

# **Lesson 3 Brief Analysis of Inverse Kinematics**

## **1. Inverse Kinematics Introduction**

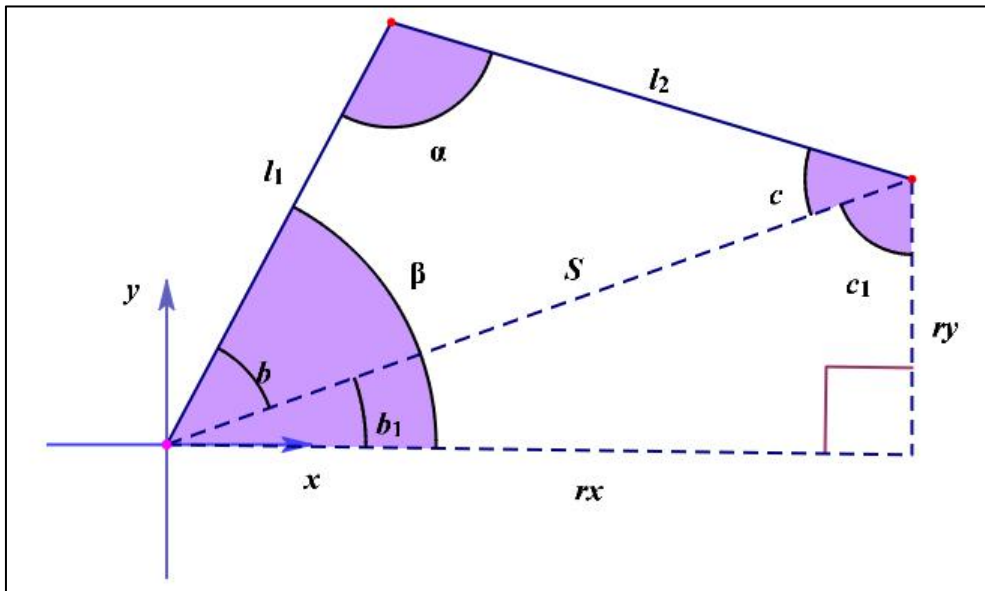
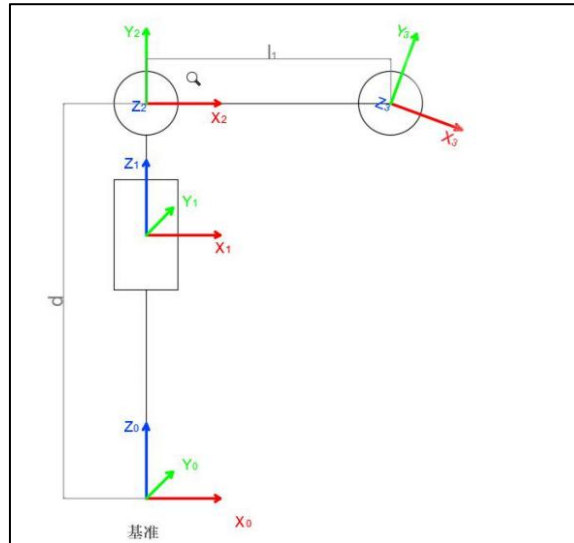
Inverse kinematics is the process of determining the parameters of the robot joints to be set to reach a desired position.

The inverse kinematics of the robotic arm is an crucial foundation for the trajectory planning and control. Whether the inverse kinematics is fast and accurate to get solution will directly affect the accuracy of the robotic arm's trajectory planning and control. Therefore, for the six-degree-of-freedom robotic arm, a fast and accurate inverse kinematics solution method is very important.

## **2. Brief Analysis of Inverse Kinematics**

For robotic arm, given the position and the orientation of the end-effector, the rotation angle of each joint can be calculated. The three-dimensional motion of robotic arm is relatively complex. To simplify the model, we remove the rotation joint of the below pan-tilt so that the kinematic analysis can be performed in a two-dimensional plane.

Performing inverse kinematic analysis generally involves a large number of matrix operations, which is a complex and computationally intensive process and therefore difficult to implement. In order to better fit our needs, we use the geometric method to analyze the robotic arm.



We can consider the MaxArm robotic arm as a special 3-link robotic arm.  
According to the above figure, we simplify the inverse motion of the robotic arm to the solution of two angles  $\alpha$ ,  $\beta$ .

Given  $l_1$  and  $l_2$ , according to  $S^2 = r_x^2 + r_y^2$

$$\angle \alpha = \arccos[(l_1^2 + l_2^2 - S^2) / 2l_1l_2]$$

$$\angle \beta = \arccos[(l_1^2 + S^2 - l_2^2) / 2l_1S]$$

$$\angle c = \pi - \angle \alpha - \angle \beta$$

$$\angle b_1 = \arcsin(r_x / S)$$

$$\angle c1 = \pi/2 - \arcsin(rx/S)$$

$$\angle \beta = \angle b1 + \angle b$$

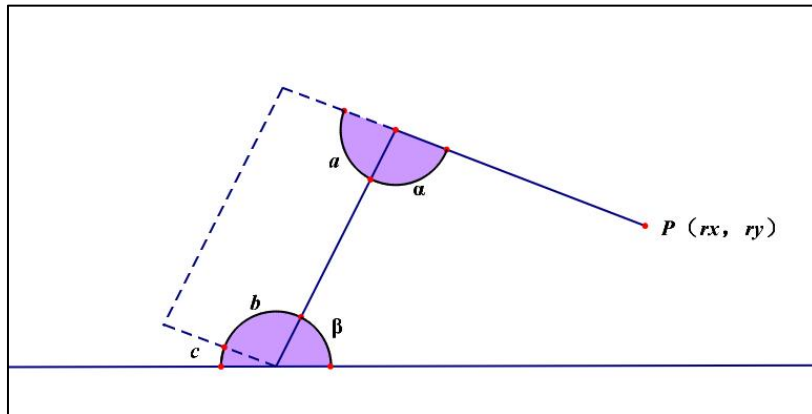
Through calculation,

$$\angle \alpha = \arccos[|l2 + l22 - (rx^2 + ry^2)/2l1l2|]$$

$$\angle \beta = \arccos[|l2 - l22 + rx^2 + ry^2/2l1 \sqrt{rx^2 + ry^2}|]$$

Calculate the angle of  $\angle \alpha$  and  $\angle \beta$ , and  $\angle \beta$  is the rotation angle of ID2.

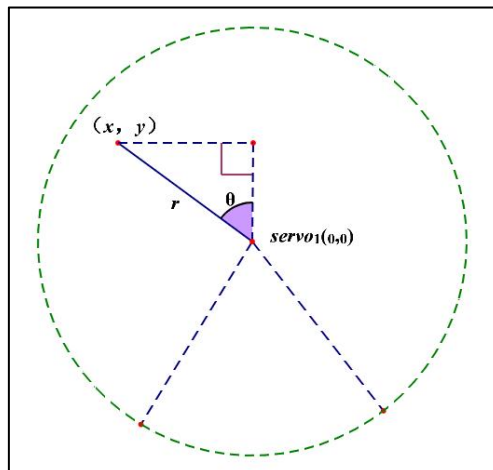
Then get from the figure above,



The angle of rotation of the ID3 servo,  $\angle c$ , is obtained by the parallelogram relationship for  $\angle \alpha$ .

$$\angle b = \angle \alpha$$

$$\angle c = 180^\circ - \angle \beta - \angle \alpha$$



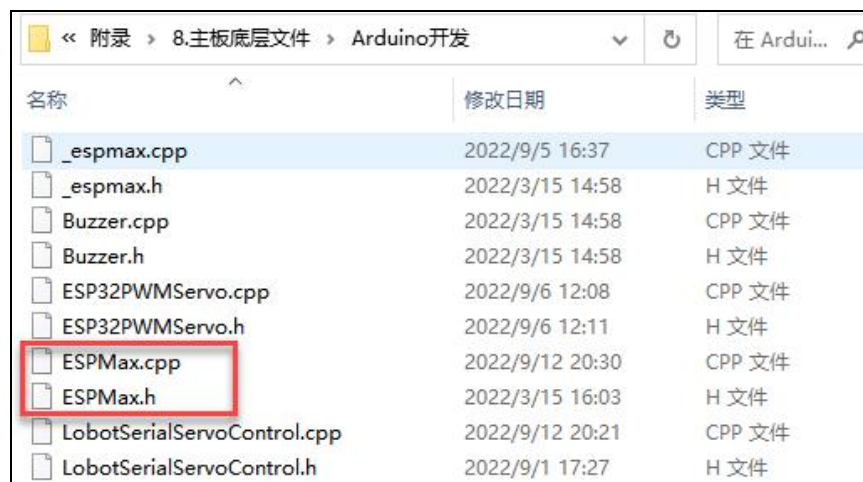
Based on the above figure, the angle of ID1 servo is  $\theta$ . Thus,

$$\theta = \arctan(x/y)$$

Finally, the final angle is determined by determining the quadrant in which the position is located based on the positive and negative relationship between x and y.

### 3. The path of inverse kinematics program

You can view the inverse kinematics encapsulation library in “Appendix/ 8. Controller Underlying Files/ Arduino Development”. The code instructions can refer to the annotation of corresponding program.



名称	修改日期	类型
_espmax.cpp	2022/9/5 16:37	CPP 文件
_espmax.h	2022/3/15 14:58	H 文件
Buzzer.cpp	2022/3/15 14:58	CPP 文件
Buzzer.h	2022/3/15 14:58	H 文件
ESP32PWMServo.cpp	2022/9/6 12:08	CPP 文件
ESP32PWMServo.h	2022/9/6 12:11	H 文件
ESPMMax.cpp	2022/9/12 20:30	CPP 文件
ESPMMax.h	2022/3/15 16:03	H 文件
LobotSerialServoControl.cpp	2022/9/12 20:21	CPP 文件
LobotSerialServoControl.h	2022/9/1 17:27	H 文件