```
robo = quadcopter();

%syms x y z;
%syms x_dot y_dot z_dot;
%syms R_x R_y R_z;
%syms w_x w_y w_z;
%xvec = [x;y;z;R_x;R_y;R_z;x_dot;y_dot;z_dot;w_x;w_y;w_z]

syms x [12 1];
xvec = x;
syms u [4 1];
uvec = u;

syms m g b;
x0 = zeros(12,1);
u0 = m*g*[1;1;1;1]/(4*b);

% Gets f(x,u) as a symbolic expression
dynamics = robo.dynamics_symbolic(xvec,uvec);
dynamics = simplify(dynamics)
```

```
dynamics =
```

$$
\begin{pmatrix}
x_7 \\
x_8 \\
x_9 \\
x_{10}\cos(x_5) + x_{12}\sin(x_5) \\
\dfrac{x_{11}\cos(x_4) - x_{12}\cos(x_5)\sin(x_4) + x_{10}\sin(x_4)\sin(x_5)}{\cos(x_4)} \\
\dfrac{x_{12}\cos(x_5) - x_{10}\sin(x_5)}{\cos(x_4)} \\
\dfrac{b\,u_1\,\sigma_2 - \mathrm{Dt}_{1,2}\,x_8 - \mathrm{Dt}_{1,3}\,x_9 - \mathrm{Dt}_{1,1}\,x_7 + b\,u_2\,\sigma_2 + b\,u_3\,\sigma_2 + b\,u_4\,\sigma_2}{m} \\
\dfrac{b\,u_1\,\sigma_1 - \mathrm{Dt}_{2,2}\,x_8 - \mathrm{Dt}_{2,3}\,x_9 - \mathrm{Dt}_{2,1}\,x_7 + b\,u_2\,\sigma_1 + b\,u_3\,\sigma_1 + b\,u_4\,\sigma_1}{m} \\
-\dfrac{\mathrm{Dt}_{3,1}\,x_7 + \mathrm{Dt}_{3,2}\,x_8 + \mathrm{Dt}_{3,3}\,x_9 + g\,m - b\,u_1\cos(x_4)\cos(x_5) - b\,u_2\cos(x_4)\cos(x_5) - b\,u_3\cos(x_4)\cos(x_5) -}{m} \\
-\dfrac{\mathrm{Dw}_{1,1}\,x_{10} + \mathrm{Dw}_{1,2}\,x_{11} + \mathrm{Dw}_{1,3}\,x_{12} - b\,u_3 + b\,u_4 - I_2\,x_{11}\,x_{12} + I_3\,x_{11}\,x_{12}}{I_1} \\
-\dfrac{\mathrm{Dw}_{2,1}\,x_{10} + \mathrm{Dw}_{2,2}\,x_{11} + \mathrm{Dw}_{2,3}\,x_{12} + b\,u_1 - b\,u_2 + I_1\,x_{10}\,x_{12} - I_3\,x_{10}\,x_{12}}{I_2} \\
-\dfrac{\mathrm{Dw}_{3,1}\,x_{10} + \mathrm{Dw}_{3,2}\,x_{11} + \mathrm{Dw}_{3,3}\,x_{12} + b\,u_1 + b\,u_2 - b\,u_3 - b\,u_4 - I_1\,x_{10}\,x_{11} + I_2\,x_{10}\,x_{11}}{I_3}
\end{pmatrix}
$$

where

$$\sigma_1 = \sin(x_5)\sin(x_6) - \cos(x_5)\cos(x_6)\sin(x_4)$$

$$\sigma_2 = \cos(x_6)\sin(x_5) + \cos(x_5)\sin(x_4)\sin(x_6)$$

```
% proof that (x0, u0) is an equilibrium point
f_eq = subs(dynamics,[xvec;uvec],[x0;u0])
```

```
f_eq =
```

$$
\begin{pmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{pmatrix}
$$

```
% Linearized A and B matrices symbolically
A = subs(jacobian(dynamics,xvec),[xvec;uvec],[x0;u0])
```

A =

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & g & 0 & -\frac{\mathrm{Dt}_{1,1}}{m} & -\frac{\mathrm{Dt}_{1,2}}{m} & -\frac{\mathrm{Dt}_{1,3}}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & -g & 0 & 0 & -\frac{\mathrm{Dt}_{2,1}}{m} & -\frac{\mathrm{Dt}_{2,2}}{m} & -\frac{\mathrm{Dt}_{2,3}}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mathrm{Dt}_{3,1}}{m} & -\frac{\mathrm{Dt}_{3,2}}{m} & -\frac{\mathrm{Dt}_{3,3}}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mathrm{Dw}_{1,1}}{I_1} & -\frac{\mathrm{Dw}_{1,2}}{I_1} & -\frac{\mathrm{Dw}_{1,3}}{I_1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mathrm{Dw}_{2,1}}{I_2} & -\frac{\mathrm{Dw}_{2,2}}{I_2} & -\frac{\mathrm{Dw}_{2,3}}{I_2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mathrm{Dw}_{3,1}}{I_3} & -\frac{\mathrm{Dw}_{3,2}}{I_3} & -\frac{\mathrm{Dw}_{3,3}}{I_3} \end{pmatrix}$$

```
B = subs(jacobian(dynamics,uvec),[xvec;uvec],[x0;u0])
```

B =

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{b}{m} & \frac{b}{m} & \frac{b}{m} & \frac{b}{m} \\ 0 & 0 & \frac{b}{I_1} & -\frac{b}{I_1} \\ -\frac{b}{I_2} & \frac{b}{I_2} & 0 & 0 \\ -\frac{b}{I_3} & -\frac{b}{I_3} & \frac{b}{I_3} & \frac{b}{I_3} \end{pmatrix}$$

```
lin_dynamics = simplify(A*xvec + B*uvec)
```

lin_dynamics =

$$\begin{pmatrix} x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ -\frac{\mathrm{Dt}_{1,1}\,x_7 + \mathrm{Dt}_{1,2}\,x_8 + \mathrm{Dt}_{1,3}\,x_9 - g\,m\,x_5}{m} \\ -\frac{\mathrm{Dt}_{2,1}\,x_7 + \mathrm{Dt}_{2,2}\,x_8 + \mathrm{Dt}_{2,3}\,x_9 + g\,m\,x_4}{m} \\ \frac{b\,u_1 - \mathrm{Dt}_{3,2}\,x_8 - \mathrm{Dt}_{3,3}\,x_9 - \mathrm{Dt}_{3,1}\,x_7 + b\,u_2 + b\,u_3 + b\,u_4}{m} \\ -\frac{\mathrm{Dw}_{1,1}\,x_{10} + \mathrm{Dw}_{1,2}\,x_{11} + \mathrm{Dw}_{1,3}\,x_{12} - b\,u_3 + b\,u_4}{I_1} \\ -\frac{\mathrm{Dw}_{2,1}\,x_{10} + \mathrm{Dw}_{2,2}\,x_{11} + \mathrm{Dw}_{2,3}\,x_{12} + b\,u_1 - b\,u_2}{I_2} \\ -\frac{\mathrm{Dw}_{3,1}\,x_{10} + \mathrm{Dw}_{3,2}\,x_{11} + \mathrm{Dw}_{3,3}\,x_{12} + b\,u_1 + b\,u_2 - b\,u_3 - b\,u_4}{I_3} \end{pmatrix}$$

```
% Using default parameters of the robot,
% gets linearized A and B
[A,B] = robo.getLinearization();
A
```

```
A = 12×12
         0         0         0         0         0         0    1.0000         0 ···
         0         0         0         0         0         0         0    1.0000
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0    9.8100         0         0         0
         0         0         0   -9.8100         0         0         0         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
    ⋮
```

```
B
```

```
B = 12×4
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
    2.0000    2.0000    2.0000    2.0000
```

```
         0         0  431.0345 -431.0345
    ⋮
```

```
q = 10;
Q = blkdiag(q*eye(6),eye(6));
R = eye(4);
K = robo.getLinearGain(Q,R)
```

```
K = 4×12
   -2.2361   -0.0000    1.5811    0.0000   -5.3957   -1.5811   -1.7204   -0.0000 ···
    2.2361    0.0000    1.5811   -0.0000    5.3957   -1.5811    1.7204    0.0000
    0.0000   -2.2361    1.5811    5.3957    0.0000    1.5811    0.0000   -1.7204
    0.0000    2.2361    1.5811   -5.3957    0.0000    1.5811    0.0000    1.7204
```