

ME455 Active Learning for Robotics Homework 4

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Problem 1

Find: $a_z(t), b_v(t), M, m_1, m_2$

From the description, the $a_x(t)$, $b_u(t)$ and p_1 are:

$$\begin{aligned}a_x(t) &= D_1 l(x(t), u(t)) \\b_u(t) &= D_2 l(x(t), u(t)) \\p_1 &= Dm(x(T))\end{aligned}$$

From equation (7), we can derive that

$$\begin{aligned}& \int_0^T [D_1 l(x(t), u(t)) \cdot z(t) + D_2 l(x(t), u(t)) \cdot v(t)] dt + Dm(x(T)) \cdot z(T) \\& + \int_0^T [z(t)^\top Q_z z(t) + v(t)^\top R_v v(t)] dt \\& = \int_0^T \{ [a_x(t) + z(t)^\top Q_z] \cdot z(t) + [b_u(t) + v(t)^\top R_v] \cdot v(t) \} dt + p_1 \cdot z(T) \\& = \int_0^T [D_1 l'(z(t), v(t)) \cdot z(t) + D_2 l'(z(t), v(t)) \cdot v(t)] dt + p_1 \cdot z(T) \\& = \int_0^T [a_z(t) \cdot z(t) + b_v(t) \cdot v(t)] dt + p_1 \cdot z(T)\end{aligned}$$

Hence, the expression for $a_z(t)$ and $b_v(t)$ are

$$\begin{aligned}a_z(t) &= a_x(t) + z(t)^\top Q_z \\&= D_1 l(x(t), u(t)) + z(t)^\top Q_z \\b_v(t) &= b_u(t) + v(t)^\top R_v \\&= D_2 l(x(t), u(t)) + v(t)^\top R_v\end{aligned}$$

Then, since we know that

$$\begin{aligned}0 &= p(t)^\top B(t) + b_v(t)^\top \\b_v(t) &= b_u(t) + v(t)^\top R_v\end{aligned}$$

We can find the expression for $v(t)$ by

$$\begin{aligned}0 &= p(t)^\top B(t) + [b_u(t) + v(t)^\top R_v]^\top \\&= p(t)^\top B(t) + b_u(t)^\top + R_v^\top v(t) \\R_v^\top v(t) &= -p(t)^\top B(t) - b_u(t)^\top \\v(t) &= -R_v^{-\top} p(t)^\top B(t) - R_v^{-\top} b_u(t)^\top\end{aligned}$$

Then since the ODE is expressed as:

$$\begin{aligned}\dot{p}(t) &= -A(t)^\top p(t) - a_z(t) \\\dot{z}(t) &= A(t)z(t) + B(t)v(t)\end{aligned}$$

The ODE can be expressed in matrix form as:

$$\begin{aligned}\begin{bmatrix} \dot{z}(t) \\ \dot{p}(t) \end{bmatrix} &= \begin{bmatrix} A(t) & 0 \\ 0 & -A(t)^\top \end{bmatrix} \begin{bmatrix} z(t) \\ p(t) \end{bmatrix} + \begin{bmatrix} -a_z(t) \\ B(t)v(t) \end{bmatrix} \\&= \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ p(t) \end{bmatrix} + \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}\end{aligned}$$

Finally the M matrix and vector m_1, m_2 can be expressed as:

$$\begin{aligned}
 M &= \begin{bmatrix} A(t) & 0 \\ 0 & -A(t)^\top \end{bmatrix} \\
 m_1 &= -a_z(t) \\
 m_2 &= B(t)v(t) \\
 &= B(t) [-R_v^{-\top} p(t)^\top B(t) - R_v^{-\top} b_u(t)^\top] \\
 &= -B(t) R_v^{-\top} [p(t)^\top B(t) + b_u(t)^\top]
 \end{aligned}$$

After we solve the BVP to find $z(t)$ and $p(t)$ we can find $v(t)$ by

$$\begin{aligned}
 \dot{z}(t) &= \frac{d}{dt} z(t) \\
 \dot{z}(t) &= A(t)z(t) + B(t)v(t) \\
 v(t) &= B^{-1}(t) [\dot{z}(t) - A(t)z(t)]
 \end{aligned}$$

Problem 2

