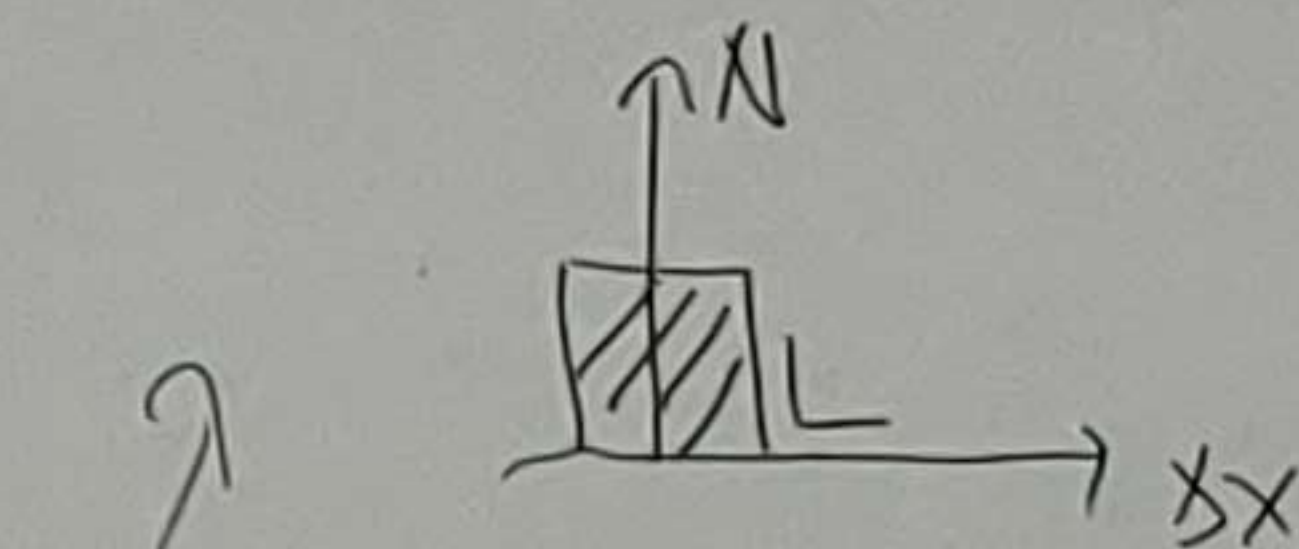
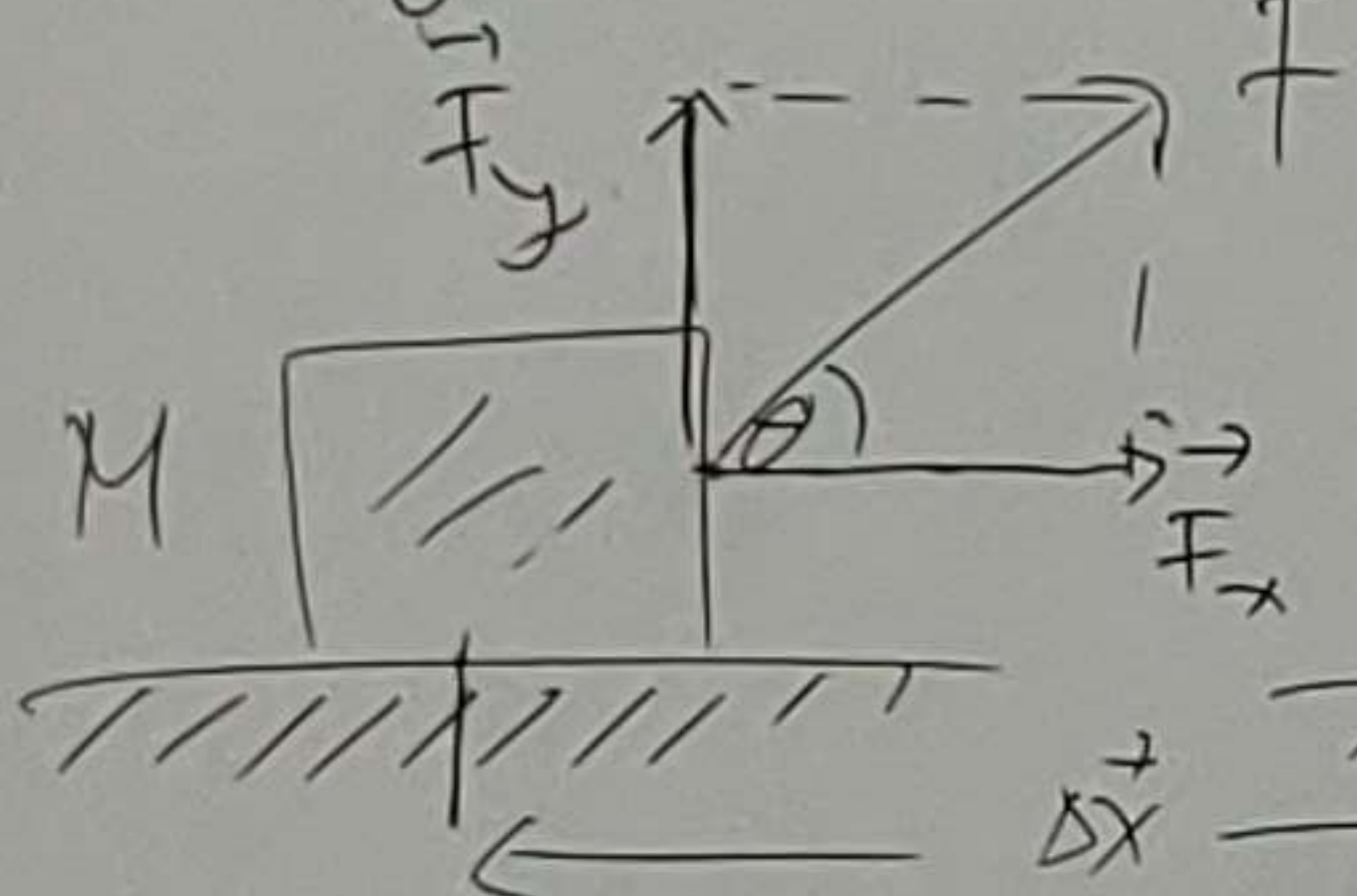


Usaha & Energi

Usaha W → skalar

Definisi :

Diam



$$W = N \Delta x \cos 90^\circ$$

$$W = 0 \text{ J}$$

Perpindahan
arah \vec{x}

→ Gaya yang segaris dengan perpindahan

$$W = \vec{F}_x \cdot \Delta \vec{x} = \vec{F} \cdot \Delta \vec{x} \cos \theta$$

Skalar Vektor Vektor

θ = sudut antara
 \vec{F} dan $\Delta \vec{x}$

$$W = \vec{F} \cdot \Delta \vec{x}$$

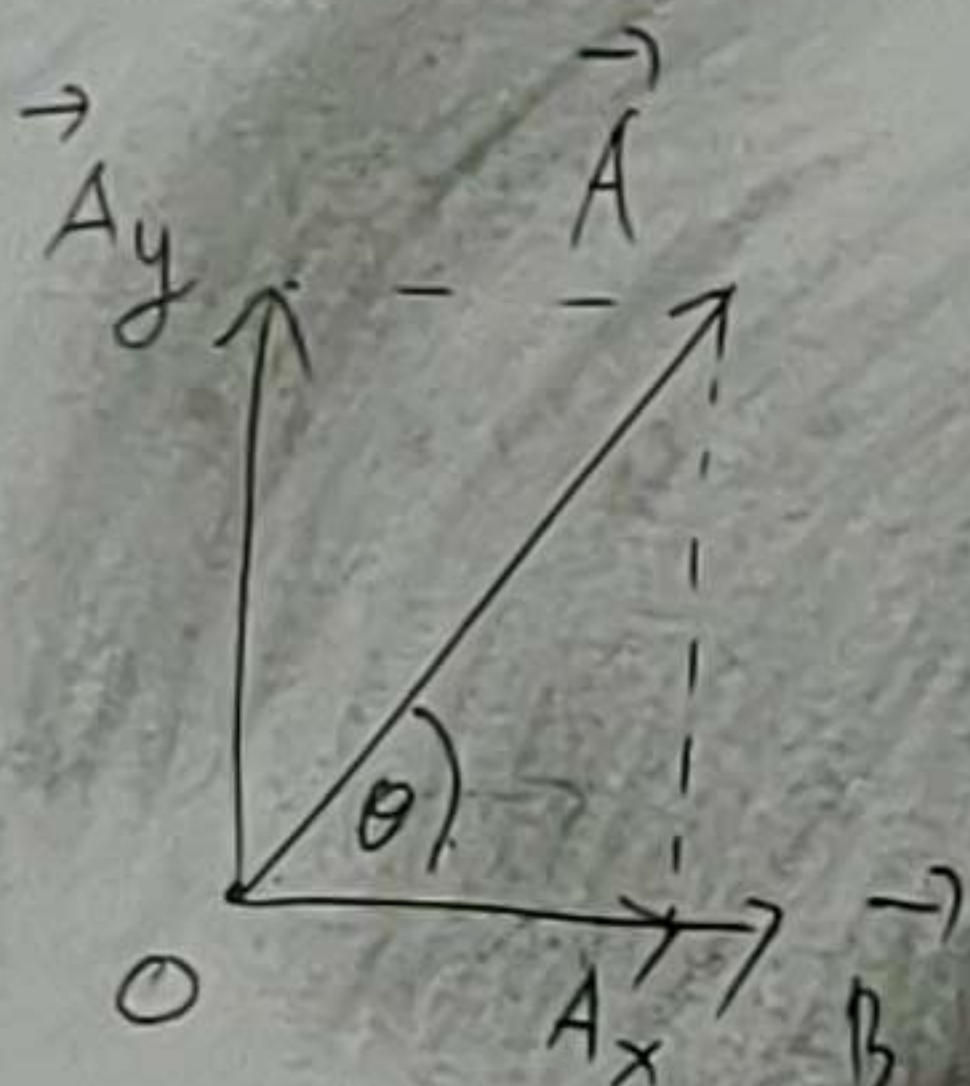
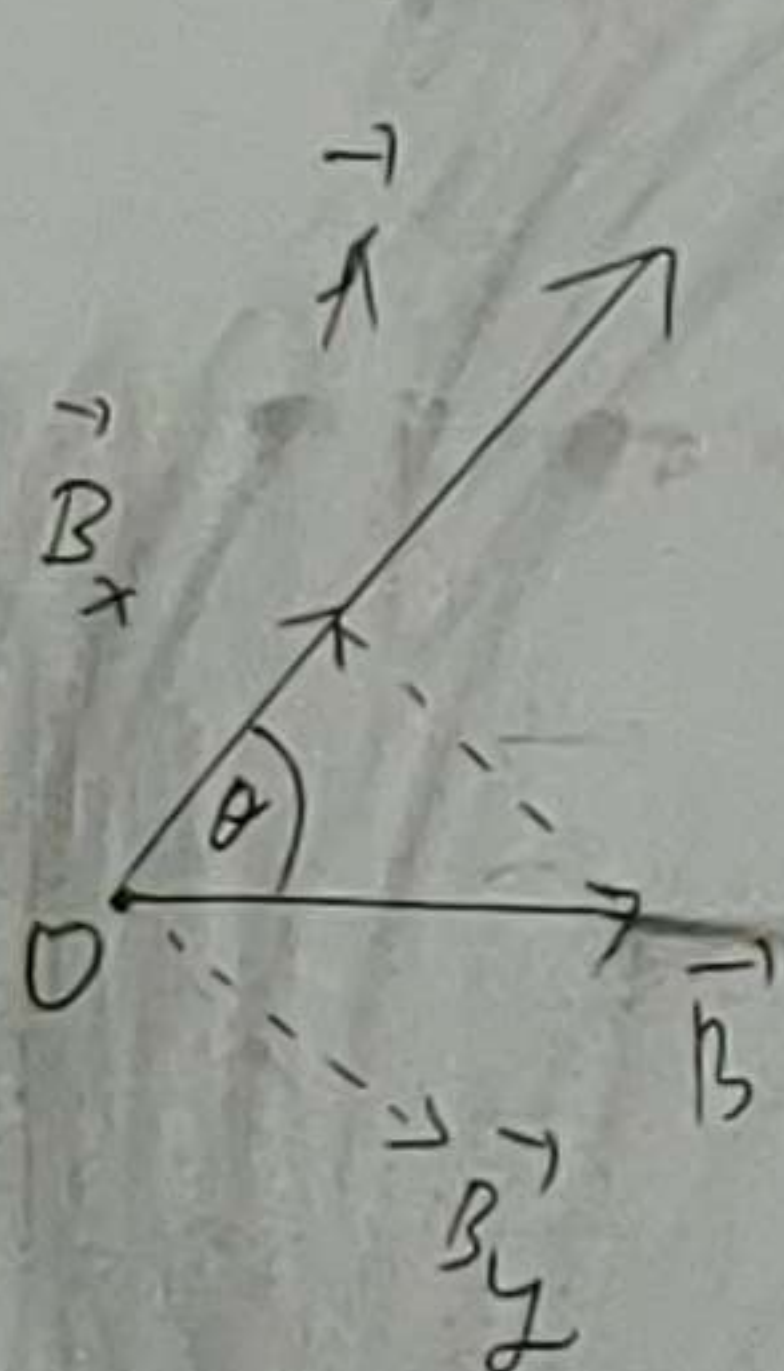
Dot product / Perkalian Vektor

Perkalian Vektor

\vec{A}
 \vec{B}

• Perkalian titik

↳ Hasil: Skalar



$$\vec{A} \cdot \vec{B} = \vec{A} B_x$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\vec{B} \cdot \vec{A} = \vec{B} A_x$$

$$\vec{B} \cdot \vec{A} = \vec{B} A \cos \theta$$

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

Vektor Satuan

↳ Vektor nilainya 1.

$$\vec{x} \rightarrow \hat{i}$$

$$\vec{y} \rightarrow \hat{j}$$

$$\vec{z} \rightarrow \hat{k}$$

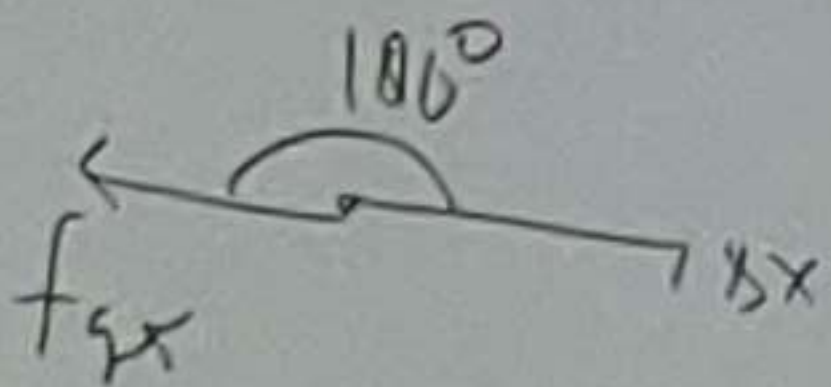
$$\hat{i} \cdot \hat{i} = 1 \cos 0^\circ = 1$$

$$\hat{j} \cdot \hat{j} = 1 \cos 0^\circ = 1$$

$$\hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = 1 \cos 90^\circ = 0$$

$$\hat{j} \cdot \hat{k} = 0$$



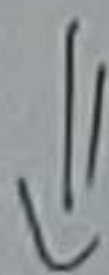
$$\begin{aligned} \textcircled{c} \quad W_{f_{gx}} &= f_{gx} \Delta x \cos 180^\circ \\ &= -f_{gx} \Delta x \end{aligned}$$

$$\textcircled{d} \quad W_F = F \Delta x \cos \theta$$

$$\begin{aligned} \textcircled{e} \quad W_T &= W_N + W_w + W_{f_{gx}} + W_F \\ W_T &= -f_{gx} \Delta x + F \Delta x \cos \theta \end{aligned}$$

$$W_T = (F \cos \theta - f_{gx}) \Delta x \quad J$$

$$\boxed{W_T = (F_x - f_{gx}) \Delta x} \quad \checkmark$$



$$W_T = \sum f_x \cdot \Delta x \quad \checkmark$$

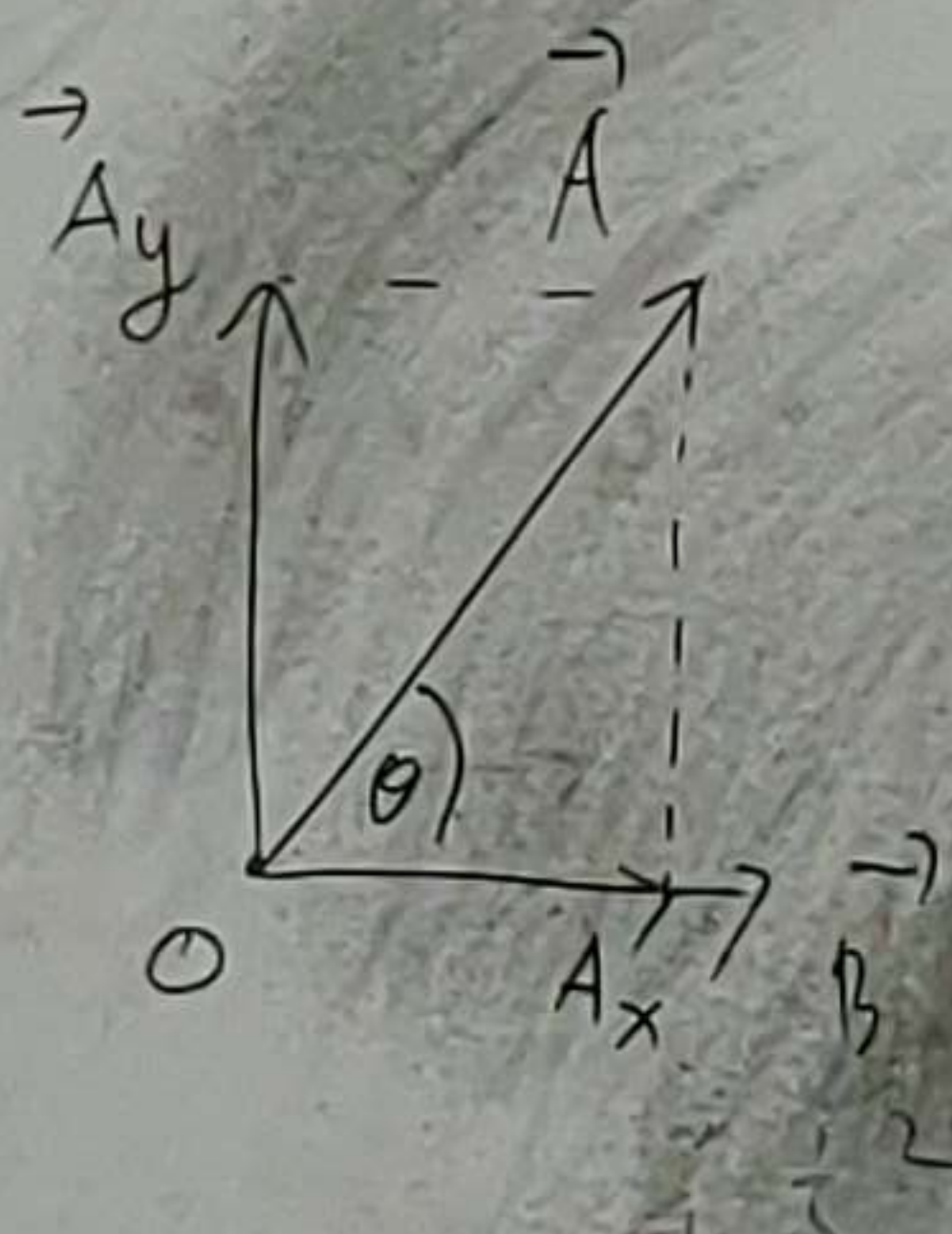
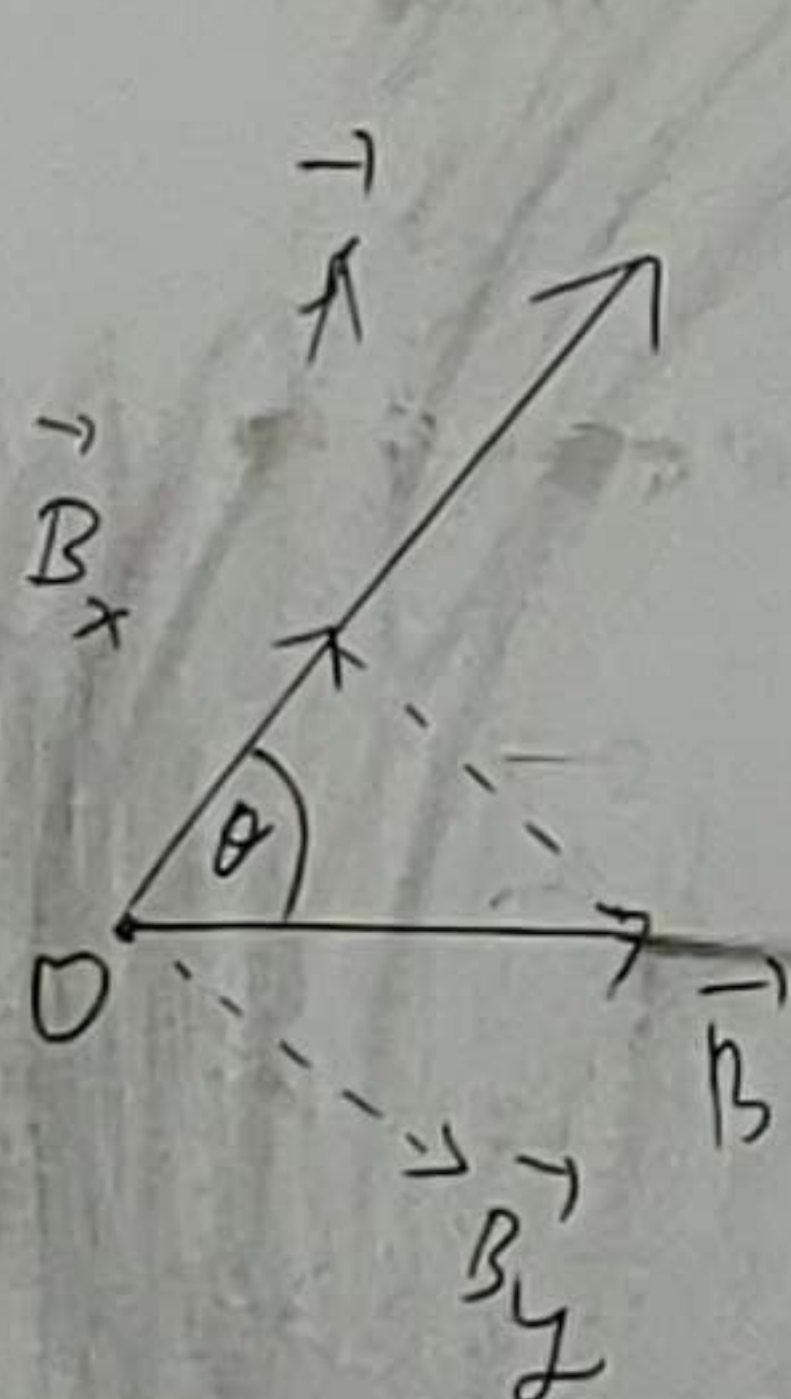
Perkalian Vektor

$$\vec{A}$$

$$\vec{B}$$

• Perkalian titik

↳ Hasil: Skalar.



$$\vec{A} \cdot \vec{B} = \vec{A} B_x$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\vec{B} \cdot \vec{A} = \vec{B} A_x$$

$$\vec{B} \cdot \vec{A} = \vec{B} A \cos \theta$$

$$\boxed{\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}}$$

Vektor Satuan

↳ Vektor nilainya 1.

$$\vec{x} \rightarrow \hat{i}$$

$$\vec{y} \rightarrow \hat{j}$$

$$\vec{z} \rightarrow \hat{k}$$

$$\hat{i} \cdot \hat{i} = 1 \cos 0^\circ = 1$$

$$\hat{j} \cdot \hat{j} = 1 \cos 0^\circ = 1$$

$$\hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = 1 \cos 90^\circ = 0$$

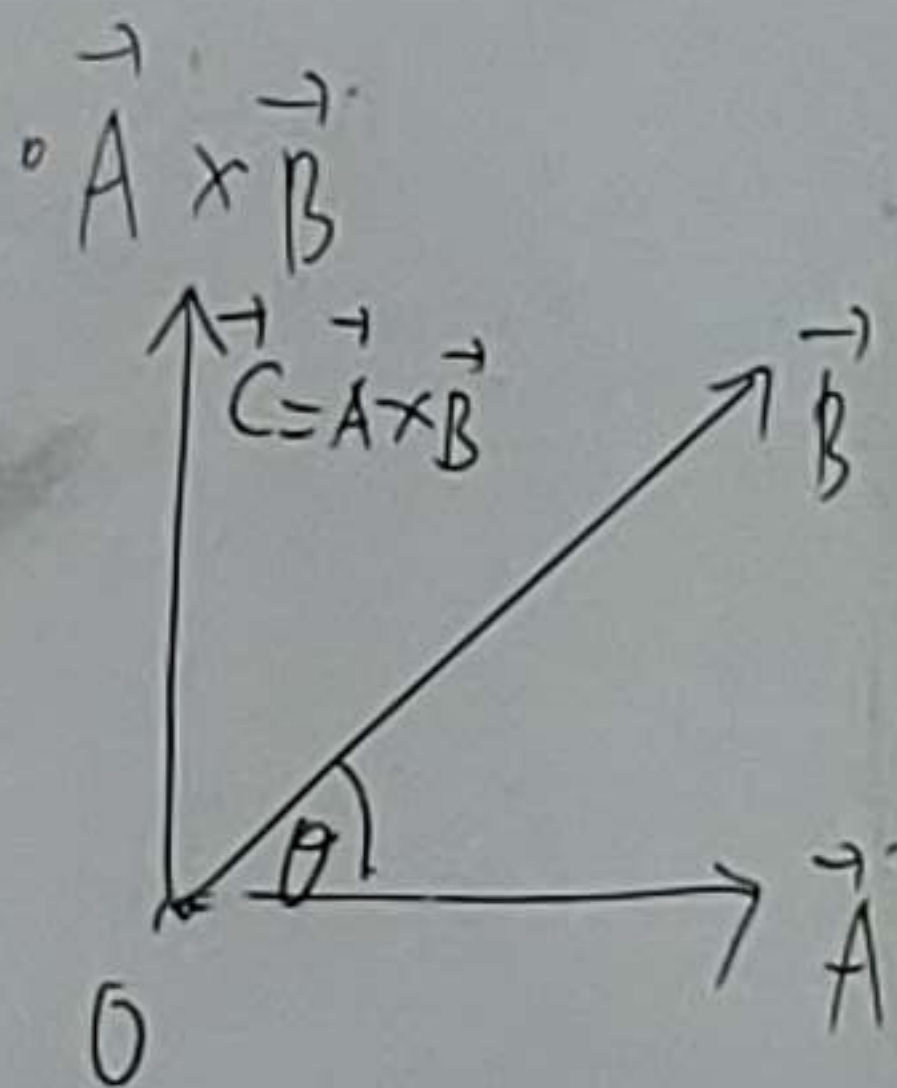
$$\hat{j} \cdot \hat{k} = 0$$

• Perkalian silang \rightarrow Vektor.

$$\begin{matrix} \vec{A} \\ \vec{B} \end{matrix} \left\{ \vec{A} \times \vec{B} \right.$$

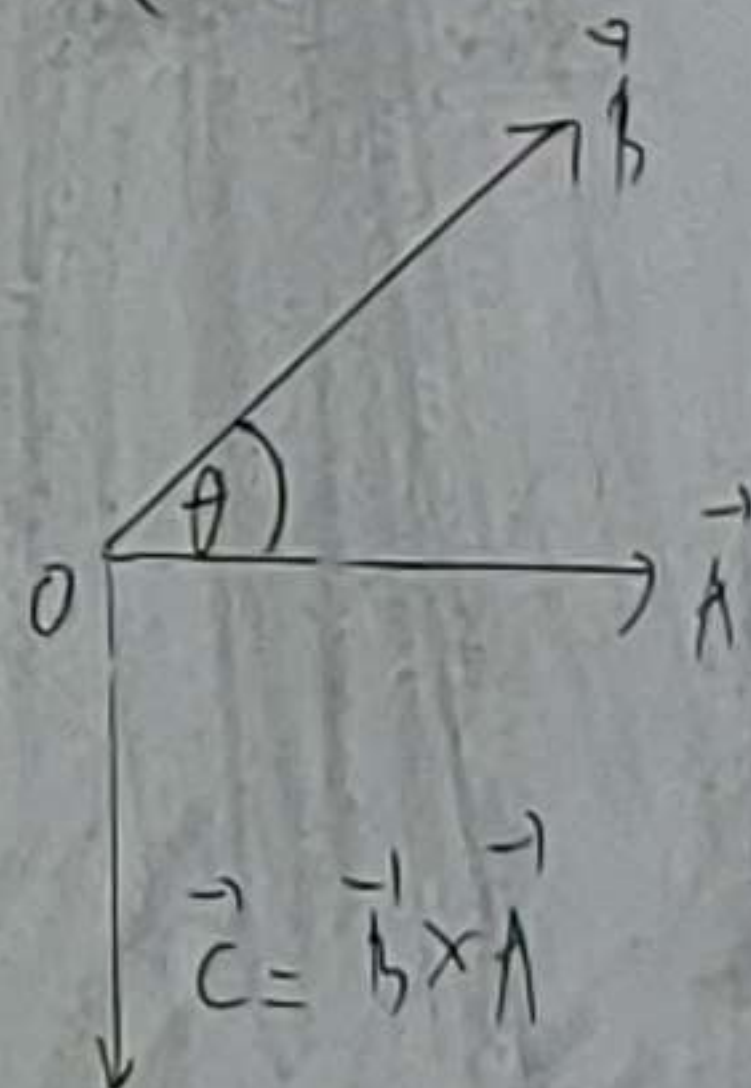
Cross

let gaya Normal $\rightarrow N$
 gaya berat
 gaya gesek
 gaya elastis



$$\vec{A} \times \vec{B} = \vec{A} \vec{B} \sin \theta$$

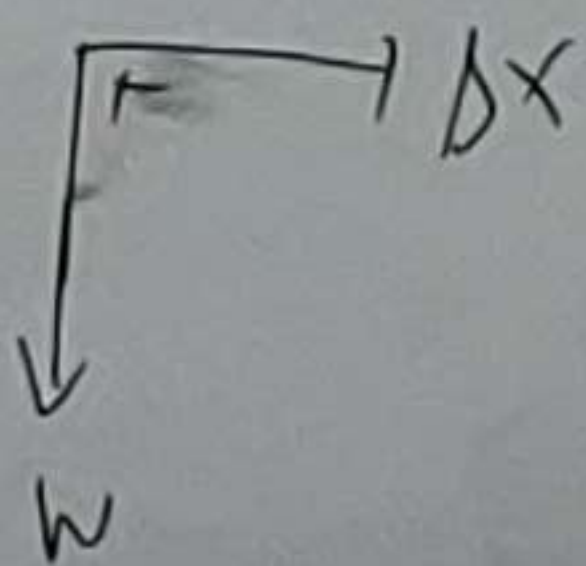
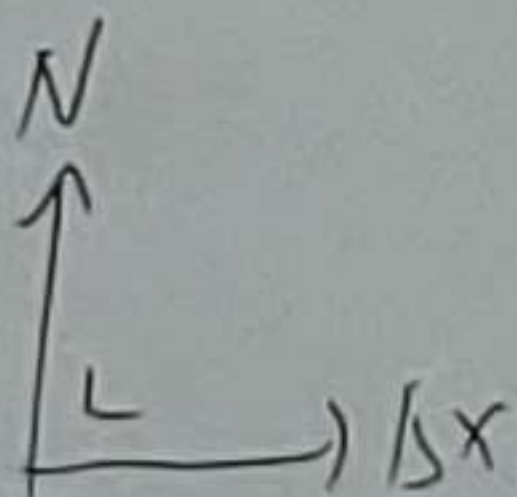
$$\vec{B} \times \vec{A}$$



$$\vec{C} = \vec{B} \times \vec{A}$$

$$\vec{B} \times \vec{A} = \vec{B} \vec{A} \sin \theta$$

$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$



ex: ① $\vec{F} = (3x - 2)$ Newton

lent usich \vec{F} dari $x=0\text{m} \rightarrow x=2\text{m}$!

Jwb: $W = \int_{x=0}^{x=2} \vec{F} \, dx = \int_{x=0}^{x=2} (3x - 2) \, dx$

$$W = \left(\frac{3}{2}x^2 - 2x \right) \Big|_{x=0}^{x=2}$$

$$W = \left(\frac{3}{2} \cdot 4 - 4 \right) - \left(\frac{3}{2} \cdot 0 - 0 \right) = 2 \, \text{J}$$

② $\vec{F} = 3\hat{i} - 2\hat{j} + 4\hat{k}$
 $\vec{x}_1 = \hat{i} + 2\hat{j} \rightarrow \vec{x}_2 = 2\hat{i} - \hat{j} + 2\hat{k}$

Tent. W ?

$$W = \vec{F} \cdot \vec{x}$$

$$W = (3\hat{i} - 2\hat{j} + 4\hat{k}) \cdot (\hat{i} - \hat{j} + 2\hat{k})$$

$$W = 3 + 6 + 8 = 17 \, \text{J} //$$

Persamaan $W = \int \vec{F} \cdot d\vec{x}$ cara

berlaku : \vec{F} konstan.

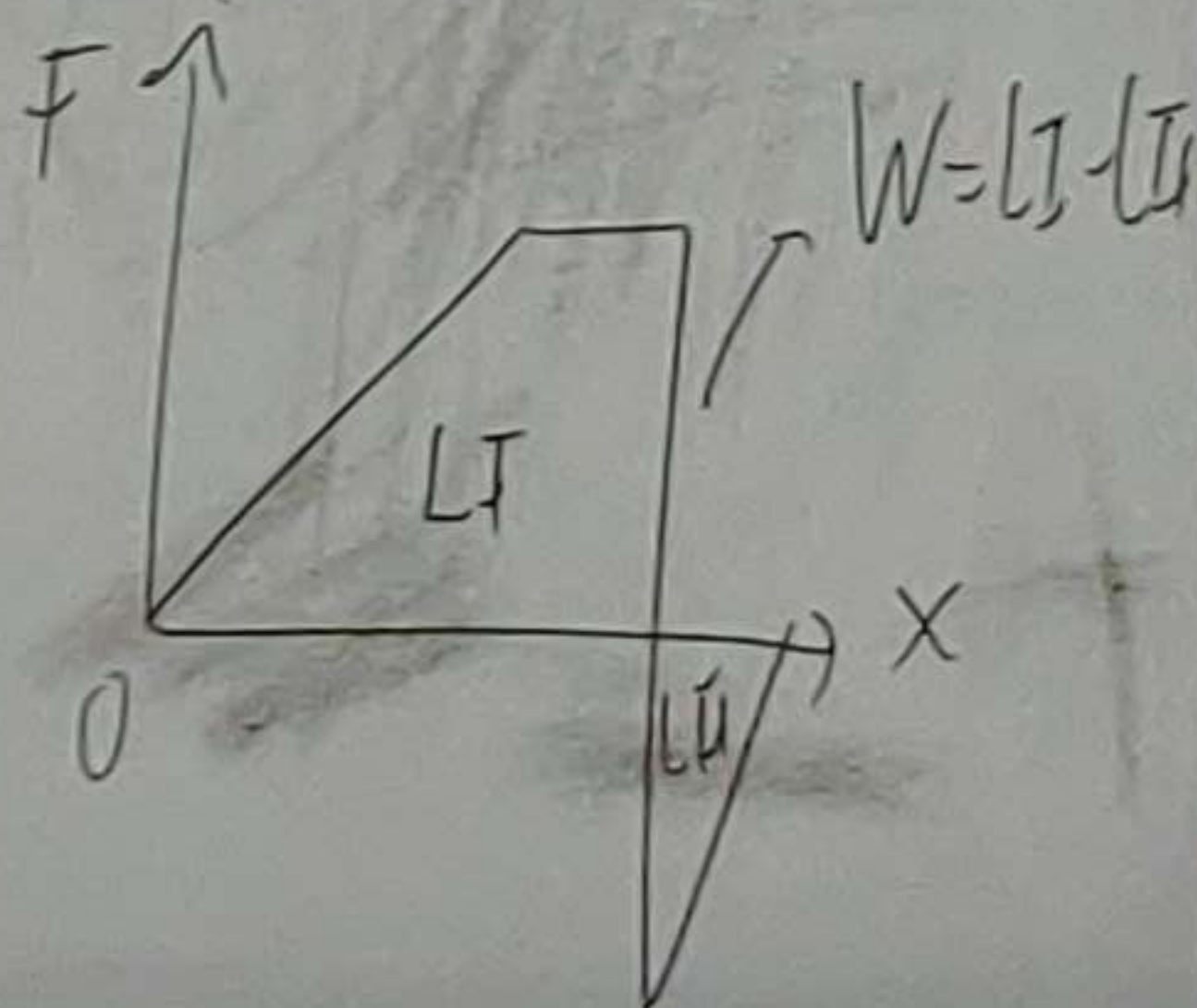
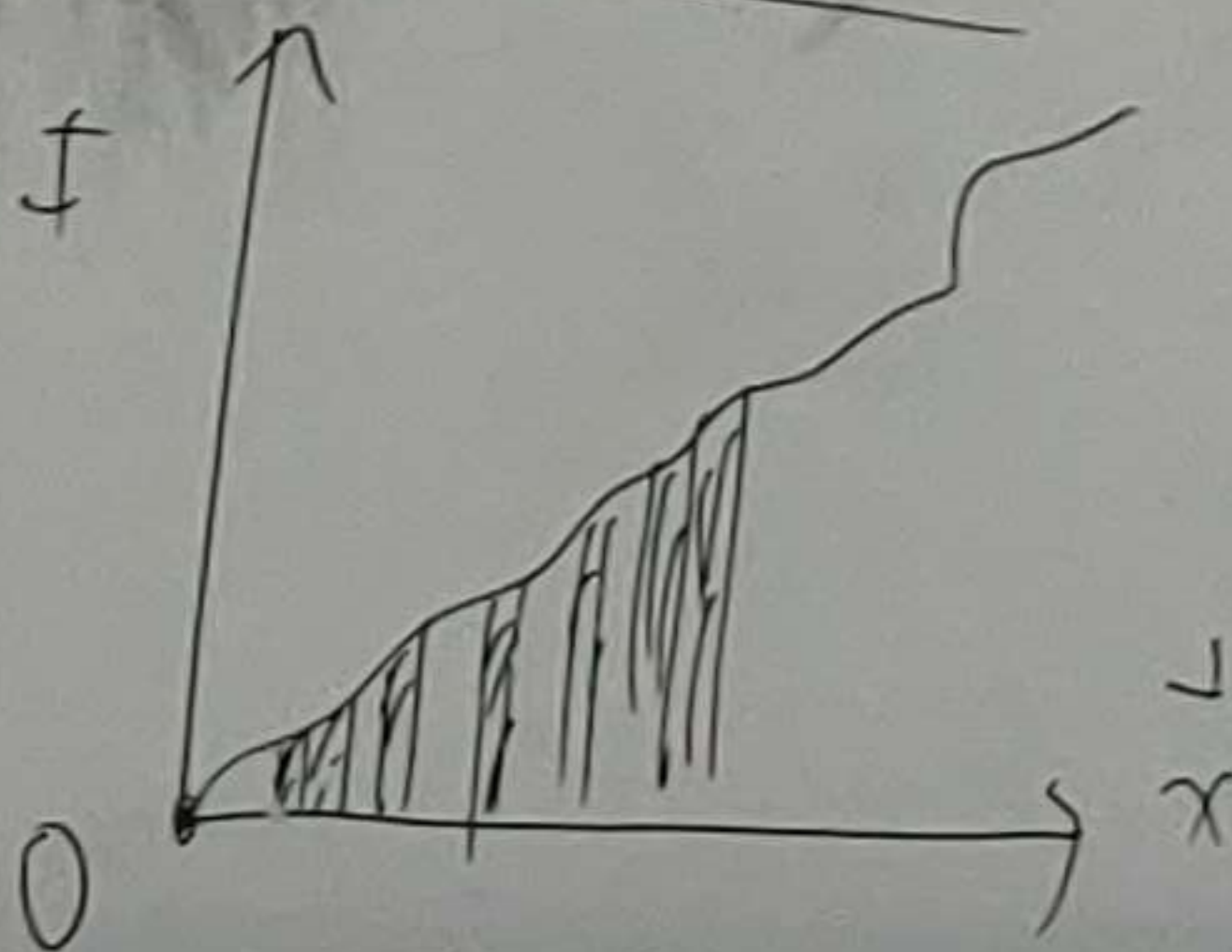
Kekal $\vec{F} \rightarrow$ tidak konstan!

$$W = \vec{F} \cdot \vec{\Delta x}$$

$$dW = F \cdot dx$$

$$\int dW = \int F \cdot dx$$

$$W = \int F \cdot dx \rightarrow \text{luas graph } F \text{ dan } x.$$



Differentiaal / Turinan \rightarrow Kalkulus

$\rightarrow y = ax^n$
 $n \rightarrow$ orde
 $a \rightarrow$ koef
 $x \rightarrow$ Variabele

\rightarrow Turinan funksie y tbd x

$\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{d}{dx}(ax^n) = n \cdot a \cdot x^{n-1}$

$y = 5t^2$

$\frac{dy}{dt} = 10t$

$y = 6$

$\frac{dy}{dt} = 0$

$y = 5x^2z$

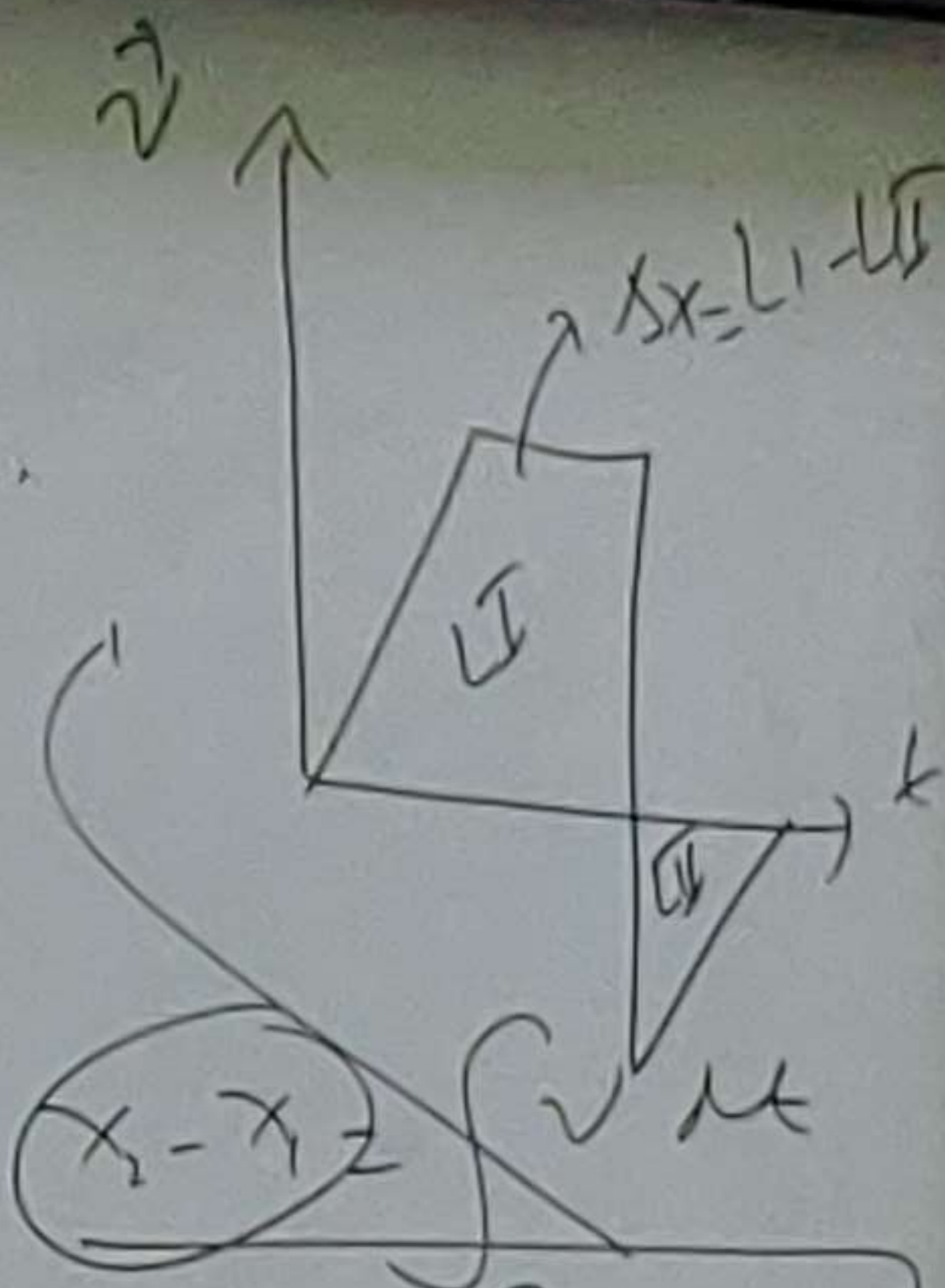
$\frac{dy}{dx} = 10xz$

\rightarrow frakke?

$\vec{v} = \left(\frac{\Delta x}{\Delta t} \right)$

\rightarrow Gemiddelde snelheid

$\vec{v} = \frac{dx}{dt} \rightarrow$ Momentane snelheid



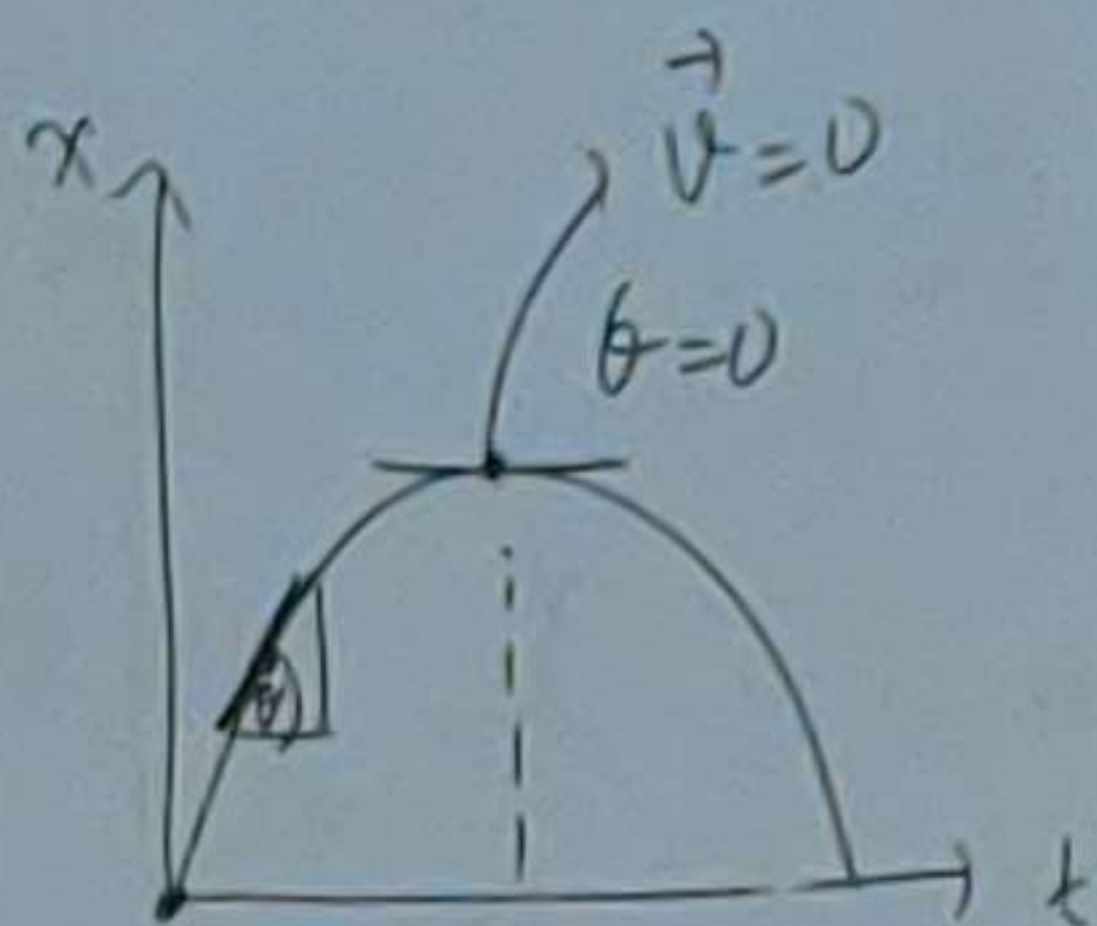
$\Delta x = \int v dt$

$\Delta x = v \cdot \Delta t$

$$= \frac{1}{2} \sqrt{t}$$

$$t$$

$$t$$



$$\rightarrow \vec{a} = \frac{d\vec{v}}{dt} \quad \leftarrow \quad \vec{a} = \frac{d\vec{v}}{dt}$$

$$\rightarrow \vec{f} = \frac{d\vec{p}}{dt} \quad \leftarrow \quad \vec{f} = \frac{d\vec{p}}{dt}$$

• Integral $\rightarrow \int$

$$\rightarrow y = ax^n$$

\rightarrow Integral fungsi y terhadap x

$$\int y \, dx = \int ax^n \, dx$$

$$= \frac{a}{n+1} x^{n+1} + C$$

$$\cdot \int 5x \, dx = \frac{5}{2} x^2$$

$$\cdot \int 2 \, dx = 2x$$

$$\int 2x^0 \, dx = \frac{2}{1} x^{0+1}$$

$$\cdot \vec{v} = \frac{dx}{dt}$$

$$dx = \vec{v} \, dt$$

$$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v \, dt$$

$$x \Big|_{x_1}^{x_2} = \int_{t_1}^{t_2} v \, dt$$

$$x_2 - x_1 = \dots$$

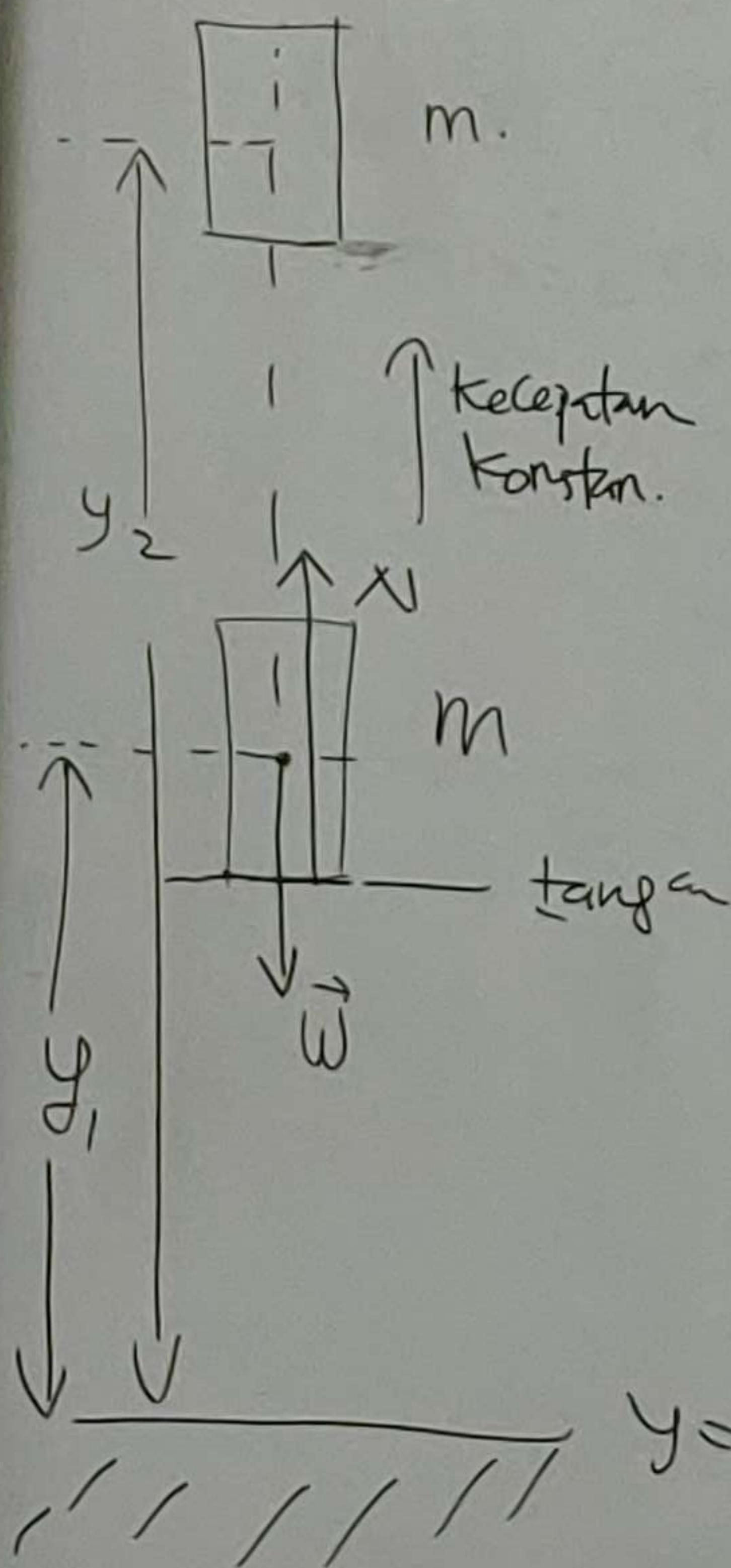
Energi Potensial ^{tersempit}

EP. pegas.

EP. grav

EP. grav.

→ usaha oleh gaya gravitasi/Berat



$$W = \vec{w} \cdot \vec{s}_y \cos \theta$$

$$W = \vec{w} \cdot \vec{s}_y \cos 180^\circ$$

$$W = - (mg) (\vec{y}_2 - \vec{y}_1)$$

$$W = - (mgy_2 - mgy_1)$$

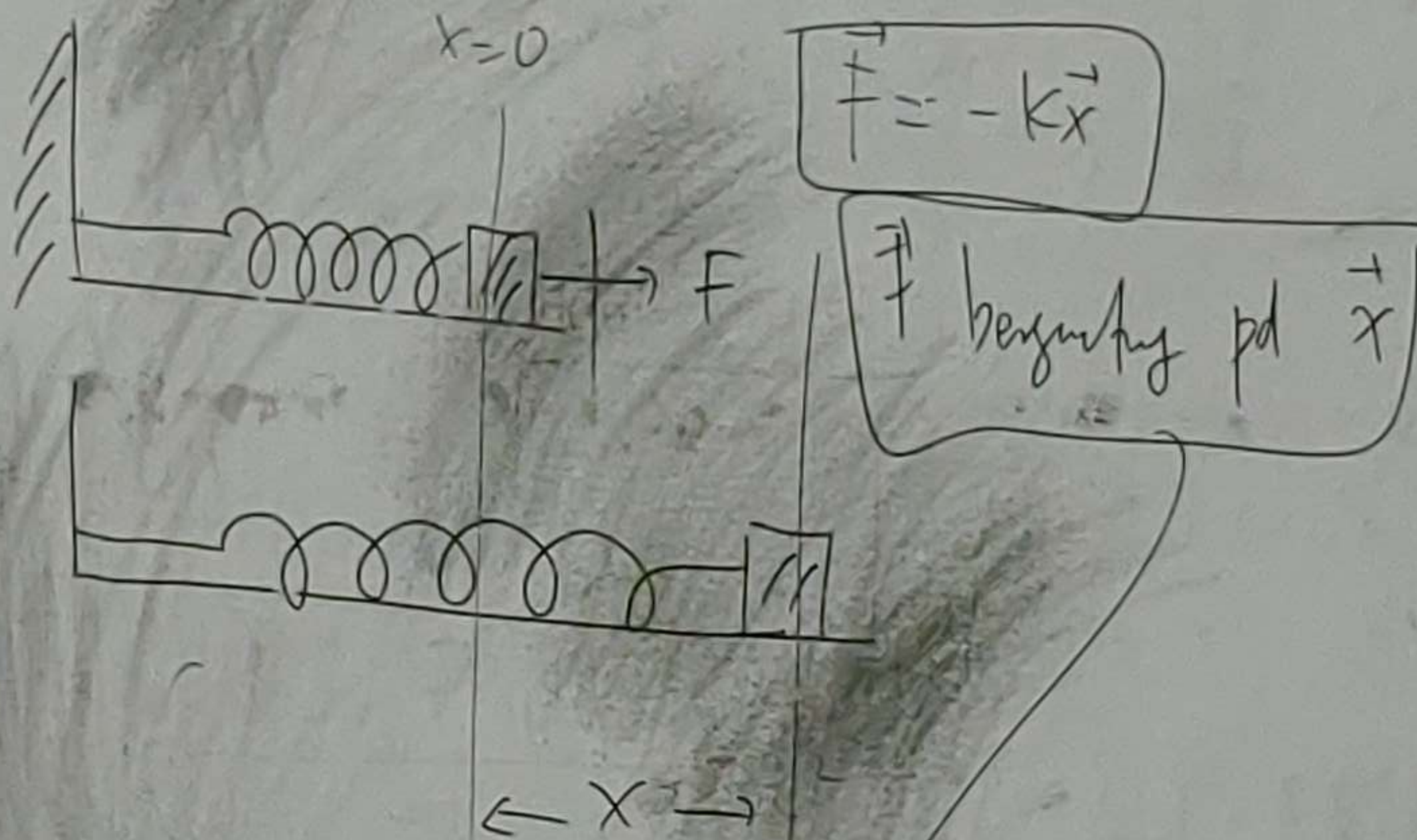
EP_{grav}

$$EP_{grav} = mgy$$

$$W = - (EP_{grav,2} - EP_{grav,1})$$

$$W = - \Delta EP \quad \checkmark$$

E.P. Pegas



Usaha dan Gay Pegas

$$W = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} -kx dx$$

$$W = - \left. \frac{k}{2} x^2 \right|_{x_1}^{x_2} = - \left[\frac{1}{2} k x_2^2 - \frac{1}{2} k x_1^2 \right]$$

Ep

$$E_{p, \text{ pegas}} = \frac{1}{2} k x^2$$

$$W = - (E_{p2} - E_{p1}) = - \Delta E_p$$

Teorema Usaha - Energi \rightarrow

$$W = - \Delta E_p$$

$$W = \Delta E_k$$

$$E_M = E_p + E_k$$

Hukum Kekekalan Energi Mekanik

Syarat : tidak bekerja gaya gesek

$$E_{M \text{ awal}} = E_{M \text{ akhir}}$$

$$\Delta E_k + \Delta E_p = 0$$

gaya gesek
 \rightarrow energi mekanik

$$\Delta E_p + \Delta E_k =$$

$$\Delta E_p + \Delta E_k =$$

Integral \rightarrow

$$\rightarrow y = ax^n$$

\rightarrow Integral fungsi

$$\int y \, dx =$$

$$= \frac{y}{n}$$

* jika gaya sudah bekerja

↳ energi mekanik berubah bentuk → panas
↳ bunyi

$$\Delta K + \Delta U = W_{\text{fry}}$$

$$\Delta K + \Delta U = - \int_{x_1}^{x_2} F_{\text{ges}} dx$$

$$= K + U$$

• Integral → \int

$$\rightarrow y = ax^n$$

→ Integral fungsi y terhadap x

$$\begin{aligned} \int y \, dx &= \int ax^n \, dx \\ &= \frac{a}{n+1} x^{n+1} + C \end{aligned}$$

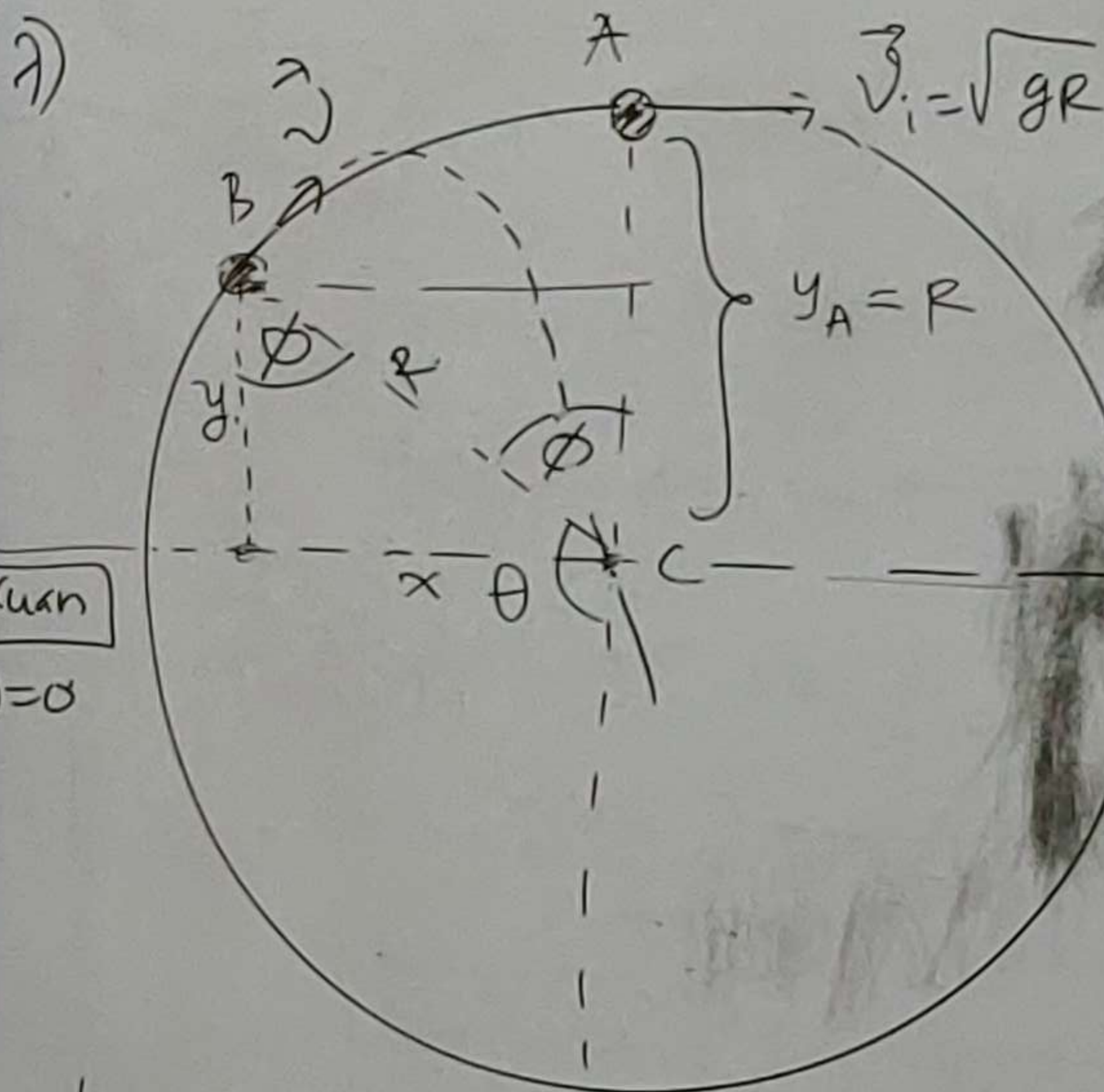
$$v = \frac{dx}{dt}$$

$$dx = v \, dt$$

$$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v \, dt$$

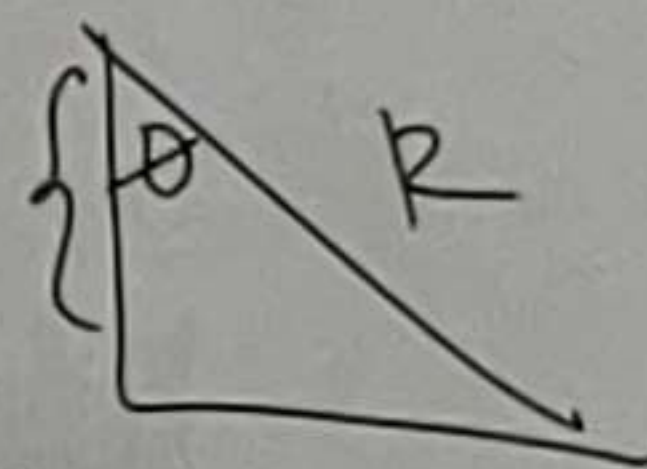
$$x \Big|_{x_1}^{x_2} = \int_{t_1}^{t_2} v \, dt$$

$$x_2 - x_1 = \dots$$



$$\theta + \phi = 180^\circ$$

$$\phi = 180^\circ - \theta.$$



Kordinat bola supra melalui pusat O , $\{x, y\}$

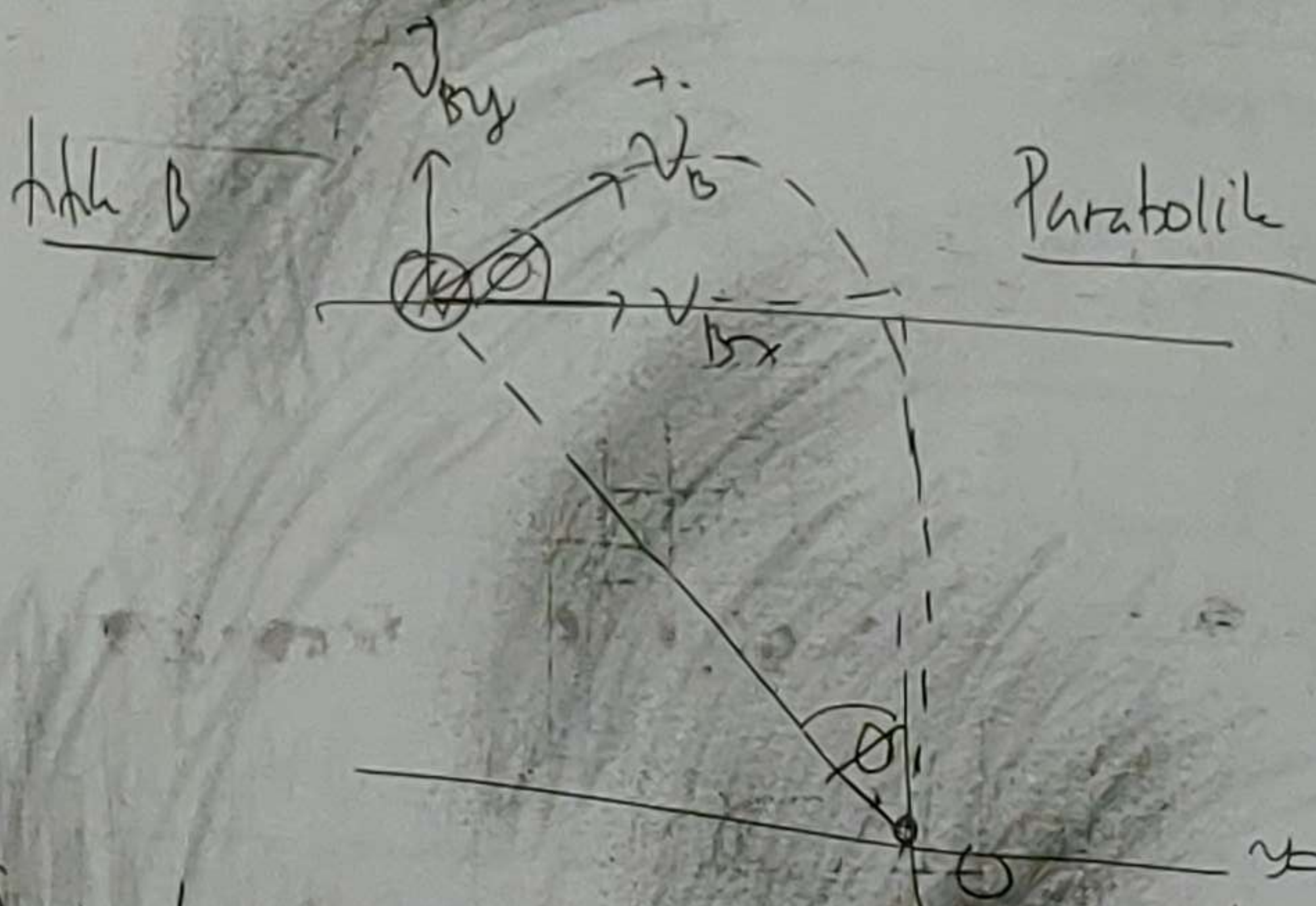
→ Ide 1 : Mencari nilai $\vec{v}_B \rightarrow$ tali putus!

$$EM_A = EM_B$$

$$EP_A + EK_A = EP_B + EK_B.$$

$$mgR + \frac{1}{2}m gR = mgR \cos \phi + \frac{1}{2}m v_B^2$$

$$\sqrt{3gR - 2gR \cos \phi} = v_B$$



Gerak sb $x \rightarrow$ G.l.B.

Gerak sb y .

$$x = v_x \cdot t$$

$$x = v_B \cos \theta \cdot t$$

$$R \sin \theta = v_B \cos \theta \cdot t$$

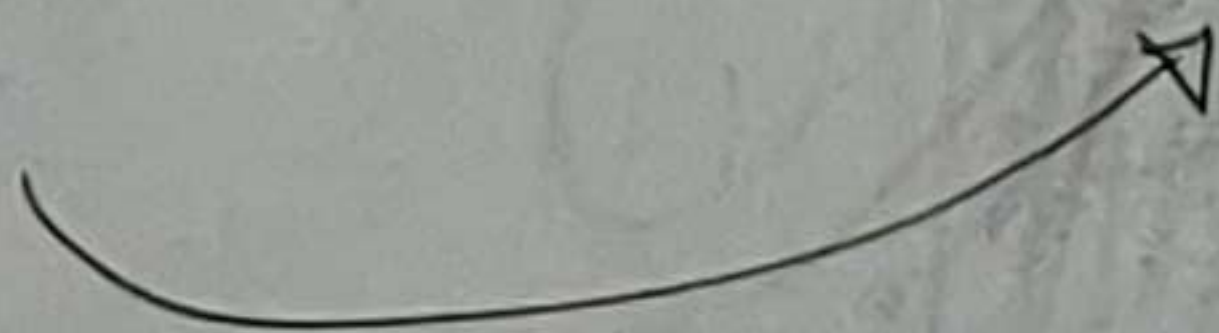
$$t = \frac{R \sin \theta}{v_B \cos \theta} \quad \text{--- (1)}$$

$$y = v_{By} \cdot t - \frac{1}{2} g t^2$$

$$y_2 - y_1 = v_{By} t - \frac{1}{2} g t^2$$

$$0 - R \cos \theta = v_B \sin \theta \cdot t - \frac{1}{2} g t^2$$

$$-R \cos \theta = v_B \sin \theta \cdot t - \frac{1}{2} g t^2 \quad \text{--- (2)}$$



$$5R \sin^2 \varphi + 45R \cos^2 \varphi - 65R \cos \varphi = 0$$

$$\boxed{\sin^2 \varphi = 1 - \cos^2 \varphi}$$

$$5R(1 - \cos^2 \varphi) + 45R \cos^2 \varphi - 65R \cos \varphi = 0$$

$$5R - 5R \cos^2 \varphi + 45R \cos^2 \varphi - 65R \cos \varphi = 0$$

$$35R \cos^2 \varphi - 65R \cos \varphi + 5R = 0$$

$$3 \cos^2 \varphi - 6 \cos \varphi + 1 = 0$$

$$a = 3$$

$$b = -6$$

$$c = 1$$

$$\cos \varphi_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{36 - 4(3)(1)}}{2(3)}$$

$$\cos \varphi_{1,2} = \frac{6 \pm \sqrt{24}}{6}$$

$$\cos \varphi_1 = \frac{6 + \sqrt{24}}{6}; \cos \varphi_2 = \frac{6 - \sqrt{24}}{6}$$

↑ M

$$\boxed{\cos \varphi_2 = 0.18}$$

$$\varphi_2 = \cos^{-1}(0.18)$$

$$\varphi_2 = 79.63^\circ$$

$$\varphi = 180^\circ - 79.63^\circ$$

$$\varphi = 100.37^\circ$$

Peris 2

$$-R \cos \theta = \frac{V_B \sin \theta \cdot R \sin \theta}{V_B \cos \theta} - \frac{1}{2} g \frac{R^2 \sin^2 \theta}{V_B^2 \cos^2 \theta}$$

$$-R \cos \theta = \frac{R \sin^2 \theta}{\cos \theta} - \frac{g R^2 \sin^2 \theta}{2 \cdot (3gR - 2gR \cos \theta) \cos^2 \theta}$$

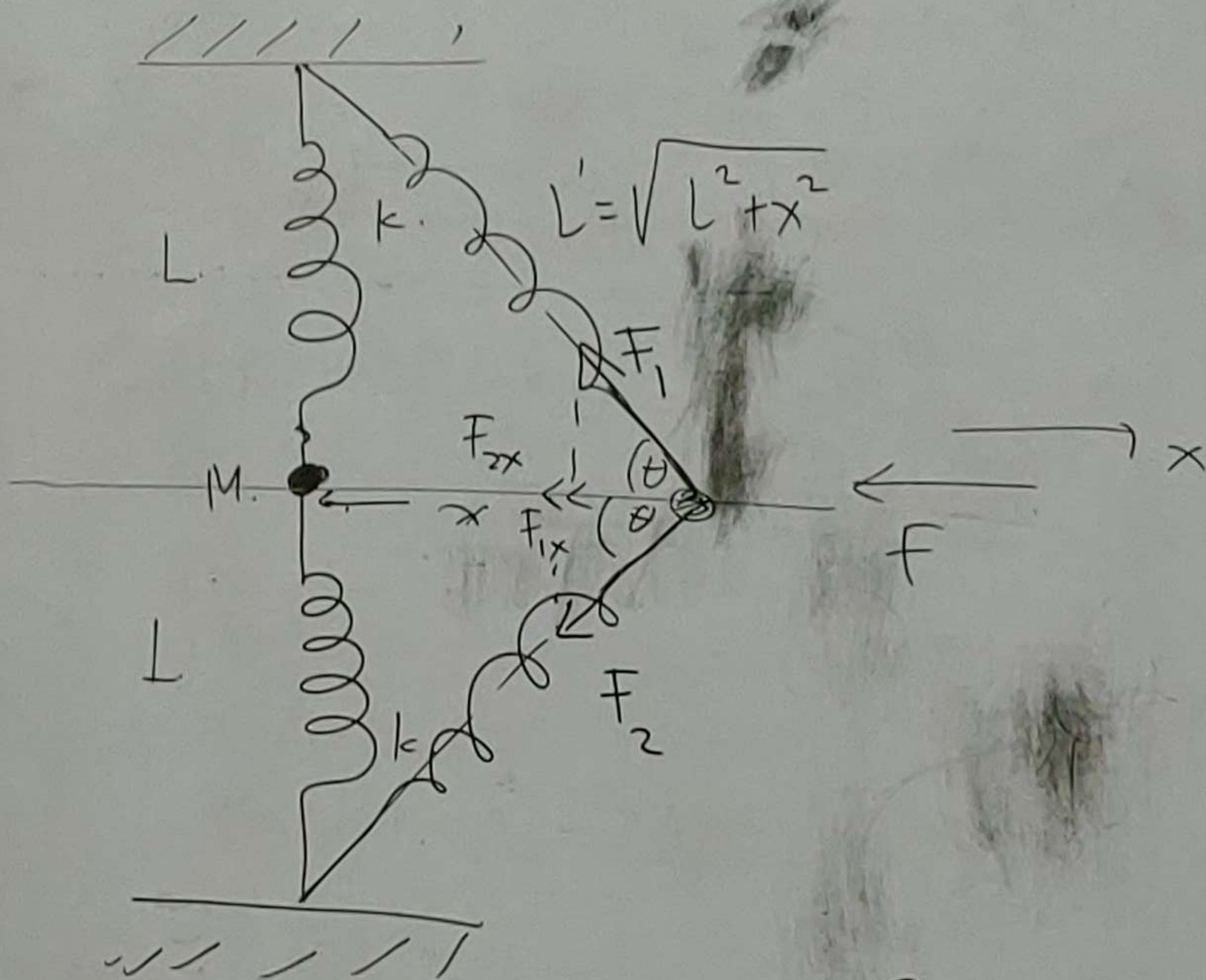
$$-\cos \theta - \frac{\sin^2 \theta}{\cos \theta} = \frac{-R g \sin^2 \theta}{(6gR - 4gR \cos \theta) \cos^2 \theta} \quad | \times \cos \theta$$

$$-\cos^2 \theta - \sin^2 \theta = \frac{-R g \sin^2 \theta \cdot \cancel{\cos \theta}}{(6gR - 4gR \cos \theta) \cos^2 \theta}$$

$$-(\sin^2 \theta + \cos^2 \theta) = \frac{-gR \sin^2 \theta}{(6gR - 4gR \cos \theta) \cos \theta}$$

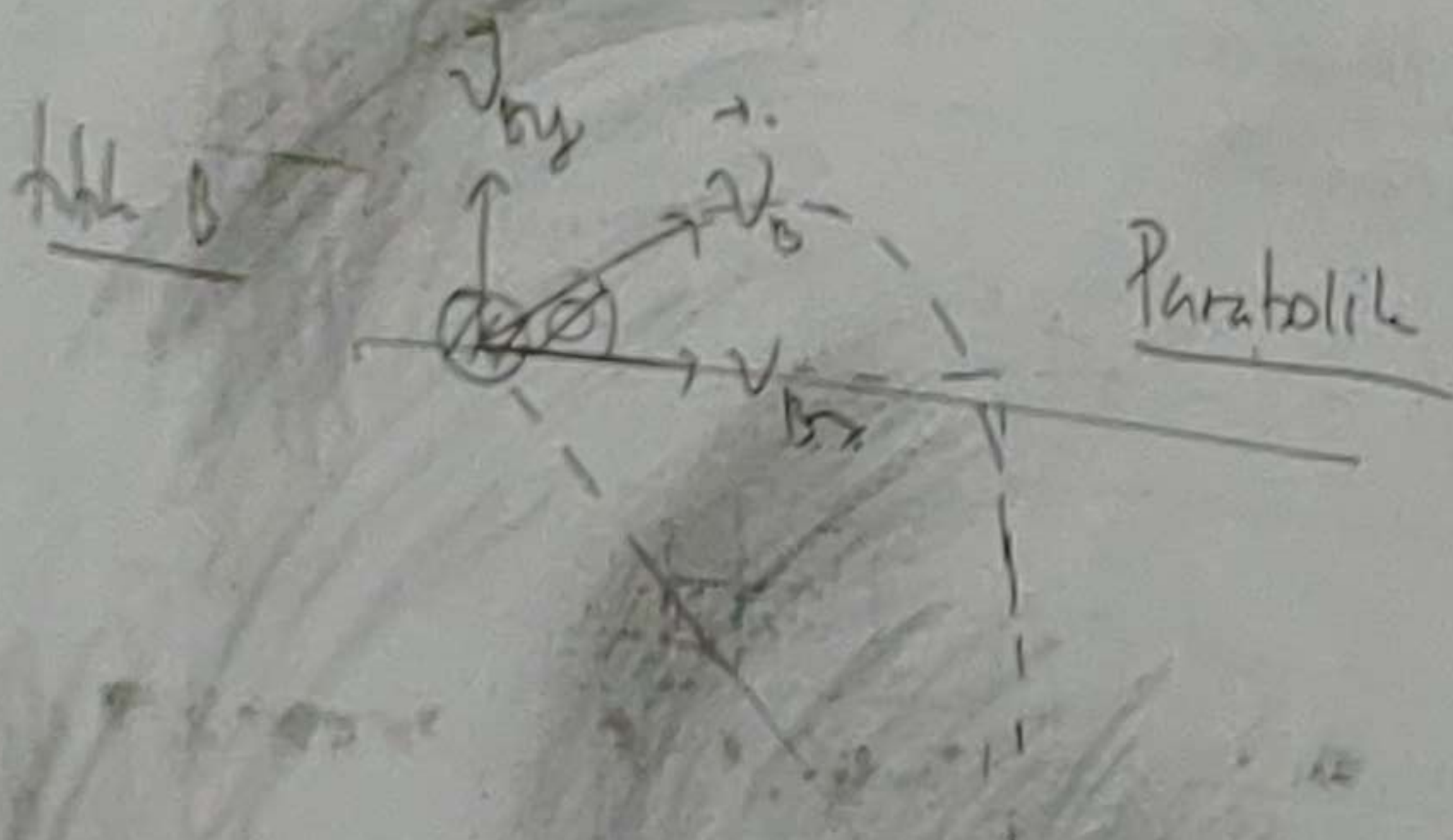
$$gR \sin^2 \theta = (6gR - 4gR \cos \theta) \cos \theta$$

$$gR \sin^2 \theta = 6gR \cos \theta - 4gR \cos^2 \theta \quad \checkmark$$



EP(x) ? $\rightarrow W = \int F_{\text{Total}} \cdot dx$

$-\Delta EP = W = - \int_{x=0}^x (F_{1x} + F_{2x}) \cdot dx$



$$E_p(x) = Kx^2 - 2KL \left(L - \sqrt{x^2 + L^2} \right) \quad J$$

$$\int \frac{x}{\sqrt{x^2 + L^2}} dx$$

$$\text{Mit: } u = x^2 + L^2$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$\int \frac{x dx}{\sqrt{x^2 + L^2}} = \int \frac{du/2}{u^{1/2}}$$

$$= \frac{1}{2} \int u^{-1/2} du$$

$$= \frac{1}{2} \cdot \frac{1}{1/2} u^{1/2}$$

$$\Rightarrow \sqrt{u} = \sqrt{x^2 + L^2} //$$