.223, 1.722, -0.430, -0.043, -0.520, 1.713)
angular error ||omega_b||: 2.844871e+00
 linear error     ||v_b||: 1.790849e+00

-------------- Iteration 30 ---------------
Joint Vector:
```

```
    1.1216    0.4270   -1.3196    2.5700   -1.4303    2.6626

SE(3) end-effector config:
   -0.5983   -0.7972   -0.0803    0.1290
    0.7825   -0.6029    0.1558    0.5450
   -0.1726    0.0303    0.9845    0.3091
         0         0         0    1.0000

          error twist V_b: (1.362, -1.250, -0.992, -1.063, 0.571, -0.072)
angular error ||omega_b||: 2.098092e+00
 linear error     ||v_b||: 1.208741e+00

-------------- Iteration 31 ---------------
Joint Vector:
    0.1143    2.0602    0.4815   -0.3669   -3.0600    2.5924

SE(3) end-effector config:
    0.8986   -0.4087    0.1596   -0.5969
    0.1731   -0.0041   -0.9849   -0.0411
    0.4032    0.9126    0.0670   -0.4480
         0         0         0    1.0000

          error twist V_b: (0.194, 0.601, -0.381, 0.238, 0.892, 0.625)
angular error ||omega_b||: 7.379325e-01
 linear error     ||v_b||: 1.114968e+00

-------------- Iteration 32 ---------------
Joint Vector:
    1.0643   -2.0437   -1.6310    2.1645    3.0955   -1.5254

SE(3) end-effector config:
    0.4833   -0.0330    0.8749   -0.2353
    0.8754    0.0355   -0.4822   -0.3682
   -0.0152    0.9988    0.0460    0.2661
         0         0         0    1.0000

          error twist V_b: (0.048, -0.281, 0.009, -0.146, 0.232, 0.033)
angular error ||omega_b||: 2.854392e-01
 linear error     ||v_b||: 2.762801e-01

-------------- Iteration 33 ---------------
Joint Vector:
    1.1157   -2.8843   -1.6099    2.1107   -3.1240   -2.3848

SE(3) end-effector config:
    0.4280    0.0114    0.9037   -0.2131
    0.9038   -0.0041   -0.4280   -0.3739
   -0.0012    0.9999   -0.0121   -0.1176
         0         0         0    1.0000

          error twist V_b: (-0.012, -0.343, 0.003, -0.155, 0.617, -0.002)
```

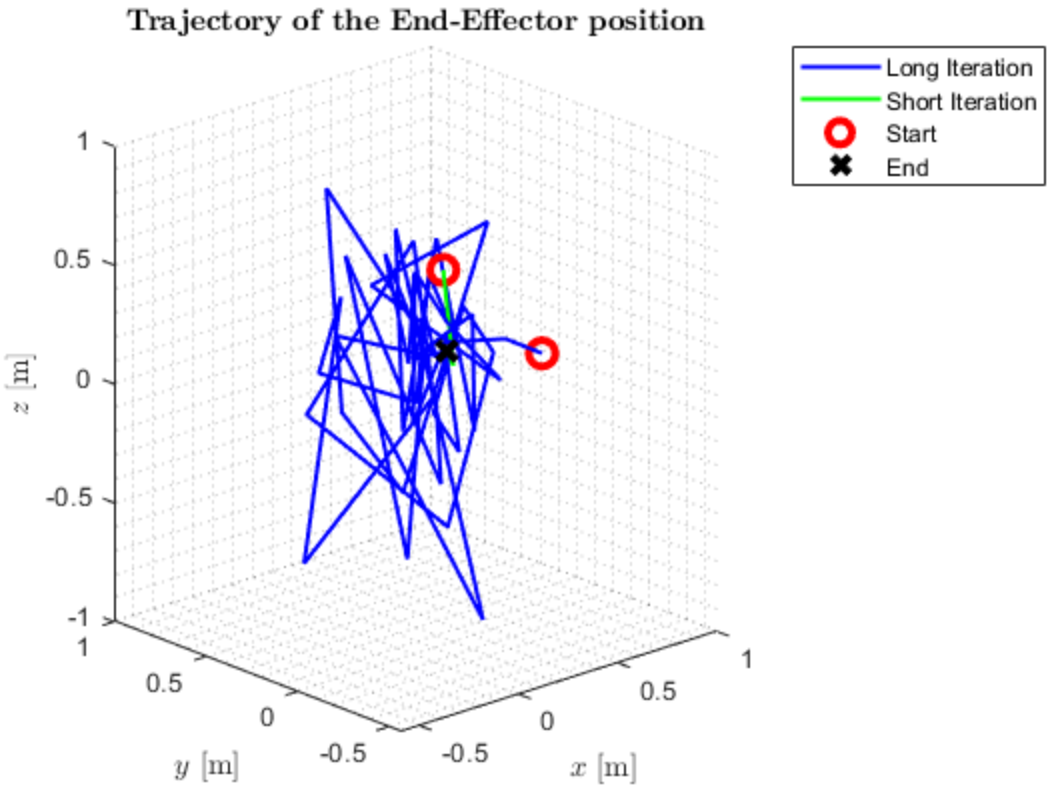
```
angular error ||omega_b||: 3.432742e-01
 linear error     ||v_b||: 6.365098e-01

--------------- Iteration 34 ---------------
Joint Vector:
    1.1318    0.0275   -0.9335   -0.5000    2.8900   -1.4060

SE(3) end-effector config:
    0.3877   -0.2245    0.8940    0.2978
    0.9218    0.0997   -0.3747    0.7039
   -0.0050    0.9694    0.2456    0.3904
         0         0         0    1.0000

          error twist V_b: (0.246, -0.386, -0.043, -1.339, 0.094, 0.190)
angular error ||omega_b||: 4.596383e-01
 linear error     ||v_b||: 1.356193e+00

--------------- Iteration 35 ---------------
Joint Vector:
    0.8566    2.1257   -0.9942   -2.8482   -3.1324   -1.6824

SE(3) end-effector config:
    0.6539   -0.0156    0.7565    0.0038
    0.7558   -0.0319   -0.6540    0.0456
    0.0343    0.9994   -0.0091   -0.6139
         0         0         0    1.0000

          error twist V_b: (-0.010, -0.072, -0.034, -0.587, 1.125, 0.144)
angular error ||omega_b||: 8.061524e-02
 linear error     ||v_b||: 1.277174e+00

--------------- Iteration 36 ---------------
Joint Vector:
    2.9433    0.0908   -2.8545   -1.5797   -2.5404    1.9206

SE(3) end-effector config:
   -0.9972   -0.0652   -0.0375    0.0046
    0.0027   -0.5289    0.8487   -0.0431
   -0.0752    0.8462    0.5276    0.2726
         0         0         0    1.0000

          error twist V_b: (0.358, -2.298, -0.619, -0.329, 0.419, -0.483)
angular error ||omega_b||: 2.406548e+00
 linear error     ||v_b||: 7.191852e-01

--------------- Iteration 37 ---------------
Joint Vector:
    1.2684   -0.5438    1.0210    0.6390   -2.5458    0.7606

SE(3) end-effector config:
    0.6510   -0.2501    0.7167    0.1413
```

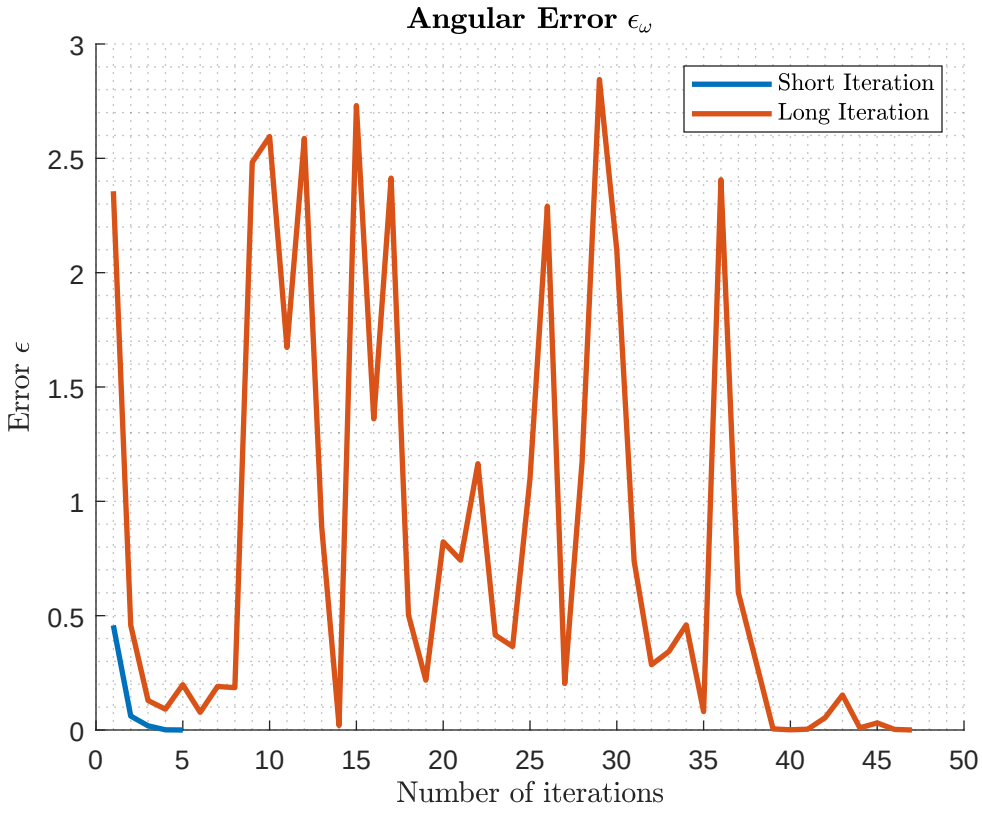
```
    0.7215    0.4974   -0.4818    0.5911
   -0.2360    0.8307    0.5042    0.1285
         0         0         0    1.0000

          error twist V_b: (0.549, -0.115, 0.218, -1.142, 0.121, 0.475)
angular error ||omega_b||: 6.017699e-01
 linear error     ||v_b||: 1.243204e+00

--------------- Iteration 38 ---------------
Joint Vector:
    1.4779    2.1651   -0.0835   -2.0927   -2.7489   -0.0597

SE(3) end-effector config:
    0.4659    0.0268    0.8844   -0.0757
    0.8835    0.0418   -0.4666   -0.4549
   -0.0495    0.9988   -0.0042   -0.7004
         0         0         0    1.0000

          error twist V_b: (0.003, -0.300, 0.050, -0.200, 1.196, -0.154)
angular error ||omega_b||: 3.040574e-01
 linear error     ||v_b||: 1.222130e+00

--------------- Iteration 39 ---------------
Joint Vector:
    1.0537   -0.9094   -0.7859    1.7113   -2.8735    0.0132

SE(3) end-effector config:
    0.7069   -0.0015    0.7073    0.0675
    0.7073    0.0045   -0.7069    0.1792
   -0.0022    1.0000    0.0042    0.7187
         0         0         0    1.0000

          error twist V_b: (0.004, -0.000, 0.002, -0.740, -0.220, 0.220)
angular error ||omega_b||: 4.768190e-03
 linear error     ||v_b||: 8.026123e-01

--------------- Iteration 40 ---------------
Joint Vector:
    0.9740   -0.5161    2.7541   -2.2333   -2.9531    0.0048

SE(3) end-effector config:
    0.7071   -0.0008    0.7071    0.0390
    0.7071    0.0004   -0.7071    0.1080
    0.0003    1.0000    0.0009   -0.1041
         0         0         0    1.0000

          error twist V_b: (0.001, -0.000, -0.000, -0.670, 0.604, 0.190)
angular error ||omega_b||: 9.037095e-04
 linear error     ||v_b||: 9.217504e-01

--------------- Iteration 41 ---------------
```

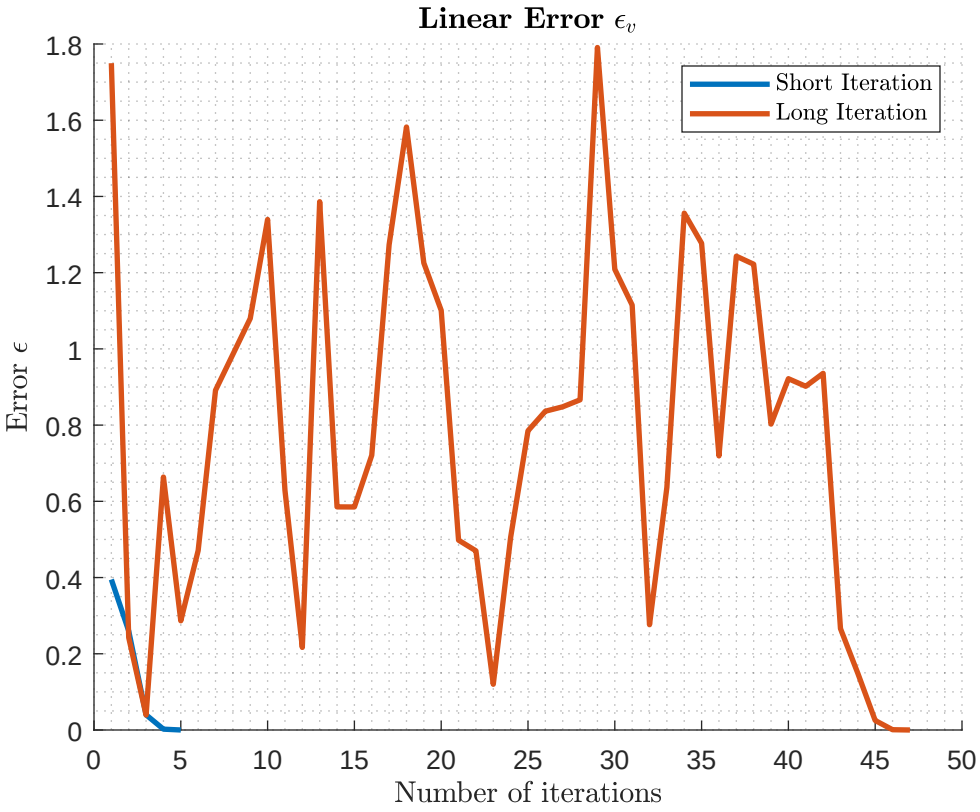
```
Joint Vector:
    0.4918   -0.6001   -1.0820    1.6938    2.8480    0.0119

SE(3) end-effector config:
    0.7071    0.0019    0.7071    0.2763
    0.7071   -0.0029   -0.7071    0.1826
    0.0007    1.0000   -0.0034    0.6233
         0         0         0    1.0000

          error twist V_b: (-0.003, 0.000, -0.001, -0.890, -0.123, 0.075)
angular error ||omega_b||: 3.457515e-03
 linear error     ||v_b||: 9.018477e-01

--------------- Iteration 42 ---------------
Joint Vector:
   -0.5843   -0.5538    2.6808   -2.1686    1.7719   -0.0435

SE(3) end-effector config:
    0.7079   -0.0039    0.7063    0.2502
    0.7055    0.0537   -0.7067   -0.0545
   -0.0351    0.9986    0.0408   -0.1120
         0         0         0    1.0000

          error twist V_b: (0.041, 0.001, 0.035, -0.715, 0.601, -0.061)
angular error ||omega_b||: 5.383302e-02
 linear error     ||v_b||: 9.358911e-01

--------------- Iteration 43 ---------------
Joint Vector:
    1.5222   -1.4922   -1.2765    2.8042   -2.4003    0.1773

SE(3) end-effector config:
    0.6994   -0.1236    0.7039   -0.0674
    0.6987   -0.0893   -0.7099   -0.3876
    0.1506    0.9883    0.0239    0.5625
         0         0         0    1.0000

          error twist V_b: (0.024, 0.002, -0.151, -0.248, -0.043, -0.085)
angular error ||omega_b||: 1.530570e-01
 linear error     ||v_b||: 2.660240e-01

--------------- Iteration 44 ---------------
Joint Vector:
    0.8560   -1.8644   -1.1571    3.0481   -3.0727    0.0356

SE(3) end-effector config:
    0.7059   -0.0077    0.7083   -0.3616
    0.7083   -0.0051   -0.7059   -0.3752
    0.0091    1.0000    0.0018    0.4479
         0         0         0    1.0000
```

```
          error twist V_b: (0.002, -0.002, -0.009, -0.045, 0.052, 0.132)
angular error ||omega_b||: 9.428401e-03
 linear error     ||v_b||: 1.486420e-01

-------------- Iteration 45 ---------------
Joint Vector:
    1.1054   -1.9785   -0.8722    2.7540   -2.8217   -0.0969

SE(3) end-effector config:
    0.7059    0.0251    0.7079   -0.2794
    0.7083   -0.0178   -0.7057   -0.4869
   -0.0051    0.9995   -0.0304    0.4945
         0         0         0    1.0000

          error twist V_b: (-0.030, -0.002, 0.005, -0.024, 0.005, -0.005)
angular error ||omega_b||: 3.082948e-02
 linear error     ||v_b||: 2.497565e-02

-------------- Iteration 46 ---------------
Joint Vector:
    1.0821   -2.0257   -0.7934    2.8122   -2.8463   -0.0073

SE(3) end-effector config:
    0.7061    0.0019    0.7081   -0.3001
    0.7081   -0.0010   -0.7061   -0.4993
   -0.0007    1.0000   -0.0020    0.4999
         0         0         0    1.0000

          error twist V_b: (-0.002, -0.001, 0.001, -0.000, 0.000, 0.001)
angular error ||omega_b||: 2.520669e-03
 linear error     ||v_b||: 7.341997e-04

-------------- Iteration 47 ---------------
Joint Vector:
    1.0829   -2.0248   -0.7948    2.8196   -2.8441    0.0001

SE(3) end-effector config:
    0.7071   -0.0000    0.7071   -0.3000
    0.7071    0.0000   -0.7071   -0.5000
    0.0000    1.0000    0.0000    0.5000
         0         0         0    1.0000

          error twist V_b: (0.000, -0.000, -0.000, -0.000, -0.000, -0.000)
angular error ||omega_b||: 1.690179e-05
 linear error     ||v_b||: 3.227473e-06


theta_vec =

    1.0829
   -2.0248
```

```
   -0.7948
    2.8196
   -2.8441
    0.0001

>>
```


Trajectory of the End-Effector position
Long Iteration
Short Iteration
Start
End
1
0.5
0
-0.5
-1
z [m]
1
0.5
0
-0.5
y [m]
1
0.5
0
-0.5
x [m]


Angular Error ϵ_ω
Short Iteration
Long Iteration
Error ϵ
Number of iterations
0
0.5
1
1.5
2
2.5
3
0
5
10
15
20
25
30
35
40
45
50


Linear Error ϵ_v
Short Iteration
Long Iteration
Error ϵ
Number of iterations
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
5
10
15
20
25
30
35
40
45
50

At some configurations, robot will not be able to move toward the desired frame, so that when trying to solve it with NR, robot will move away from a configuration rather than moving toward the desired frame. And also, when robot moves closer to the singularity points, robot will be difficult to move in some direction, hence it will make the robot difficult to converge for a starting configuration that is far away from its desired one.