CS1138

Machine Learning

Lecture : Linear Regression

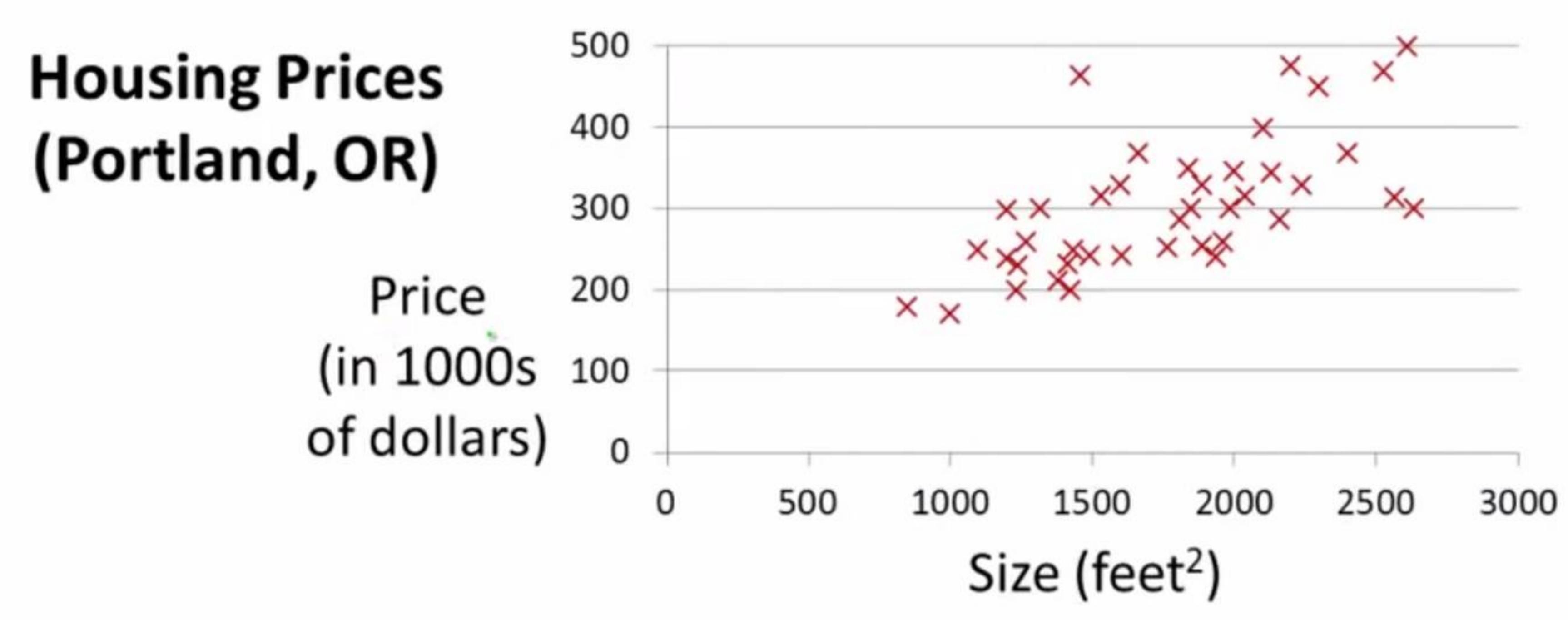
(Slide Credits: Andrew Ng)

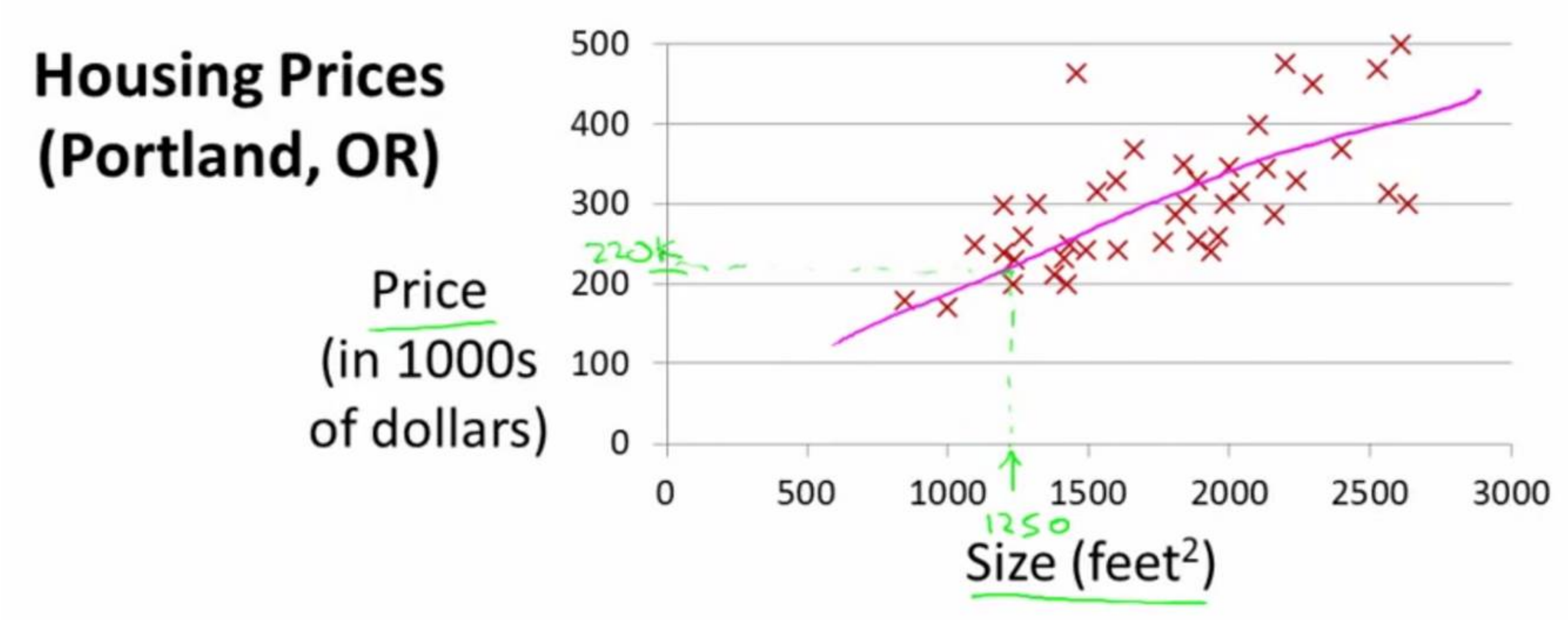
Arpan Gupta

- **Supervised Learning**

  Given the "right answer" for each example in the data.

  - **Regression**
    - Predict real-valued output
  - **Classification**
    - Discrete valued output


Housing Prices
(Portland, OR)
Price
(in 1000s
of dollars)
500
400
300
200
100
0
0
500
1000
1500
2000
2500
3000
Size (feet²)



Housing Prices
(Portland, OR)
Price
(in 1000s
of dollars)
500
400
300
220k
200
100
0
0
500
1000
1500
2000
2500
3000
1250
Size (feet²)

Training set of housing prices (Portland, OR)

Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

Notation:

- **m** = Number of training examples
- **x**'s = "input" variable / features
- **y**'s = "output" variable / "target" variable

Training set of housing prices (Portland, OR)

Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

$m = 47$

Notation:

- **m** = Number of training examples
- **x**'s = "input" variable / features
- **y**'s = "output" variable / "target" variable

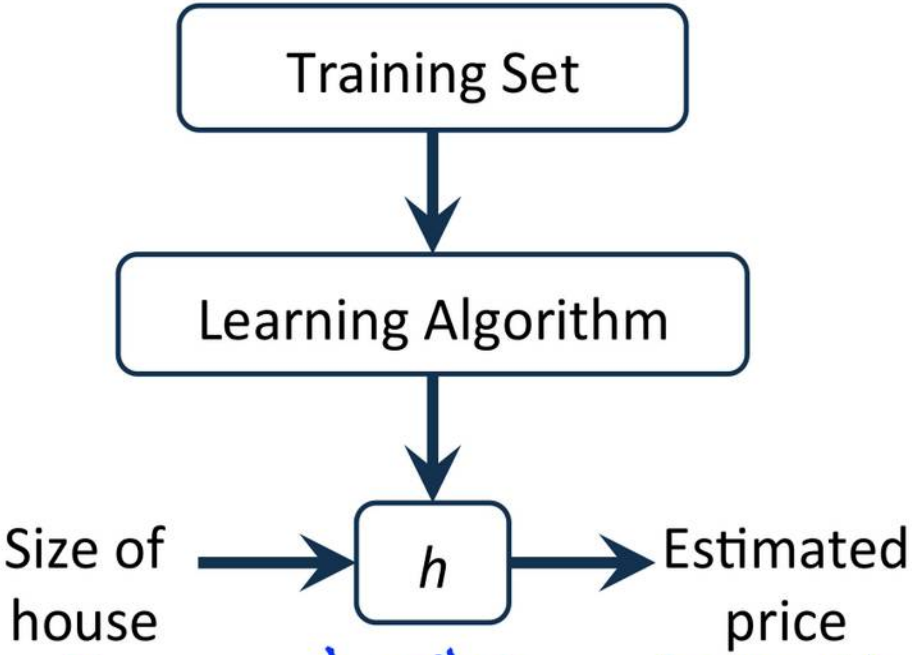
(x, y) - one training example

$(x^{(i)}, y^{(i)})$ - i^{th} training example

$x^{(1)} = 2104$

$x^{(2)} = 1416$

$y^{(1)} = 460$



x

hypothesis

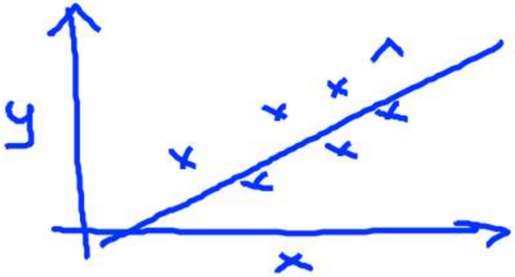
(estimated value of y)

h maps from x's to y's.

How do we represent *h* ?

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Shorthand: $h(x)$



$h(x) = \theta_0 + \theta_1 x$

Linear regression with one variable. (x)

Univariate linear regression.

└ one variable

Andrew Ng

Linear Regression with one variable

Training Set

Size in feet² (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
...	...

47 samples

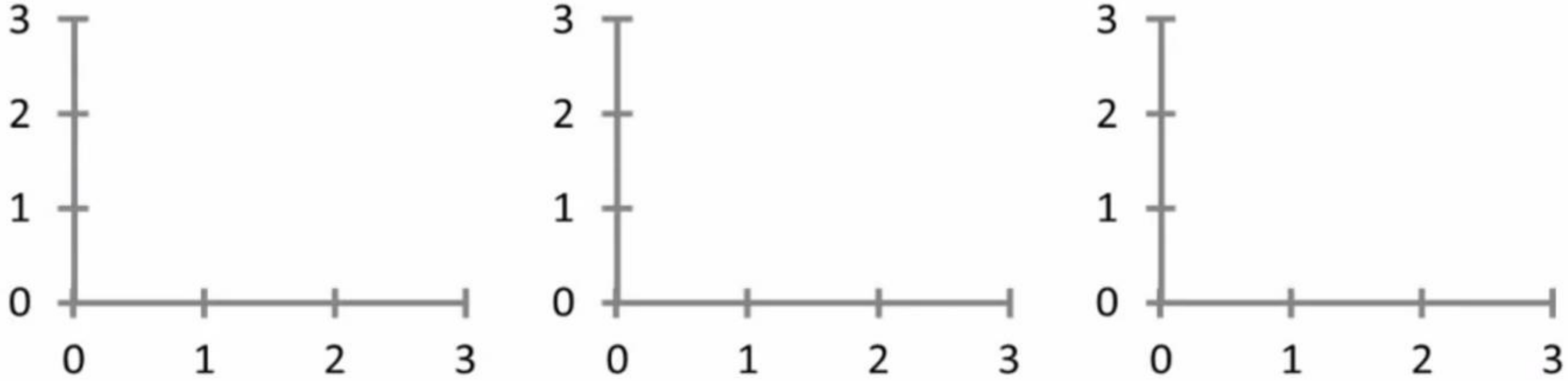
Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

θ_i's: Parameters

How to choose θ_i's ?

How will the models look like?

$$h_\theta(x) = \theta_0 + \theta_1 x$$

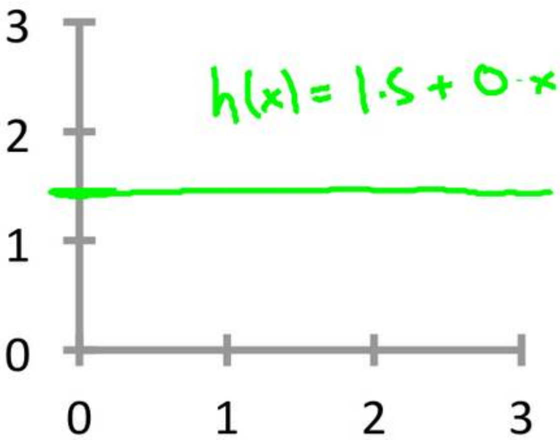


$\theta_0 = 1.5$
$\theta_1 = 0$

$\theta_0 = 0$
$\theta_1 = 0.5$

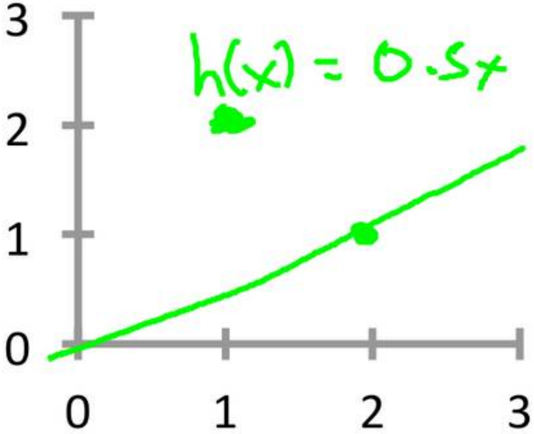
$\theta_0 = 1$
$\theta_1 = 0.5$

$$\underline{h_\theta(x)} = \theta_0 + \theta_1 x$$



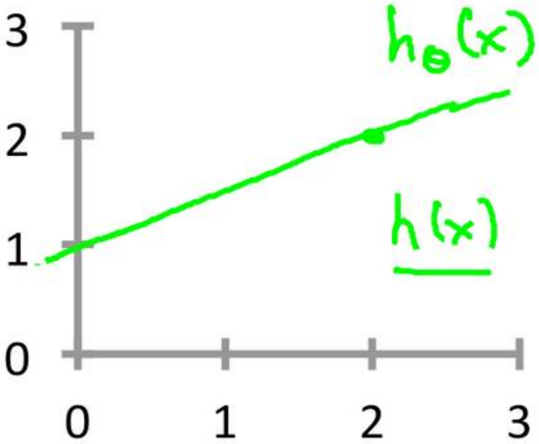
$\theta_0 = 1.5$

$\theta_1 = 0$



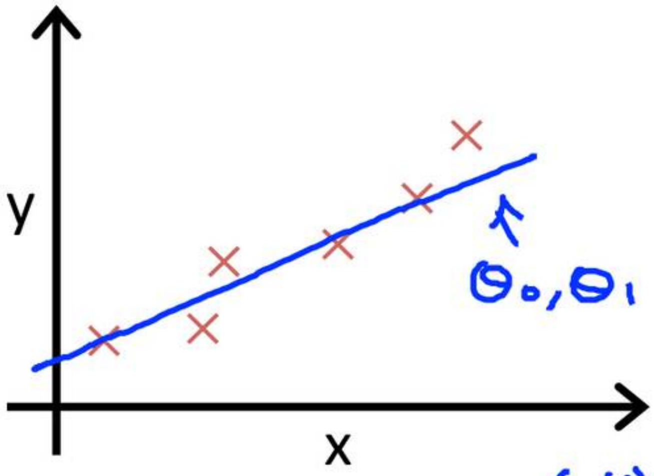
$\theta_0 = 0$

$\theta_1 = 0.5$



$\theta_0 = 1$

$\theta_1 = 0.5$



$(x^{(i)}, y^{(i)})$

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ is close to y for our training examples (x, y)

x, y

training examples

$$\underset{\theta_0, \theta_1}{\text{minimize}} \ \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$$h_\theta(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$$\underset{\theta_0, \theta_1}{\text{minimize}} \ \underbrace{J(\theta_0, \theta_1)}_{\text{Cost function}}$$

Squared error function

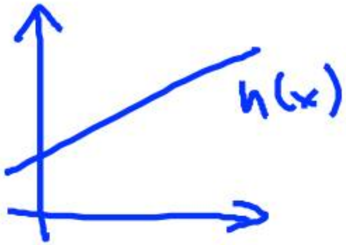
Andrew Ng

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters:

$$\theta_0, \theta_1$$



Cost Function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

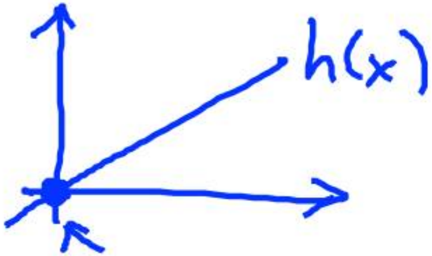
Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

Simplified

$$h_\theta(x) = \theta_1 x$$

$\theta_0 = 0$

$$\theta_1$$



$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$\theta, x^{(i)}$

$$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$$

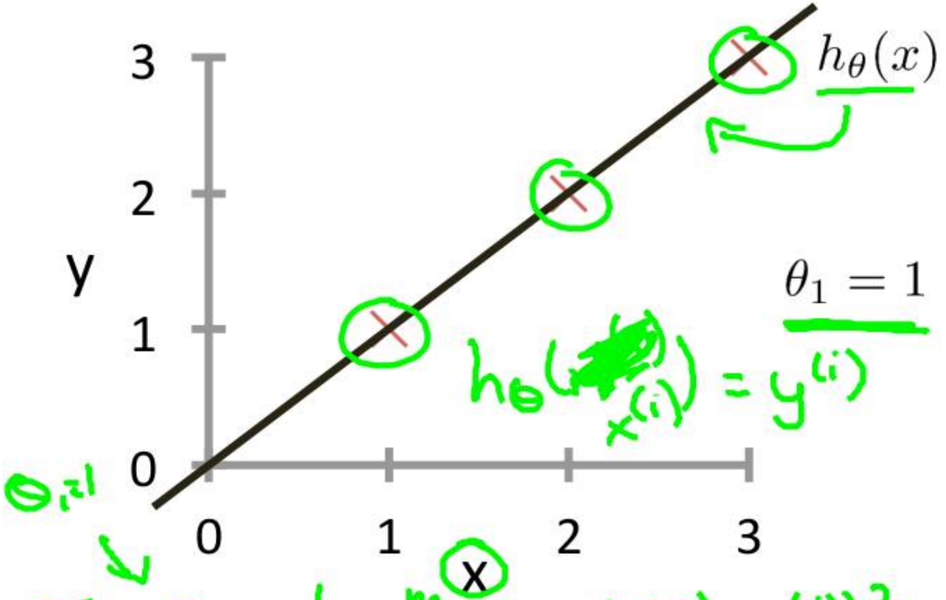
Andrew Ng

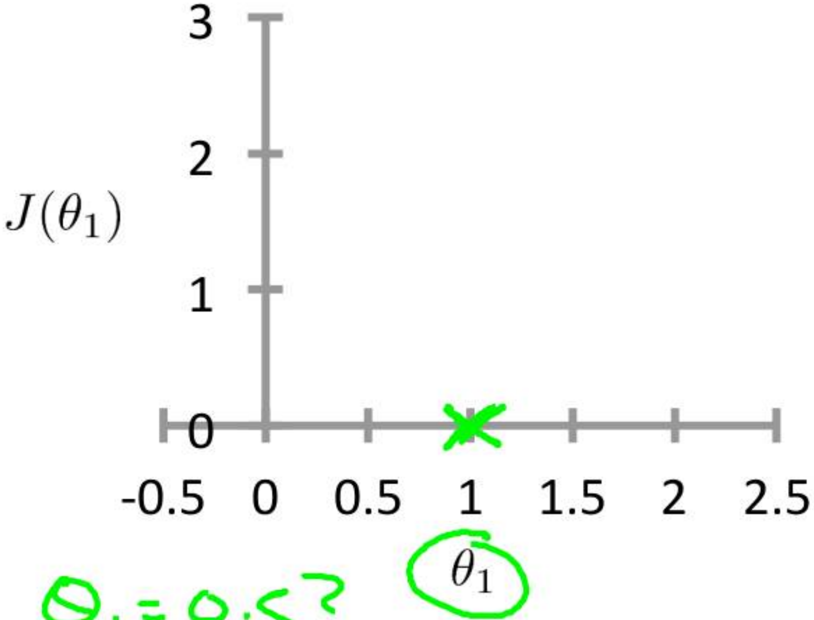
$\rightarrow h_\theta(x)$

(for fixed θ_1, this is a function of x)

$\rightarrow J(\theta_1)$

(function of the parameter θ_1)





$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

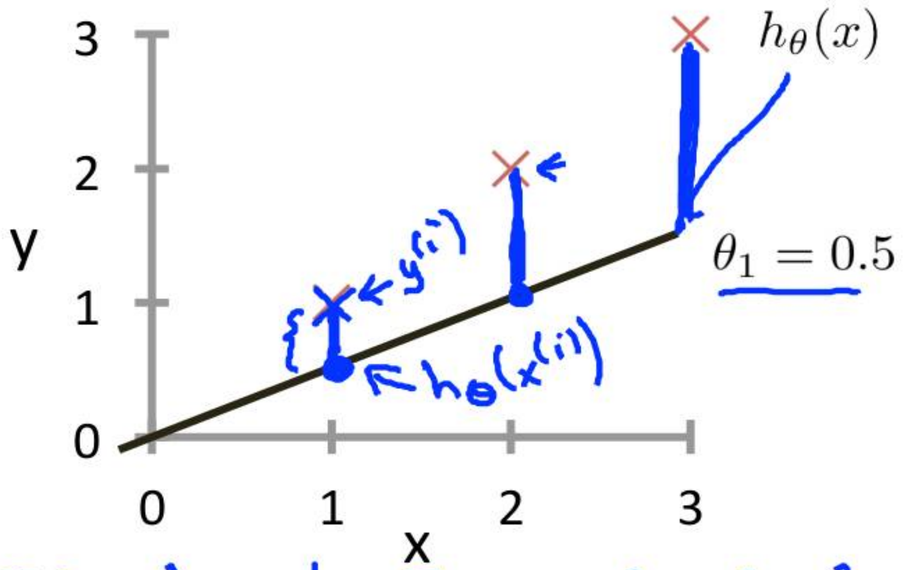
$$= \frac{1}{2m}\sum_{i=1}^{m}\left(\theta_1 x^{(i)} - y^{(i)}\right)^2 = \frac{1}{2m}\left(0^2+0^2+0^2\right) = 0$$

$\theta_1 = 0.5?$

$J(1) = 0$

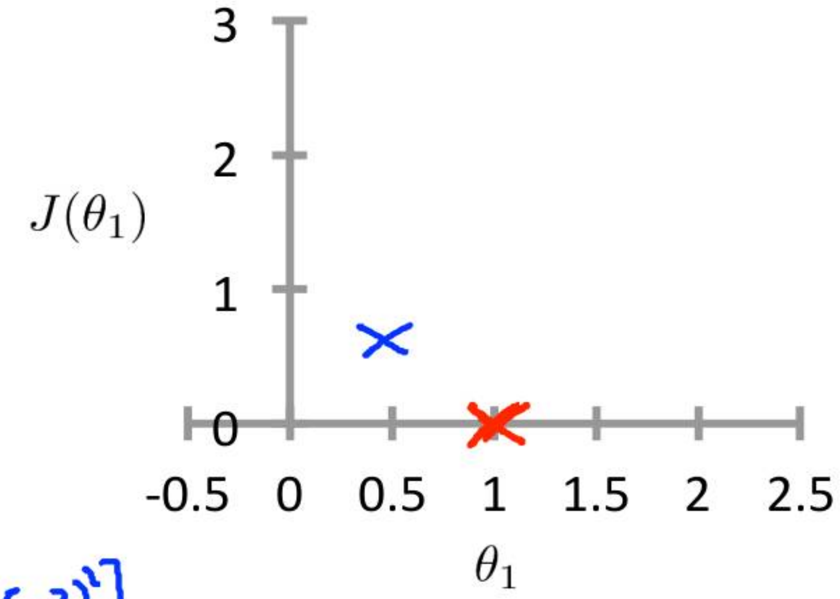
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



$J(\theta_1)$

(function of the parameter θ_1)



$$J(0.5) = \frac{1}{2m}\left[(0.5-1)^2 + (1-2)^2 + (1.5-3)^2\right]$$

$$= \frac{1}{2\times 3}(3.5) = \frac{3.5}{6} \approx 0.58$$

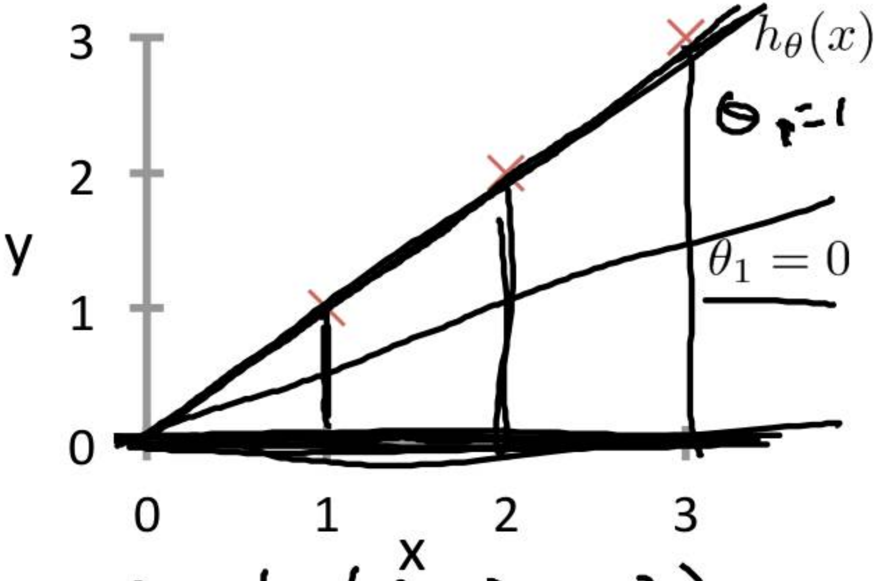
$\theta_1 = 0$?

$J(0) = $?

Andrew Ng

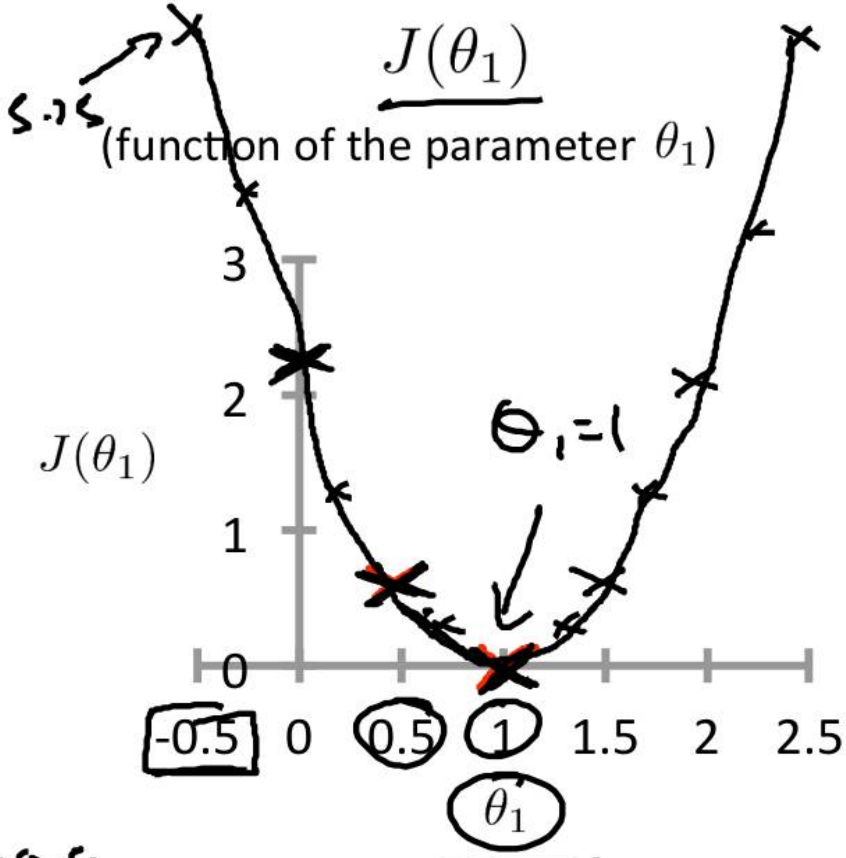
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



$J(\theta_1)$

(function of the parameter θ_1)



$J(0) = \frac{1}{2m}(1^2 + 2^2 + 3^2)$

$= \frac{1}{6} \cdot 14 \approx 2.3$

$h(x) = -0.5x$

$\boxed{\underset{\theta_1}{\text{minimize}}\ J(\theta_1)}$

h(x)

Andrew Ng

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

Parameters: θ_0, θ_1

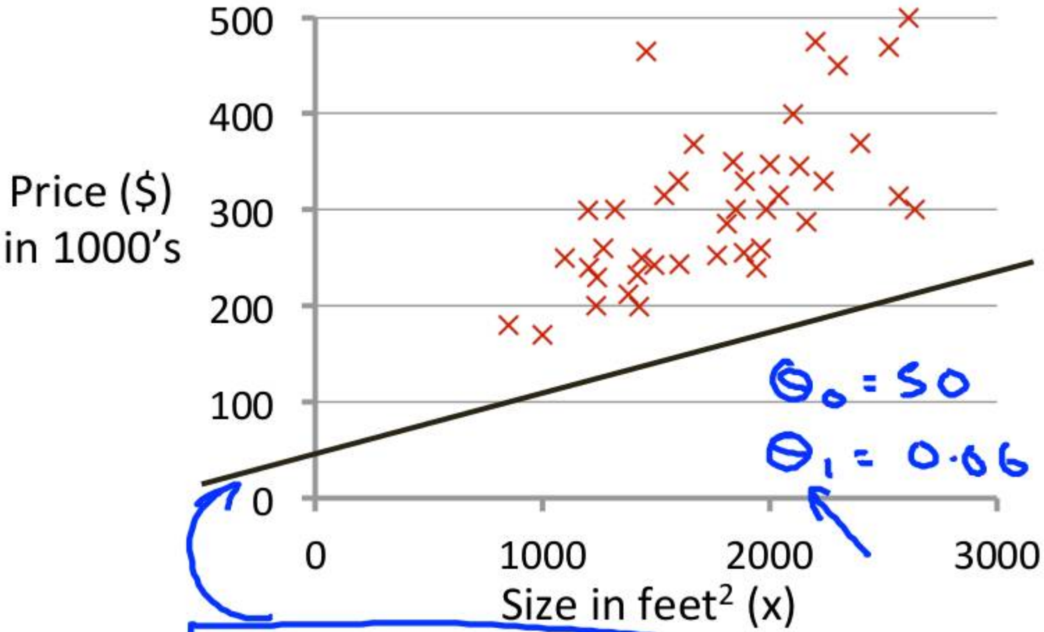
Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

Andrew Ng

$$h_\theta(x)$$

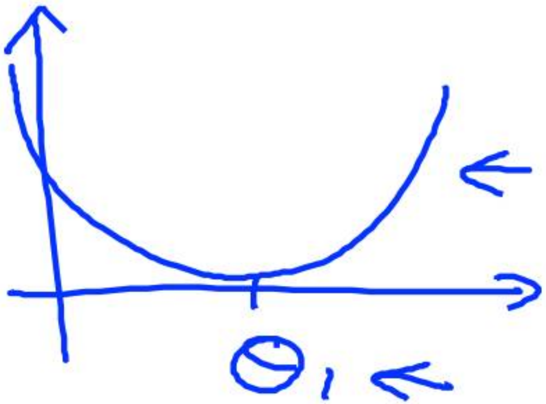
(for fixed θ_0, θ_1, this is a function of x)



$$\boxed{h_\theta(x) = 50 + 0.06x}$$

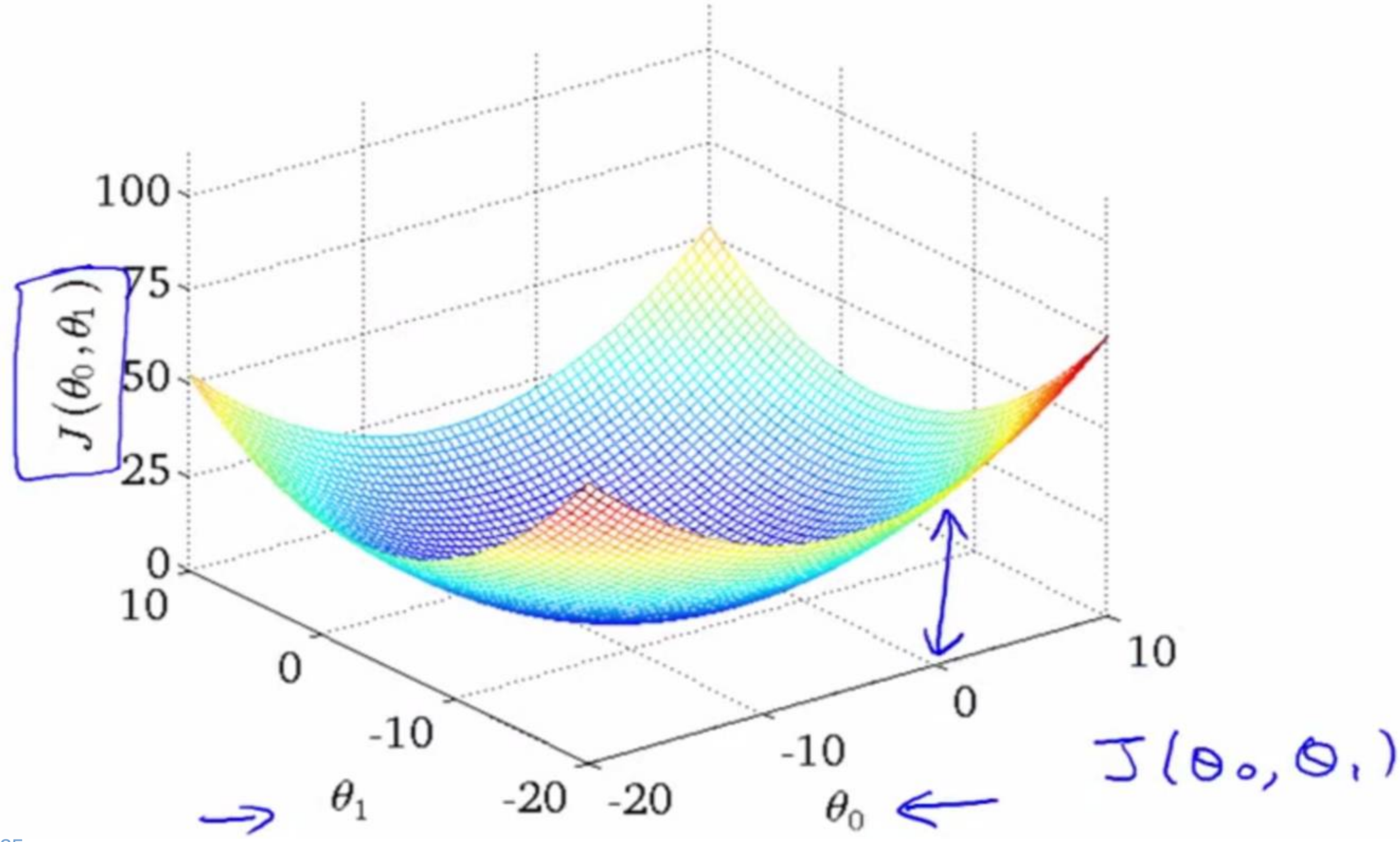
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



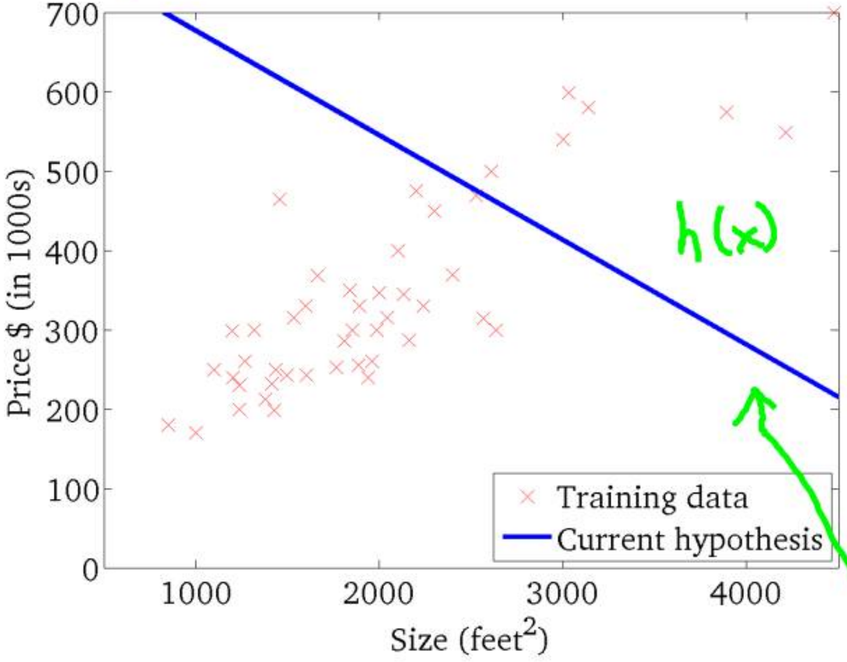
$$\boxed{\theta_0, \theta_1}$$

Andrew Ng


100
75
50
25
0
$J(\theta_0, \theta_1)$
10
0
-10
-20
θ_1
-20
-10
0
10
θ_0
$J(\theta_0, \theta_1)$

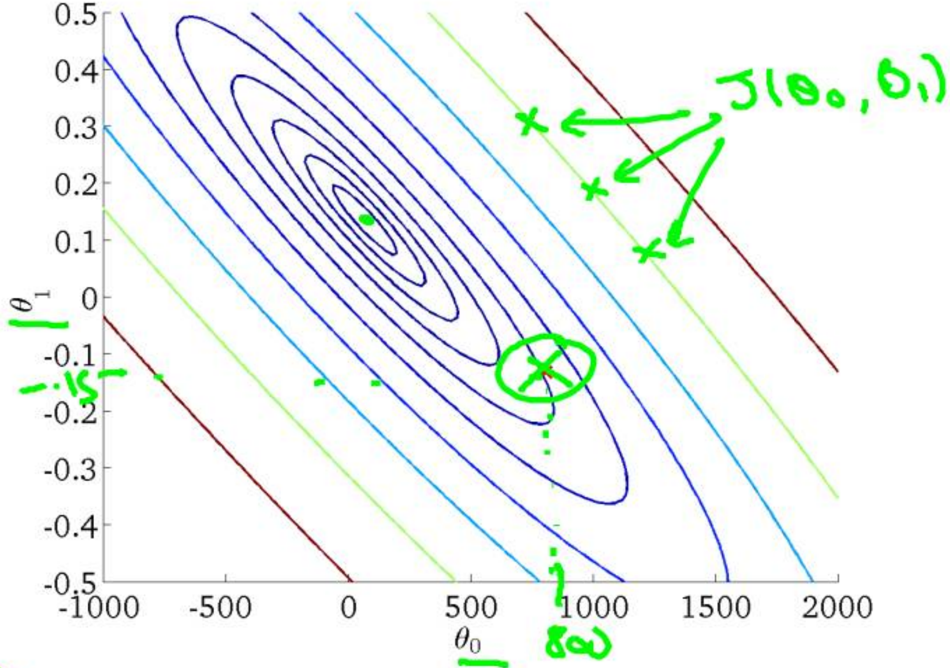
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



θ_0, θ_1

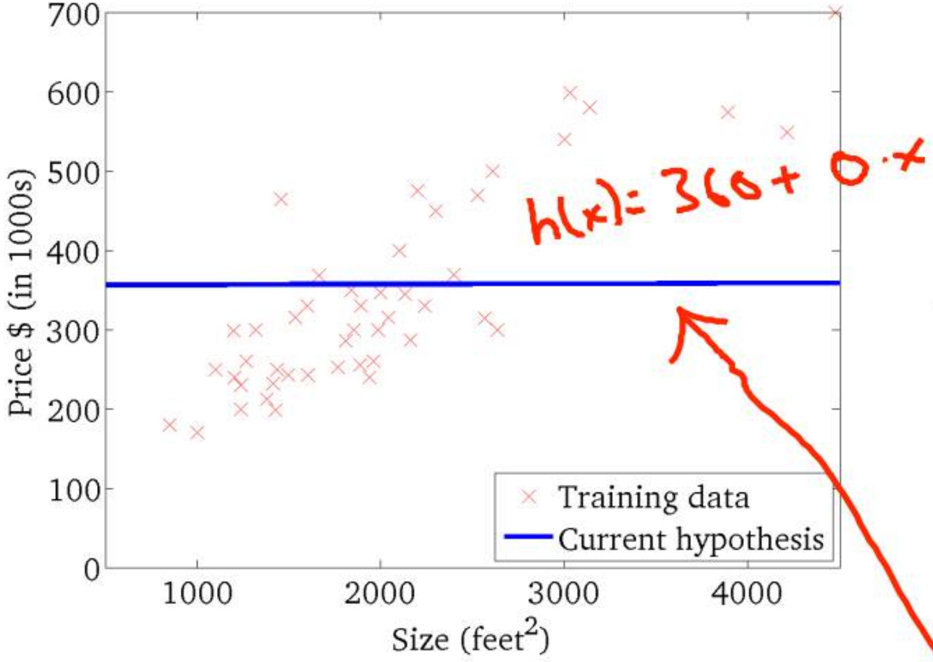
Andrew Ng

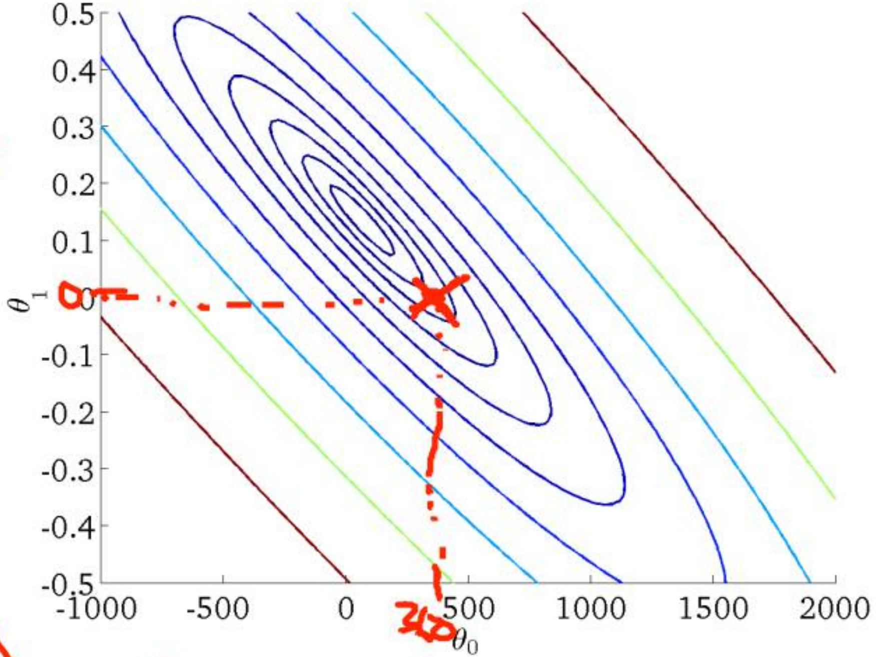
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)

$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



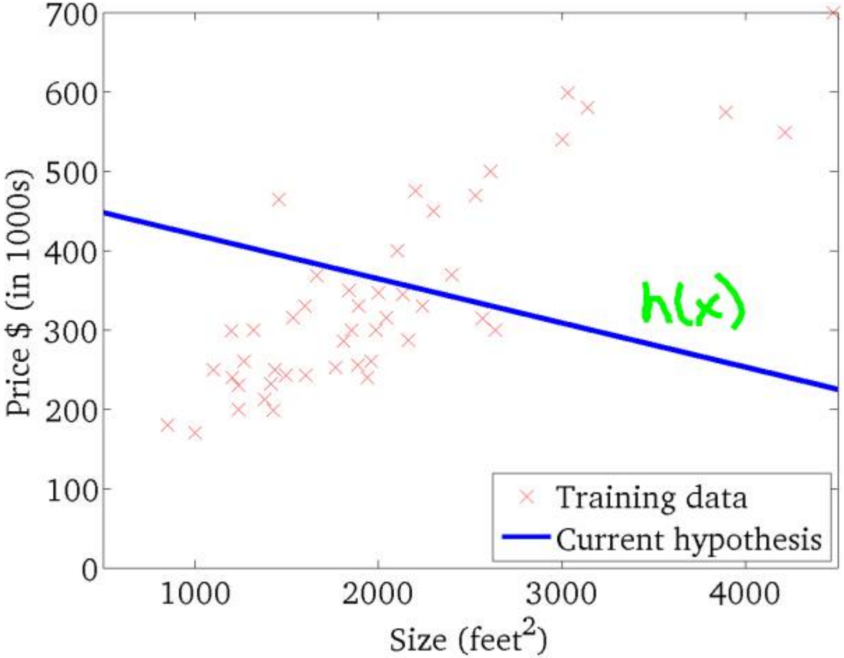


$\theta_0 = 360$
$\theta_1 = 0$

Andrew Ng

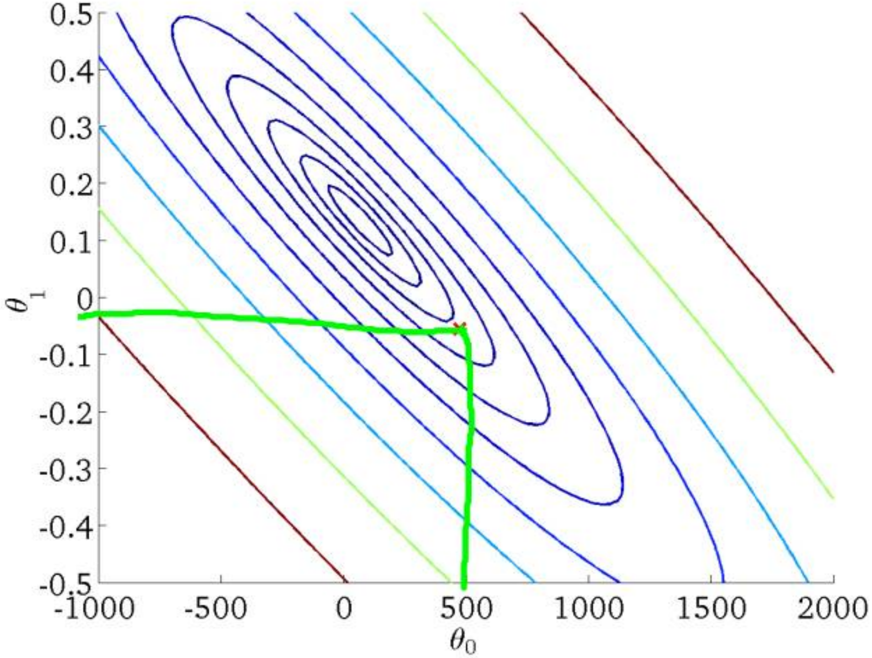
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

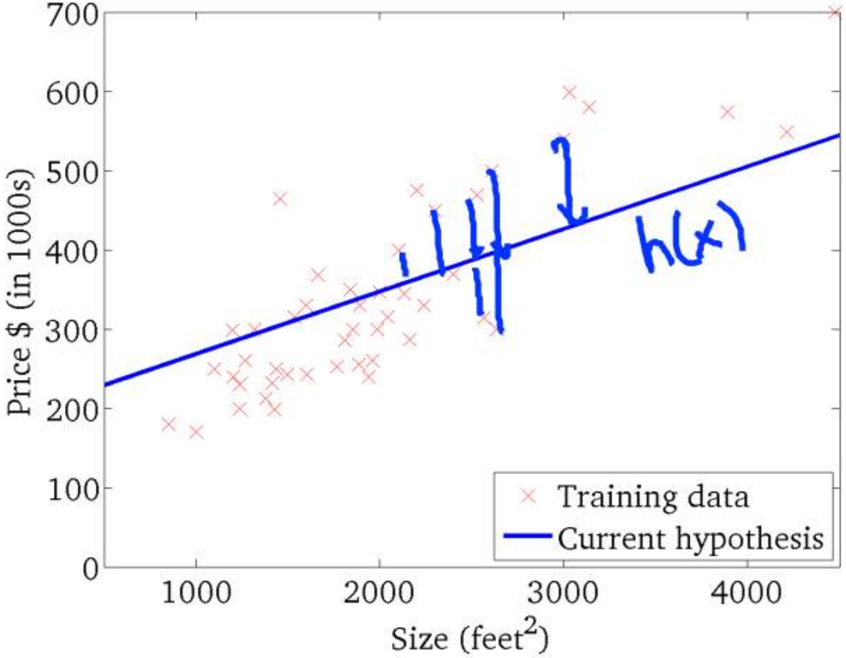
(function of the parameters θ_0, θ_1)



Andrew Ng

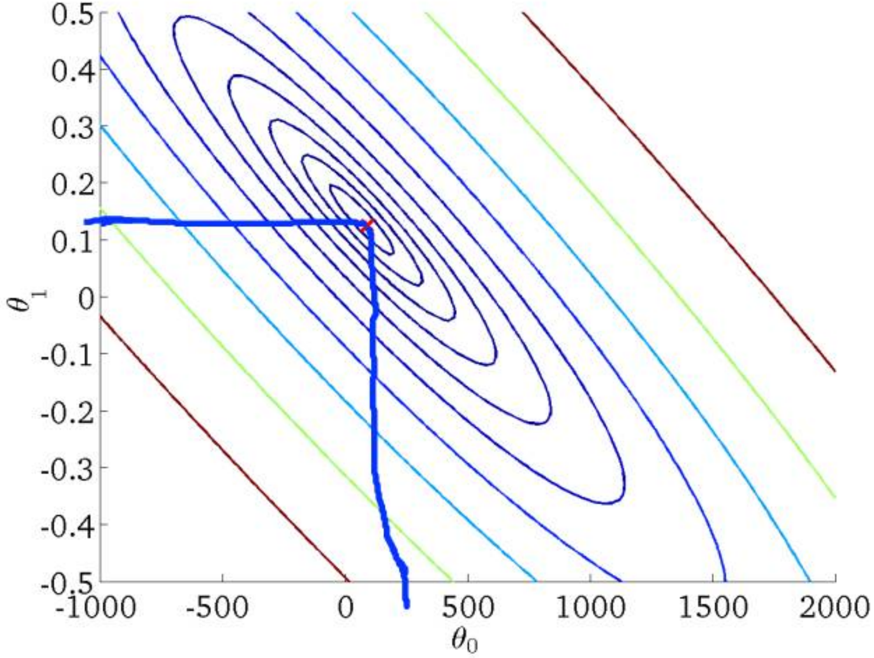
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



Andrew Ng

End of Lecture