Name:	**Student ID:**

SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

50.007 Machine Learning, Fall 2021
Midterm Exam

Date: 5 November 2021
Time: 17:00 - 20:00

Instructions:

1. This is an open-book exam.
2. Write your name and student ID at the top of this page.
3. The problems are not necessarily in order of difficulty. We recommend that you scan through all the questions first, and then decide on the order to answer them.
4. Problem 1 and Problem 2 were already provided through the link.
5. Write your answers in the space provided.
6. You may access the Internet.
7. **You may NOT communicate via any means with anyone.**

For staff's use:

Problem 1	/7
Problem 2	/12
Problem 3	/4
Problem 4	/6
Problem 5	/5
Problem 6	/6
Problem 7	/5
Problem 8	/5
Total	**/50**

Problem 3: Classification (4 Points)

(a) Consider data points from a 2-d space where each point is of the form $x = (x_1, x_2)$. You are given the same dataset as in homework 1, problem 1, with two positive examples: (1, 1) and (2, 2), and two negative examples (-1, 1) and (1, -1). For the hypothesis space, inside or outside of a (a, b)-centered circle with radius r, find the parameters (where, a, b, r are the parameters) of the classifier (a member of the hypothesis space) that can correctly classify all the examples in the dataset, or explain why no such classifier exists. If a classifier is possible then graphically represent all the examples of the dataset along with the classifier parameters. *(2 points)*

(b) We know that *"The perceptron update rule converges after a finite number of mistakes when the training examples are linearly separable through origin."* Consider a training example $x^{(t)}$, from a dataset that is linearly separable through the origin, which has been initially misclassified. Prove that the perceptron update rule ($\theta^{(k+1)} = \theta^{(k)} + y^{(t)}x^{(t)}$) does indeed attempt to correctly classify the training example by increasing the value of $y^{(t)}(\theta \cdot x^{(t)})$. *(2 points)*

Problem 4: Regression (6 Points)

A candy factory is preparing for Christmas and they want to optimize their production pricing. They currently produce 3 types of candy: Apple, Banana and Chocolate. They have data from 10 previous years regarding their production price, the price at which they sold the candies, and the quantities that were bought. They also have data about the total number of candies of these 3 types that were on the market in the past 2 years, and they know the retail price charged for competitor candies in the last 10 years (but not the quantities that were sold). They want you to help them predict the amount of candy of each category that they will be able to sell, as a function of all the data available (production price, selling price, retail price of competitors and total number of candies in the market). For each of the following algorithm, explain how it works, how would the algorithm perform and any drawbacks. Also, if the algorithm can be improved by any pre-processing of the data, please state so.

1. Linear Regression *(2 points)*

2. Polynomial Regression *(2 points)*

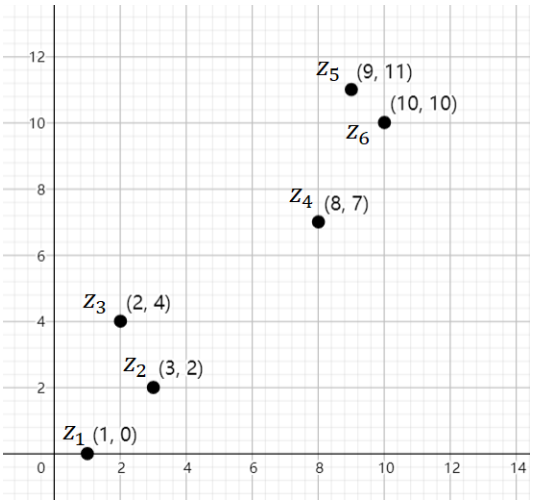
3. Ridge Regression *(2 points)*

Problem 5: K-Means (5 Points)

Suppose we want to use k-means algorithm to perform clustering on the dataset

$$S = \{z_1, z_2, z_3, z_4, z_5, z_6\}$$

which is also given below:



In the initialization step, z_1 is used as the first cluster center C_1 and z_2 as the second cluster center C_2. k is set to 2.

a) Suppose we simulate k-means algorithm for **ONE iteration**. We first calculate the Euclidean distance between cluster centers and each point in the dataset S, we then perform cluster assignment. Please fill your results in the table given below for one iteration. *(2 points)*

Data		Distance to		Cluster Assignment
i	z_i	$C_1 = (1,0)$	$C_2 = (3,2)$	
1	(1,0)			
2	(3,2)			
3	(2,4)			
4	(8,7)			
5	(9,11)			
6	(10,10)			

b) How many iterations we need for **convergence**? Please list the **final** cluster assignments. *(2 points) Please note that you don't need to list all the iterations.*

c) Suppose we want to cluster a new data point $z_7 = (11, 8)$ after the k-means algorithm is converged. Which cluster is z_7 assigned to? *(1 point)*

Problem 6: SVM (6 points)

1. In the Dual formulation of with SVM soft margin, why is Lagrange multiplier α upper bounded by C (in lecture slides)? (1 points)

2. What's the "kernel trick" in SVM and how is it useful? Please provide an example (a particular task) in which we need to use "kernel trick". (1 points)

Indicate whether the following functions are valid SVM kernels, and explain your answers (i.e. provide a formal proof to justify your answer). *Hint: Please remember the properties of kernel functions that we discussed in the lectures.*

3. $K(x, x') = 16$ *(1 points)*

Is this a kernel? _____ (*yes* or *no*).
Explanations:

4. $K(x, x') = (x \cdot x' + 9)$ *(1 points)*

Is this a kernel? _____ (*yes* or *no*).
Explanations:

5. $K(x, x') = (x \cdot x')^2 - 8$ *(1 points)*

Is this a kernel? _____ (*yes* or *no*).
Explanations:

6. $K(x, x') = (x \cdot x)^4 + (x' \cdot x')^2$ *(1 points)*

Is this a kernel? _____ (*yes* or *no*).
Explanations:

Problem 7: Logistic Regression (5 points)

In our lectures, we have studied how to formulate the decision boundary of a classifier based on logistic regression. Please answer the following questions qualitatively.

(a) Please explain a scenario you need to use logistic regression to solve a particular problem. *(2 points)*

Clearly state the problem, the data that you have (labeled/unlabeled), input-output to your logistic regression, and why you would prefer using logistic regression over SVM. Please do not use more than 6-7 sentences.

(b) Suppose you trained a logistic classifier with a hypothesis function as given below:

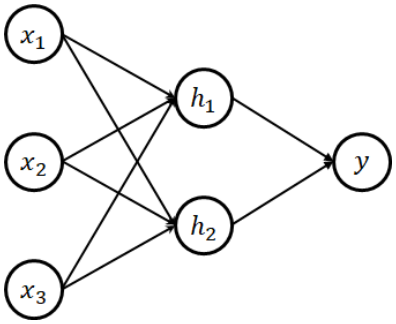
$$h_\theta(x) = g(\theta_0 + x_1\theta_1 + x_2\theta_2 + x_1^2\theta_3 + x_2^2\theta_4)$$

Then, you obtained $\theta = [-1 \quad 0 \quad 0 \quad 4 \quad 9]^T$ using gradient descent algorithm. Please **formulate and draw** the decision boundary of your classifier. Note that this is a binary classification problem, which means class label y can be 0 or 1. *(2 points)*

(c) After training completed, if you test your logistic regression with a new data point, would your decision boundary change? Please explain your answer. *(1 points)*

Problem 8: Neural Networks (5 points)

As can be seen below, we have a simple neural network, which includes an input layer, a single hidden layer and an output. Please note that the parameters of the network (weight matrices) are already estimated. The weight matrix W that connects input to the hidden layer is $\begin{bmatrix} -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$. The weight matrix V that connects the hidden layer to the output is $\begin{bmatrix} 1 & 2 \end{bmatrix}$. ReLu function is used as the activation function for the hidden and the output layer.



Let's assume that you are given the following input data:

$$x = (x_1, x_2, x_3) = (2, 1, 2)$$

What is the output value of this neural network given this input? Please also report the output of h_1 and h_2. Explain your answer, and provide step-by-step formulation.

Important note: 1) Please ignore the bias term, and 2) during the lecture (week 6), we have studied the mathematical formulation of a neural network.