ME455 Active Learning for Robotics Homework 4

Author: Jingkun (Allen) Liu

Problem 1

Find: $a_z(t), b_v(t), M, m_1, m_2$

From the description, the $a_x(t)$, $b_u(t)$ and p_1 are:

$$a_x(t) = D_1 l(x(t), u(t))$$
$$b_u(t) = D_2 l(x(t), u(t))$$
$$p_1 = Dm(x(T))$$

From equation (7), we can derive that

$$\int_0^T \left[D_1 l(x(t), u(t)) \cdot z(t) + D_2 l(x(t), u(t)) \cdot v(t) \right] dt + Dm(x(T)) \cdot z(T)$$

$$+ \int_0^T \left[z(t)^\top Q_z z(t) + v(t)^\top R_v v(t) \right] dt$$

$$= \int_0^T \left\{ \left[a_x(t) + z(t)^\top Q_z \right] \cdot z(t) + \left[b_u(t) + v(t)^\top R_v \right] \cdot v(t) \right\} dt + p_1 \cdot z(T)$$

$$= \int_0^T \left[D_1 l'(z(t), v(t)) \cdot z(t) + D_2 l'(z(t), v(t)) \cdot v(t) \right] dt + p_1 \cdot z(T)$$

$$= \int_0^T \left[a_z(t) \cdot z(t) + b_v(t) \cdot v(t) \right] dt + p_1 \cdot z(T)$$

Hence, the expression for $a_z(t)$ and $b_v(t)$ are

$$a_z(t) = a_x(t) + z(t)^\top Q_z$$
$$= D_1 l(x(t), u(t)) + z(t)^\top Q_z$$
$$b_v(t) = b_u(t) + v(t)^\top R_v$$
$$= D_2 l(x(t), u(t)) + v(t)^\top R_v$$

Then, since we know that

$$0 = p(t)^\top B(t) + b_v(t)^\top$$
$$b_v(t) = b_u(t) + v(t)^\top R_v$$

We can find the expression for $v(t)$ by

$$0 = p(t)^\top B(t) + \left[b_u(t) + v(t)^\top R_v \right]^\top$$
$$= p(t)^\top B(t) + b_u(t)^\top + R_v^\top v(t)$$
$$R_v^\top v(t) = -p(t)^\top B(t) - b_u(t)^\top$$
$$v(t) = -R_v^{-\top} p(t)^\top B(t) - R_v^{-\top} b_u(t)^\top$$

Then since the ODE is expressed as:

$$\dot{p}(t) = -A(t)^\top p(t) - a_z(t)$$
$$\dot{z}(t) = A(t) z(t) + B(t) v(t)$$

The ODE can be expressed in matrix form as:

$$\begin{bmatrix} \dot{z}(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} A(t) & 0 \\ 0 & -A(t)^\top \end{bmatrix} \begin{bmatrix} z(t) \\ p(t) \end{bmatrix} + \begin{bmatrix} B(t) v(t) \\ -a_z(t) \end{bmatrix}$$

$$= \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ p(t) \end{bmatrix} + \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$$
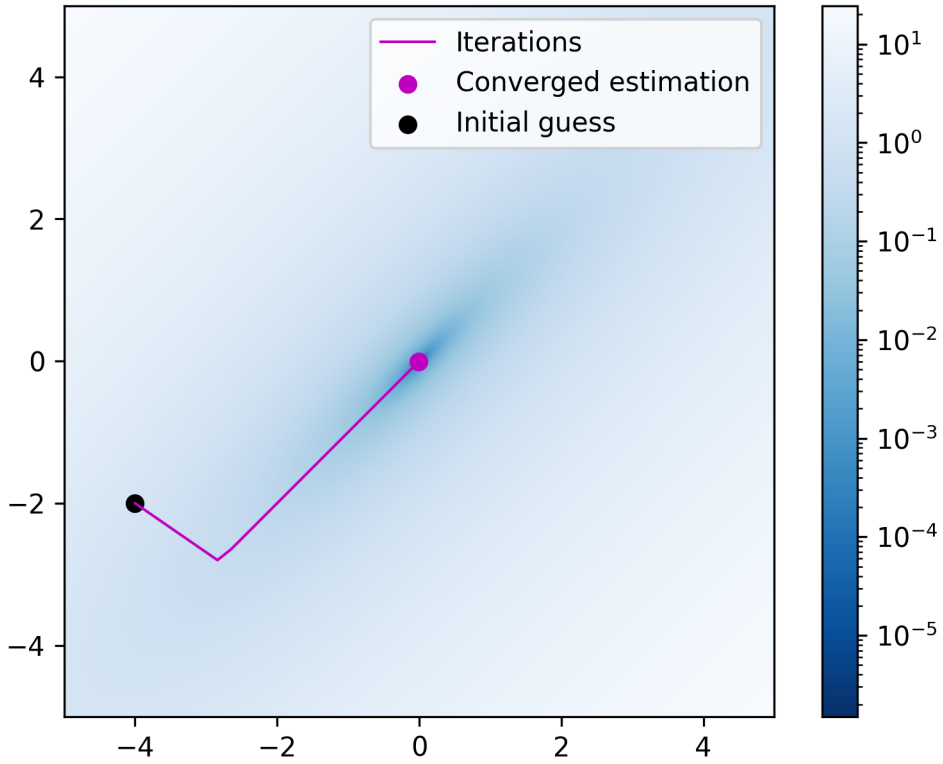
Finally the M matrix and vector m_1, m_2 can be expressed as:

$$M = \begin{bmatrix} A(t) & 0 \\ 0 & -A(t)^\top \end{bmatrix}$$

$$m_1 = B(t)v(t)$$
$$= B(t)\left[-R_v^{-\top}p(t)^\top B(t) - R_v^{-\top}b_u(t)^\top\right]$$
$$= -B(t)R_v^{-\top}\left[p(t)^\top B(t) + b_u(t)^\top\right]$$
$$m_2 = -a_z(t)$$
$$= -a_x(t) - z(t)^\top Q_z$$

After we solve the BVP to find $z(t)$ and $p(t)$ we can find $v(t)$ by using the expression for $v(t)$ derived above:

$$v(t) = -R_v^{-\top}p(t)^\top B(t) - R_v^{-\top}b_u(t)^\top$$
$$= -R_v^{-\top}\left[p(t)^\top B(t) + b_u(t)^\top\right]$$

Problem 2

Problem 3