0103

National University of Computer and Emerging Sciences

FAST School of Computing **Spring-2023** **Islamabad Campus**

AI-2002 Artificial Intelligence BS(CS)

Serial No:

Final Exam
Total Time: 3 Hours
Total Marks: 85

Signature of Invigilator

Monday, 22nd May, 2023

Course Instructors

Mr. Saad Salman, Dr. Labiba Fahad,

Mr. Owais Idrees

Asadullah Nawaz	20I-0761	C	Asad
Student Name	Roll No.	Course Section	Student Signature

DO NOT OPEN THE QUESTION BOOK OR START UNTIL INSTRUCTED.

Instructions:

1. Attempt on question paper. Attempt all of them. Read the question carefully, understand the question, and then attempt it.
2. No additional sheet will be provided for rough work. Use the back of the last page for rough work.
3. If you need more space write on the back side of the paper and clearly mark question and part number etc.
4. After asked to commence the exam, please verify that you have **Fourteen (14)** different printed pages including this title page. There are total of **5** questions.
5. Calculator sharing is strictly prohibited.
6. Use permanent ink pens only. Any part done using soft pencil will not be marked and cannot be claimed for rechecking.

	Q-1	Q-2	Q-3	Q-4	Q-5	Total
Marks Obtained	7	20	15	20	15	77
Total Marks	15	20	15	20	15	85

Scanned with CamScanner

Question 1 [15 Marks]

Apply Decision Tree classifier on the give data

- Please draw a box around all the information gain values.
- No marks will be given without drawing the tree. In case of partial solution, you must draw partial tree.
- Calculations are must. Any solution without calculation will not be graded.

	Feature1	Feature2	Feature3	Feature4	Class
1	Red	Small	Square	Yes	A
2	Blue	Large	Rectangle	No	B
3	Green	Medium	Circle	Yes	A
4	Green	Small	Triangle	No	C
5	Red	Large	Square	Yes	B
6	Blue	Small	Rectangle	No	A
7	Green	Medium	Circle	Yes	A
8	Red	Small	Triangle	No	C
9	Blue	Medium	Square	Yes	B
10	Green	Large	Circle	No	A
11	Blue	Small	Rectangle	Yes	C
12	Red	Medium	Triangle	No	A
13	Green	Large	Circle	Yes	B
14	Red	Small	Square	No	A
15	Blue	Large	Rectangle	Yes	C

$$S_{Dataset} = -\left[\frac{7}{15}\log\frac{7}{15} + \frac{4}{15}\log\frac{4}{15} + \frac{4}{15}\log\frac{4}{15}\right] = 0.9654$$

$$S_{F_1} = -\left[\frac{5}{15}\left[\frac{3}{5}\log\frac{3}{5} + \frac{1}{5}\log\frac{1}{5} + \frac{1}{5}\log\frac{1}{5}\right] + \frac{5}{15}\left[\frac{2}{5}\log\frac{2}{5} + \frac{2}{5}\log\frac{2}{5} + \frac{1}{5}\log\frac{1}{5}\right] + \frac{5}{15}\left[\frac{3}{5}\log\frac{3}{5} + \frac{1}{5}\log\frac{1}{5} + \frac{1}{5}\log\frac{1}{5}\right]\right] = 0.8967$$

$$S_{F_2} = -\left[\frac{6}{15}\left[\frac{3}{6}\log\frac{3}{6} + \frac{3}{6}\log\frac{3}{6}\right] + \frac{5}{15}\left[\frac{3}{5}\log\frac{3}{5} + \frac{1}{5}\log\frac{1}{5} + \frac{1}{5}\log\frac{1}{5}\right] + \frac{4}{15}\left[\frac{3}{4}\log\frac{3}{4} + \frac{1}{4}\log\frac{1}{4}\right]\right] = 0.6772$$

$$S_{F_3} = -\left[\frac{4}{15}\left[\frac{2}{4}\log\frac{2}{4} + \frac{2}{4}\log\frac{2}{4}\right] + \frac{4}{15}\left[\frac{2}{4}\log\frac{2}{4} + \frac{1}{4}\log\frac{1}{4} + \frac{1}{4}\log\frac{1}{4}\right] + \frac{4}{15}\left[\frac{3}{4}\log\frac{3}{4} + \frac{1}{4}\log\frac{1}{4}\right] + \frac{3}{15}\left[\frac{2}{3}\log\frac{2}{3} + \frac{1}{3}\log\frac{1}{3}\right]\right] = 0.673$$

$$S_{F_4} = -\left[\frac{8}{15}\left[\frac{3}{8}\log\frac{3}{8} + \frac{3}{8}\log\frac{3}{8} + \frac{2}{8}\log\frac{2}{8}\right] + \frac{7}{15}\left[\frac{4}{7}\log\frac{4}{7} + \frac{2}{7}\log\frac{2}{7} + \frac{1}{7}\log\frac{1}{7}\right]\right] = 0.9313$$

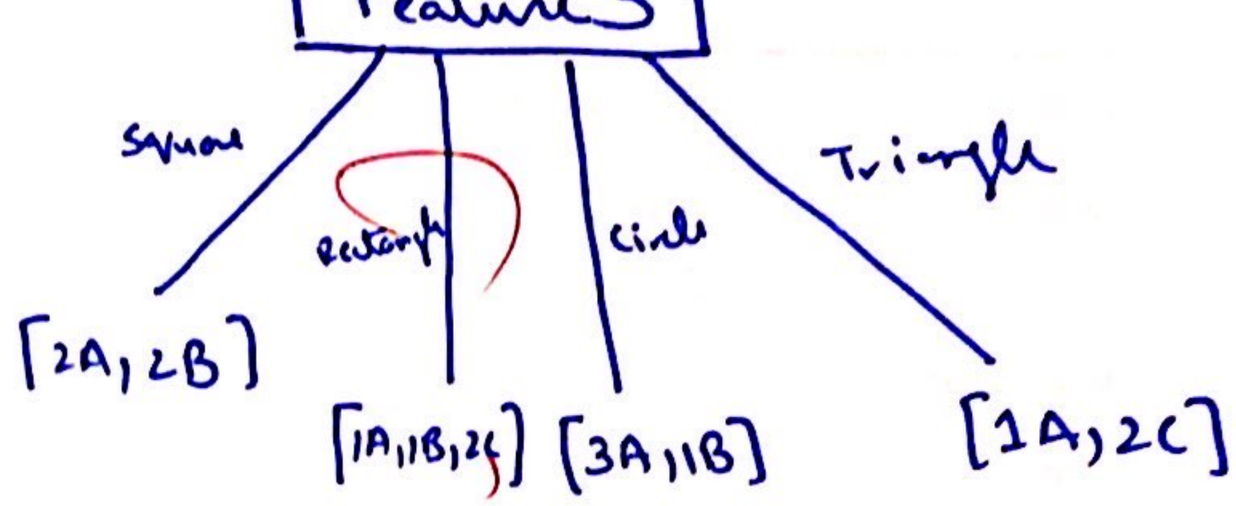
Scanned with CamScanner

$Gain(F_1) = 0.9654 - 0.8967 =$ 0.0687

$Gain(F_2) = 0.9654 - 0.6772 =$ 0.2882

$Gain(F_3) = 0.9654 - 0.673 =$ 0.2924 ✓

$Gain(F_4) = 0.9654 - 0.9313 =$ 0.0341



* For Feature 3 (square)

	~~F1~~ F1	F2	F4	Class
1	Red	Small	Yes	A
5	Red	Large	Yes	B
9	Blue	Medium	Yes	B
14	Red	Small	No	A

$S_{DATASET} = -\left[\frac{2}{4}\log\frac{2}{4} + \frac{2}{4}\log\frac{2}{4}\right] = 1$

$S_{F_1} = -\left[\frac{3}{4}\left[\frac{2}{3}\log\frac{2}{3} + \frac{1}{3}\log\frac{1}{3}\right] + \frac{1}{4}[0]\right] = 0.6887$

$S_{F_2} = -\left[\frac{2}{4}[0] + \frac{1}{4}[0] + \frac{1}{4}[0]\right] = 0$

$S_{F_4} = -\left[\frac{3}{4}\left[\frac{2}{3}\log\frac{2}{3} + \frac{1}{3}\log\frac{1}{3}\right] + \frac{1}{4}[0]\right] = 0.6887$

$Gain(F_1) = 1 - 0.6887 =$ 0.3113

$Gain(F_2) = 1 - 0 =$ 1 ✓

$Gain(F_4) = 1 - 0.6887 =$ 0.3113

* For Feature 3 (Rectangle)

	F1	F2	F4	Class
2	Blue	Large	No	B
6	Blue	Small	No	A
11	Blue	Small	Yes	C
15	Blue	Large	Yes	C

Scanned with CamScanner

$S_{DATASET} = -\left[\frac{1}{4}\log\frac{1}{4} + \frac{1}{4}\log\frac{1}{4} + \frac{2}{4}\log\frac{2}{4}\right] = 0.9464$

$S(F_1) = -\left[\frac{4}{4}\left[\frac{1}{4}\log\frac{1}{4} + \frac{1}{4}\log\frac{1}{4} + \frac{2}{4}\log\frac{2}{4}\right] + 0\right] = 0.9464$

$S(F_2) = -\left[\frac{2}{4}\left[\frac{1}{2}\log\frac{1}{2} + \frac{1}{2}\log\frac{1}{2}\right] + \frac{2}{4}\left[\frac{1}{2}\log\frac{1}{2} + \frac{1}{2}\log\frac{1}{2}\right]\right] = 0.6309$

$S(F_4) = -\left[\frac{2}{4}\left[\frac{1}{2}\log\frac{1}{2} + \frac{1}{2}\log\frac{1}{2}\right] + \frac{2}{4}[0]\right] = 0.3155$

$Gain(F_1) = 0.9464 - 0.9464 = \boxed{0}$

$Gain(F_2) = 0.9464 - 0.6309 = \boxed{0.3155}$

$Gain(F_4) = 0.9464 - 0.3155 = \boxed{0.6309}$ ✓

* For feature 3 (Circle)

	F_1	F_2	F_4	Class
3	Green	Medium	Yes	A
7	Green	Medium	Yes	A
10	Green	Large	No	A
13	Green	Large	Yes	B

$S_{DATASET} = -\left[\frac{1}{4}\log\frac{1}{4} + \frac{3}{4}\log\frac{3}{4}\right] = 0.8113$

$S_{F_1} = \frac{4}{4}\left[\frac{3}{4}\log\frac{3}{4} + \frac{1}{4}\log\frac{1}{4}\right] + 0\right] = 0.8113$

$S_{F_2} = -\left[\frac{2}{4}[0] + \frac{2}{4}[-1]\right] = 0.5$

$S_{F_4} = -\left[\frac{3}{4}\left[\frac{2}{3}\log\frac{2}{3} + \frac{1}{3}\log\frac{1}{3}\right] + \frac{1}{4}[0]\right] = 0.6887$

$Gain(F_1) = 0.8113 - 0 = \boxed{0}$

$Gain(F_2) = 0.8113 - 0.5 = \boxed{0.3113}$ ✓

$Gain(F_4) = 0.8113 - 0.6887 = \boxed{0.1226}$

Scanned with CamScanner

* For Feature 3 (Triangle)

	F_1	F_2	F_4	Class
4	Green	Small	No	C
8	Red	Small	No	C
12	Red	Medium	No	A

$$S_{DATASET} = -\left[\frac{2}{3}\log\frac{2}{3} + \frac{1}{3}\log\frac{1}{3}\right] = 0.9183$$

$$S_{F_1} = -\left[\frac{1}{3}[0] + \frac{2}{3}[-1]\right] = 0.6667$$

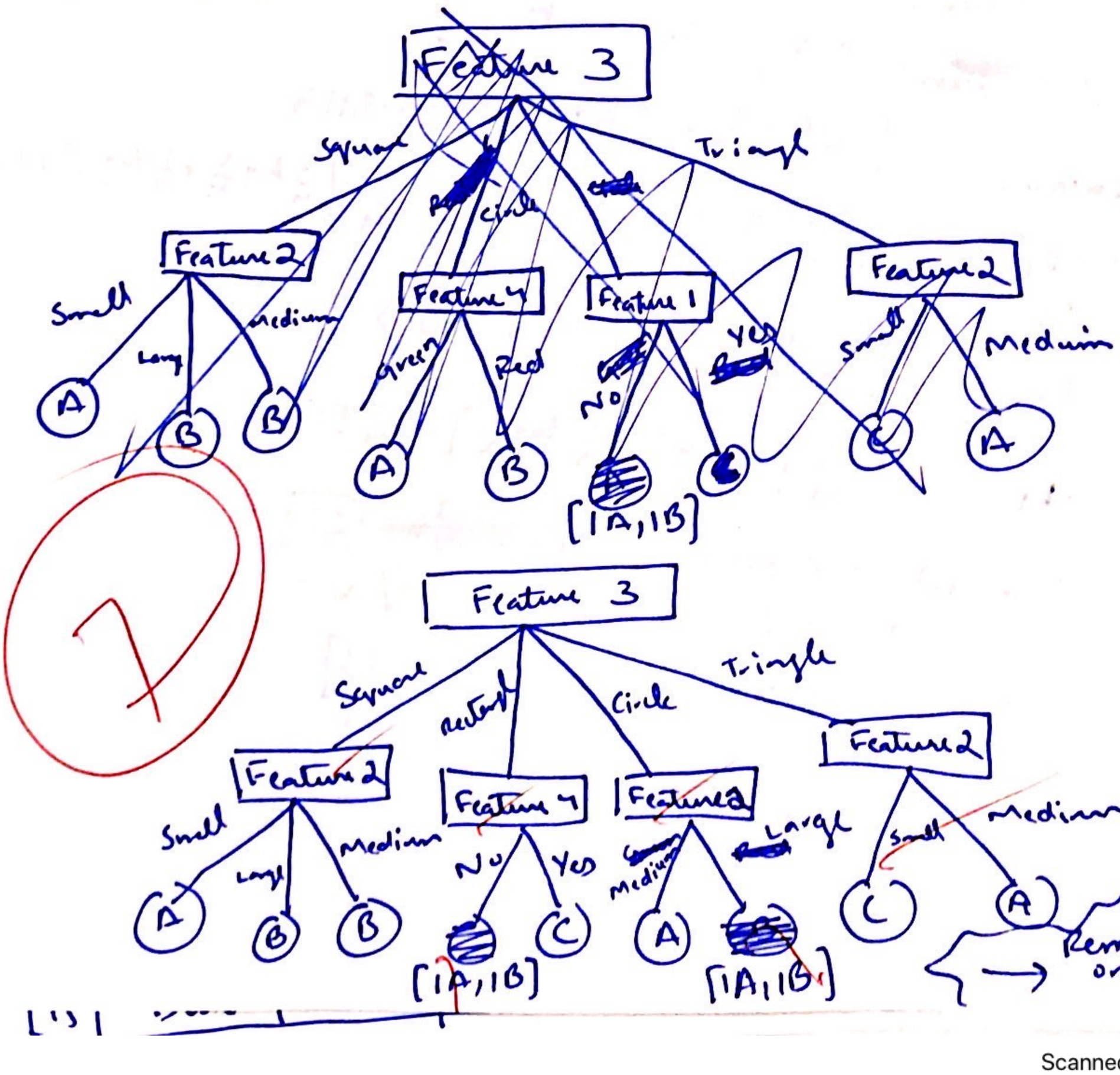
$$S_{F_2} = -\left[\frac{2}{3}[0] + \frac{1}{3}[0]\right] = 0$$

$$S_{F_3} = -\left[\frac{3}{3}\left[\frac{2}{3}\log\frac{2}{3} + \frac{1}{3}\log\frac{1}{3}\right] + 0\right] = 0.9183$$

$$Gain(F_1) = 0.9183 - 0.6667 = \boxed{0.2516}$$

$$Gain(F_2) = 0.9183 - 0 = \boxed{0.9183}$$

$$Gain(F_3) = 0.9183 - 0.9183 = \boxed{1}$$

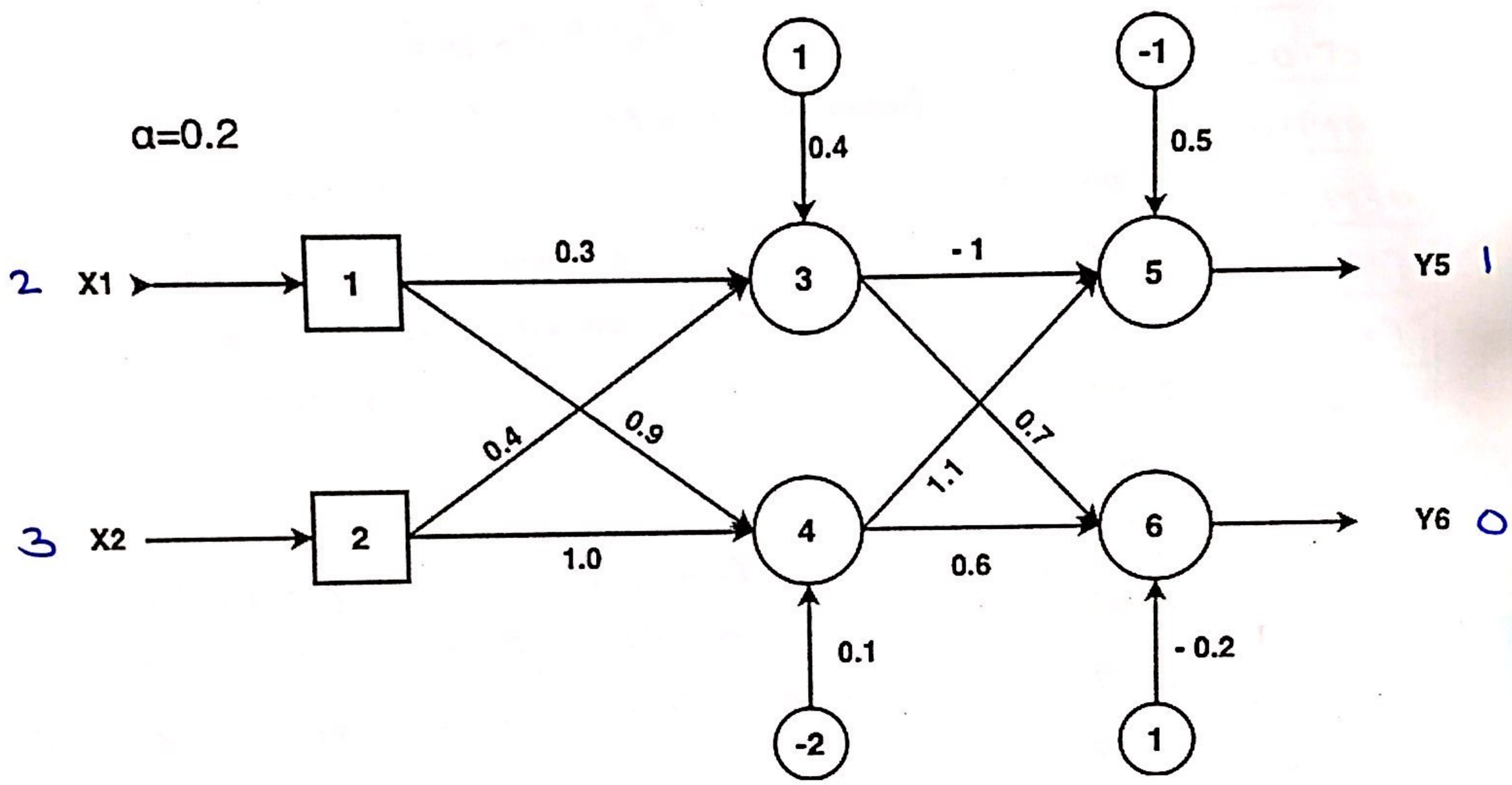


Scanned with CamScanner

Question 2 [20 Marks]

Consider the neural network below. Perform one forward pass and fix the error and update the weight for the given inputs and outputs. You need to be accurate in your calculations. Please perform calculations up to 4 decimal places. This shows the complete working of the algorithm; fill in the table on the right, no marks will be awarded without that. Direct answers will not get any marks.

Inputs: X1 = 2, X2 = 3,
Output: Y5 = 1, Y6 = 0,
Alpha = 0.2
Activation function in the neurons is sigmoid function.



$y_3 = sig(2(0.3) + 3(0.4) + (1)(0.4)) = 0.9002$

$y_4 = sig(3(1) + 2(0.9) + (-2)(0.1)) = 0.9900$

$y_5 = sig(0.9002(-1) + 0.9900(1.1) + (-1)(0.5)) = 0.4228$

$y_6 = sig(0.9002(0.7) + 0.9900(0.6) + (1)(-0.2)) = 0.7358$

$e_5 = 1 - 0.4228 = 0.5772$

$e_6 = 0 - 0.7358 = -0.7358$

Scanned with CamScanner

$\delta_5 = y_5(1-y_5)e^5$

$= (0.4228)(1-0.4228)(0.5772) = 0.1409$

$\delta_6 = y_6(1-y_6)e^6$

$= (0.7358)(1-0.7358)(-0.7358) = -0.1430$

$\delta_3 = y_3(1-y_3)[(-1)\delta_5 + (0.7)\delta_6]$

$= (0.9002)(1-0.9002)(-0.1409 + (0.7)(-0.1430))$

$= -0.0217$

$\delta_4 = y_4(1-y_4)[(1.1)\delta_5 + (0.6)\delta_6]$

$= (0.9900)(1-0.9900)[(1.1)(0.1409) + (0.6)(-0.1430)]$

$= 0.0007$

Neuron 5

$\Delta w_{45} = \alpha(y_4)(\delta_5) = (0.2)(0.9900)(0.1409) = 0.0279$

$\Delta w_{35} = \alpha(y_3)(\delta_5) = (0.2)(0.9002)(0.1409) = 0.0254$

$\Delta\theta_5 = \alpha(-1)(\delta_5) = (0.2)(-1)(0.1409) = -0.0282$

Neuron 6

$\Delta w_{46} = \alpha(y_4)(\delta_6) = (0.2)(0.9900)(-0.1430) = -0.0283$

$\Delta w_{36} = \alpha(y_3)(\delta_6) = (0.2)(0.9002)(-0.1430) = -0.0257$

$\Delta\theta_6 = \alpha(1)(\delta_6) = (0.2)(1)(-0.1430) = -0.0286$

Neuron 3

$\Delta w_{13} = \alpha(x_1)(\delta_3) = (0.2)(2)(-0.0217) = -0.0087$

$\Delta w_{23} = \alpha(x_2)(\delta_3) = (0.2)(3)(-0.0217) = -0.0130$

$\Delta\theta_3 = \alpha(1)(\delta_3) = (0.2)(1)(-0.0217) = -0.0043$

Neuron 4

$\Delta w_{14} = \alpha(x_1)(\delta_4) = (0.2)(2)(0.0007) = 0.0003$

$\Delta w_{24} = \alpha(x_2)(\delta_4) = (0.2)(3)(0.0007) = 0.0004$

$\Delta\theta_4 = \alpha(-2)(\delta_4) = (0.2)(-2)(0.0007) = -0.0003$

	Value
Y3	0.9002
Y4	0.9900
Y5	0.4228
Y6	0.7358
E5	0.5772
E6	-0.7358
δ5	0.1409
δ6	-0.1430
δ3	-0.0217
δ4	0.0007
ΔW45	0.0279
ΔW35	0.0254
Δ05	-0.0282
ΔW46	-0.0283
ΔW36	-0.0257
Δ06	-0.0286
ΔW13	-0.0087
ΔW23	-0.0130
Δ03	-0.0043
ΔW14	0.0003
ΔW24	0.0004
Δ04	-0.0003

Scanned with CamScanner

Question 3 [15 Marks]

Suppose you are given the following data pairs. You will simulate the k-means algorithm to identify TWO clusters in the data.

Data #	x	y
1	1.90	0.97
2	1.76	0.84
3	2.32	1.63
4	2.31	2.09
5	1.14	2.11
6	5.02	3.02
7	5.74	3.84
8	2.25	3.47
9	4.71	3.60
10	3.17	4.96

Suppose you are given initial assignment cluster center as {cluster1: #1}, {cluster2: #10} – the first data point is used as the first cluster center and the 10-th as the second cluster center.
Please simulate the k-means (k=2) algorithm for TWO iteration. What are cluster assignments after TWO iteration? (Fill in the table below)
Assume k-means uses Euclidean distance. Calculate distance value up to 4 decimal places. In case of tie, you can assign the data point to cluster #1.

Iteration (1)

	C1 (1.90, 0.97)	C2 (3.17, 4.96)	
1	0	4.6706	C1
2	0.1910	4.355	C1
3	0.7823	3.4368	C1
4	1.1927	2.9961	C1
5	1.3701	3.4991	C1
6	3.7332	2.6807	C2
7	4.7940	2.8034	C2
8	2.5244	1.7511	C2
9	3.8488	2.0545	C2
10	4.1872	0	C2

Scanned with CamScanner

Recalculate centroids

$C_1 = \left[\frac{1.90+1.76+2.32+2.31+1.14}{5}, \frac{0.97+0.84+1.63+2.09+2.11}{5}\right]$

$= (1.886, 1.528)$

$C_2 = \left[\frac{5.02+5.74+2.25+4.71+5.17}{5}, \frac{3.02+3.84+3.47+3.60+4.96}{5}\right]$

$= (4.178, 3.778)$

Data#	Cluster Assignment after 1st Iteration	Cluster Assignment after 2nd Iteration
1	C1	C1
2	C1	C1
3	C1	C1
4	C1	C1
5	C1	C1
6	C2	C2
7	C2	C2
8	C2	C2
9	C2	C2
10	C2	C2

Iteration (2)

	$C_1(1.886, 1.528)$	$C_2(4.178, 3.778)$	
1	0.5582	3.6158	C_1
2	0.6994	3.8051	C_1
3	0.4458	2.8401	C_1
4	0.7040	2.5177	C_1
5	0.9462	3.4658	C_1
6	3.4710	1.1329	C_2
7	4.4943	1.5632	C_2
8	1.9758	1.9524	C_2
9	3.5026	0.561	C_2
10	3.6643	1.5534	C_2

Centroids remains same as clusters didn't change

Scanned with CamScanner

Question 4[5+5+5+5=20 Marks]

There are a total of 6 questions in the exam of AI-2002 and the course staff has yet to write the test. Each question will cover a topic and the exam format is given below:

- Q1. Search
- Q2. Games
- Q3. CSPs
- Q4. MDPs
- Q5. True/False
- Q6. Short Answer

20/20

There are 7 people on the course staff: Brad, Donahue, Ferguson, Judy, Kyle, Michael, and Nick. Each of them is responsible to work with Prof. Abbeel on one question. (But a question could end up having more than one staff person, or potentially zero staff assigned to it.) However, the staff are pretty quirky and want the following constraints to be satisfied:

i. Donahue (D) will not work on a question together with Judy (J).

ii. Kyle (K) must work on either Search, Games or CSPs

iii. (M) is very odd, so he can only contribute to an odd-numbered question.

iv. Nick (N) must work on a question that's before Michael (M)'s question.

v. Kyle (K) must work on a question that's before Donahue (D)'s question

vi. Brad (B) does not like grading exams, so he must work on True/False.

vii. Judy (J) must work on a question that's after Nick (N)'s question.

viii. If Brad (B) is to work with someone, it cannot be with Nick (N).

ix. Nick (N) cannot work on question 6.

x. Ferguson (F) cannot work on questions 4, 5, or 6

xi. Donahue (D) cannot work on question 5.

xii. Donahue (D) must work on a question before Ferguson (F)'s question.

Scanned with CamScanner

a) We will model this problem as a constraint satisfaction problem (CSP). Our variables correspond to each of the staff members, J, F, N, D, M, B, K, and the domains are the questions 1, 2, 3, 4, 5, 6. After applying the unary constraints, what are the resulting domains of each variable?

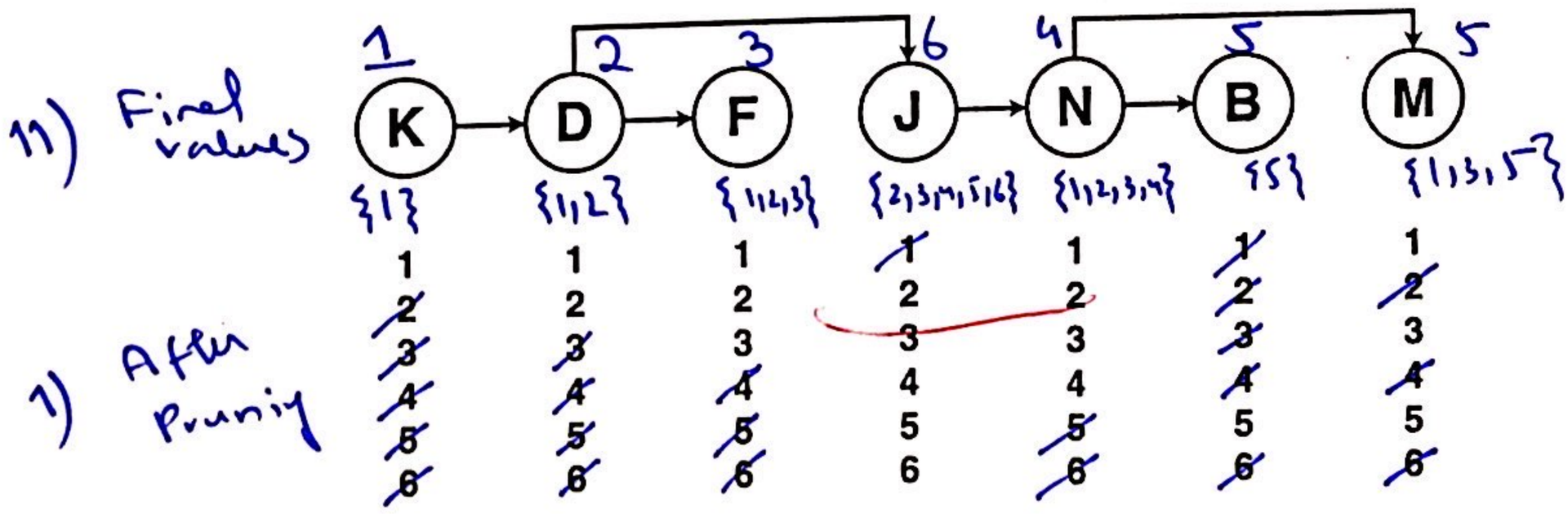
	1	2	3	4	5	6
B	✓	✓	✓	✓	✓	✓
D	✓	✓	✓	✓	✓	✓
F	✓	✓	✓	✓	✓	✓
J	✓	✓	✓	✓	✓	✓
K	✓	✓	✓	✓	✓	✓
N	✓	✓	✓	✓	✓	✓
M	✓	✓	✓	✓	✓	✓

done on paper

b) If we apply the Minimum Remaining Value (MRV) heuristic, which variable should be assigned first?

c) Let's proceed with assigning Michael first. For value ordering we use the Least Constraining Value (LCV) heuristic, where we use Forward Checking to compute the number of remaining values in other variables domains. What ordering of values is prescribed by the LCV heuristic? Include your work |i.e., include the resulting filtered domains that are different for the different values.

d) Realizing this is tree-structured, we decide to solve the tree-structured CSPs. Below is the linearized version of the tree-structured CSP graph:



I. From right to left in the above figure, perform Domain Pruning after applying all the constraints. Write the values that remain in each domain below each node in the figure.

II. From left to right, assign a value to each node for the solution. If there is more than one possible assignment, choose the highest value.

Final values
K = 1 , J = 6 , M = 5
D = 2 , N = 4
F = 3 , B = 5

Scanned with CamScanner

a)

	1	2	3	4	5	6	
B					✓		(1)
D	✓	✓	✓	✓		✓	(5)
F	✓	✓	✓				(3)
J	✓	✓	✓	✓	✓	✓	(6)
K	✓	✓	✓				(3)
N	✓	✓	✓	✓	✓		(5)
M	✓		✓		✓		(3)

b) If we apply MRV, Brad must be assigned first, since it has minimum remaining value of 1

c) Assigning value to michael:

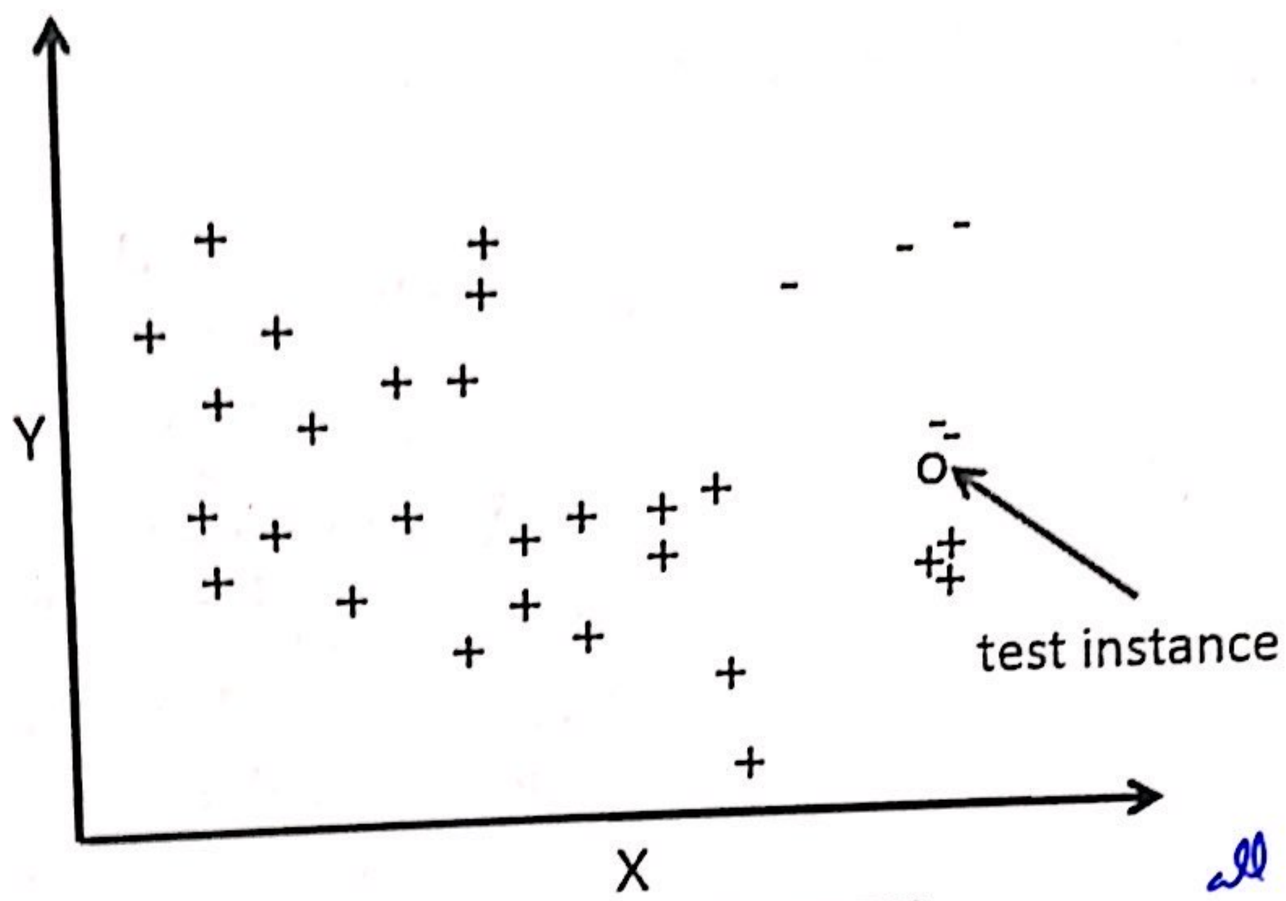
Assign 1	Assign 3	Assign 5
only Nick's domain will change	only Nick's domain will change	only Nick's domain will change
B = {5}	B = {5}	B = {5}
D = {1,2,3,4,6}	D = {1,2,3,4,6}	D = {1,2,3,4,6}
F = {1,2,3}	F = {1,2,3}	F = {1,2,3}
J = {1,2,3,4,5,6}	J = {1,2,3,4,5,6}	J = {1,2,3,4,5,6}
K = {1,2,3}	K = {1,2,3}	K = {1,2,3}
N = { } → all removed	N = {1,2} → 3,4,5 removed	N = {1,2,3,4} → 5 removed

Hence, LCV, order of assigment should be 5 then 3 then 1.

Scanned with CamScanner

Question 5[3+3+3+3+3=15 Marks]

A KNN classifier assigns a test instance the majority class associated with its K nearest training instances. Distance between instances is measured using Euclidean distance. Suppose we have the following training set of positive (+) and negative (-) instances and a single test instance (o). All instances are projected onto a vector space of two real-valued features (X and Y). Answer the following questions. Assume "unweighted" KNN (every nearest neighbor contributes equally to the final vote).



15/15

(a) How is training performed in KNN?

There is no training in KNN, only data is required when there is a test point. The training complexity is O(1). all the calculations are done at test (O(nmn))

(b) What would be the class assigned to this test instance for K=1

For K=1, the nearest neighbor to test instance is (-) so we will assign it (-) class.

(c) What would be the class assigned to this test instance for K=3

For K=3, the nearest neighbors to test instance are (-), (-) and (+), so we will assign majority i.e (-) class to test instance.

(d) What would be the class assigned to this test instance for K=5

For K=5, the nearest neighbors to test instance are (-), (-), (+), (+), (+), so we will assign majority i.e (+) class to test instance.

(e) Setting K to a large value seems like a good idea. We get more votes! Given this particular training set, would you recommend setting K = 11? Why or why not?

No, setting K=11 is not a good idea since this particular dataset since the dataset only has 5 (-) instances while all other are (+) instances. So with K=11, wherever we place test point, the majority will be of (+) instances and irrespective of position, the test point will always be assigned (+) class. Hence, this is not a suitable K value for given data.

Scanned with CamScanner