MODEL IDENTIFICATION AND KALMAN FILTERING FOR QUADROTOR

Nugraha Setya Ardi – 10714522- 952035

Estimation and learning in aerospace

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Tasks

1. IDENTIFY THE MODEL GIVEN DATASET.

2. USE THE IDENTIFIED MODEL TO ESTIMATE VELOCITY USING DT-DT KALMAN FILTERING.

TASK 1: MODEL IDENTIFICATION

Task 1 - Model Identification

- DATA SET:

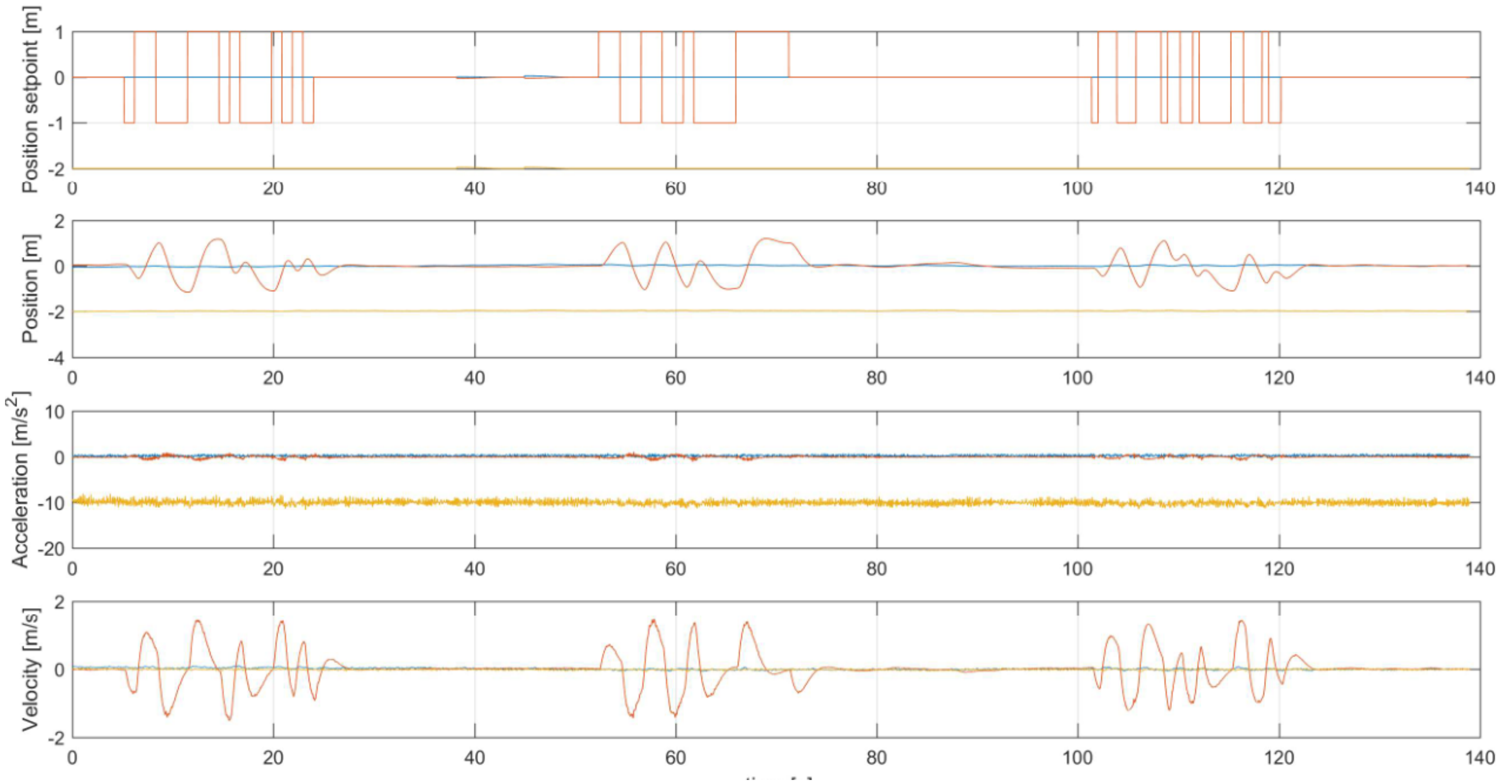
1. Position setpoint in NED frame as input.

2. Position in NED frame as output.

3. Acceleration in body frame as output.

4. Velocity in NED frame.

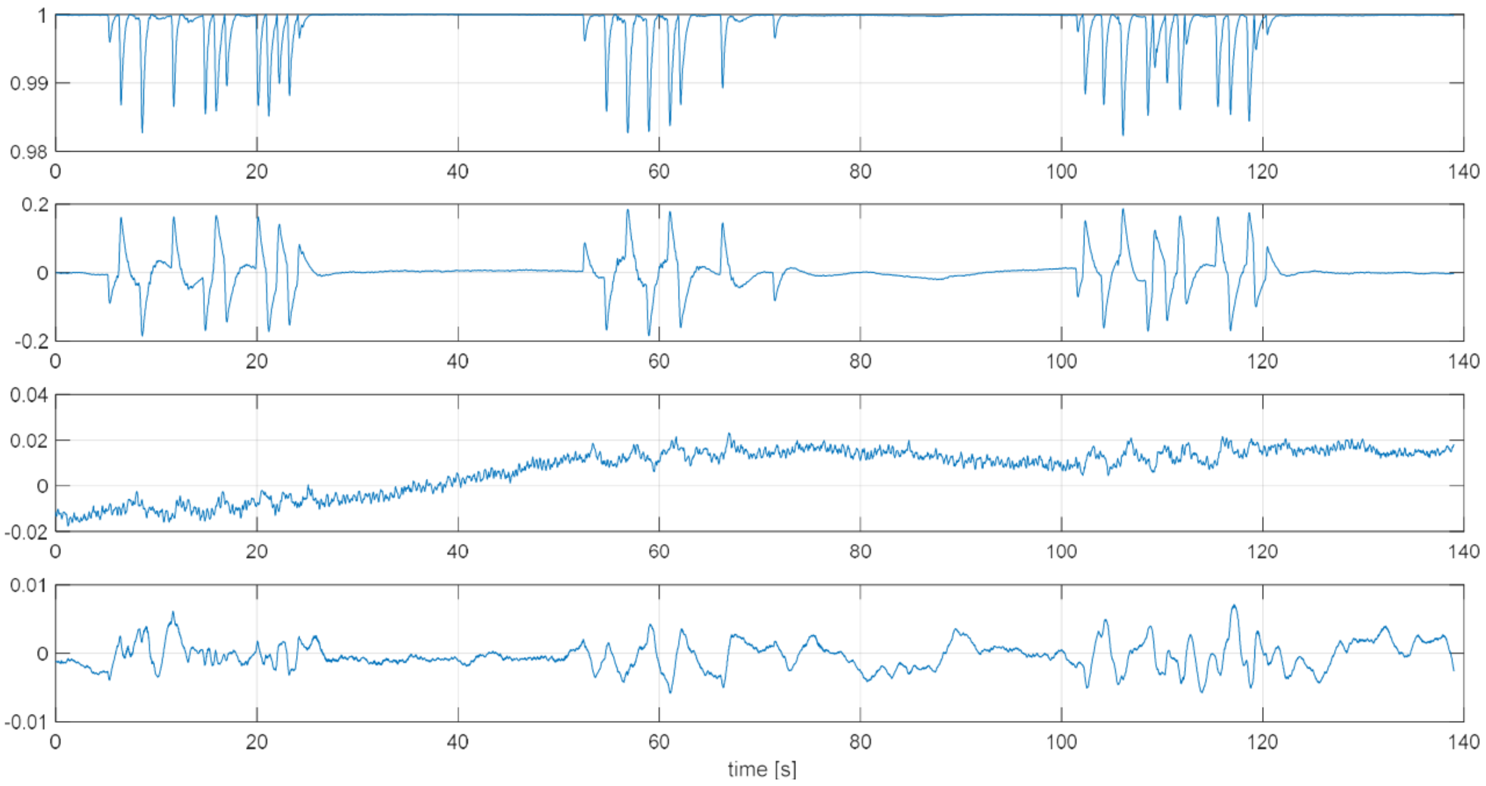
- All sampled at 100 Hz.



Task 1 - Model Identification

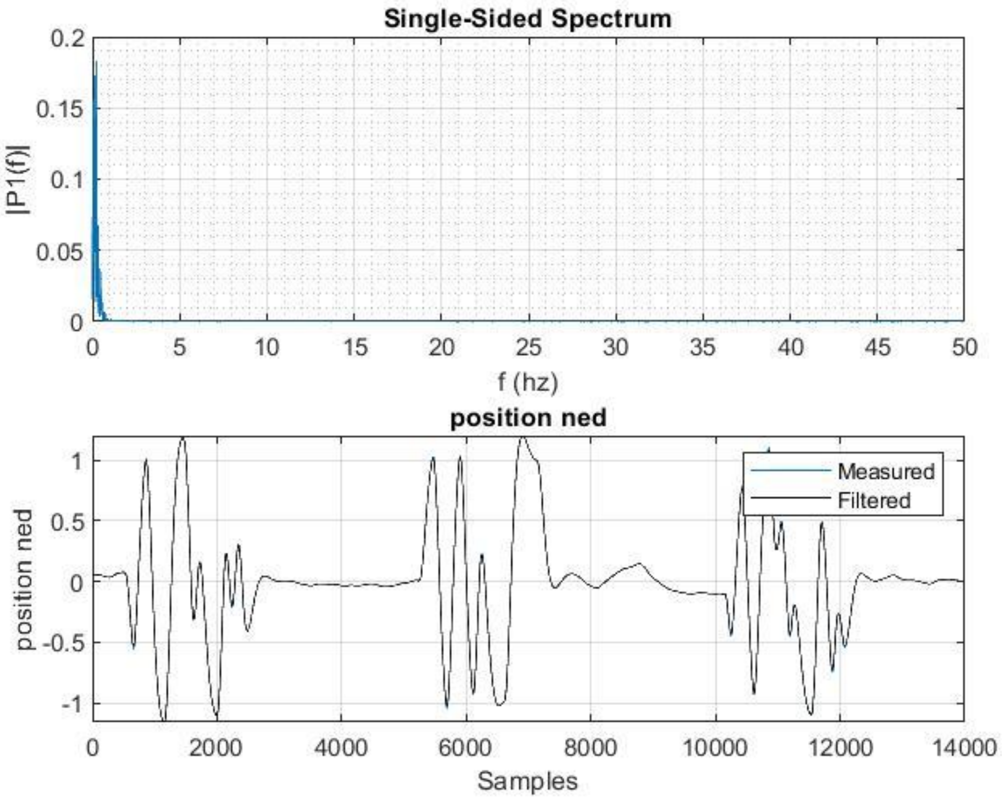
- DATA SET:

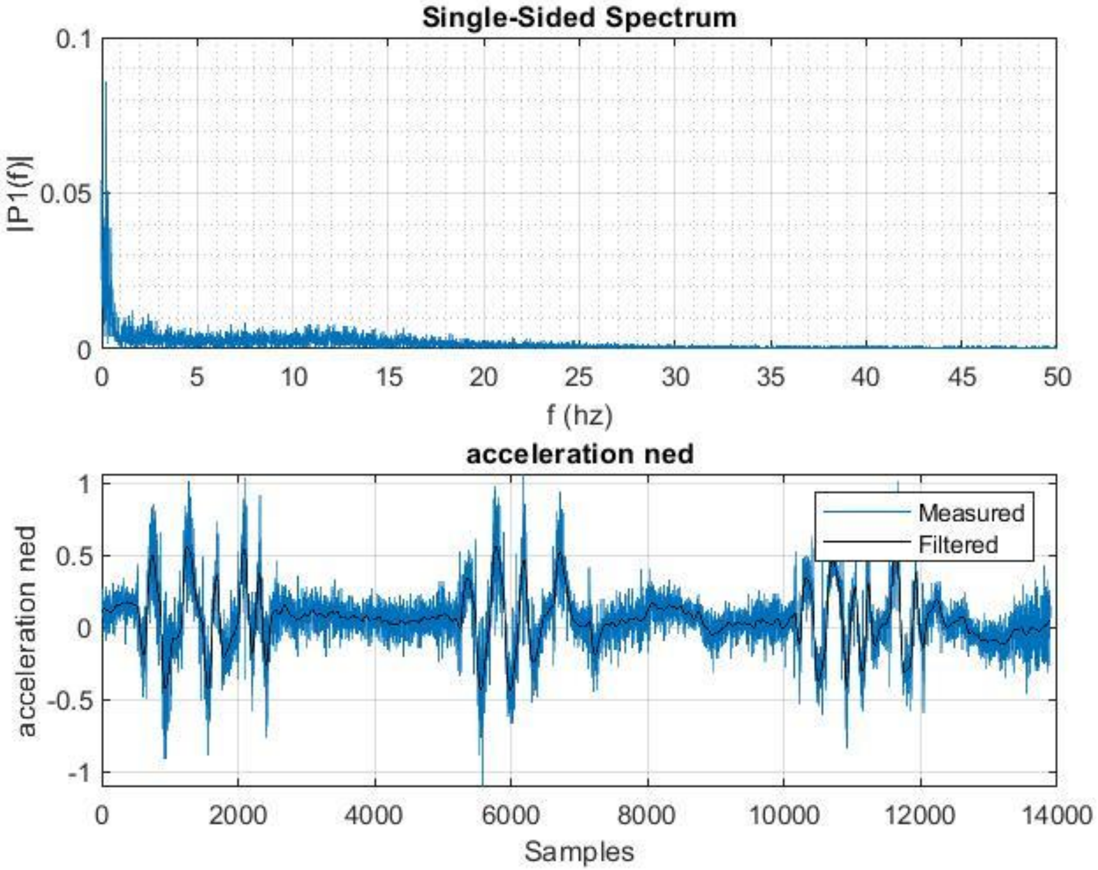
Quaternion body to NED. The first element is the scalar part. Sampled at 100 Hz.



Model Identification Steps

Data preparation	- FFT to determine the cut-off frequency. - Smoothing. - Training and validation dataset.
Model identification using training dataset	PBSID with VARX and VARMAX
Validation using validation dataset and verify the results (VAF)	Quantify the results using VAF.

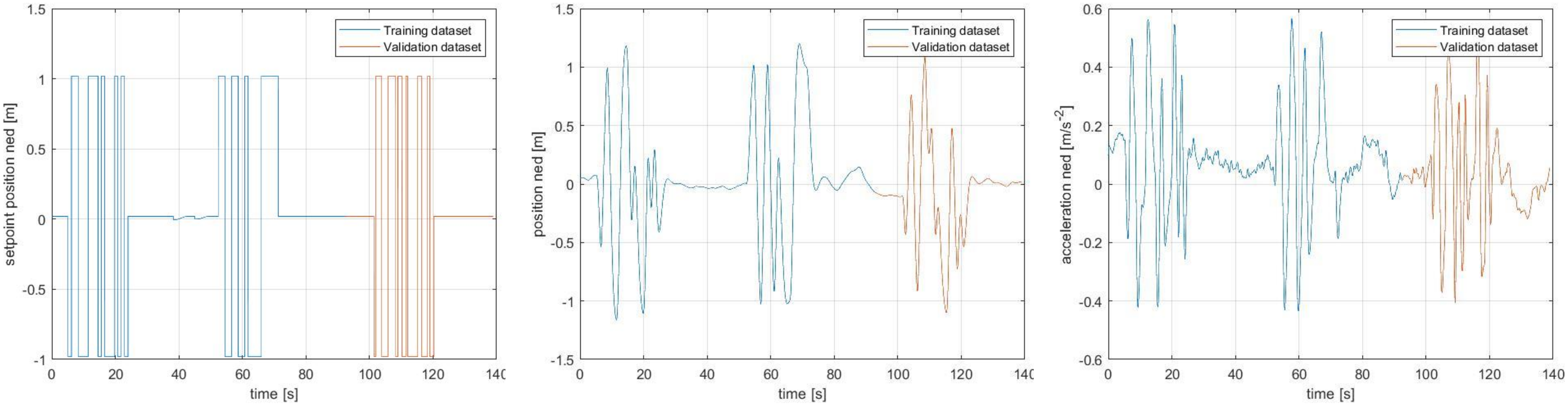




DATA PREPARATION: FFT & SMOOTHING

SMOOTHING: <u>BUTTERWORTH 2ND ORDER</u>

DATA PREPARATION: TRAINING SET AND VALIDATION SET



MODEL IDENTIFICATION: SUBSPACE MODEL IDENTIFICATION (SMI)

WHY SUBSPACE IDENTIFICATION?

- No need of non-linear optimization techniques, only linear algebra (SVD) + Ricatti
- No need to impose onto the system a canonical form
- Computationally efficient and robust
- This method can be applied equally to MIMO and SISO.

SOME CONS OF SUBSPACE IDENTIFICATION

Statistical analysis is difficult

No physical model representation

SMI:

Normal subspace methods are not consistent if there is feedback so we use a specialized method for closed-loop systems

Existence of correlations between the external unmeasurable noise and the control inputs.

Future inputs dependency on **past outputs/noise.**

PBSID

PREDICTOR BASED SUBSPACE IDENTIFICATION METHOD

PBSID

$$\mathcal{S}\begin{cases} x_{k+1} &= Ax_k + Bu_k + Ke_k \\ y_k &= Cx_k + Du_k + e_k, \end{cases}$$

PROBLEM: Given input sequence u_k and output y_k, over time $k = \{0, \dots, N-1\}$ find **A, B, C, D**, and **K**.

ASSUMPTIONS:

- System is observable
- Noise sequence e_k is white
- Input sequence u_k has sufficient excitation
- Feedback loop does not have direct feedthrough

No other assumptions on **correlation between the input and noise sequence** --> Possibility to apply the algorithm in **CLOSED LOOP**

PBSID

- Rewrite the state-space in Kalman predictor form

$$\mathcal{S}\begin{cases} x_{k+1} &= Ax_k + Bu_k + Ke_k \\ y_k &= Cx_k + Du_k + e_k, \end{cases} \longrightarrow \begin{cases} x_{k+1} &= \tilde{A}x_k + \tilde{B}u_k + Ky_k, \\ y_k &= Cx_k + Du_k + e_k, \end{cases}$$

$$\tilde{A} = A - KC \quad \tilde{B} = B - KD$$

- Introduce the extended controllability and observability matrix

$$\mathcal{K}^p = [\bar{A}^{p-1}\widetilde{B}_0 \ \dots \ \widetilde{B}] \qquad \Gamma^p = \begin{bmatrix} C \\ C\bar{A} \\ \vdots \\ C\bar{A}^{p-1} \end{bmatrix}$$

- Past window and the future window vectors

$$\bar{y}_{k-p} = \begin{bmatrix} y_{k-p} \\ y_{k-p+1} \\ \vdots \\ y_{k-1} \end{bmatrix}, \quad \bar{y}_k = \begin{bmatrix} y_k \\ y_{k+1} \\ \vdots \\ y_{k+f-1} \end{bmatrix}$$

$f \leq p$

----> Same for *u* and *e*

PBSID

- Defining $z(k) = \left[u^T(k)\ y^T(k)\right]$

Same for *X* and E

$$\bar{Z} = \left[\bar{z}_0, \quad \cdots, \quad \bar{z}_{N-p}\right]$$

$$Y = \left[y_p, \quad \cdots, \quad y_{N-1}\right]$$

- Rewrite the problem in matrix notation

$$X^{p,f} \simeq \mathcal{K}^p \bar{Z}^{p,f}$$

$$Y^{p,f} \simeq C\mathcal{K}^p \bar{Z}^{p,f} + DU^{p,f} + E^{p,f}$$

$FOR\ f = p$

$$\min_{C\mathcal{K}^p, D} \|Y^{p,p} - C\mathcal{K}^p \bar{Z}^{p,p} - DU^{p,p}\|_F.$$

- To estimate the state sequence $X^{p,p}$ and retrieve the order of the system use SVD of the projection $\Gamma^p \mathcal{K}^p \bar{Z}^{p,p} = U\Sigma V^T$

- Then estimate matrix C from least squares problem: $\min_C \|Y^{p,p} - \widehat{D}U^{p,p} - C\widehat{X}^{p,p}\|_F$

PBSID

- Estimation of the innovation data matrix

$$E_N^{p,f} = Y^{p,p} - \widehat{C}\widehat{X}^{p,p} - \widehat{D}U^{p,p}$$

- A,B and K can be obtained by solving least squares problem

$$\min_{A,B,K} \|\widehat{X}^{p+1,p} - A\widehat{X}^{p,p-1} - BU^{p,p-1} - KE^{p,p-1}\|_F.$$

PBSID – VARX and VARMAX model set

VARX

$$\begin{cases} x_{k+1} &= \tilde{A}x_k + \tilde{B}u_k + Ky_k \\ y_k &= Cx_k + Du_k + e_k, \end{cases}$$

$$\epsilon_{k|k-1} = y_k - \hat{y}_{k|k-1}$$

- The one-step ahead predictor is linear in the Markov parameters (Computational better)
- One-step ahead prediction error has truncation error and noise error.
- If past window is small, the truncation error leads to biased estimation of state sequence
- Optimal solution for noise is when p->∞ because $\bar{G}_p \rightarrow \bar{G}$ and $\bar{H}_p \rightarrow \bar{H}$ and truncation becomes small

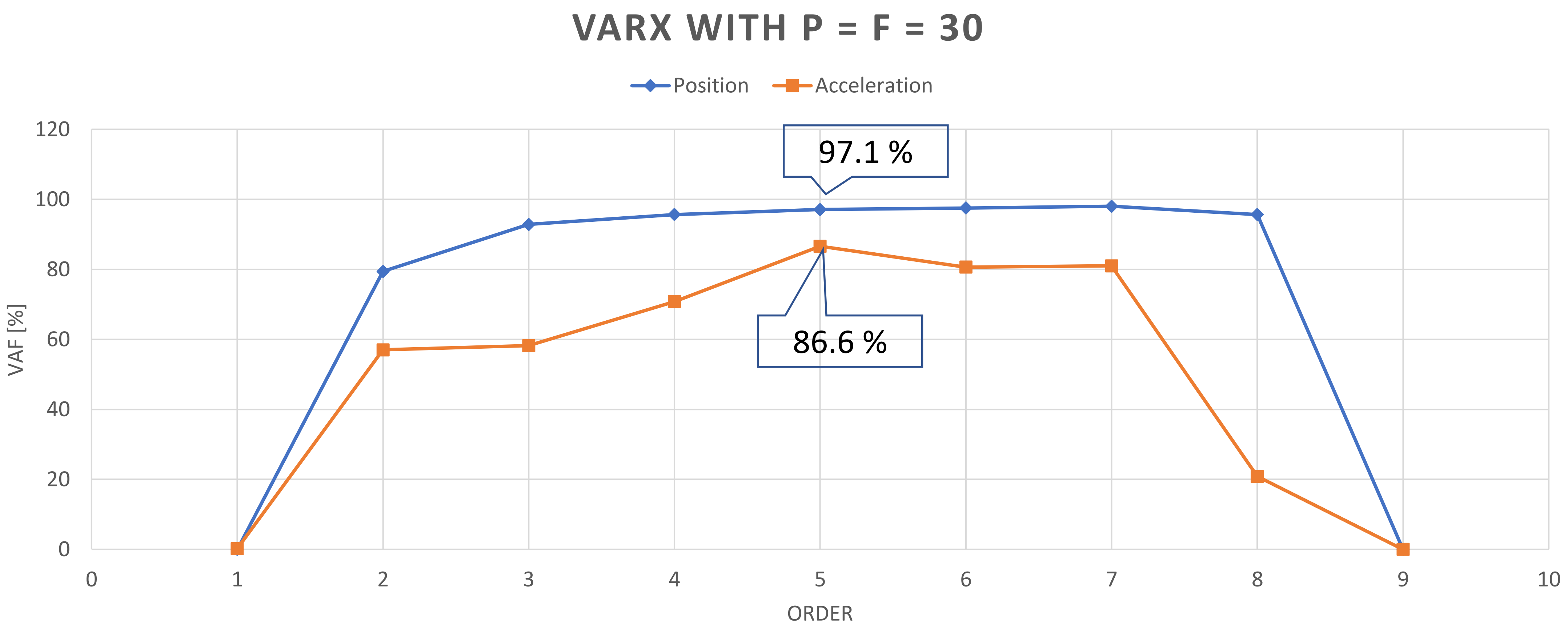
VARMAX

$$\begin{cases} x_{k+1} &= \bar{A}x_k + \bar{B}u_k + My_k + \bar{K}e_k \\ y_k &= Cx_k + Du_k + e_k, \end{cases}$$

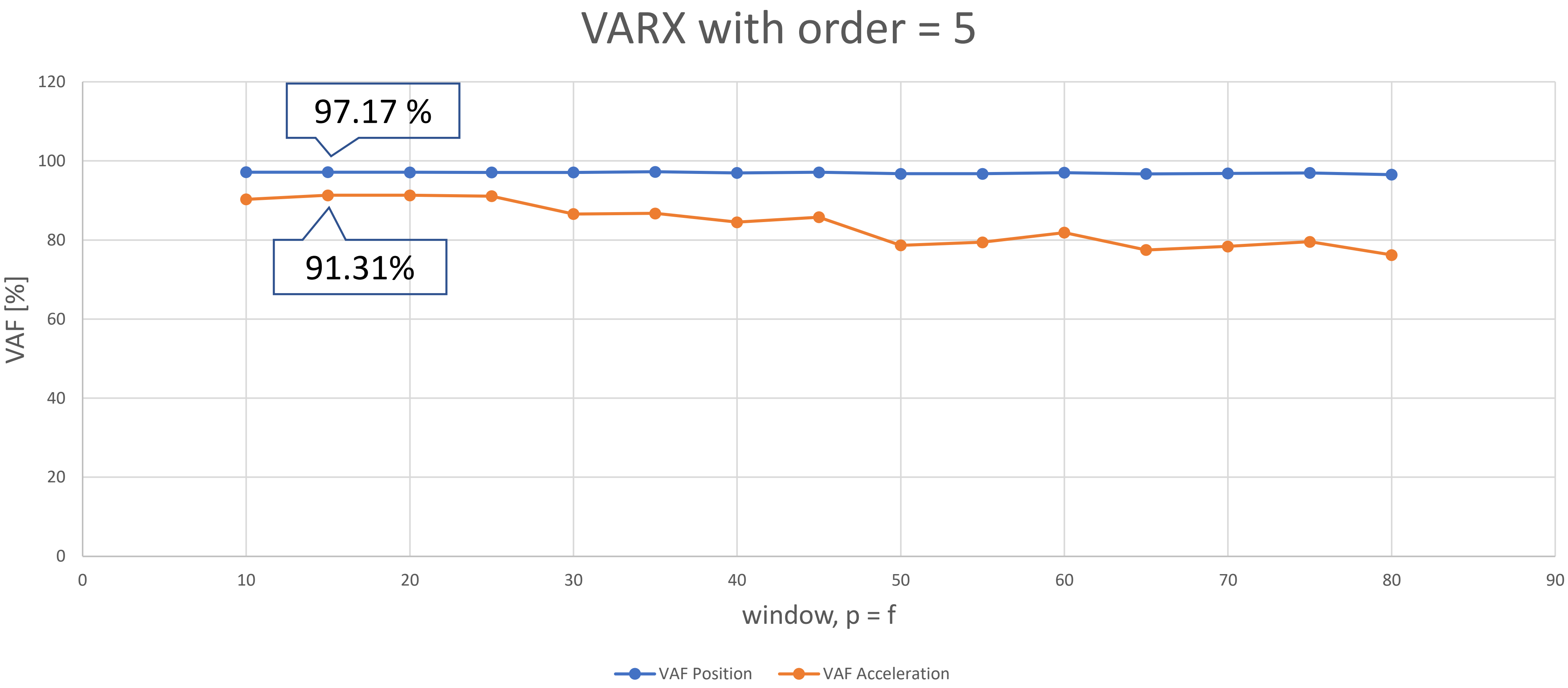
$$y_k = G(z)\,u_k + H(z)\,e_k.$$

- Introduces another observer matrix to create additional freedom for the optimizer.
- The one-step ahead predictor is no longer linear but extended least squares still gives efficient solution.
- For finite case error now only contains noise term, therefore for $p > n$ $\bar{G}_p = \bar{G}$ and $\bar{H}_p = \bar{H}$ no approximation is needed without truncation error.
- Lower past window for asymptotical consistent estimates. Beneficial when p is restricted.

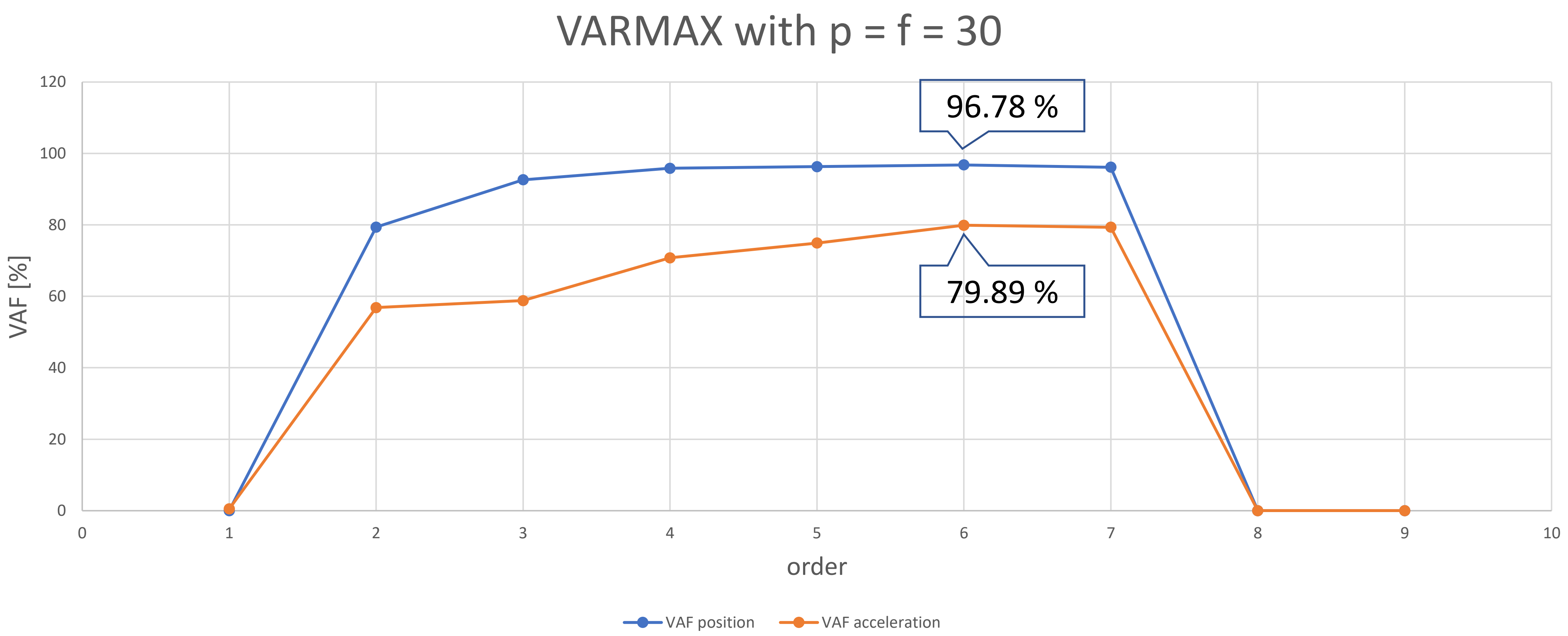
RESULTS: VARX



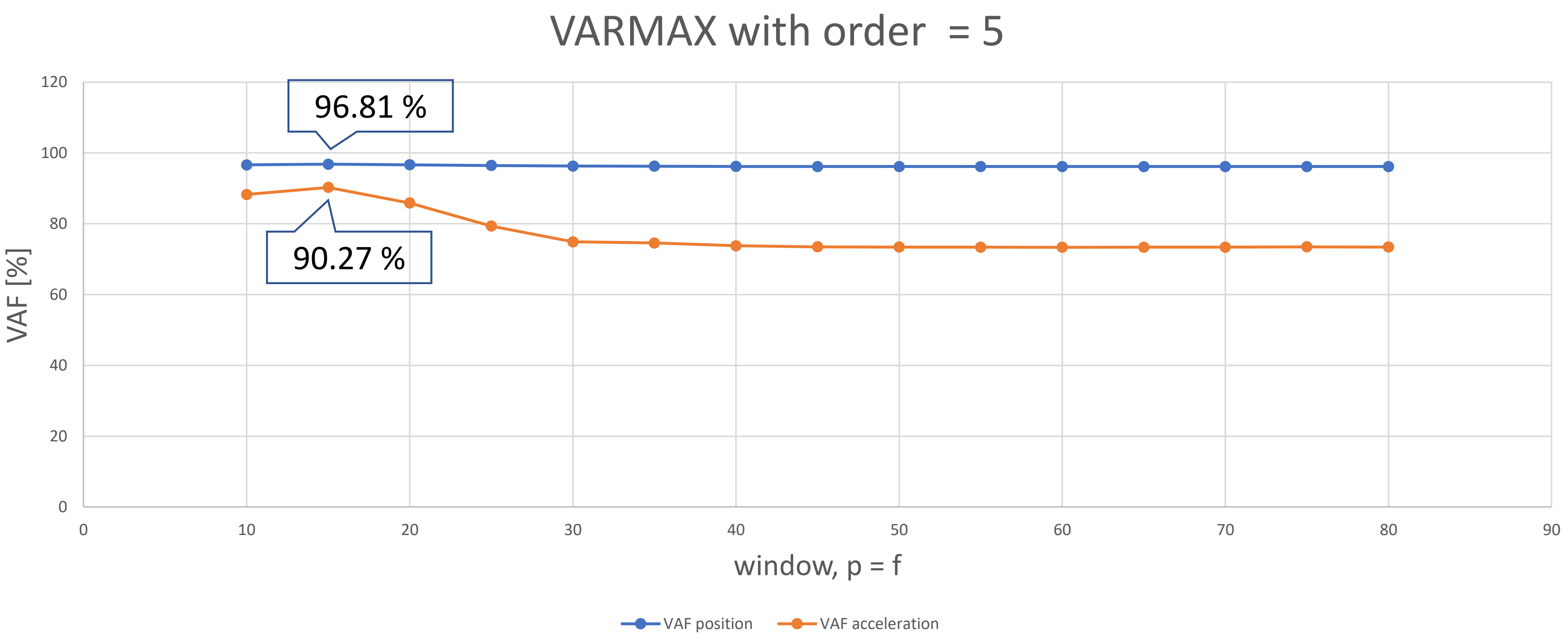
RESULTS: VARX



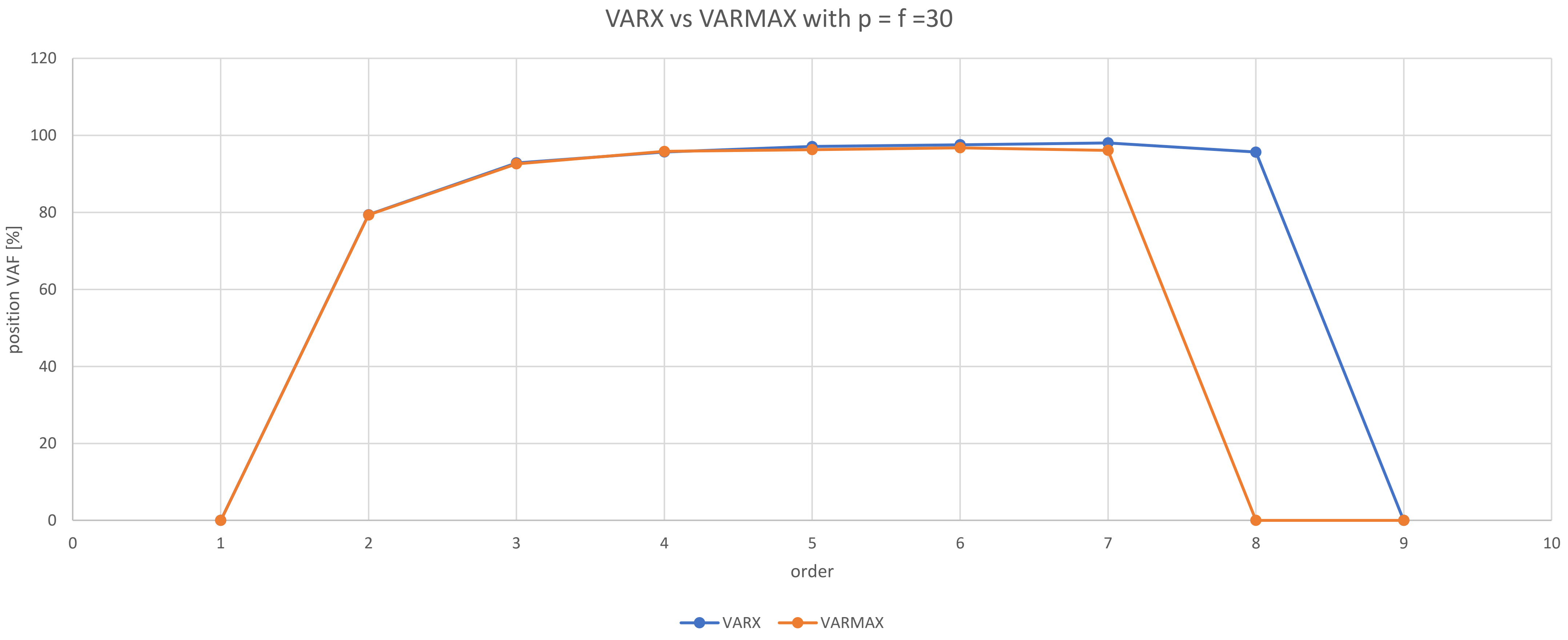
RESULTS: VARMAX



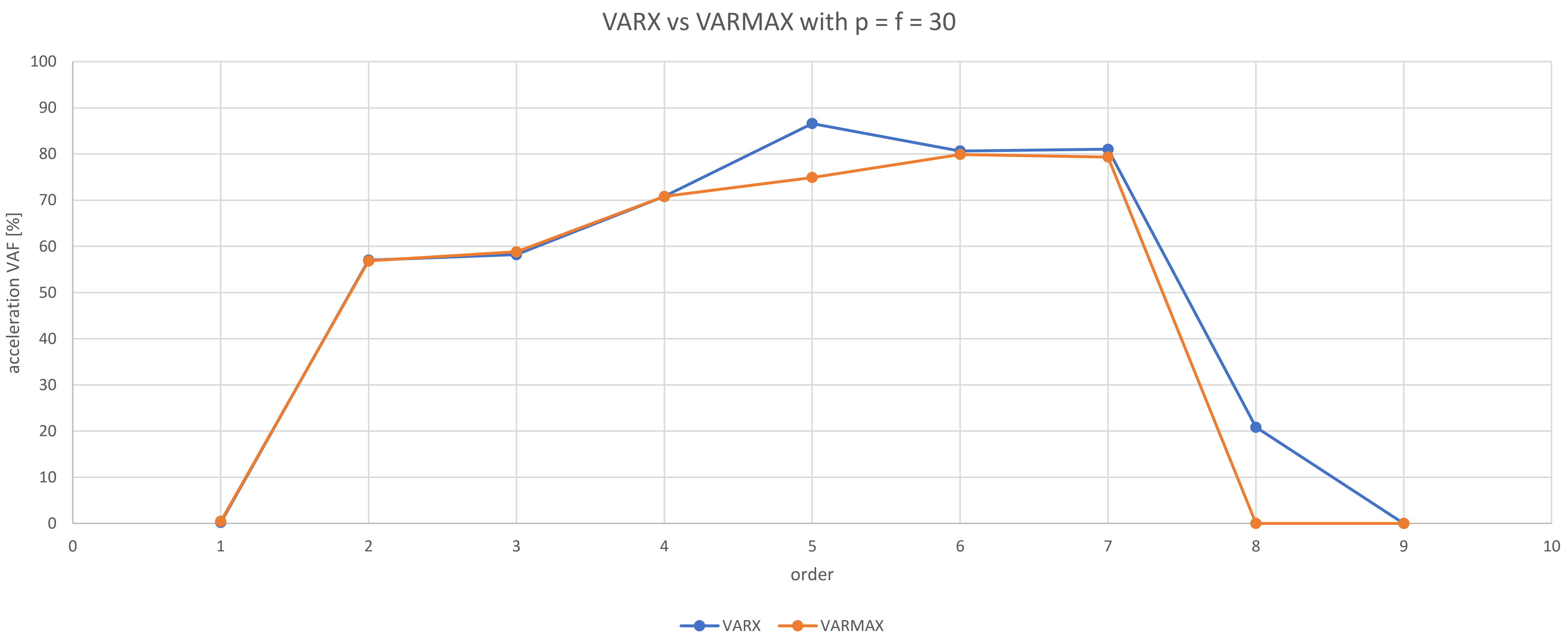
RESULTS: VARMAX



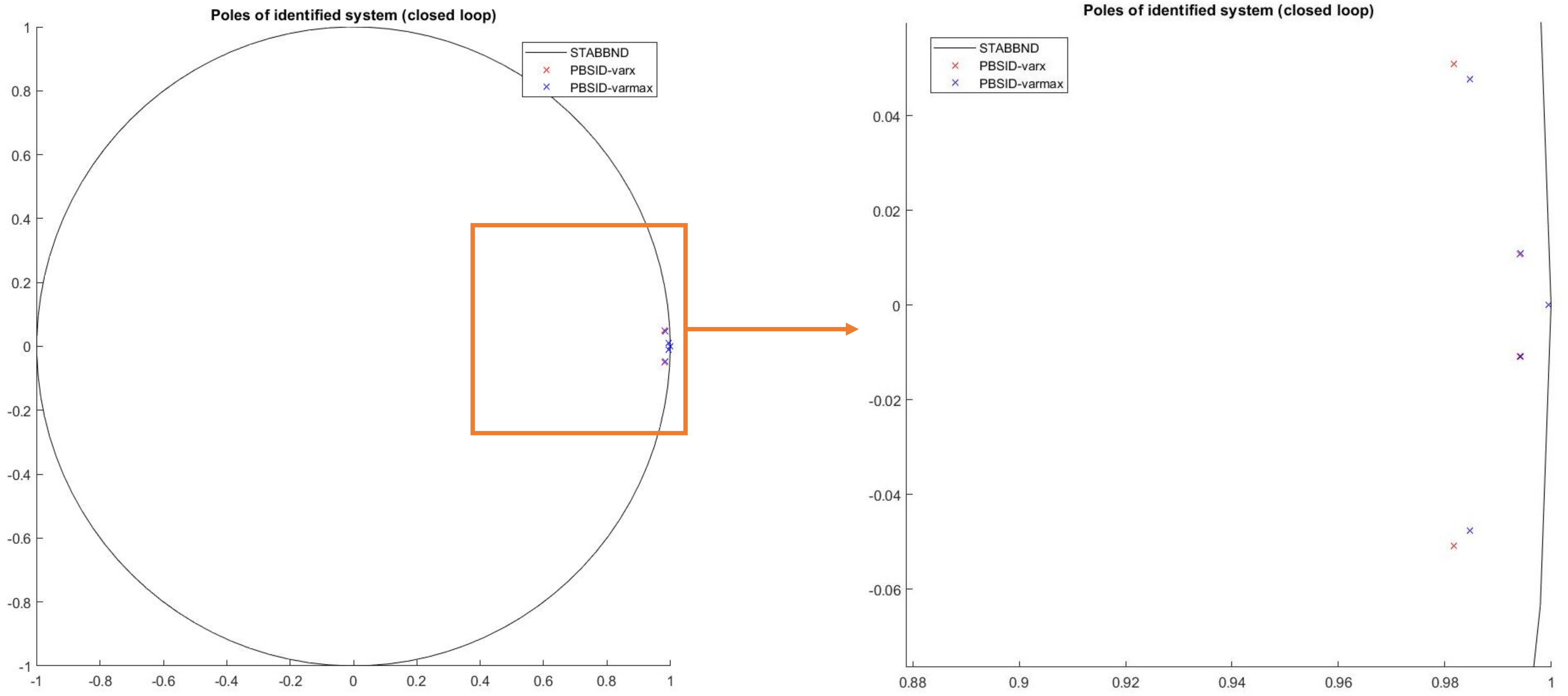
RESULT: POSITION, VARX vs VARMAX



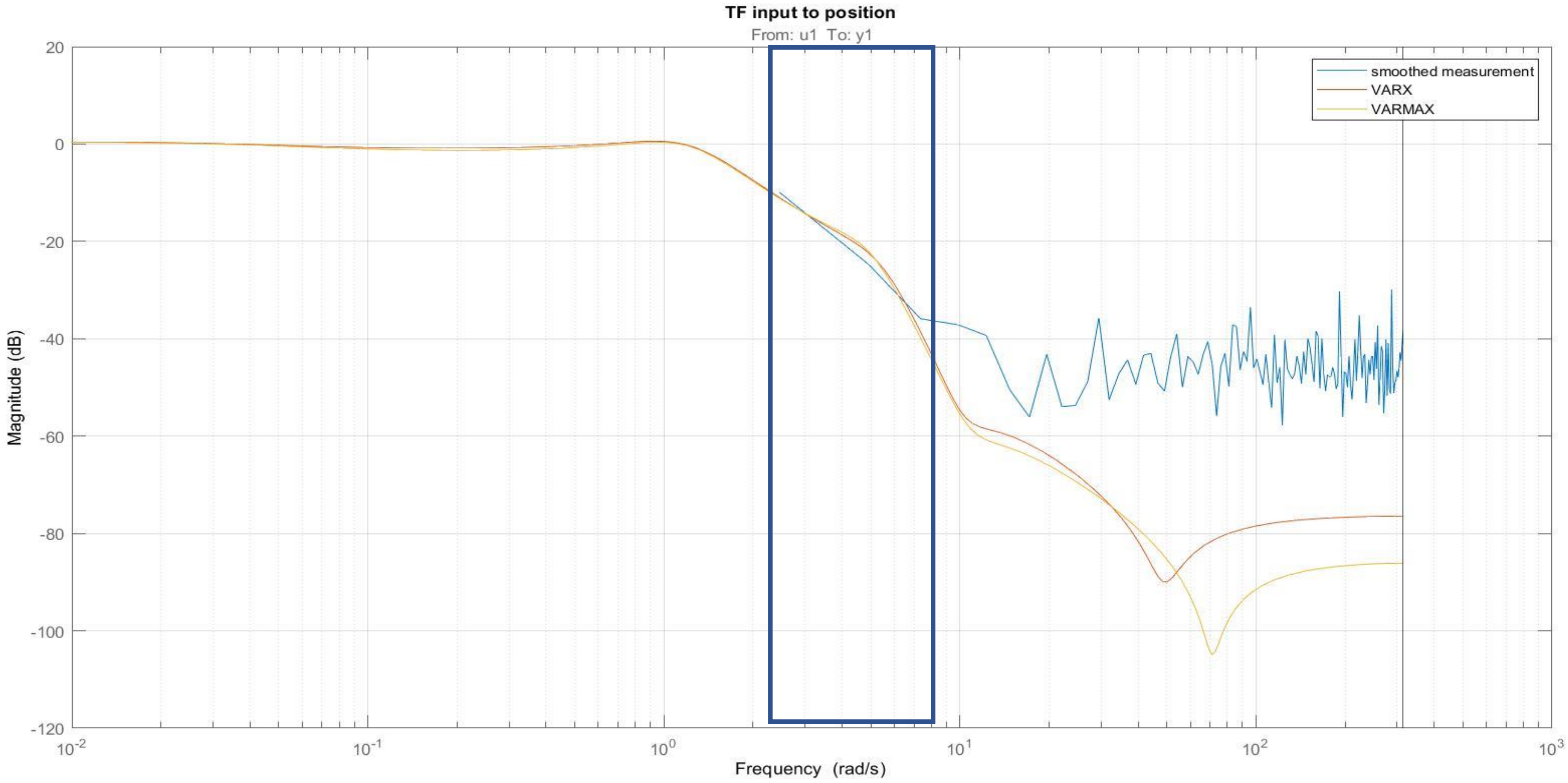
RESULT: ACCELERATION, VARX vs VARMAX



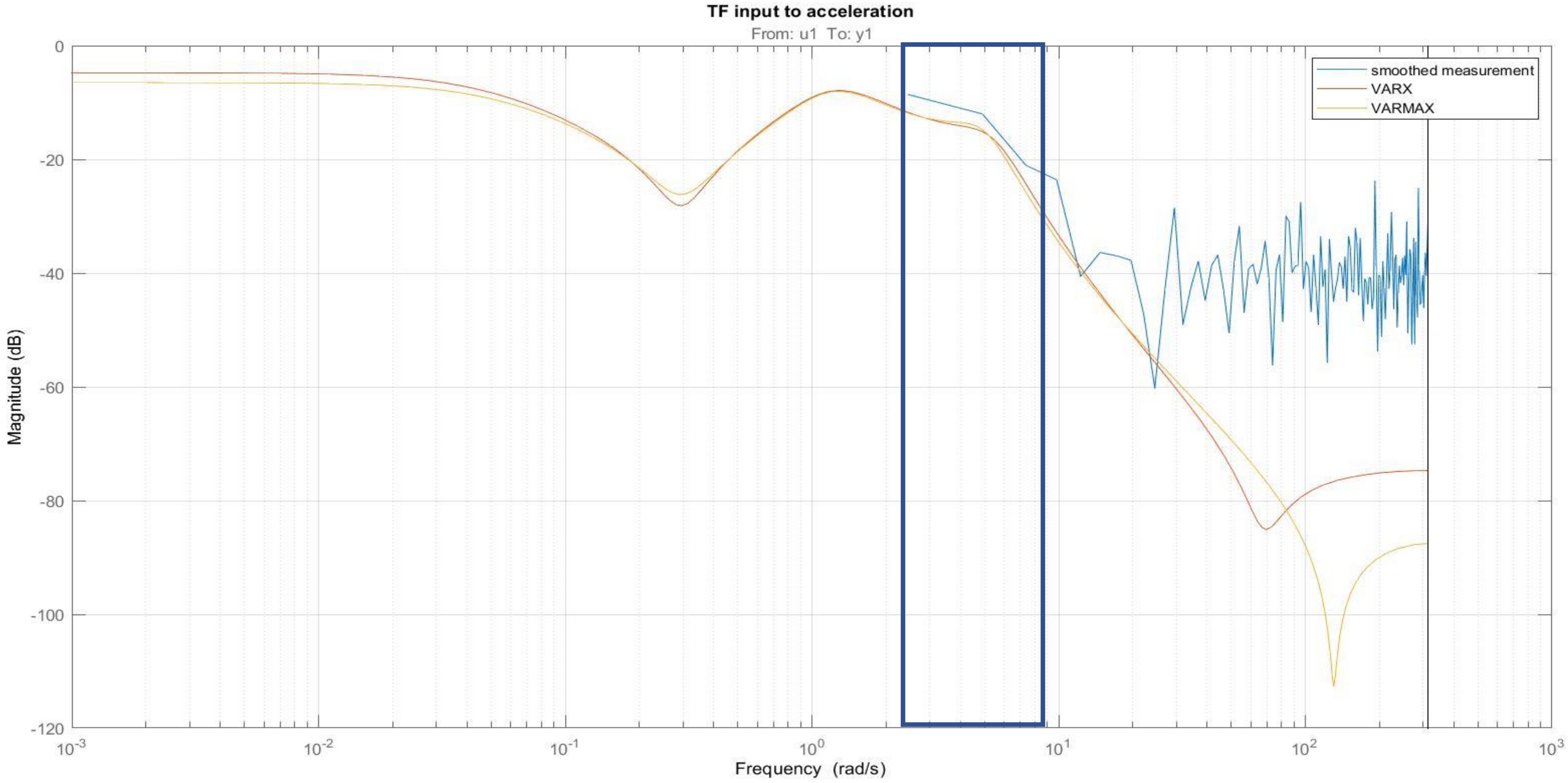
POLES OF IDENTIFIED MATRICE **A**:



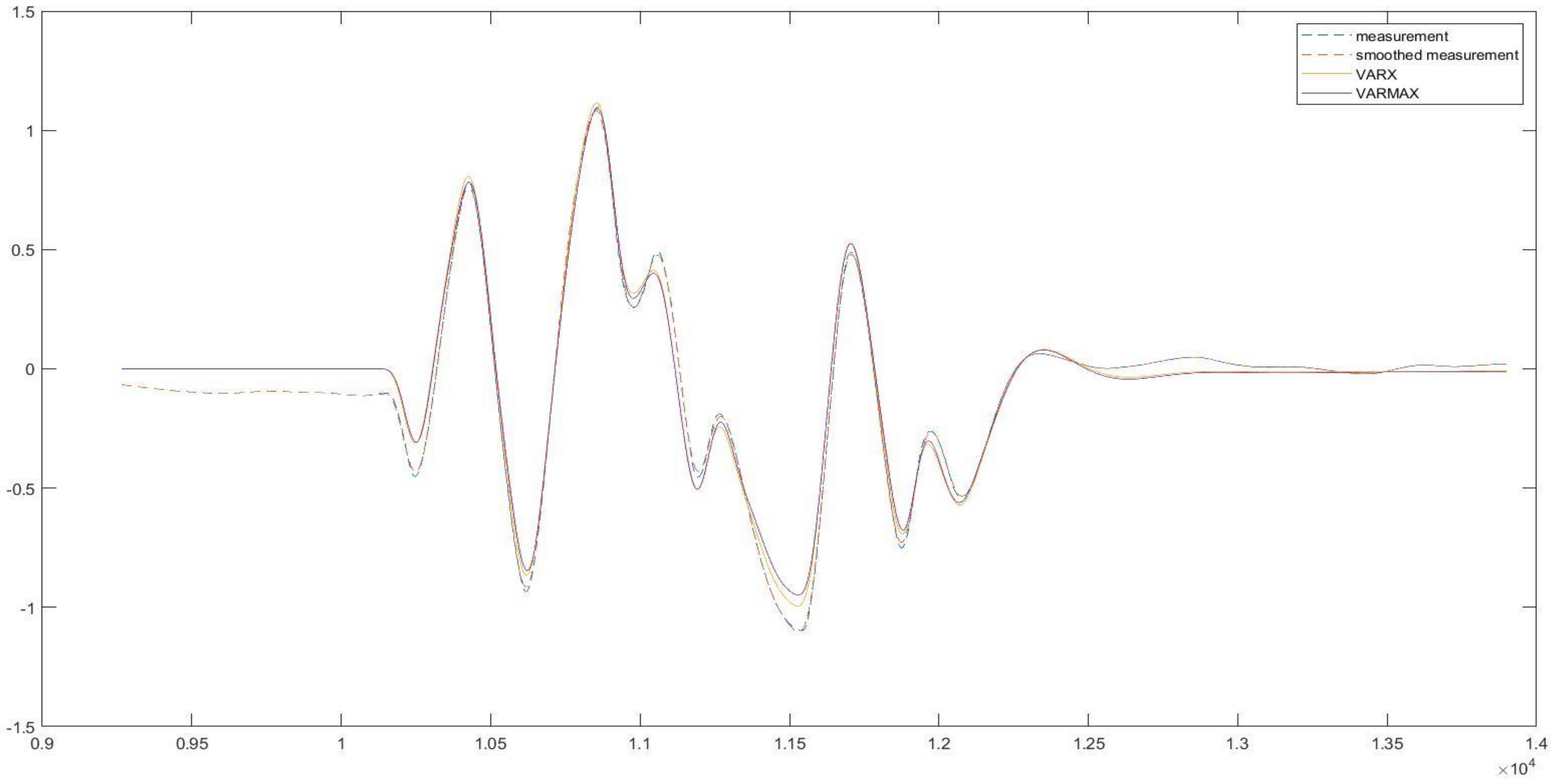
BODEMAG: INPUT TO POSITION (order 5, p = f = 15)



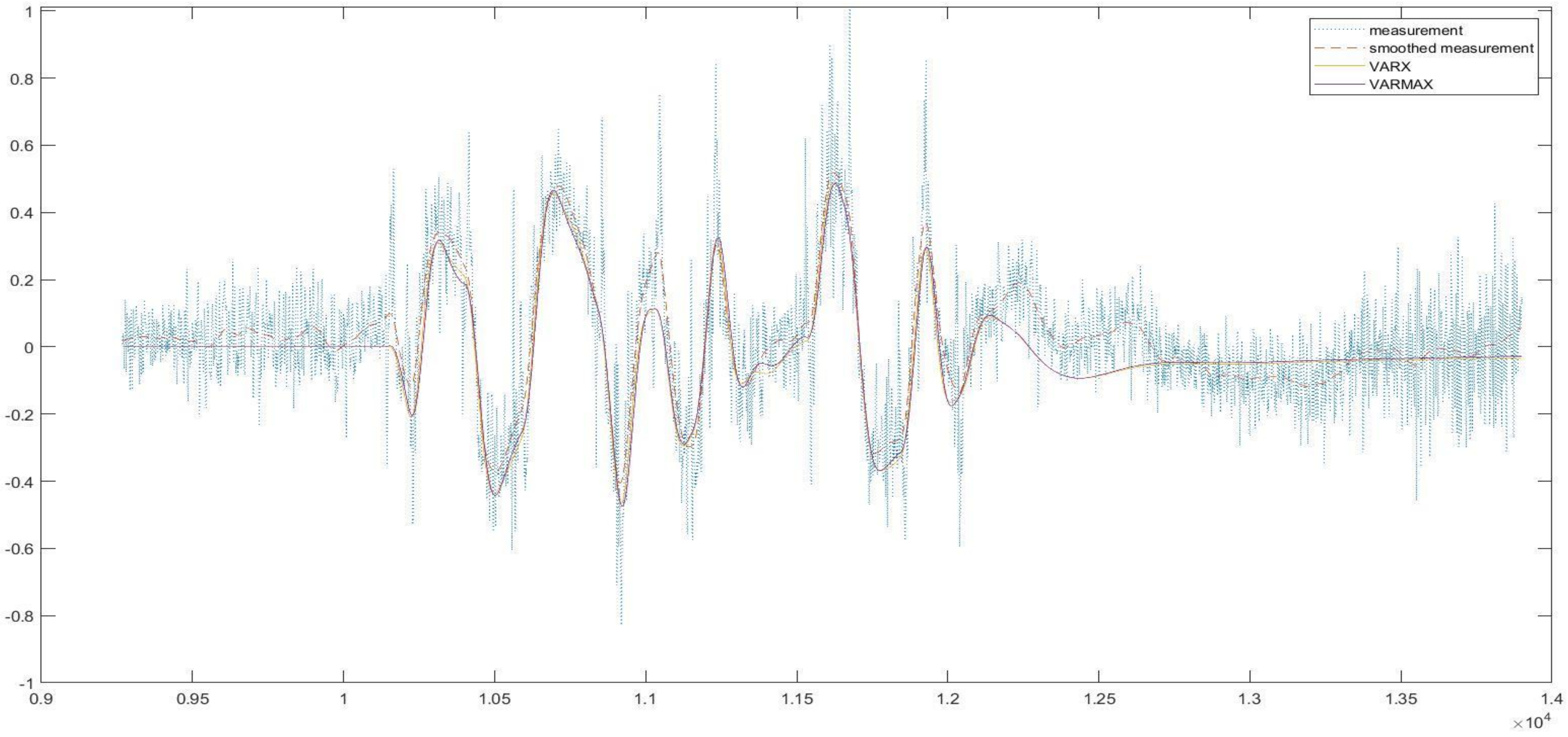
BODEMAG: INPUT TO ACCELERATION (order 5, p = f = 15)



VARX & VARMAX, order 5, p = f = 15 (position)



VARX & VARMAX, order 5, p = f = 15 (acceleration)



TASK 2: KALMAN FILTERING USING THE IDENTIFIED MODEL

DT-DT PREDICTOR/CORRECTOR FORM

Motion model: $\mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{G}_{k-1}\underbrace{\mathbf{u}_{k-1}}_{input} + \underbrace{\mathbf{w}_{k-1}}_{noise}$

Measurement model: $\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \underbrace{\mathbf{v}_k}_{noise}$

$\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k)$

Measurement Noise

$\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_k)$

Process or Motion Noise

1 Prediction

$$\check{\mathbf{x}}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{G}_{k-1}\mathbf{u}_{k-1}$$
$$\check{\mathbf{P}}_k = \mathbf{F}_{k-1}\hat{\mathbf{P}}_{k-1}\mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1}$$

2a Optimal Gain

$$\mathbf{K}_k = \check{\mathbf{P}}_k\mathbf{H}_k^T(\mathbf{H}_k\check{\mathbf{P}}_k\mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

2b Correction

$$\hat{\mathbf{x}}_k = \check{\mathbf{x}}_k + \mathbf{K}_k(\mathbf{y}_k - \mathbf{H}_k\check{\mathbf{x}}_k)$$
$$\hat{\mathbf{P}}_k = (\mathbf{1} - \mathbf{K}_k\mathbf{H}_k)\check{\mathbf{P}}_k$$

DT-DT PREDICTOR/CORRECTOR FORM

Where $\boldsymbol{F} = \boldsymbol{I} + \boldsymbol{A}.\Delta t$ and **A, B,** and **C** are the identified model

$$\boldsymbol{G} = \boldsymbol{B}$$

$$\boldsymbol{H} = \boldsymbol{C}$$

Since state estimates do not have any physical representations (because of PBSID), then we cannot recover velocity directly from state estimates. Therefore, we build it from measurement estimates and using a simple formula to obtain velocity.

$$\widehat{\boldsymbol{y}} = \boldsymbol{H}\widehat{\boldsymbol{x}} + \boldsymbol{D}\boldsymbol{u} = \begin{bmatrix} \hat{r} \\ \hat{a} \end{bmatrix} \longrightarrow \boxed{\hat{v}_t = \frac{\hat{r}_{t+1} - \hat{r}_t}{\Delta t}}$$

DATA FOR KALMAN FILTERING

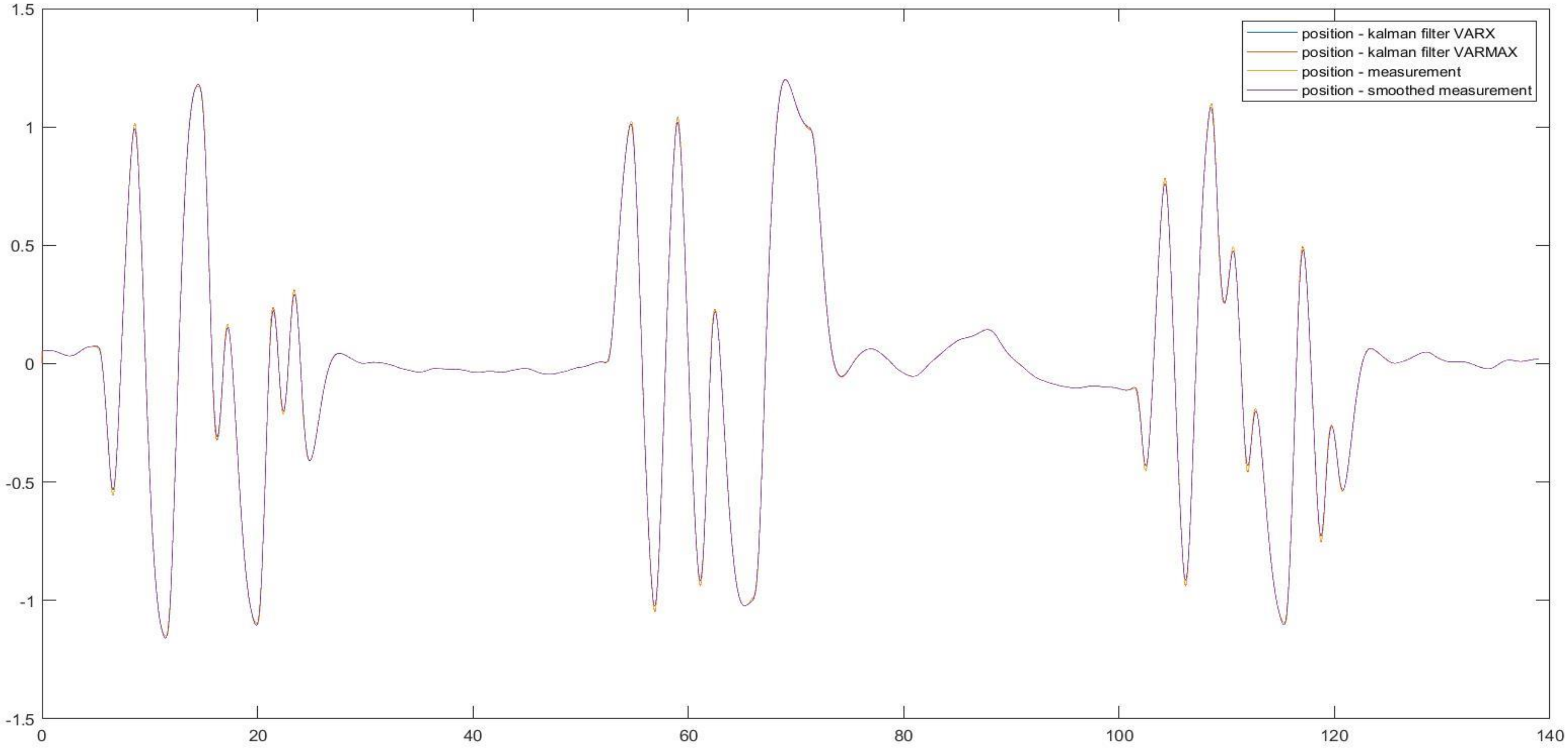
```
• x0 = zeros(order,1);          % initial state
• P = eye(order);               % initial state covariance
• dt = 0.01;                    % time step

• F = eye(order)+ A*dt;
• G = B;
• H = C;
• Q = 0.1*eye(order);           % process noise
• R = [1 0;0 1000];             % measurement noise
```

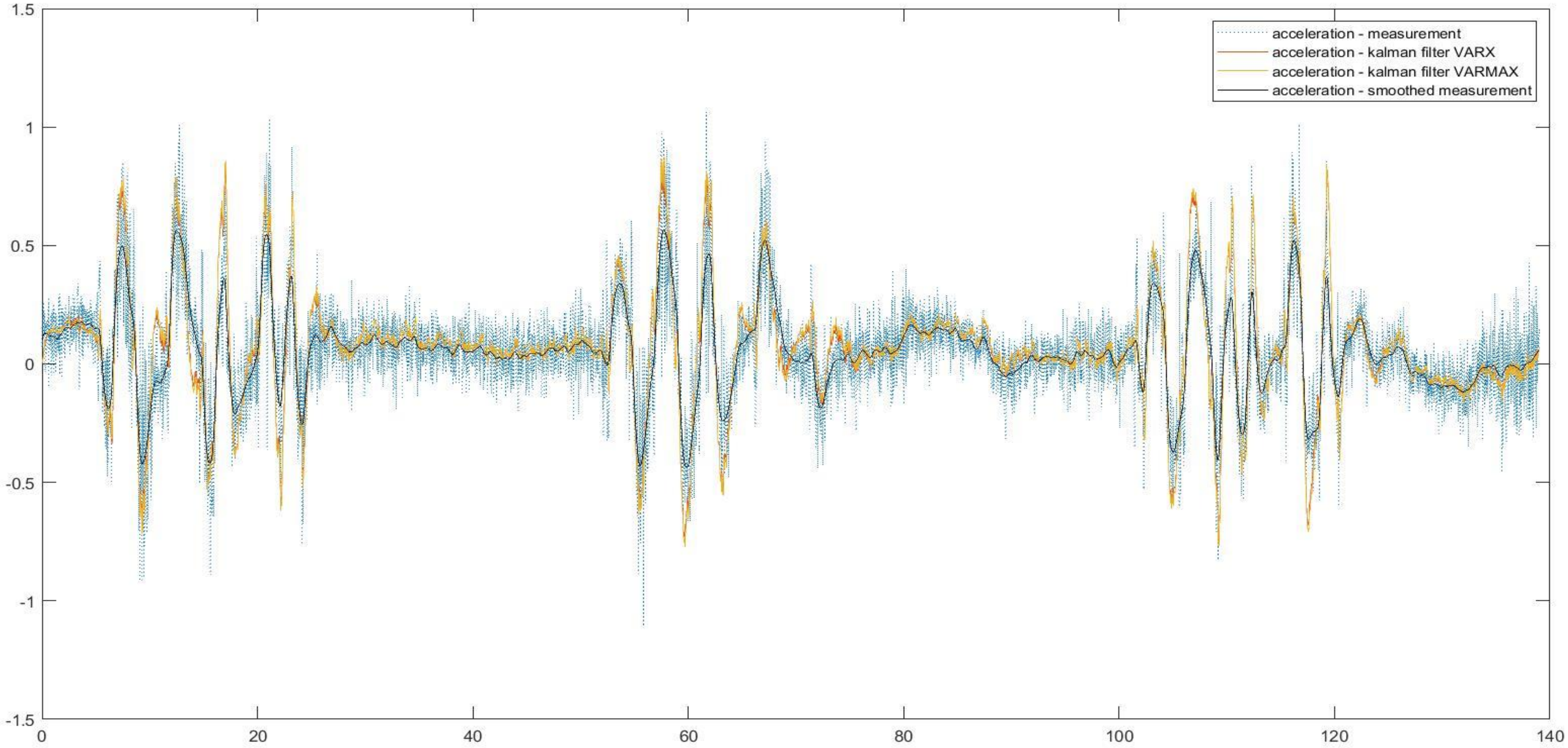
LOOPING PROCESS

```
for i=2:length(tspan)
    xhat(:,1) = x0; % initial state
    %prediction
    xmin = F*xhat(:,i-1) + Bi*u(i);
    P = F*P*F'+Q;
    %correction
    K = P*H'*inv(H*P*H'+R);
    xhat(:,i) = xmin + K*(ym(i,:)'-H*xmin);
    P = (eye(order)-K*H)*P;
    %output estimate
    yhat(:,i) = H*xhat(:,i) + Di*u(i);
    %lateral velocity ned
    velocity(i-1) = (yhat(1,i)-yhat(1,i-1))/dt;
end
```

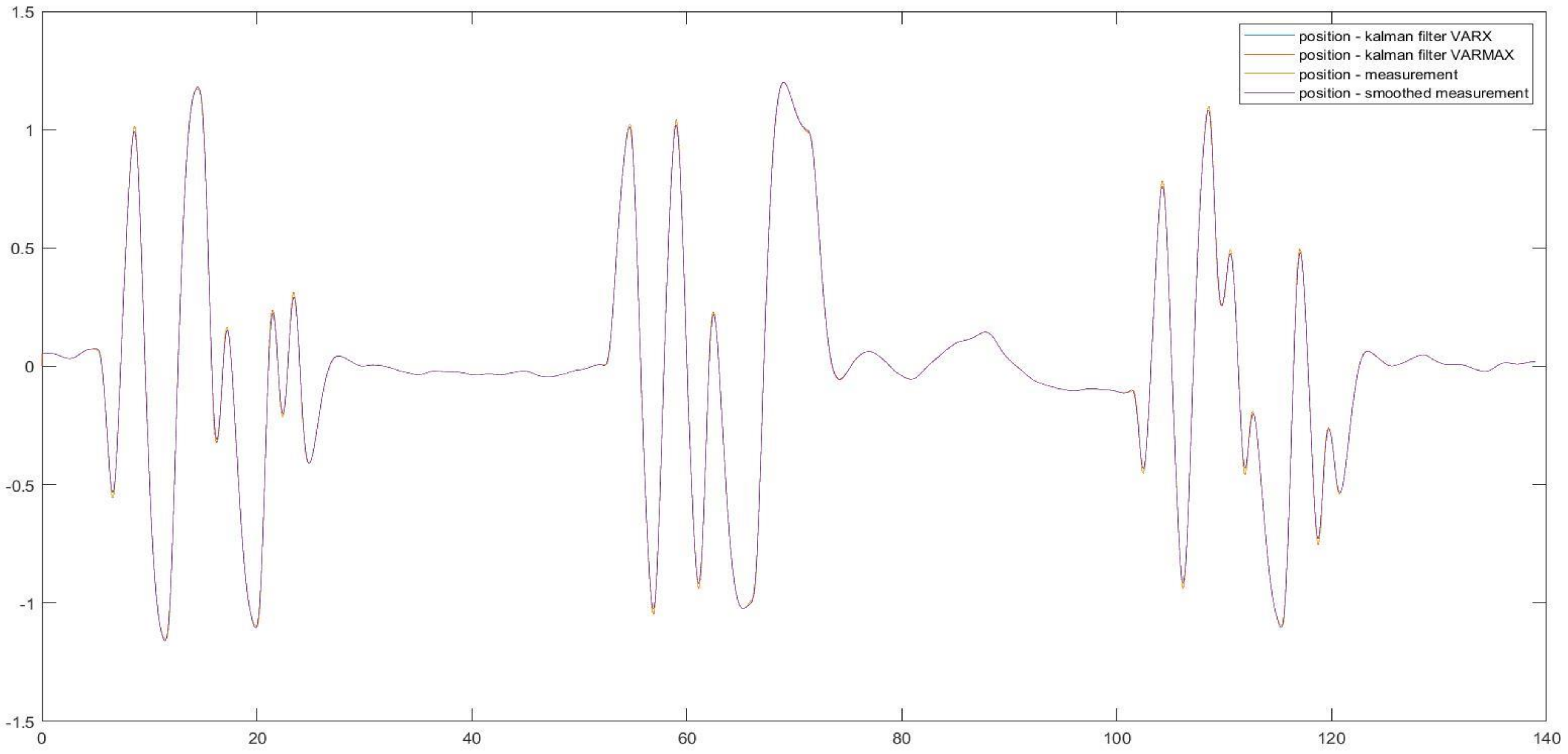
RESULT: POSITION ESTIMATES



RESULT: ACCELERATION ESTIMATES



RESULT: VELOCITY ESTIMATES



THANK YOU

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