MODEL IDENTIFICATION AND KALMAN FILTERING FOR QUADROTOR

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Estimation and learning in aerospace

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Tasks

1. IDENTIFY THE MODEL GIVEN DATASET.

2. USE THE IDENTIFIED MODEL TO ESTIMATE VELOCITY USING DT-DT KALMAN FILTERING.

TASK 1: MODEL IDENTIFICATION

Task 1 - Model Identification

- DATA SET:

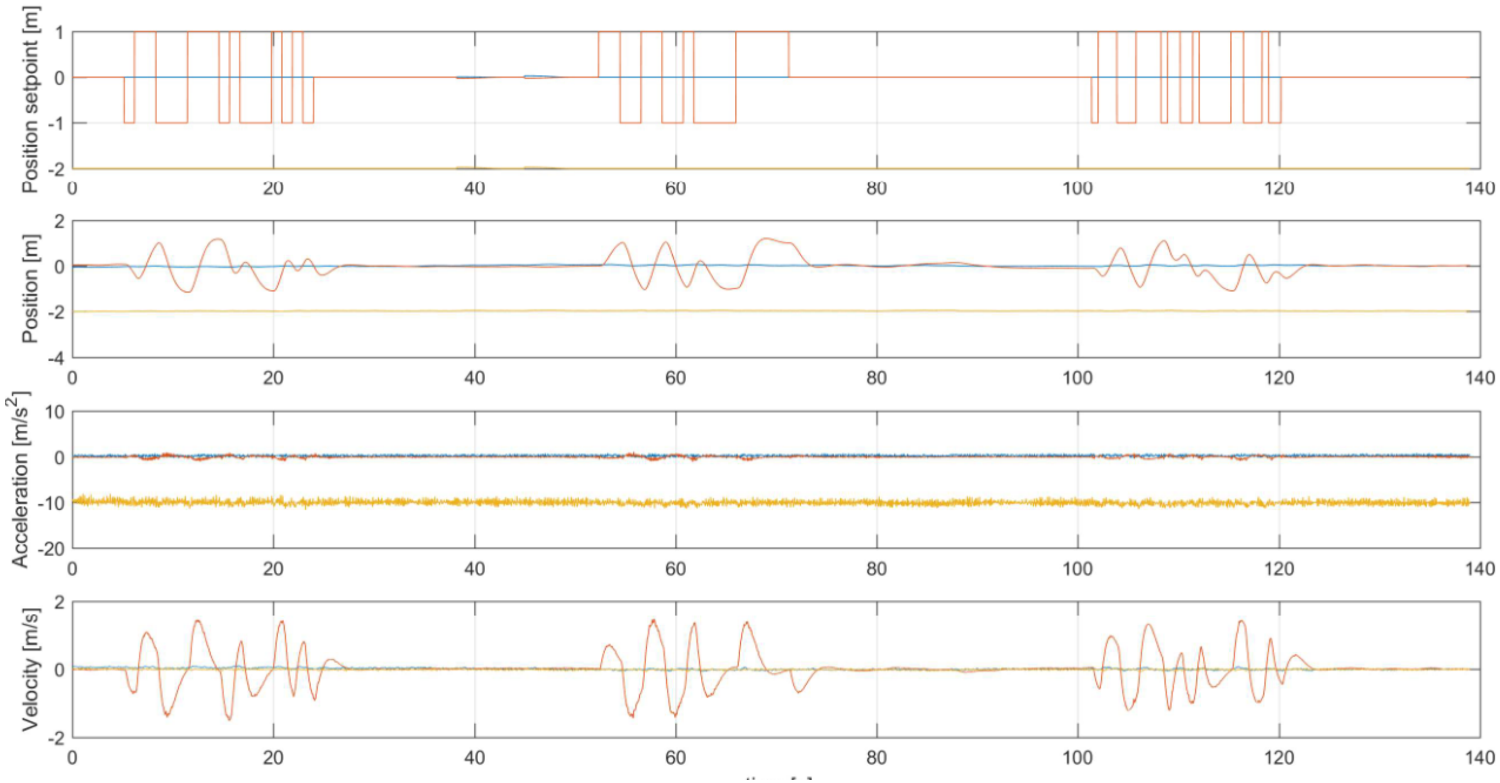
1. Position setpoint in NED frame as input.

2. Position in NED frame as output.

3. Acceleration in body frame as output.

4. Velocity in NED frame.

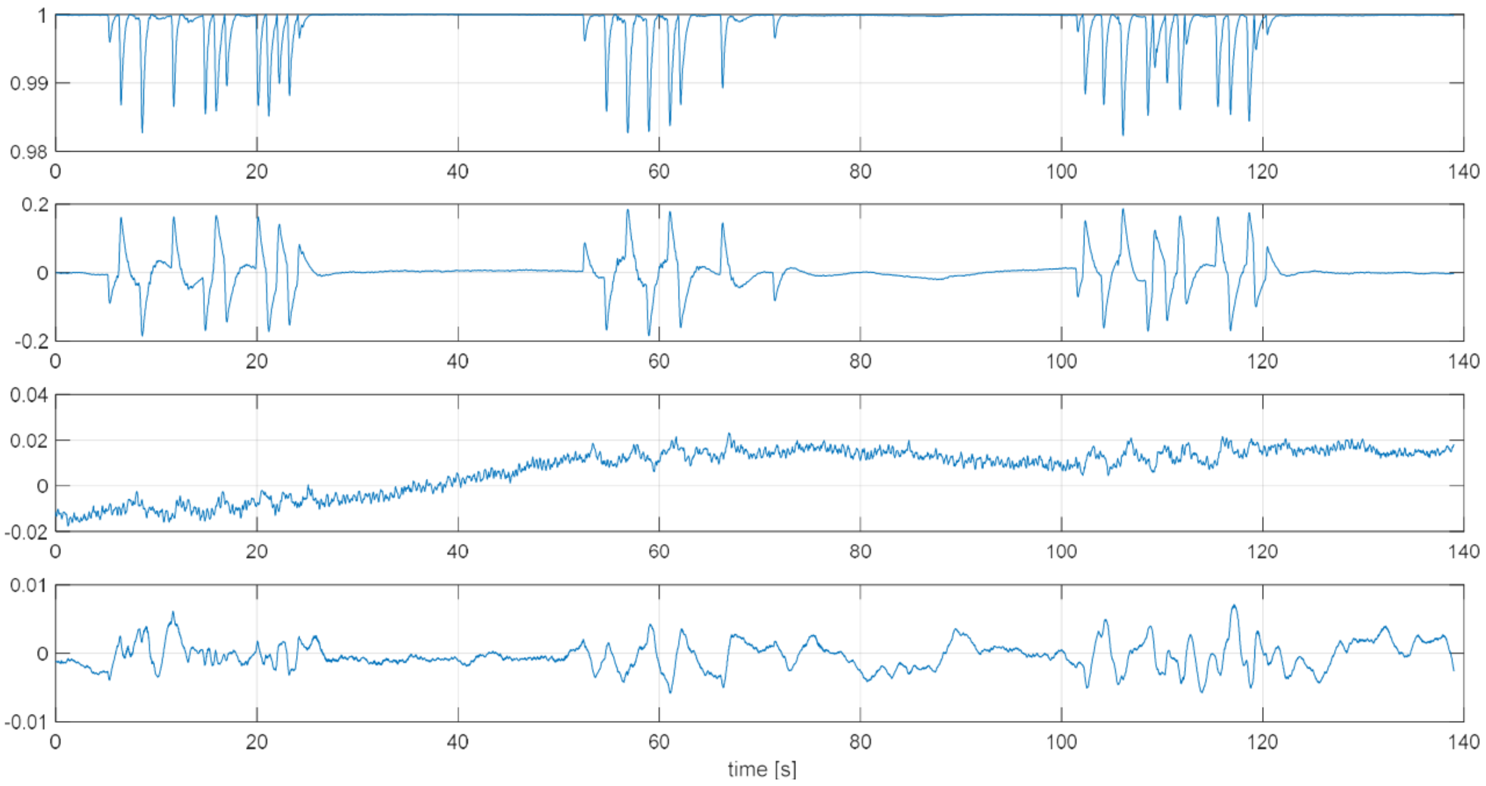
- All sampled at 100 Hz.



Task 1 - Model Identification

- DATA SET:

Quaternion body to NED. The first element is the scalar part. Sampled at 100 Hz.



Model Identification Steps

Data preparation

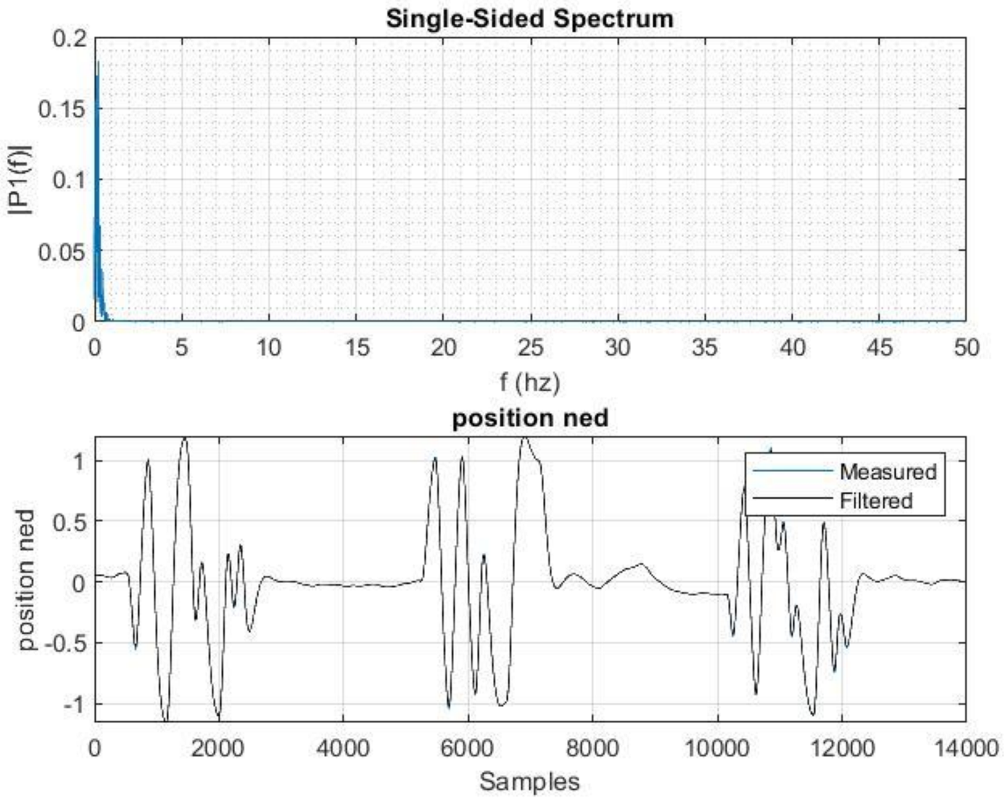
- FFT to determine the cut-off frequency.
- Smoothing.
- Training and validation dataset.

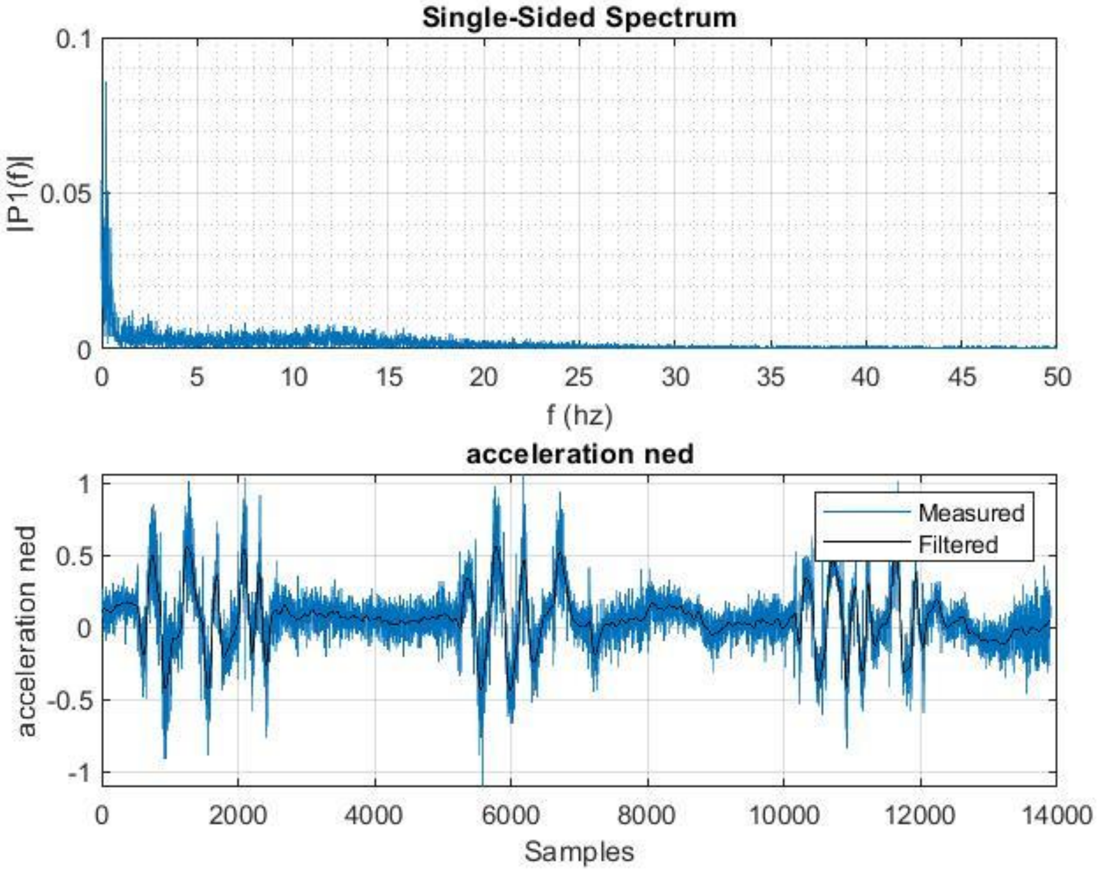
Model identification using training dataset

PBSID with VARX and VARMAX

Validation using validation dataset and verify the results (VAF)

Quantify the results using VAF.

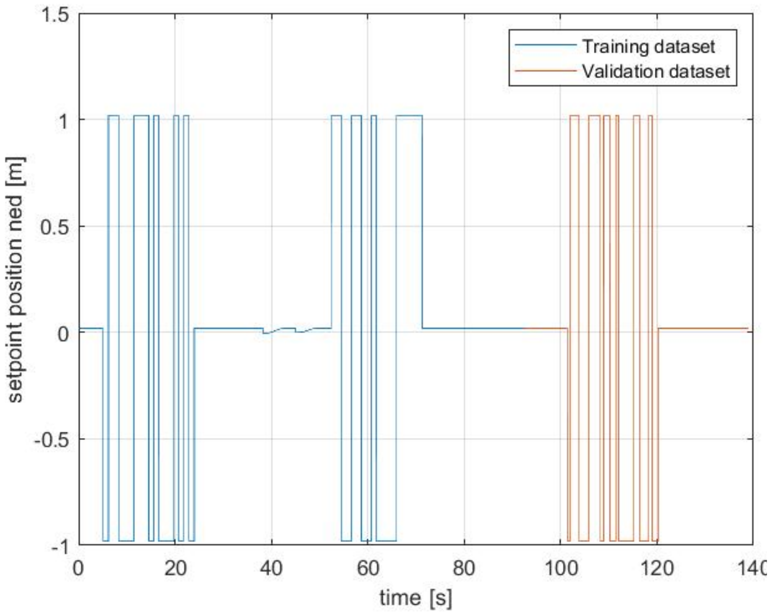


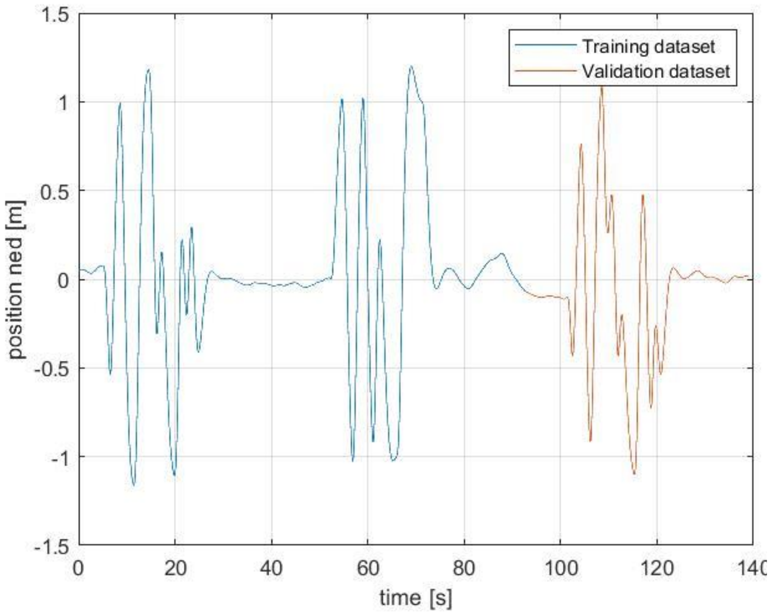


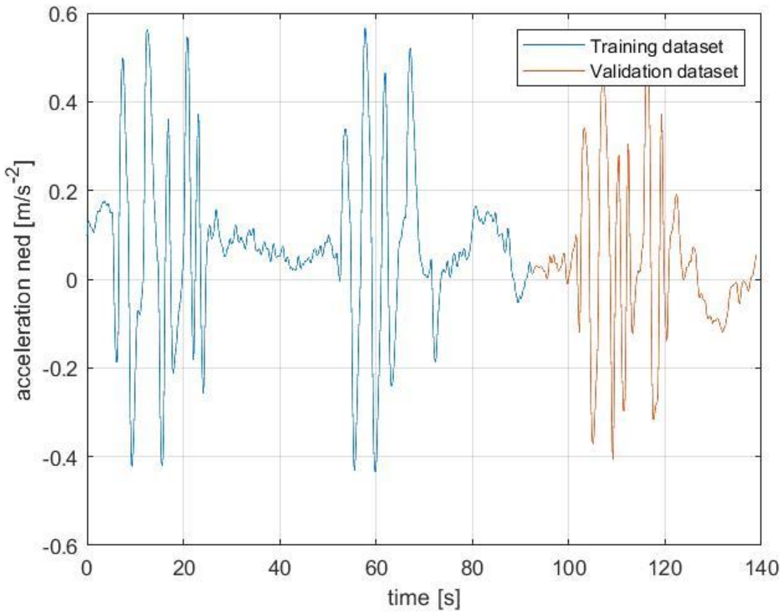
DATA PREPARATION: FFT & SMOOTHING

SMOOTHING: <u>BUTTERWORTH 2ND ORDER</u>

DATA PREPARATION: TRAINING SET AND VALIDATION SET







MODEL IDENTIFICATION: SUBSPACE MODEL IDENTIFICATION (SMI)

WHY SUBSPACE IDENTIFICATION?

- No need of non-linear optimization techniques, only linear algebra (SVD) + Ricatti
- No need to impose onto the system a canonical form
- Computationally efficient and robust
- This method can be applied equally to MIMO and SISO.

SOME CONS OF SUBSPACE IDENTIFICATION

Statistical analysis is difficult

No physical model representation

SMI:

Normal subspace methods are not consistent if there is feedback so we use a specialized method for closed-loop systems

PBSID

PREDICTOR BASED SUBSPACE IDENTIFICATION METHOD

Existence of correlations between the external unmeasurable noise and the control inputs.

Future inputs dependency on **past outputs/noise.**

PBSID

$$\mathcal{S}\begin{cases} x_{k+1} &= Ax_k + Bu_k + Ke_k, \\ y_k &= Cx_k + Du_k + e_k, \end{cases}$$

PROBLEM: Given input sequence u_k and output y_k, over time $k = \{0, \dots, N-1\}$ find **A, B, C, D**, and **K**.

ASSUMPTIONS:

- System is observable
- Noise sequence e_k is white
- Input sequence u_k has sufficient excitation
- Feedback loop does not have direct feedthrough

No other assumptions on **correlation between the input and noise sequence** --> Possibility to apply the algorithm in **CLOSED LOOP**

PBSID

- Rewrite the state-space in Kalman predictor form

$$\mathcal{S}\begin{cases} x_{k+1} &= Ax_k + Bu_k + Ke_k \\ y_k &= Cx_k + Du_k + e_k, \end{cases} \longrightarrow \begin{cases} x_{k+1} &= \tilde{A}x_k + \tilde{B}u_k + Ky_k, \\ y_k &= Cx_k + Du_k + e_k, \end{cases}$$

$$\tilde{A} = A - KC \quad \tilde{B} = B - KD$$

- Introduce the extended controllability and observability matrix

$$\mathcal{K}^p = [\bar{A}^{p-1}\widetilde{B}_0 \ \ldots \ \widetilde{B}] \qquad \Gamma^p = \begin{bmatrix} C \\ C\bar{A} \\ \vdots \\ C\bar{A}^{p-1} \end{bmatrix}$$

- Past window and the future window vectors

$$\bar{y}_{k-p} = \begin{bmatrix} y_{k-p} \\ y_{k-p+1} \\ \vdots \\ y_{k-1} \end{bmatrix}, \quad \bar{y}_k = \begin{bmatrix} y_k \\ y_{k+1} \\ \vdots \\ y_{k+f-1} \end{bmatrix}$$

$f \leq p$

----> Same for *u* and *e*

PBSID

- Defining $z(k) = \left[u^T(k)\ y^T(k)\right]$

Same for *X* and E

$$\bar{Z} = \left[\bar{z}_0, \quad \cdots, \quad \bar{z}_{N-p}\right]$$

$$Y = \left[y_p, \quad \cdots, \quad y_{N-1}\right]$$

- Rewrite the problem in matrix notation

$$X^{p,f} \simeq \mathcal{K}^p \bar{Z}^{p,f}$$

$$Y^{p,f} \simeq C\mathcal{K}^p \bar{Z}^{p,f} + DU^{p,f} + E^{p,f}$$

$FOR\ f = p$

$$\min_{C\mathcal{K}^p, D} \|Y^{p,p} - C\mathcal{K}^p \bar{Z}^{p,p} - DU^{p,p}\|_F.$$

- To estimate the state sequence $X^{p,p}$ and retrieve the order of the system use SVD of the projection $\Gamma^p \mathcal{K}^p \bar{Z}^{p,p} = U\Sigma V^T$
- Then estimate matrix C from least squares problem: $\min_C \|Y^{p,p} - \widehat{D}U^{p,p} - C\widehat{X}^{p,p}\|_F$

PBSID

- Estimation of the innovation data matrix

$$E_N^{p,f} = Y^{p,p} - \widehat{C}\widehat{X}^{p,p} - \widehat{D}U^{p,p}$$

- A,B and K can be obtained by solving least squares problem

$$\min_{A,B,K} \|\widehat{X}^{p+1,p} - A\widehat{X}^{p,p-1} - BU^{p,p-1} - KE^{p,p-1}\|_F.$$

PBSID – VARX and VARMAX model set

VARX

$$\begin{cases} x_{k+1} &= \tilde{A}x_k + \tilde{B}u_k + Ky_k \\ y_k &= Cx_k + Du_k + e_k, \end{cases}$$

$$\epsilon_{k|k-1} = y_k - \hat{y}_{k|k-1}$$

- The one-step ahead predictor is linear in the Markov parameters (Computational better)
- One-step ahead prediction error has truncation error and noise error.
- If past window is small, the truncation error leads to biased estimation of state sequence
- Optimal solution for noise is when p->∞ because $\bar{G}_p \rightarrow \bar{G}$ and $\bar{H}_p \rightarrow \bar{H}$ and truncation becomes small

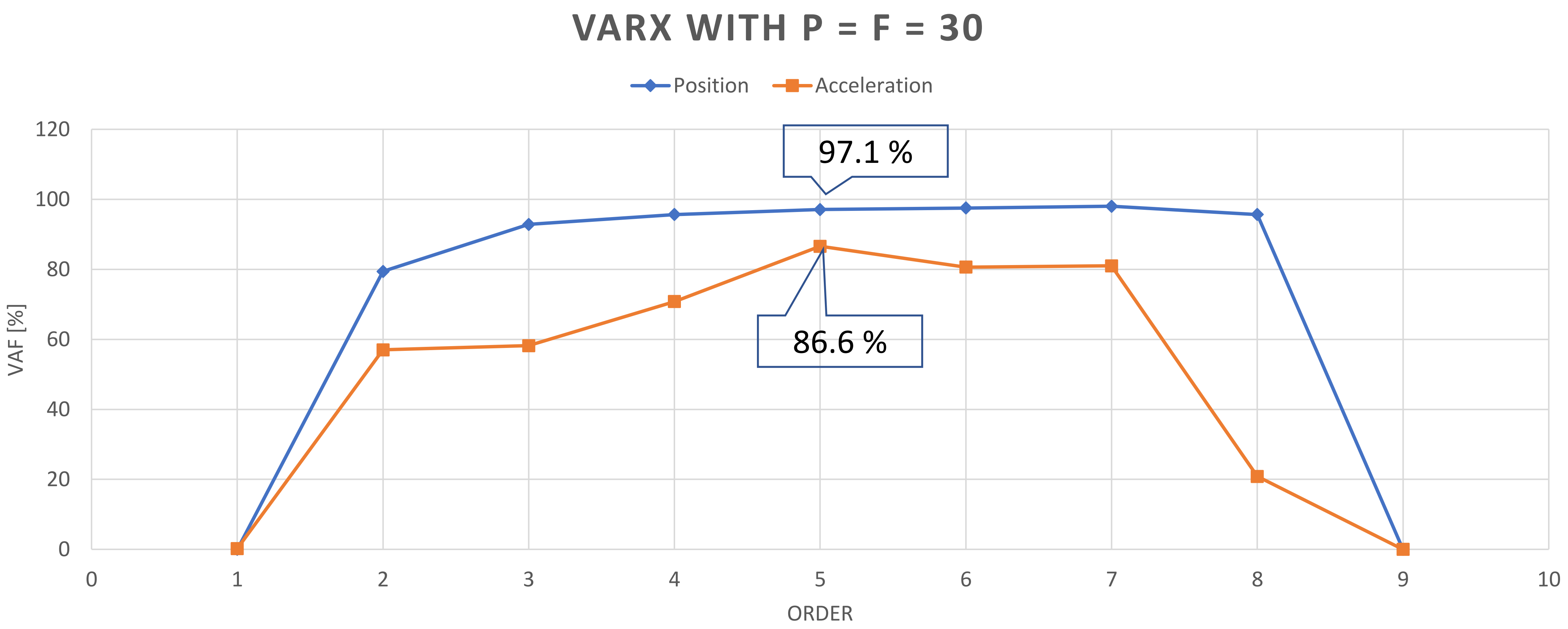
VARMAX

$$\begin{cases} x_{k+1} &= \bar{A}x_k + \bar{B}u_k + My_k + \bar{K}e_k \\ y_k &= Cx_k + Du_k + e_k, \end{cases}$$

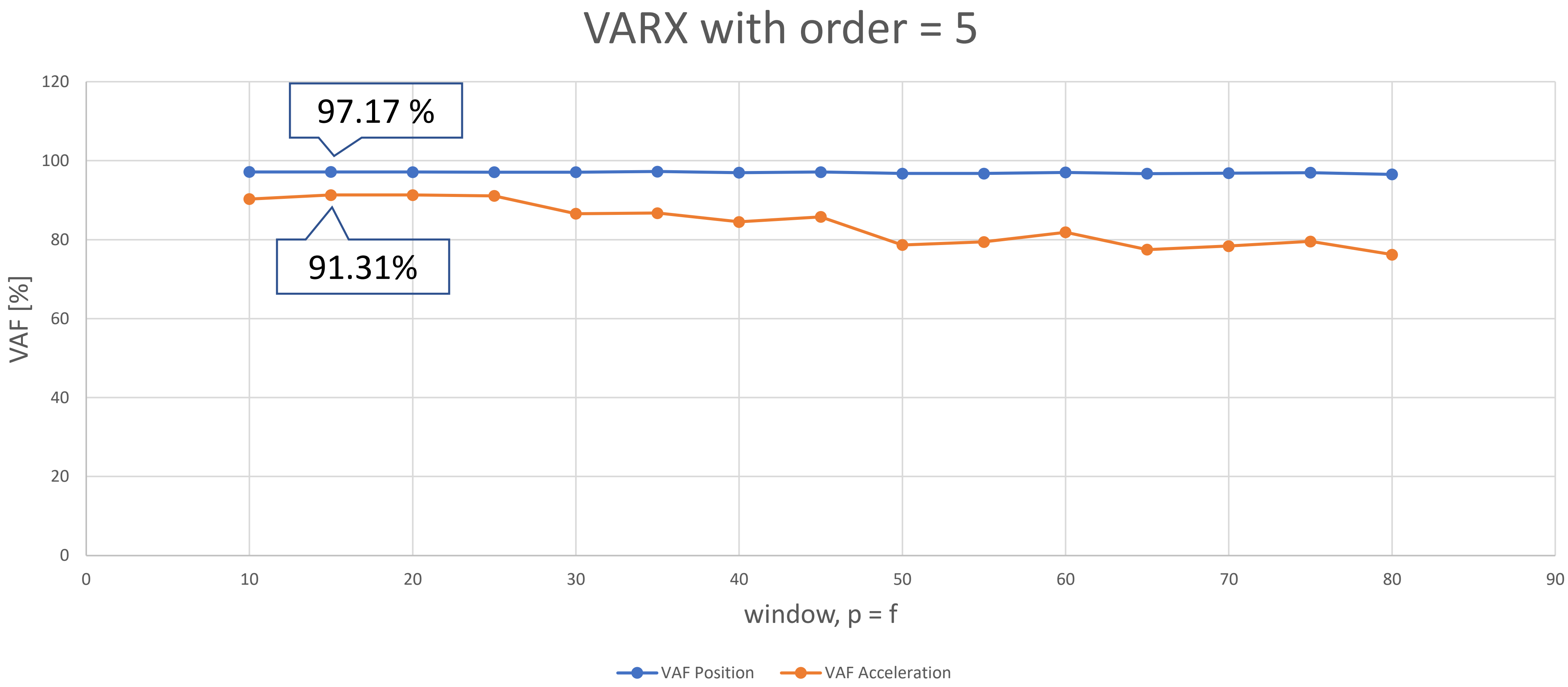
$$y_k = G(z)\,u_k + H(z)\,e_k.$$

- Introduces another observer matrix to create additional freedom for the optimizer.
- The one-step ahead predictor is no longer linear but extended least squares still gives efficient solution.
- For finite case error now only contains noise term, therefore for $p > n$ $\bar{G}_p = \bar{G}$ and $\bar{H}_p = \bar{H}$ no approximation is needed without truncation error.
- Lower past window for asymptotical consistent estimates. Beneficial when p is restricted.

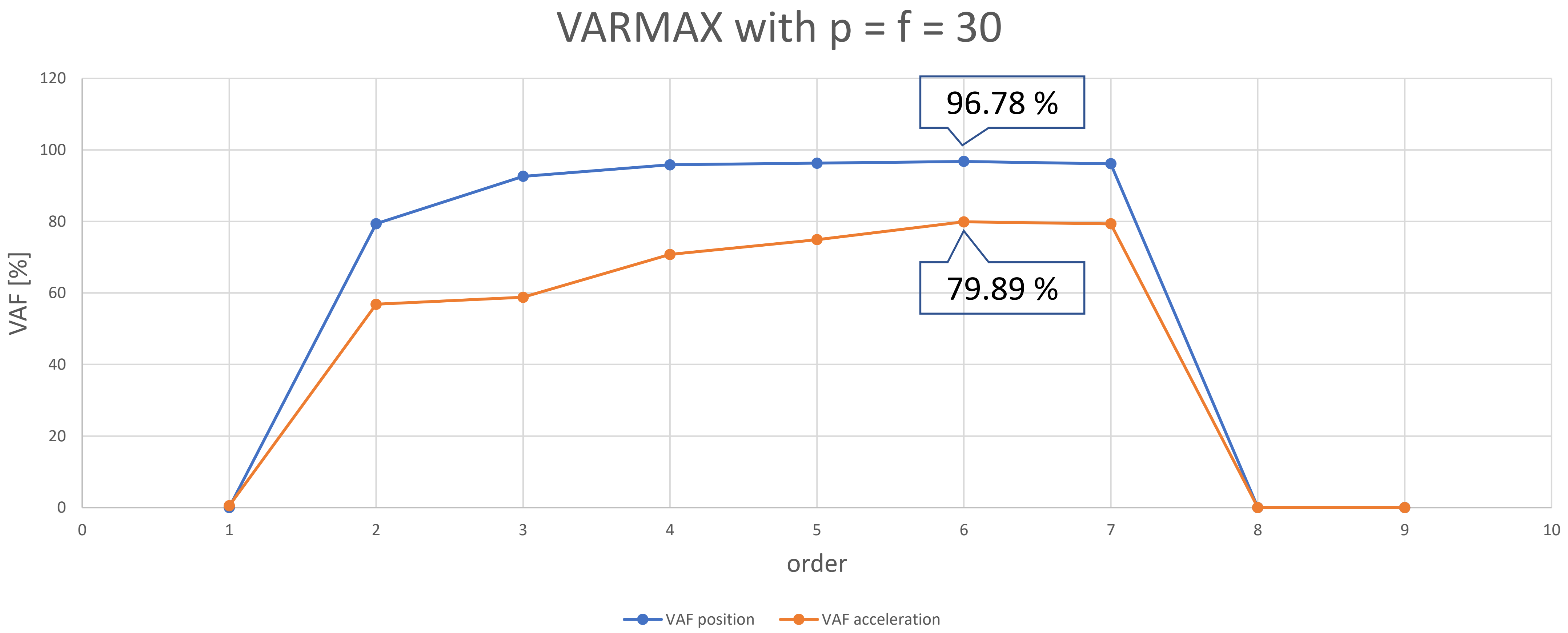
RESULTS: VARX



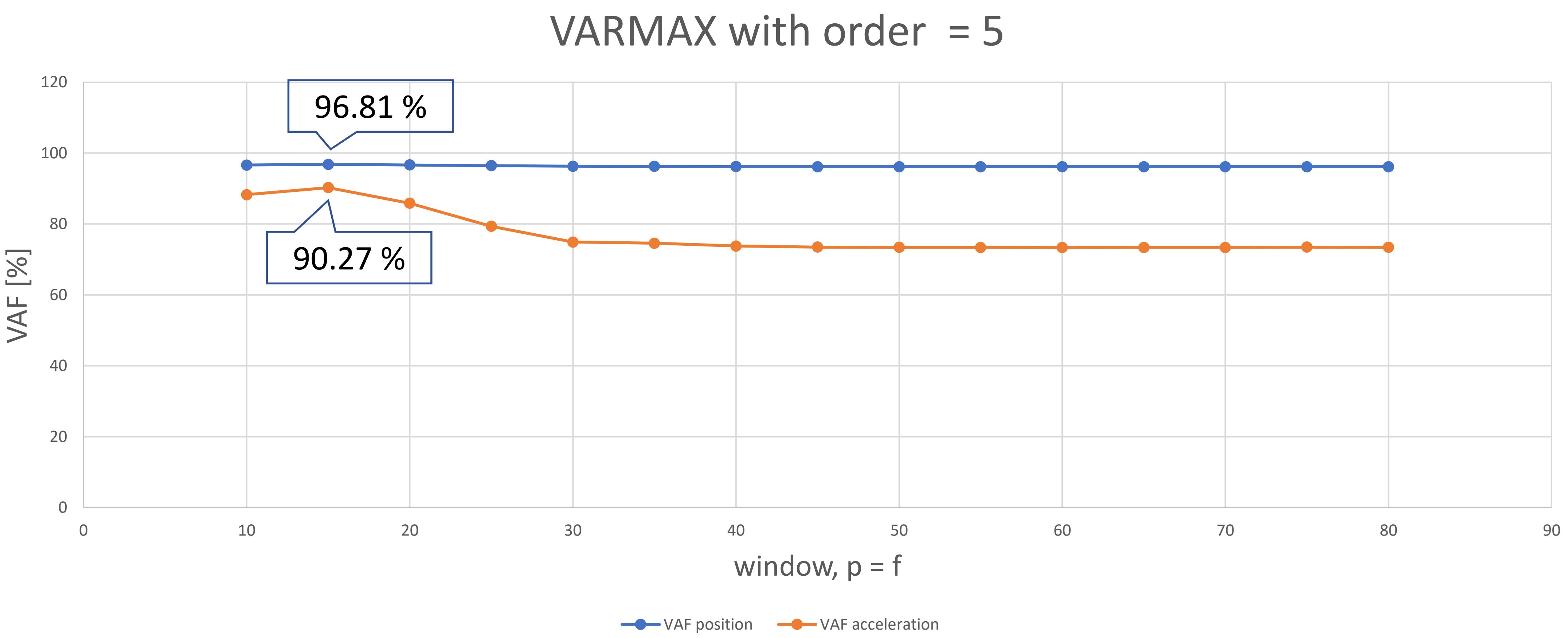
RESULTS: VARX



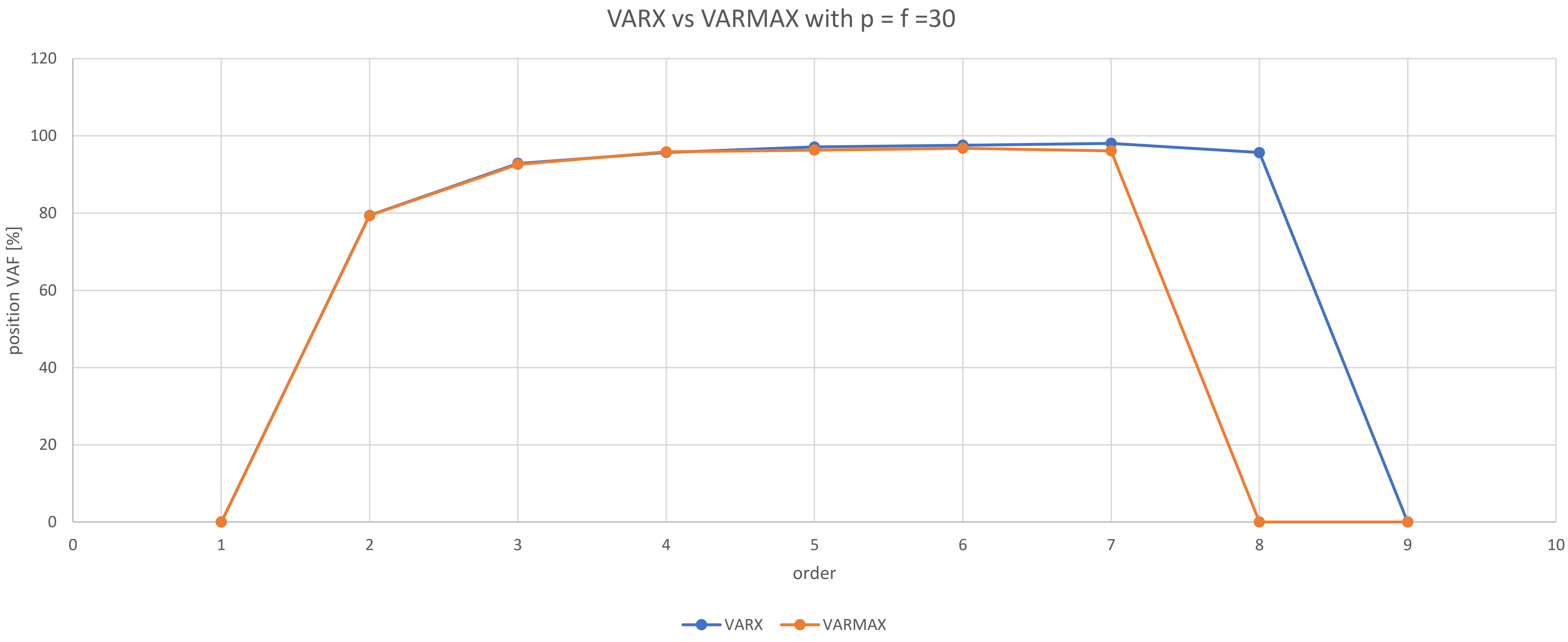
RESULTS: VARMAX



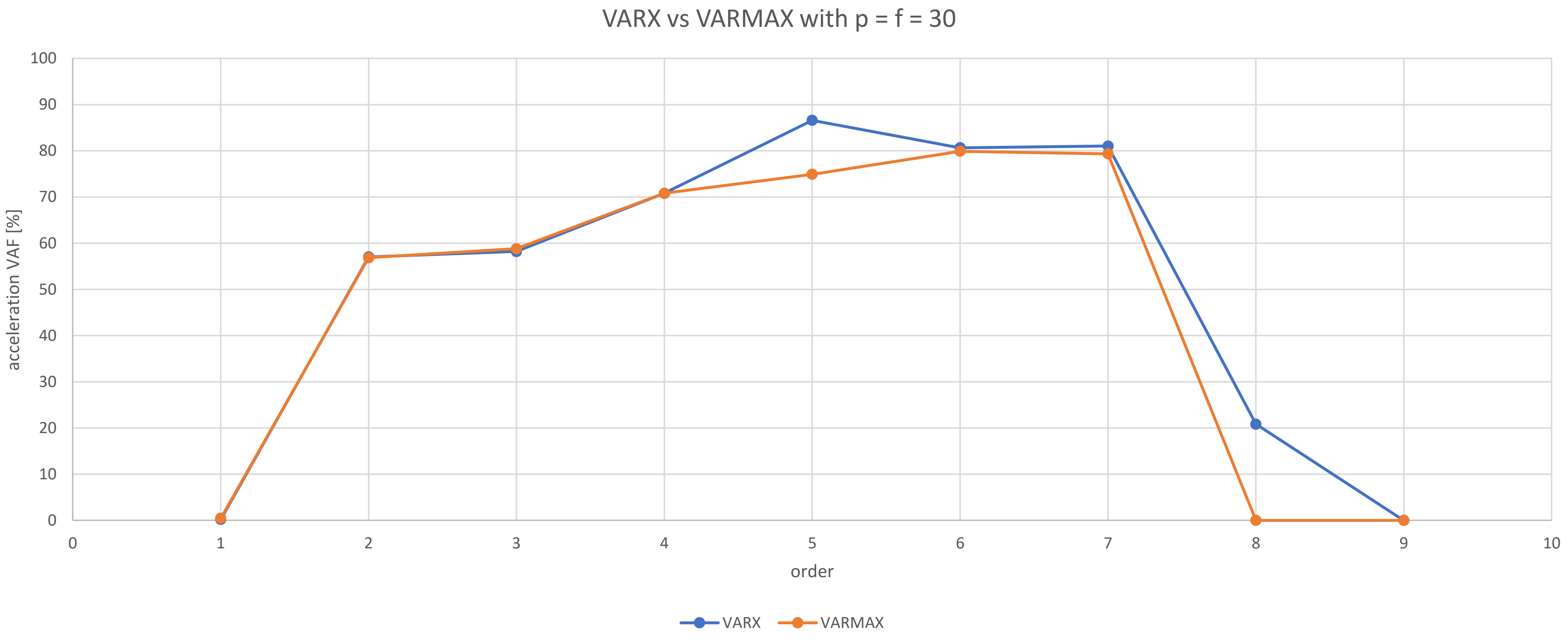
RESULTS: VARMAX



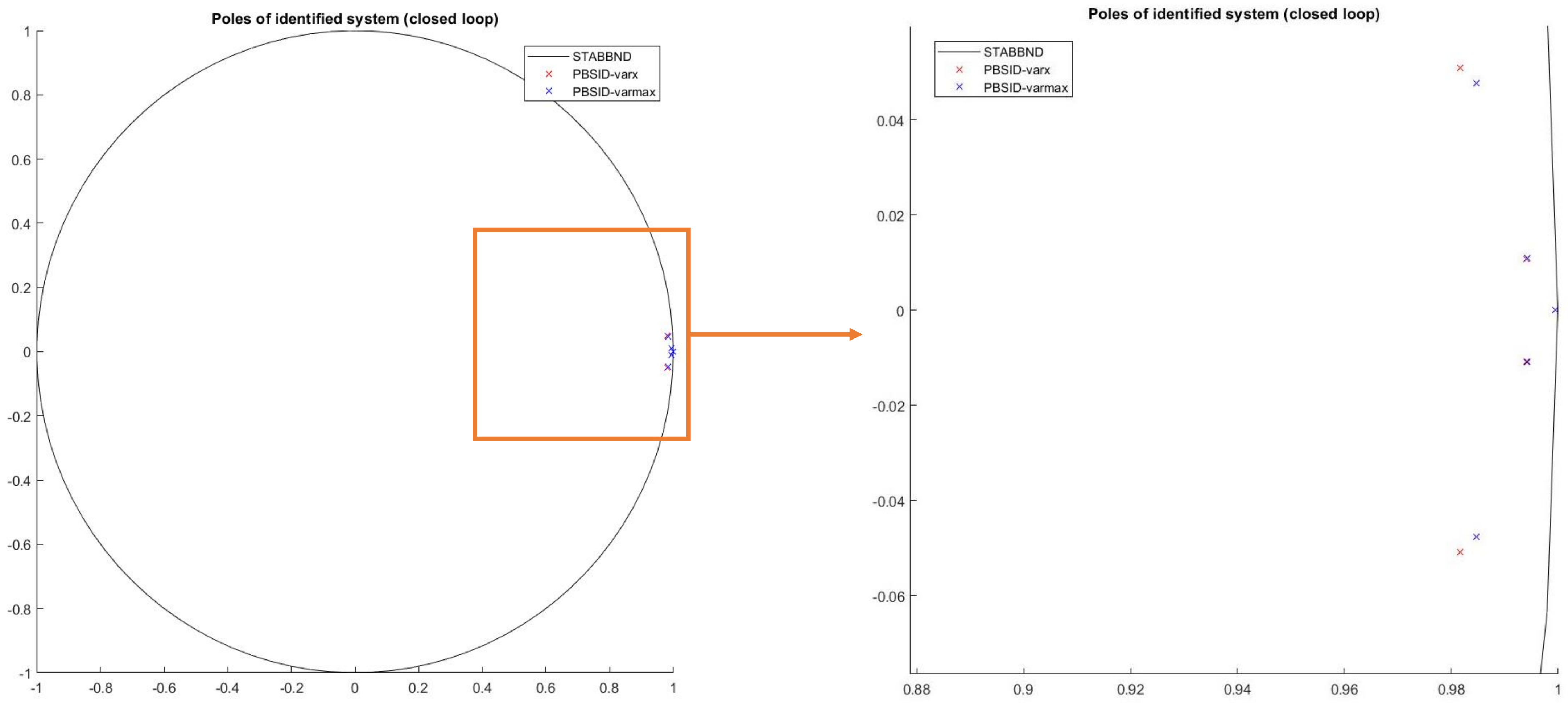
RESULT: POSITION, VARX vs VARMAX



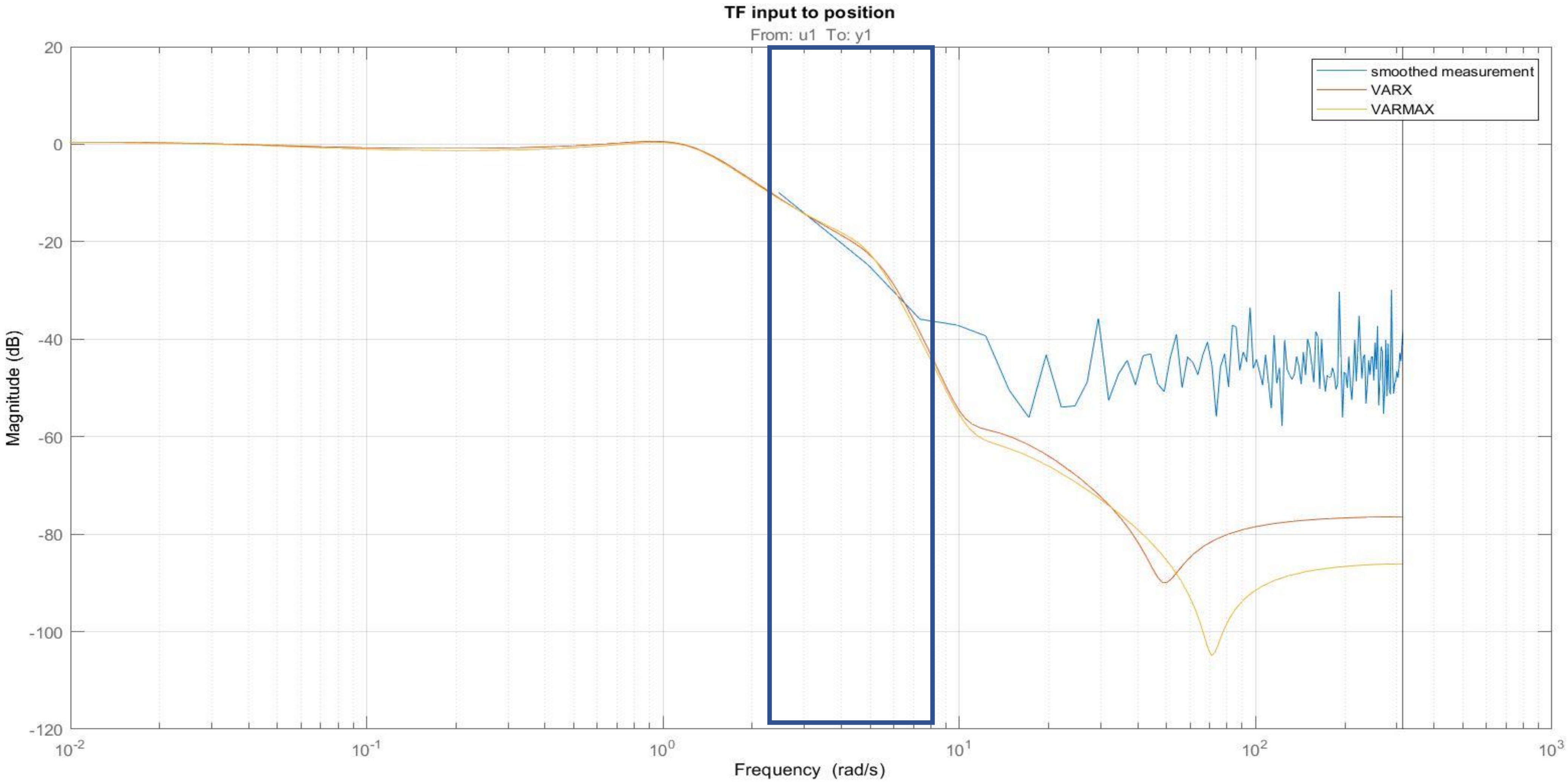
RESULT: ACCELERATION, VARX vs VARMAX



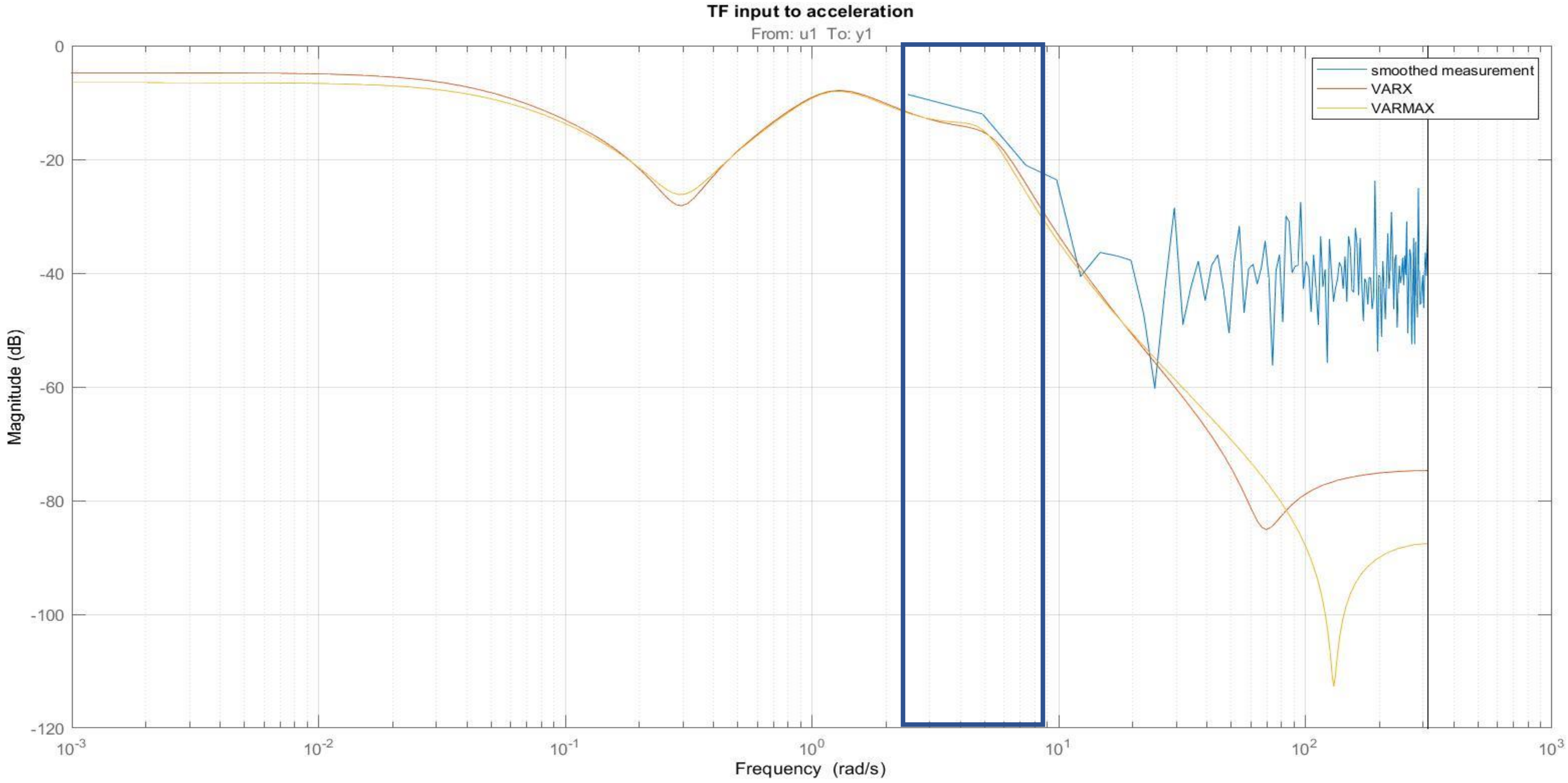
POLES OF IDENTIFIED MATRICE **A**:



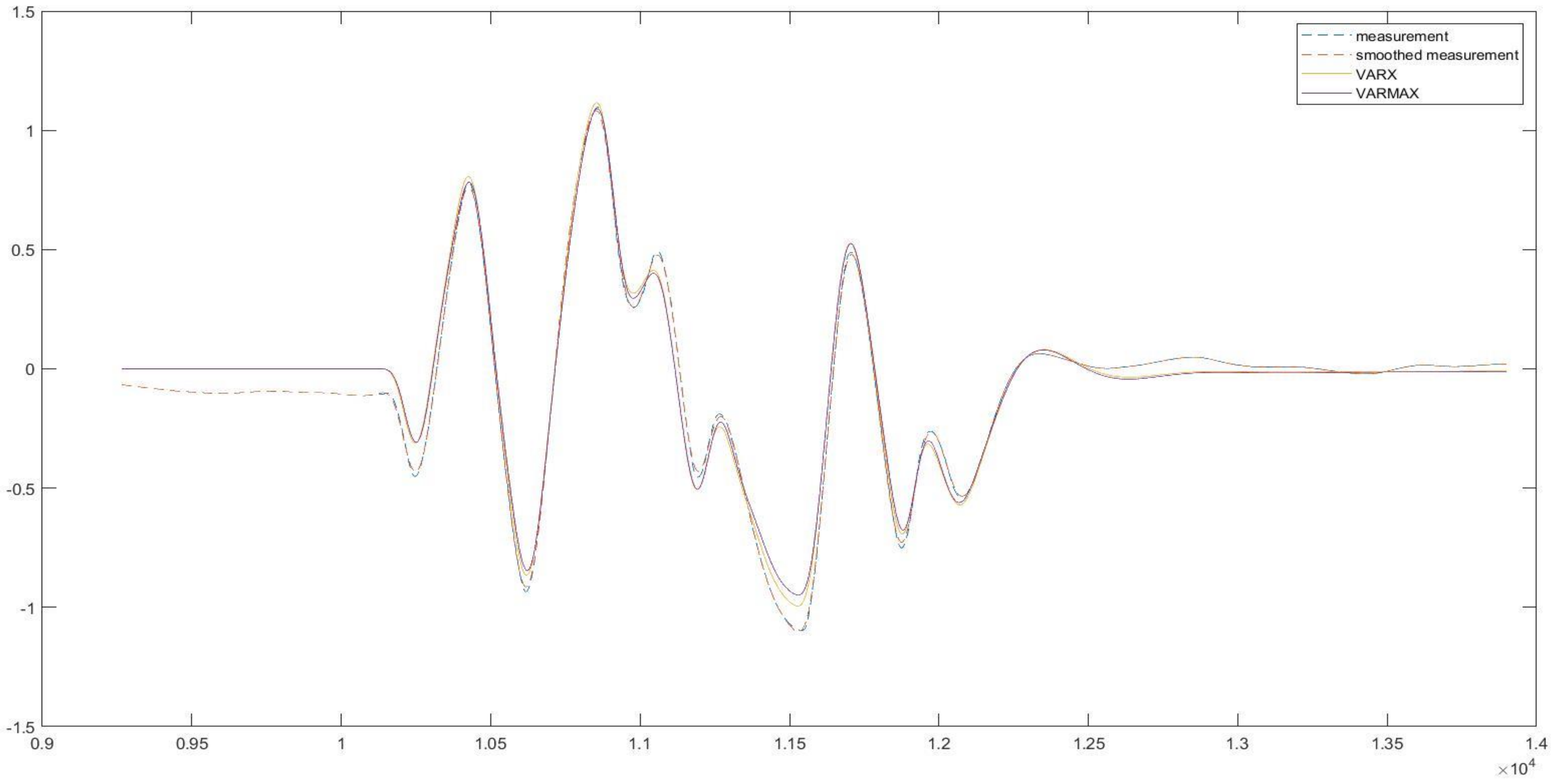
BODEMAG: INPUT TO POSITION (order 5, p = f = 15)



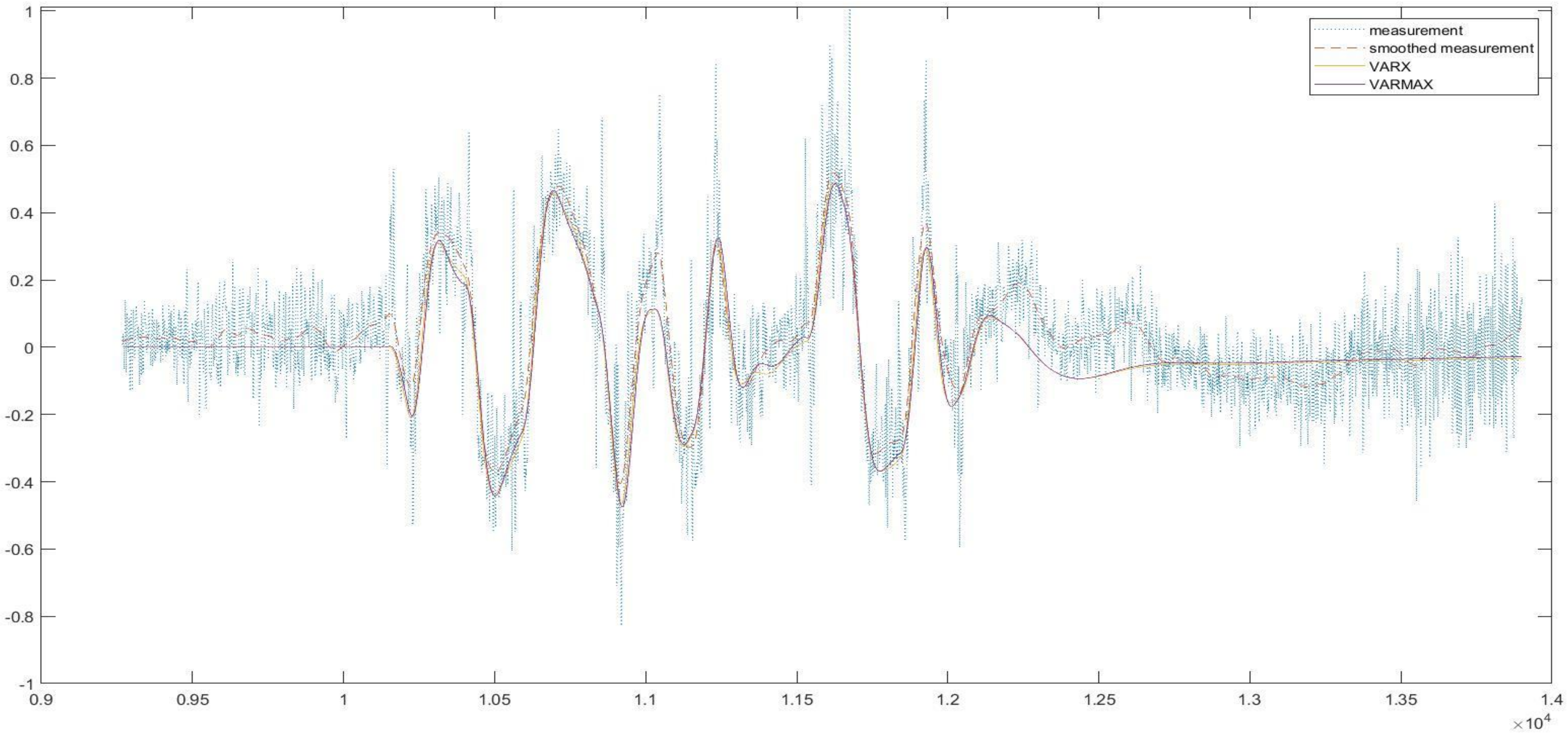
BODEMAG: INPUT TO ACCELERATION (order 5, p = f = 15)



VARX & VARMAX, order 5, p = f = 15 (position)



VARX & VARMAX, order 5, p = f = 15 (acceleration)



TASK 2: KALMAN FILTERING USING THE IDENTIFIED MODEL

DT-DT PREDICTOR/CORRECTOR FORM

Motion model: $\mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{G}_{k-1}\underbrace{\mathbf{u}_{k-1}}_{\text{input}} + \underbrace{\mathbf{w}_{k-1}}_{\text{noise}}$

Measurement model: $\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \underbrace{\mathbf{v}_k}_{\text{noise}}$

$\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k)$

Measurement Noise

$\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_k)$

Process or Motion Noise

1 Prediction

$$\check{\mathbf{x}}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{G}_{k-1}\mathbf{u}_{k-1}$$
$$\check{\mathbf{P}}_k = \mathbf{F}_{k-1}\hat{\mathbf{P}}_{k-1}\mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1}$$

2a Optimal Gain

$$\mathbf{K}_k = \check{\mathbf{P}}_k\mathbf{H}_k^T(\mathbf{H}_k\check{\mathbf{P}}_k\mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

2b Correction

$$\hat{\mathbf{x}}_k = \check{\mathbf{x}}_k + \mathbf{K}_k(\mathbf{y}_k - \mathbf{H}_k\check{\mathbf{x}}_k)$$
$$\hat{\mathbf{P}}_k = (\mathbf{1} - \mathbf{K}_k\mathbf{H}_k)\check{\mathbf{P}}_k$$

DT-DT PREDICTOR/CORRECTOR FORM

Where **F = A, G = B, H = C,** and **A, B,** and **C** are matrices for the identified model.

Since state estimates do not have any physical representations (because of PBSID), then we cannot recover velocity directly from state estimates. Therefore, we build it from measurement estimates and using a simple formula to obtain velocity.

$$\hat{\boldsymbol{y}} = \boldsymbol{H}\hat{\boldsymbol{x}} + \boldsymbol{D}\boldsymbol{u} = \begin{bmatrix} \hat{r} \\ \hat{a} \end{bmatrix} \longrightarrow \boxed{\hat{v}_t = \frac{\hat{r}_t - \hat{r}_{t-1}}{\Delta t}}$$

(backward difference)

DATA FOR KALMAN FILTERING

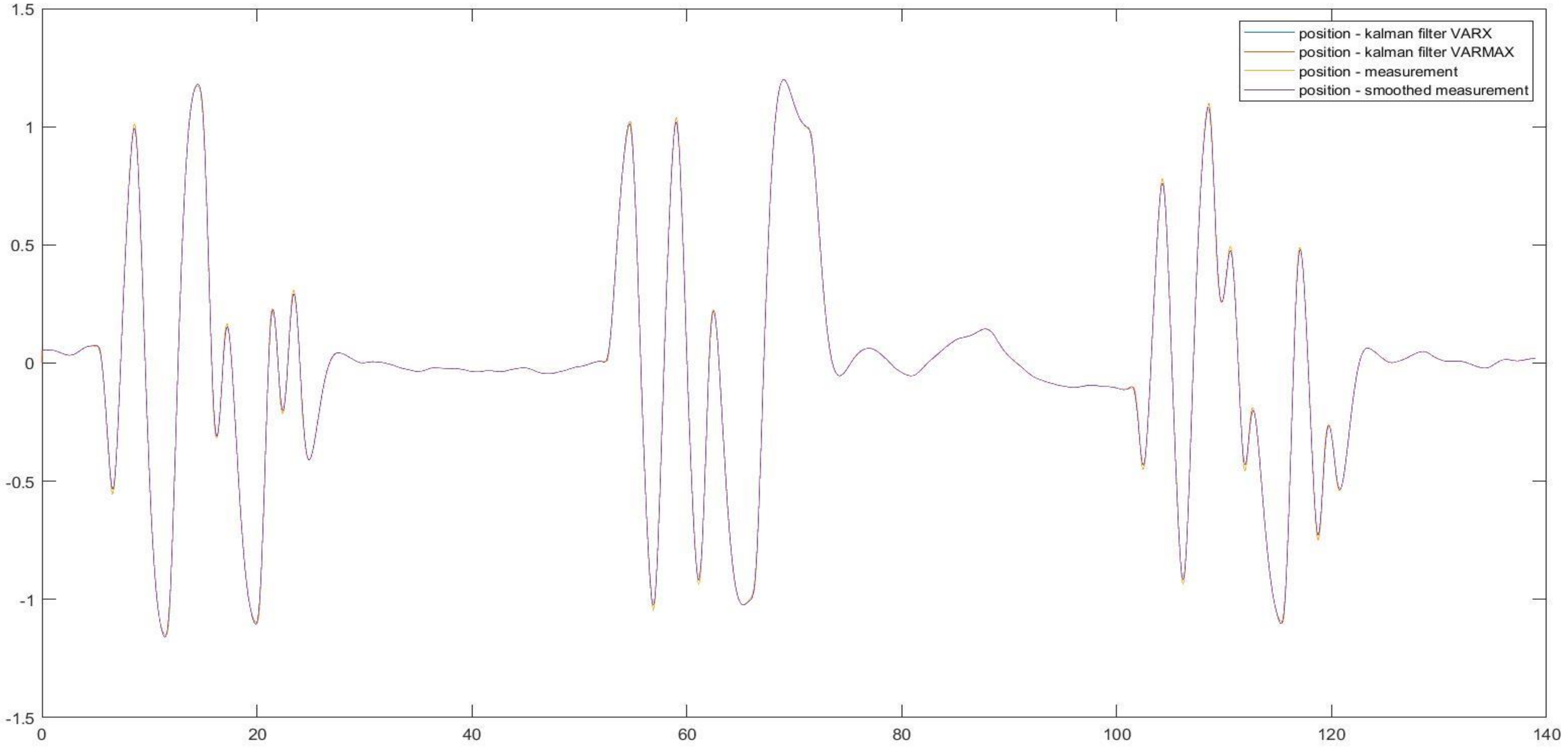
```
x0 = zeros(order,1);          % initial state
P = eye(order);               % initial state covariance
dt = 0.01;                    % time step

F = A;
G = B;
H = C;
Q = 0.1*eye(order);           % process noise
R = [1 0;0 1000];             % measurement noise
```

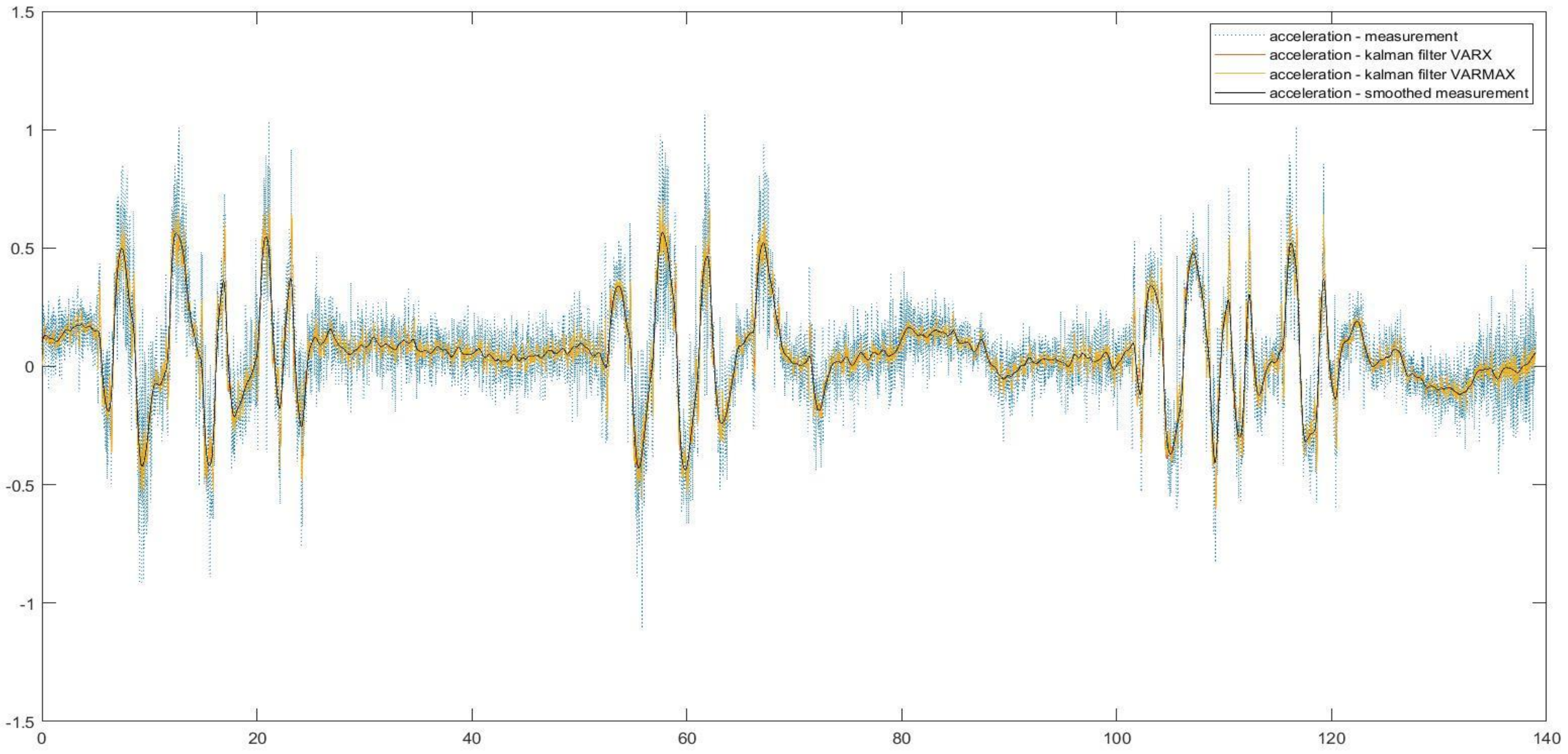
LOOPING PROCESS

- for i=2:length(tspan)
- xhat(:,1) = x0; % initial state
- %prediction
- xmin = F*xhat(:,i-1) + Bi*u(i-1);
- P = F*P*F'+Q;
- %correction
- K = P*H'*inv(H*P*H'+R);
- xhat(:,i) = xmin + K*(ym(i,:)'-H*xmin);
- P = (eye(order)-K*H)*P;
- %output estimate
- yhat(:,i) = H*xhat(:,i) + Di*u(i);
- %lateral velocity ned
- velocity(i-1) = (yhat(1,i)-yhat(1,i-1))/dt;
- end

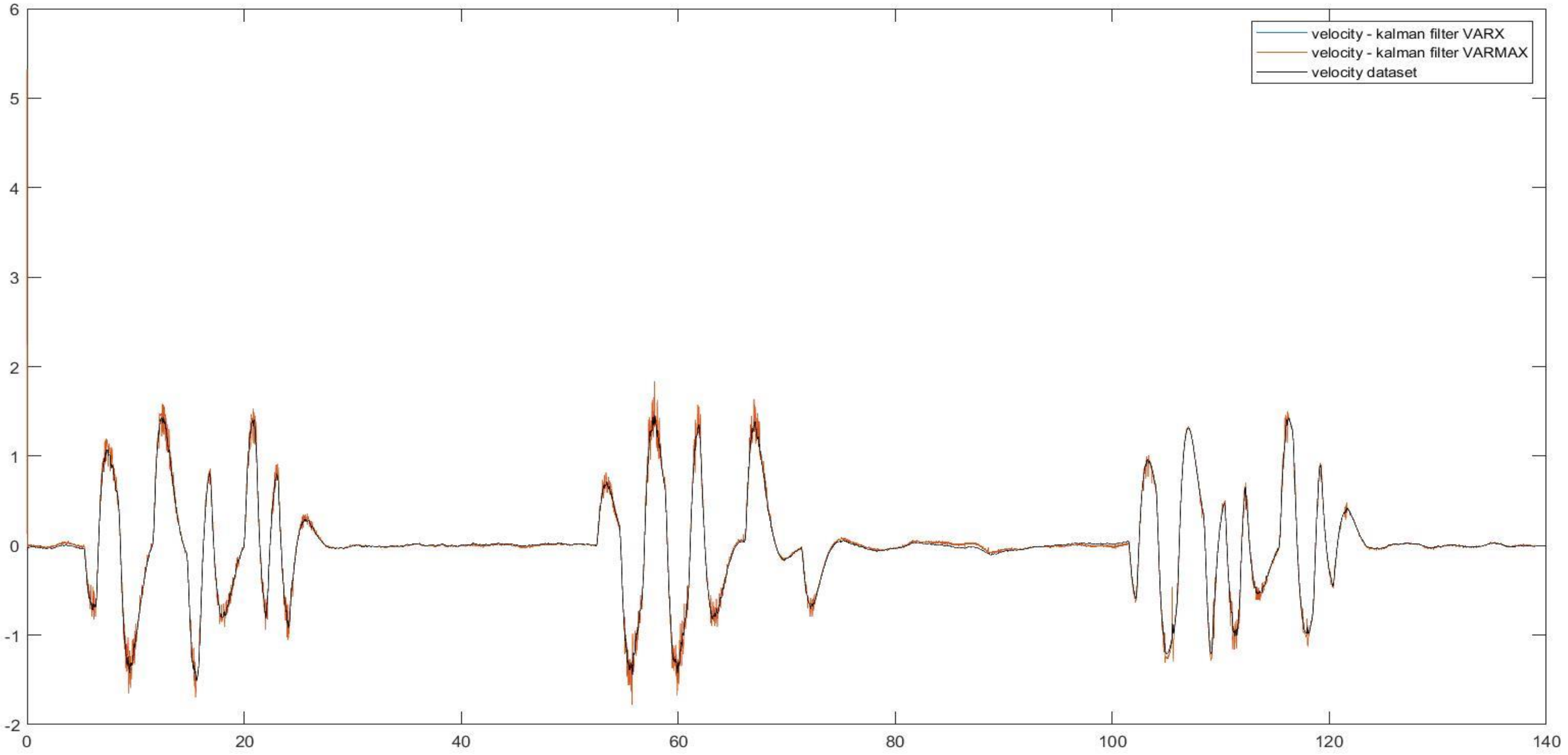
RESULT: POSITION ESTIMATES



RESULT: ACCELERATION ESTIMATES



RESULT: VELOCITY ESTIMATES



THANK YOU

REFERENCES

- Chiuso A. (2006).*The role of autoregressive modelling in predictor-based subspace identification*. Automatica 43 (2007) 1034-1048
- Chiuso A., Picci G. (2004). *Consistency Analysis of some closed-loop subspace identification methods*. Automatica 41 (2005) 377-391
- Wingerden J.W., Verhaegen, M. (2010). *VARMAX-based closed-loop subspace model identification*. IEEE Conference on Decision and Control.
- Del Cont, D., Giurato, M., Riccardi, F., and Lovera, M. (2017). *Ground effect analysis for a quadrotor platform*. In 4th CEAS Specialist Conference on Guidance, Navigation & Control.
- Katayama T. (2005). *Subspace Methods for System Identification*. Springer