

Time Limit: 30 minutes.

Instructions: For this test, work in teams of up to four to solve 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside their appropriate space on your team answer sheet will be graded.

No calculators.

1. Freddy colors a 2×2 grid of squares using red, yellow, green, and blue, with each square colored solid. He does not want to use the same color for adjacent squares. How many ways can Freddy color the grid?
2. Fazbear is writing his dreaded dissertation on the prevalence of bears in German and Polish heraldry. If he writes at a pace of 12 words per minute for half an hour and then at a pace of 17 words per minute for the next twenty minutes, what was his average writing speed, in words per minute?
3. Eric is selling refrigerators to afford his Nueva tuition. He can pay off his debt alone by selling for 12 hours. After he has been selling for 5 hours, he calls his friend Jason, who starts selling golden apples at his own house. Working together, Eric's tuition is fully paid off after another 3 hours. If Jason were to pay off his own tuition, which is the same as Eric's, how long would he have to sell his apples for alone?
4. There is a magical coin that never lands on heads more than three times in a row and never lands on tails more than two times in a row. This coin can only be flipped ten times before it will disappear forever. How many different flip sequences of heads and tails can be attained?
5. In triangle $\triangle ABC$, let D , E , and F be the feet of the altitude from vertices A , B , and C to sides BC , AC , and AB , respectively. If $AD = 10$, $BE = 9$, and $CD = 8$, find the area of $\triangle ABC$.
6. Three children want to distribute 12 indistinguishable pieces of candy. Alice wants at least 1 piece of candy, Bob wants at least 2 pieces of candy, and Carol wants at least 3 pieces of candy. How many ways can the pieces of candy be distributed such that each child gets their desired amount of candy?
7. An infinite sequence $a_1, a_2, a_3 \dots$ is defined recursively so that $a_1 = 2 - \sqrt{3}$, $a_2 = 2 + \sqrt{3}$, and

$$a_{n+2} = \frac{2(a_n + a_{n+1})(1 - a_n a_{n+1})}{1 + (a_n a_{n+1})^2 - (a_n + a_{n+1})^2}$$

- for all positive integers n . Compute the value of $(|a_{2023}| + |a_{2024}| + |a_{2025}|)^2$.
8. Daniel, Ethan, and Felix have lunch at Karl's Kitchen and agree to split the \$100 bill. If Daniel insists on paying either \$7, \$11, \$13, or \$17, Ethan insists on paying in \$8 bills, and Felix insists on paying an amount between \$37 and \$73, in how many ways can the friends split the bill?
 9. Let p , q , and r be the three positive roots of the polynomial $x^3 - 10x^2 + 31x - 29$. What is the area of the triangle with side lengths p , q , and r , given that it exists?
 10. There are two breadboards on the table, with one containing 99 red LEDs, numbered from 2 to 100, and one containing 99 blue LEDs, likewise numbered from 2 to 100. Initially, all of the LEDs are turned off. When a button is pressed, the following algorithm is executed:
 - A robot writes down the minimal integer k between 2 and 100, inclusive, such that the blue LED numbered k is off.
 - Each red LED whose number is a multiple of k is toggled: LEDs that are currently off are turned on, and LEDs that are currently on are turned off.
 - Each blue LED whose number is a multiple of k and is currently off is turned on.

Let m be the minimal number of presses of the button for all blue LEDs to turn on, and let n be the number of red LEDs that are turned on when this occurs. Compute $100m + n$.