$$y' = e^{y-x} + \mu y, \quad y(0) = -\mu$$

$$y(x,\mu) = y_0(x) + \mu y_1 + \mu^2 y_2 + \dots$$

$$y_0 = y(x,0)$$

$$y_1(x) = \frac{\partial y(x,\mu)}{\partial \mu}\Big|_{\mu=0} \qquad y_2(x) = \frac{1}{2}\frac{\partial^2 y}{\partial \mu^2}\Big|_{\mu=0}, \ \dots$$

$$y_0'(x) = y_x'(x,0) \qquad y_1'(x) = \frac{\partial}{\partial \mu}\, y_x'(x,\mu)\Big|_{\mu=0}, \ \dots$$

$$y_0' = e^{y_0 - x} \qquad y_1' = e^{y_0 - x}y_1 + y_0 \qquad y_2' = e^{y_0 - x}y_2 + y_1 + \frac{1}{2}e^{y_0-x}y_1^2, \ \dots$$

$$y_0(0) = y_2(0) = \dots = 0 \qquad y_1(0) = -1$$

$$\Rightarrow e^{-y_0} = e^{-x} + c_1 \Rightarrow c_1 = 0 \Rightarrow y_0 = x$$

$$y_1' = c_2 e^x - x - 1 \Rightarrow c_2 = 0 \Rightarrow y_1' = -x - 1$$

$$\Rightarrow y(x,\mu) = x - \mu(x+1) + \frac{\mu^2}{2}(e^x - x^2 - 2x - 1) + \dots$$

$$y' = y + xe^y \quad (y(0) = 0 \Rightarrow c_0 = 0)$$
$$y' = y + x \cdot e^y \quad (y(0))$$
$$y'(0) = 0$$
$$y''(0) = 0 + e^0 + 0 \cdot e^0 \cdot 0 = 1$$
$$y'''(0) = 1 + 0 + 0 + 0 = 1$$
$$y^{IV}(0) = 1 + 0 + 1 + 0 + 1 + 0 + 1 + 0 = 4$$
$$y = 0 + 0 \cdot \frac{x}{1!} + 1 \cdot \frac{x^2}{2!} + 1 \cdot \frac{x^3}{3!} + 4 \cdot \frac{x^4}{4!}$$
$$y = \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{6}$$

$$(1-x)y'' - 2y' + y = 0$$

$$\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} - \sum_{n=2}^{\infty} n(n+1)a_n x^{n-1} - 2a_1 + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\Rightarrow \sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2} x^n - \sum_{n=1}^{\infty}(n+1)(n+2)a_{n+1}x^n - 2a_1 + \sum_{n=0}^{\infty} a_n x^n = 0$$

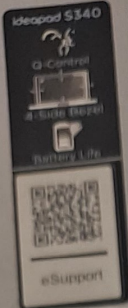$$\Rightarrow 2a_2 - 2a_1 + a_0 = 0 \qquad (n+2)(n+1)(c_{n+2} - a_{n+1}) + a_n = 0$$

$$a_1 = 0 \quad a_0 = 1 \qquad a_2 = -\frac{1}{2} \quad a_3 = -\frac{1}{2} \quad a_4 = -\frac{4}{24}, \dots$$

$$y_1 = 1 - \frac{x^2}{2} - \frac{x^3}{2} - \frac{11}{24}x^4 \Rightarrow y_2 = x + x^2 + \frac{5}{6}x^3 + \frac{3}{4}x^4 + \dots$$

$$y' = 4\mu x - y^2 \quad y(1) = 1$$
$$y(x, \mu) = y_0 + \mu y_1 + \mu^2 y_2 + \cdots$$
$$y_0' = -y_0^2 \quad y_1' = 4x - 2y_0 y_1 \quad y_2' = -y_1^2 - 2y_0 y_2 \cdots$$
$$y_0(1) = 1 \quad y_1(1) = 0 \quad y_2(1) = 0 \cdots$$
$$y_0 = \frac{1}{x} \quad y_1 = x^2 - \frac{1}{x^2} \quad y_2 = -\frac{x^5}{7} + \frac{2x}{3} + \frac{1}{x^3} - \frac{32}{21x^2} + \cdots$$

$$y = \frac{1}{x} + \mu\left(x^2 - \frac{1}{x^2}\right) + \mu^2\left(-\frac{x^5}{7} + \frac{2x}{3} - \frac{32}{21x^2} + \frac{1}{x^3}\right) + \cdots$$

N 1113

$$x^2 y'' + 2xy' - (x^2 + 2x + 2)y = 0$$
$$(r^2 + r - 2)a_0 = 0 \quad r(r+3)a_1 - 2a_0 = 0$$
$$((n+r)(n+r+1) - 2)a_n - a_{n-2} - 2a_{n-1} = 0 \quad n = 2, 3 \ldots$$
$$a_0^2 + a_1^2 \neq 0 \Rightarrow (r-1)r(r+2)(r+3) = 0$$
$$r_1 = 1 \quad r_2 = 0 \quad r_3 = -2 \quad r_4 = -3$$

пусть $r = 1$ $a_0 = 1 \Rightarrow a_1 = \frac{1}{2}$ $a_2 = \frac{1}{5}$ $a_3 = \frac{1}{20}$ $a_4 = \frac{3}{280}, \ldots$

$$y_1 = x + \frac{x^2}{2} + \frac{x^3}{5} + \frac{x^4}{20} + \frac{3x^5}{280} + \cdots$$

пусть $r = -2$ $a_0 = 1$ $a_1 = -1$ $a_2 = \frac{1}{2}$

$$\Rightarrow a_1 = \frac{2}{r^2 + 3r} \quad a_2 = \frac{r^2 + 3r + 4}{(r^2 + 3r)(r^2 + 5r + 4)} \quad a_3 = \frac{4(r+2)}{(r^2 + 3r)(r^2 + 5r + 4)\ldots}$$

$$a_1 = -1 \quad a_2 = \frac{1}{2} \quad a_3 = 0 \quad y_2 = \frac{1}{x^2} - \frac{1}{x} + \frac{1}{2} + \frac{x^2}{8} + \frac{x^3}{46} + \frac{7x^4}{120} + \cdots$$