

# ME455 Active Learning for Robotics Homework 4

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## Problem 1

**Find:**  $a_z(t), b_v(t), M, m_1, m_2$

From the description, the  $a_x(t)$ ,  $b_u(t)$  and  $p_1$  are:

$$\begin{aligned}a_x(t) &= D_1 l(x(t), u(t)) \\b_u(t) &= D_2 l(x(t), u(t)) \\p_1 &= Dm(x(T))\end{aligned}$$

From equation (7), we can derive that

$$\begin{aligned}& \int_0^T [D_1 l(x(t), u(t)) \cdot z(t) + D_2 l(x(t), u(t)) \cdot v(t)] dt + Dm(x(T)) \cdot z(T) \\& + \int_0^T [z(t)^\top Q_z z(t) + v(t)^\top R_v v(t)] dt \\& = \int_0^T \{ [a_x(t) + z(t)^\top Q_z] \cdot z(t) + [b_u(t) + v(t)^\top R_v] \cdot v(t) \} dt + p_1 \cdot z(T) \\& = \int_0^T [D_1 l'(z(t), v(t)) \cdot z(t) + D_2 l'(z(t), v(t)) \cdot v(t)] dt + p_1 \cdot z(T) \\& = \int_0^T [a_z(t) \cdot z(t) + b_v(t) \cdot v(t)] dt + p_1 \cdot z(T)\end{aligned}$$

Hence, the expression for  $a_z(t)$  and  $b_v(t)$  are

$$\begin{aligned}a_z(t) &= a_x(t) + z(t)^\top Q_z \\&= D_1 l(x(t), u(t)) + z(t)^\top Q_z \\b_v(t) &= b_u(t) + v(t)^\top R_v \\&= D_2 l(x(t), u(t)) + v(t)^\top R_v\end{aligned}$$

Then, since we know that

$$\begin{aligned}0 &= p(t)^\top B(t) + b_v(t)^\top \\b_v(t) &= b_u(t) + v(t)^\top R_v\end{aligned}$$

We can find the expression for  $v(t)$  by

$$\begin{aligned}0 &= p(t)^\top B(t) + [b_u(t) + v(t)^\top R_v]^\top \\&= p(t)^\top B(t) + b_u(t)^\top + R_v^\top v(t) \\R_v^\top v(t) &= -p(t)^\top B(t) - b_u(t)^\top \\v(t) &= -R_v^{-\top} p(t)^\top B(t) - R_v^{-\top} b_u(t)^\top\end{aligned}$$

Then since the ODE is expressed as:

$$\begin{aligned}\dot{p}(t) &= -A(t)^\top p(t) - a_z(t) \\\dot{z}(t) &= A(t)z(t) + B(t)v(t)\end{aligned}$$

The ODE can be expressed in matrix form as:

$$\begin{aligned}\begin{bmatrix} \dot{z}(t) \\ \dot{p}(t) \end{bmatrix} &= \begin{bmatrix} A(t) & 0 \\ 0 & -A(t)^\top \end{bmatrix} \begin{bmatrix} z(t) \\ p(t) \end{bmatrix} + \begin{bmatrix} B(t)v(t) \\ -a_z(t) \end{bmatrix} \\&= \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ p(t) \end{bmatrix} + \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}\end{aligned}$$

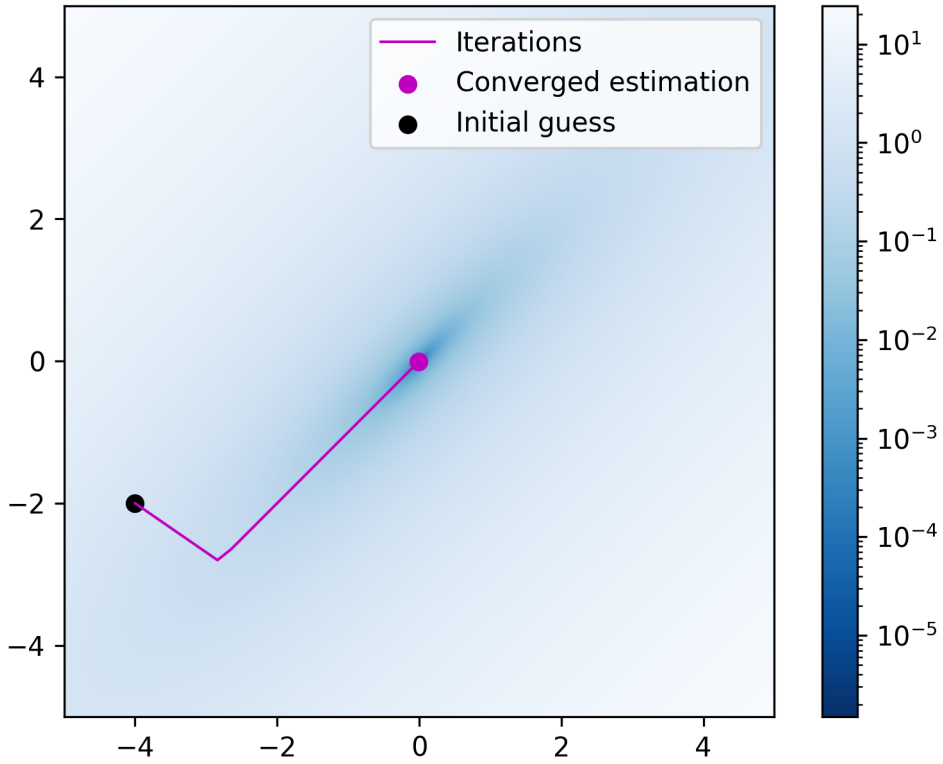
Finally the  $M$  matrix and vector  $m_1, m_2$  can be expressed as:

$$\begin{aligned}
 M &= \begin{bmatrix} A(t) & 0 \\ 0 & -A(t)^\top \end{bmatrix} \\
 m_1 &= B(t)v(t) \\
 &= B(t) \left[ -R_v^{-\top} p(t)^\top B(t) - R_v^{-\top} b_u(t)^\top \right] \\
 &= -B(t) R_v^{-\top} \left[ p(t)^\top B(t) + b_u(t)^\top \right] \\
 m_2 &= -a_z(t) \\
 &= -a_x(t) - z(t)^\top Q_z
 \end{aligned}$$

After we solve the BVP to find  $z(t)$  and  $p(t)$  we can find  $v(t)$  by using the expression for  $v(t)$  derived above:

$$\begin{aligned}
 v(t) &= -R_v^{-\top} p(t)^\top B(t) - R_v^{-\top} b_u(t)^\top \\
 &= -R_v^{-\top} \left[ p(t)^\top B(t) + b_u(t)^\top \right]
 \end{aligned}$$

## Problem 2



### Problem 3

