
Multiphase Systems and Technologies

Lecture notes

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Chapter 1

Introduction

A **homogeneous system** is a system in which chemical and physical properties are independent of the position (uniform).

a **heterogeneous system** is a system in which chemical and physical properties depend on the position. A particular set of heterogeneous systems are multiphase systems.

A **multiphase system** can be thought as a heterogeneous system coming from the union of homogeneous systems. Each homogeneous portion is called **phase**.

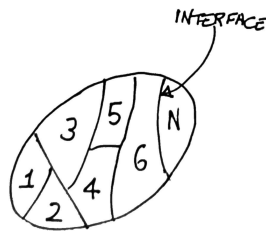


Figure 1.1: A multiphase system.

Phases are separated by boundaries, called **interfaces**. Interface are usually:

- deformable (moving);
- diathermal (diabatic);
- permeable to chemical species.

From the point of view of energy, an interface usually allows the exchange of work, heat and chemical energy.

1.1 Thermodynamic equilibrium

1.1.1 How many phases can coexist at equilibrium?

Let's consider a system with M phases and r chemical components (chemical species). Then let's write equilibrium equations:

- **Mechanical equilibrium:** for planar interfaces¹ pressure p is uniform

$$p^{(1)} = p^{(2)} = \dots = p^{(M)}$$

- **Thermal equilibrium:** temperature T is uniform

$$T^{(1)} = T^{(2)} = \dots = T^{(M)}$$

- **Chemical equilibrium:** we have $r \times M$ equations, chemical potential is the same throughout the heterogeneous system

$$\begin{aligned} \mu_1^{(1)} &= \mu_1^{(2)} = \dots = \mu_1^{(M)} \\ \mu_2^{(1)} &= \mu_2^{(2)} = \dots = \mu_2^{(M)} \\ &\vdots \\ \mu_r^{(1)} &= \mu_r^{(2)} = \dots = \mu_r^{(M)} \end{aligned}$$

This system of equations states the thermodynamic equilibrium and contains $M \times (r+2)$ variables and $(M-1) \times (r+2)$ equations.

Each one of the M phases is an homogeneous system by itself, hence each phase has to satisfy the Gibbs-Duhem Equation to fulfill the first and second principles of thermodynamics:

$$SdT - Vdp + \sum_{i=1}^r N_i d\mu_i = 0$$

The real number of equations now becomes $(M-1) \times (r+2) + M$.

Given that the well-posedness of a system arises when # Variables \geq # Equations,

$$\begin{aligned} \# \text{ Equations} &\leq \# \text{ Variables} \\ (M-1)(r+2) + M &\leq M(r+2) \\ M(r+2) - r - 2 + M &\leq M(r+2) \\ M &\leq r+2 \end{aligned}$$

¹The number of equations is independent of the shape of the interface, but for the sake of simplicity we will consider flat boundaries. We will extend these considerations when dealing with bubbles or drops.

Examples

In the case $r = 1$ (single component system),

$$r = 1 \implies M \leq 3 \quad \begin{cases} M = 3 \rightarrow \text{three phase system (triple state)} \\ M = 2 \rightarrow \text{two phase system} \\ M = 1 \rightarrow \text{single phase system} \end{cases}$$

In the case $r = 2$ (two component system), $r = 2 \implies M \leq 4$. An example could be moist air, given that we approximate dry air as if it was made by a single component. In a lab environment could be possible to observe a mixture of condensed air, condensed water and moist air.

1.1.2 How many quantities are required to describe equilibrium?

Let's now derive the number of independent variables (also known as *degrees of freedom*):

$$\begin{aligned} f &= \# \text{ Variables} - \# \text{ Equations} \\ &= M(r + 2) - (M - 1)(r + 2) - M \\ &= M(r + 2) - M(r + 2) + r + 2 - M \\ &= r - M + 2 \end{aligned}$$

Examples

For a single component system ($r = 1$) the degrees of freedom are $f = 3 - M$. Three cases:

- $M = 3 \rightarrow f = 0$ meaning that we cannot set any quantity among temperature, pressure and chemical potential;
- $M = 2 \rightarrow f = 1$ meaning we are only allowed to set one variable, typically the temperature or the pressure (*saturation condition*);
- $M = 1 \rightarrow f = 2$ meaning we are free to set temperature and pressure independently.

1.2 Basic quantities

We start by considering two-phase gas-liquid² or vapour-liquid flows³. To study fluid dynamics of a single phase system we can write EDPs that are able to give the distribution of all the quantities related to the flow field at any point inside the pipeline.

In the case of more than one phase flowing in a pipe, no analytical solutions are available, and moreover the numerical methods effective for single phase flows fail to properly simulate all the relevant quantity.

For design purposes, we need a simplified approach and hence we will adopt a *one-dimensional axial flow model* that lumps the information pertaining the distribution of the velocity of the phases using cross section averages. We are only interested in determining the behaviour of the relevant quantities along the pipe axis.

1.2.1 Total Volume Flow Rate

Volume Flow Rate is the rate at which volume flows in the pipe. Its unit is $[\frac{\text{m}^3}{\text{s}}]$ in SI units.

$$Q = Q_g + Q_l \quad (1.1)$$

where subscript g stands for *gas* whereas l stands for *liquid*.

1.2.2 Average Volume Quality

Based on the definition of Total Volume Flow Rate we can define a quantity related to phase fractions. If we divide each member of Eq. 1.1 by Q ,

$$1 = \frac{Q_g}{Q} + \frac{Q_l}{Q}$$

We now define $\overline{x_v}$ (cross-section average) as follows:

$$\overline{x_v} = \frac{Q_g}{Q}$$

Average Volume Quality is adimensional.

²Two components, $M \leq 4$.

³One component, $M \leq 3$.

1.2.3 Total Mass Flow Rate

If we call the phase densities ρ_g and ρ_l (in SI units $[\frac{\text{kg}}{\text{m}^3}]$) we can also define Mass Flow Rates: $\Gamma_g = \rho_g Q_g$ for the gas and $\Gamma_l = \rho_l Q_l$ for the liquid.

Since mass is additive, we can write the Total Mass Flow Rate (in SI units $[\frac{\text{kg}}{\text{s}}]$):

$$\Gamma = \Gamma_g + \Gamma_l \quad (1.2)$$

Recall that instead of phase densities we can use their reciprocal, *specific volumes*:

$$v_g = \frac{1}{\rho_g}$$

$$v_l = \frac{1}{\rho_l}$$

1.2.4 Average Mass Quality

Dividing each member of Eq. 1.2 by Γ :

$$1 = \frac{\Gamma_g}{\Gamma} + \frac{\Gamma_l}{\Gamma}$$

We define \bar{x} , the Average Mass Quality:

$$\bar{x} = \frac{\Gamma_g}{\Gamma}$$

Mass quality is adimensional.

1.2.5 Relationship between \bar{x} and \bar{x}_v

Owing to the difference in densities (or specific volumes), the numerical values of mass quality \bar{x} and volume quality \bar{x}_v may differ greatly.

Starting from volume quality,

$$\begin{aligned} \bar{x}_v &= \frac{Q_g}{Q} = \frac{Q_g}{Q_g + Q_l} = \frac{1}{1 + \frac{Q_l}{Q_g}} = \\ &= \frac{1}{1 + \frac{\Gamma_l \rho_g}{\rho_l \Gamma_g}} = \\ &= \frac{1}{1 + \frac{\rho_g (1-\bar{x})}{\rho_l \bar{x}}} \end{aligned}$$

In a more elegant form:

$$\frac{1 - \bar{x}_v}{\bar{x}_v} = \frac{\rho_g}{\rho_l} \frac{1 - \bar{x}}{\bar{x}}$$

Generally $\rho_g \ll \rho_l$, hence $\bar{x}_v \gg \bar{x}$. For a pure liquid-vapour mixture $\rho_g - \rho_l$ lowers with T (or p) and at a critical point $\rho_g = \rho_l$.

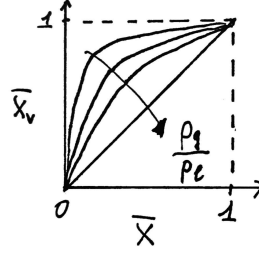


Figure 1.2: Volume Quality vs Mass Quality.

1.2.6 Bulk Density

The ratio between the mass flow rate and the volume flow rate has the dimensions of a density and is called bulk density. It is the density that would pertain to an equivalent single-phase fluid flowing with the same mass and volume flow rates of the two-phase mixture.

Referring to the cross-section average, the bulk density $\bar{\rho}_b$ is such that:

$$\bar{\rho}_b = \frac{\Gamma}{Q}$$

Let's specialize the numerator in the previous equation:

$$\bar{\rho}_b = \frac{\Gamma_g + \Gamma_l}{Q} = \frac{\rho_g Q_g + \rho_l Q_l}{Q} = \rho_g \bar{x}_v + \rho_l (1 - \bar{x}_v)$$

On the other hand, we can specialize the denominator:

$$\bar{\rho}_b = \frac{\Gamma}{Q_g + Q_l} = \frac{\Gamma}{v_g \Gamma_g + v_l \Gamma_l} = \frac{1}{v_g \bar{x} + v_l (1 - \bar{x})} = \frac{1}{\bar{v}_b}$$

where \bar{v}_b is the *bulk specific volume*.

Bulk density and bulk specific volume are only apparent quantities of the flowing mixture and cannot be considered if we're interested in the real mass or volume content in the pipeline. This is because the two phases do not travel with the same velocities (phase slippage).