A further analysis of the role of heterogeneity in coevolutionary spatial games

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HIGHLIGHTS

- Heterogeneity of link weights itself does not favour cooperation.
- Promotion of cooperation is actually induced by the utilities overlap.
- Abstention and link weights act as a mechanism to weaken the benefits of defecting.

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ABSTRACT

Heterogeneity has been studied as one of the most common explanations of the puzzle of cooperation in social dilemmas. A large number of papers have been published discussing the effects of increasing heterogeneity in structured populations of agents, where it has been established that heterogeneity may favour cooperative behaviour if it supports agents to locally coordinate their strategies. In this paper, assuming an existing model of a heterogeneous weighted network, we aim to further this analysis by exploring the relationship (if any) between heterogeneity and cooperation. We adopt a weighted network which is fully populated by agents playing both the Prisoner's Dilemma or the Optional Prisoner's Dilemma games with coevolutionary rules, i.e., not only the strategies but also the link weights evolve over time. Surprisingly, results show that the heterogeneity of link weights (states) on their own does not always promote cooperation; rather cooperation is actually favoured by the increase in the number of overlapping states and not by the heterogeneity itself. We believe that these results can guide further research towards a more accurate analysis of the role of heterogeneity in social dilemmas.

1. Introduction

Issues regarding the emergence of cooperation and altruism in structured populations have puzzled scientists in a large range of domains. In this context, methods of Statistical Physics combined with concepts of both Graph Theory and Evolutionary Game Theory [1,2] have been used as simple and powerful tools to describe and analyse the conflict of interest between individuals and groups [3]. In those models, agents are arranged on graphs in such a way that their interactions are restricted to their immediate neighbours [4,5]. Over the last two decades, it has been shown that different topologies such as lattices [6], scale-free graphs [7–9], small-world graphs [10,11], cycle graphs [12], star-like graphs [13] and bipartite graphs [14,15] have a considerable impact on the evolution of cooperation, which also favours the formation of different

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patterns and phenomena [16,17]. However, the vast majority of these studies adopt static networks, which are not suitable for modelling scenarios in which both the game strategies and the network itself are subject to evolution [18–23]. Thus, the use of dynamic networks represents a natural upgrade of the traditional spatial games [24].

The Prisoner's Dilemma (PD) is still the most often used game in this field. In this game, an agent can either cooperate (*C*) or defect (*D*), obtaining a payoff that depends on the other's agent choice [25]. However, in many scenarios, agents have the freedom to decide whether to participate in the game. Games such as the Optional Prisoner's Dilemma (OPD) [26,27] and the Voluntary Public Goods game [28,29] incorporate this concept of voluntary participation by adding a third strategy to the game, allowing agents to not only cooperate or defect but also to abstain (*A*) from a game interaction. Research has shown that the presence of abstainers in the population can actually protect cooperators against exploitation [30,31].

Studies on weighted networks have attracted much attention as such networks enable the representation of the strength of each connection, which is essential information in a wide range of real-world scenarios including biological networks and social media. Recently, both the Prisoner's Dilemma and the Optional Prisoner's Dilemma games have been explored in the context of dynamic weighted networks, which lead to a coevolutionary scenario where not only the game strategies, but also the link weights, evolve over time [32–36]. Moreover, it has been shown that the use of dynamic weighted networks can increase heterogeneity of states (i.e., the number of possible utilities in the network), which in turn induces the promotion of cooperation. In fact, previous work has also discussed the effects of heterogeneity on the evolution of cooperation [9,37–44], however, the specific conditions that increase the diversity of link weights in the dynamic weighted networks remain unclear. Also, a number of questions regarding the evolutionary dynamics of the network itself remain to be answered, such as:

- How the link weights between agents evolve over time?
- How two parameters of the model (Δ and δ) affect the link weight variance?
- Is there an optimum value of the two parameters Δ and δ?
- Why higher values of δ promote cooperation best?
- Why the Coevolutionary Optional Prisoner's Dilemma game performs better than the Coevolutionary Prisoner's Dilemma game in adverse scenarios?
- Does the value of Δ affect the convergence speed in scenarios of full dominance of cooperation?

Thus, this work aims to answer these questions by analysing the micro–macro behaviour of a population of agents playing both the Coevolutionary Prisoner's Dilemma (CPD) and the Coevolutionary Optional Prisoner's Dilemma (COPD) game, i.e., the classical PD and OPD games in a dynamic weighted network. The remainder of this paper is organized as follows. Section 2 describes the Monte Carlo simulation and the coevolutionary games adopted. Section 3 features the results. Finally, Section 4 summarizes our findings and outlines future work.

2. Methodology

This work adopts a weighted lattice grid with periodic conditions (i.e., a toroid) fully populated with $N = 102 \times 102$ agents playing a coevolutionary game. Each agent on site *x* interacts only with its eight immediate neighbours (i.e., $k = 8$, Moore neighbourhood). Both the Coevolutionary Prisoner's Dilemma (CPD) game [34] and the Coevolutionary Optional Prisoner's Dilemma (COPD) game [32] are considered.

Initially, each edge linking agents has the same weight $w = 1$, which will adaptively change according to their interaction. Also, each agent (*x*) is initially assigned to a strategy with equal probability. For the CPD game, each agent can be designated either as a cooperator ($s_x = C$) or defector ($s_x = D$), while in the COPD game, agents can also be designated as abstainer ($s_x = A$). Thus, strategies ($s_x = C, D, A$) can be denoted by a unit vector respectively as follows:

$$C = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \; D = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \; A = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{1}$$

The games are characterized by the payoff obtained according to the pairwise interaction of agent *x* and its neighbour *y*. Accordingly, the agent *x* may receive a reward $\pi_{xy}(C, C) = R$ for mutual cooperation; a punishment $\pi_{xy}(D, D) = P$ for mutual defection; $\pi_{xy}(D, C) = T$ for successful defection (i.e., there is a temptation to defect); $\pi_{xy}(C, D) = S$ for unsuccessful cooperation (well-known as the sucker's payoff); or the loner's payoff (*L*), which is obtained when one or both agents abstain (i.e., $\pi_{xy}(A, C) = \pi_{xy}(A, D) = \pi_{xy}(C, A) = \pi_{xy}(D, A) = \pi_{xy}(A, A) = L$). We adopt a weak version of both games, where the payoff $R = 1$, $T = b \, (1 < b < 2)$, $L = l \, (0 < l < 1)$ and $S = P = 0$ without destroying the nature of the dilemma [6]. Thus, the payoff matrix π is given by:

$$\pi = \begin{pmatrix} 1 & 0 & l \\ b & 0 & l \\ l & l & l \end{pmatrix}, \tag{2}$$

where:

$$\pi_{xy}(s_x, s_y) = s_x^T \pi s_y. \tag{3}$$

The utility u_{xy} of agent x with its neighbour y is calculated as follows:

$$u_{xy} = w_{xy}\pi_{xy},\tag{4}$$

where w_{xy} represents the symmetric link weight of their interaction, i.e., $w_{xy} = w_{yx}$.

A number of Monte Carlo (MC) simulations are carried out to explore the micro-macro behaviour of both the strategies and the weighted network itself. Each MC simulation comprises the following elementary steps. First an agent x is randomly selected to play the coevolutionary game with its $k = 8$ neighbours, obtaining an accumulated utility expressed as:

$$U_x = \sum_{y \in \Omega_x} u_{xy},\tag{5}$$

where Ω_x denotes the set of neighbours of the agent x. Second, the agent x updates all the link weights in Ω_x by comparing each utility u_{xy} with the average accumulated utility (i.e., $\bar{U}_x = U_x/k$) as follows:

$$w_{xy} = \begin{cases} w_{xy} + \Delta & \text{if } u_{xy} > \bar{U}_x, \\ w_{xy} - \Delta & \text{if } u_{xy} < \bar{U}_x, \\ w_{xy} & \text{otherwise,} \end{cases}\tag{6}$$

where Δ is a constant such that $0 \leq \Delta \leq \delta$. In line with previous work [32,34,45], the link weight is corrected to satisfy $1 - \delta \leq w_{xy} \leq 1 + \delta$, where δ ($0 \leq \delta < 1$) defines the weight heterogeneity. Note that when $\Delta = 0$ or $\delta = 0$, the link weight remains constant ($w = 1$), which decays in the classical scenario for static networks, i.e., only the strategies evolve. Finally, the agent x updates its strategy by comparing its current accumulated utility U_x (i.e., considering the updated weights) with the accumulated utility of one randomly selected neighbour (U_y) such that, if $U_y > U_x$ agent x copies s_y with a probability proportional to the utility difference as follows:

$$p(s_x \leftarrow s_y) = \frac{U_y - U_x}{k(T - P)},\tag{7}$$

otherwise, agent x keeps its strategy for the next step.

In one Monte Carlo step (MCS), each agent is selected once on average, which means that the number of inner steps in each MCS is equal to the population size. Simulations are run for a sufficiently long thermalization time (10^6 MCS). Furthermore, to alleviate the effect of randomness and to ensure proper accuracy in the approach, the final results are obtained by averaging 10 independent runs. It is noteworthy that due to the introduction of the weight factor (w) and the quenched heterogeneities via δ, the model is prone to evolve into frozen patterns which represent quenched spatial randomness where a Griffiths phase [46] can emerge. As has been discussed in previous studies [40,47], the evolutionary dynamics in these scenarios tend to be very slow, which introduces some technical difficulties in classifying the final stationary state. This is because the transition of the clusters of the subordinate strategy into the dominant strategy requires that a large number of the subordinate agents swap their strategies in a short period of time, which is an occurrence that is very difficult.

3. Results

In this section, we present some of the relevant experimental results obtained when simulating a population of agents playing both the Prisoner's Dilemma and the Optional Prisoner's Dilemma game on weighted networks.

3.1. Exploring the coevolutionary rules

As discussed in previous research [32], one interesting property of the ratio Δ/δ is that for any combination of both parameters Δ and δ, if their ratio is the same, then the number of states is also the same. For instance, the pairs ($\Delta = 0.02$, $\delta = 0.2$) and ($\Delta = 0.08$, $\delta = 0.8$) have both 21 possible link weights (states).

Despite the fact that the interval between the maximum and minimum link weights increases as we increase δ, intuition may lead us to believe that given two scenarios with the same payoff matrix (i.e., Eq. (2) for the same temptation to defect and loner's payoff) and the same number of states (i.e., the same ratio Δ/δ), the outcome would be the same. Surprisingly, previous research has shown that it is not true [32,34]. Actually, it has been discussed that higher values of δ promote cooperation best, even if the number of states remains the same. Fig. 1 illustrates this scenario, where the average fraction of cooperation of the last 10^3 steps for a wider range of settings (i.e., the ratio Δ/δ, the loner's payoff l and the temptation to defect b) are considered for different values of δ (i.e., $\delta = \{0.2, 0.4, 0.8\}$). In fact, the outcomes of the different scenarios are very different to each other and we can observe that as δ increases, the number of cases in which cooperation is the dominant strategy also increases. This phenomenon is still unexplained and exploring the properties that cause this discrepancy may lead to a complete understanding of the presented coevolutionary model. In any case, it is noteworthy that even for small values of δ, the Coevolutionary Optional Prisoner's Dilemma (COPD) game is still much more beneficial for the emergence of cooperation than the traditional OPD [30] or the Coevolutionary Prisoner's Dilemma (CPD) [34].

As is also shown in Fig. 1, we investigate the influence of the ratio Δ/δ in the link weight variance (i.e., link weight heterogeneity) at the last Monte Carlo steps. Note that as the link weight (w) is always within the range $[1 - \delta, 1 + \delta]$, the
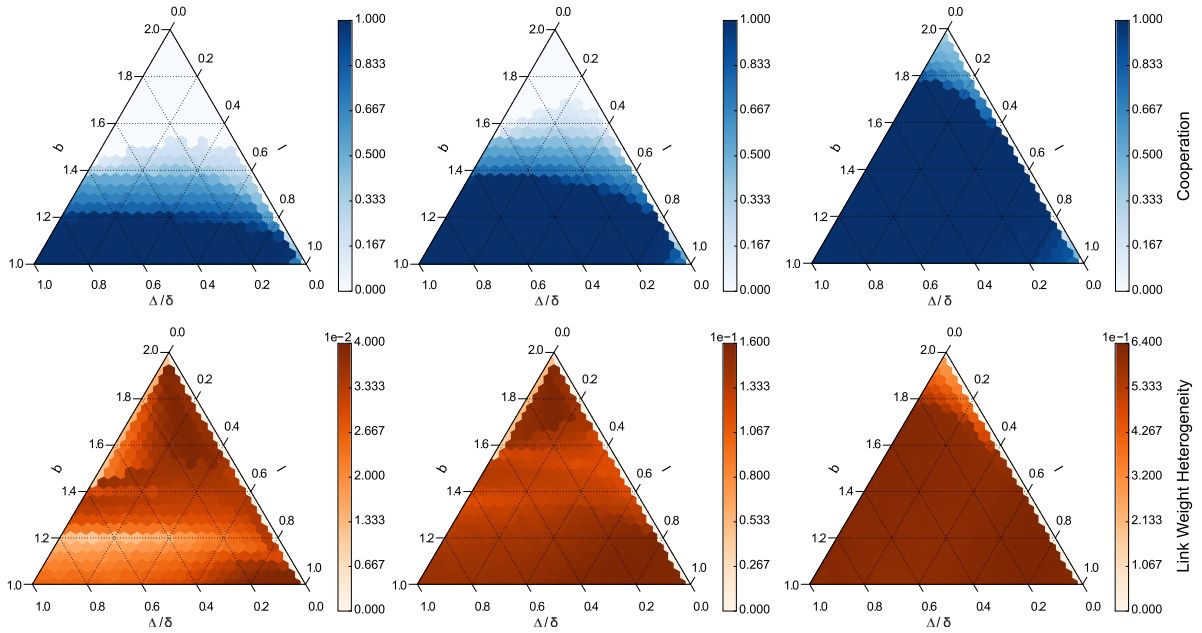
Fig. 1. Ternary diagrams showing the average fraction of cooperation (top) and the link weight heterogeneity (bottom) for different values of b (i.e., temptation to defect), l (i.e., the loner's payoff) and Δ/δ for $\delta = \{0.2, 0.4, 0.8\}$ (from left to right). The diagrams allow us to investigate the effects of different parameter settings for fixed values of δ. The diagrams at the top suggest that higher values of δ may promote cooperation best, while the ones at the bottom show that the correlation between cooperation and heterogeneity does not seem to hold for all values of δ.
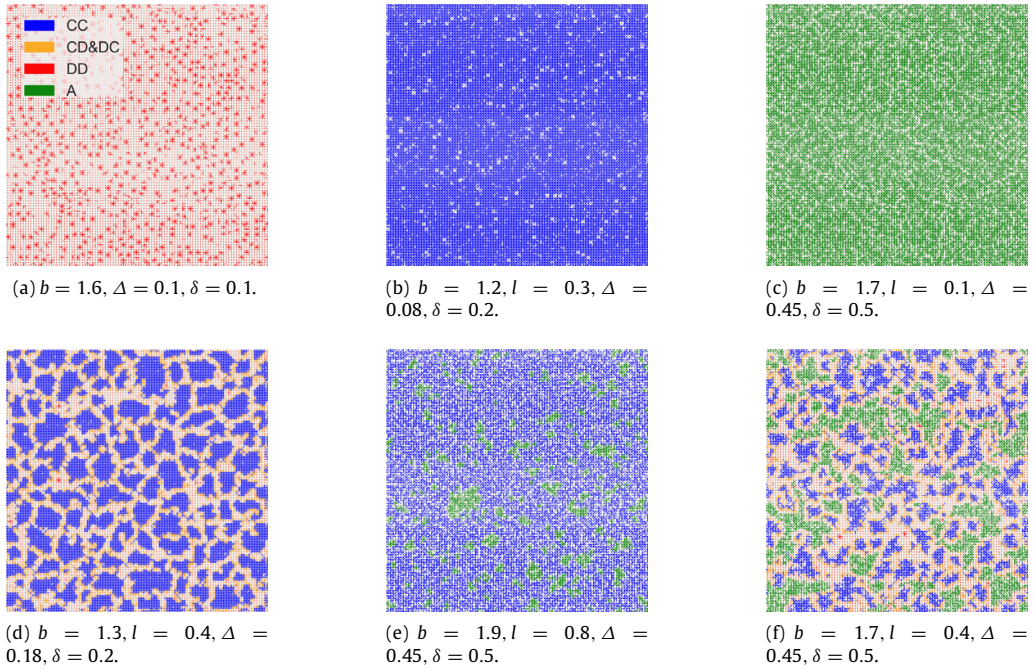
maximum link weight variance is defined by δ^2. These experiments reveal that the link weight variance is not uniform for all environmental settings. Also, although higher values of δ promote higher heterogeneity, we can see that the correlation between cooperation and heterogeneity is not necessarily true for all values of δ. For instance, the link weight variance for $\delta = 0.8$ is usually maximum when cooperation is the dominant strategy. However, this does not hold for both $\delta = 0.4$ and $\delta = 0.2$.

Fig. 2 shows the typical distributions of the pairs of strategies (i.e., the edges) in the stationary or quasi-stationary states. In these snapshots we use colours to differentiate the types of edges and opacity to differentiate the link weights, where, for each scenario, a transparent edge means that the weight is at minimum (i.e., $w = 1 - \delta$) and a bright edge means that the weight is at maximum (i.e., $w = 1 + \delta$). As expected, considering that $l > 0$, the pattern *all D* (Fig. 2a) is only possible in the CPD game. Moreover, for the CPD game, it is also possible to observe the patterns *all C* (Fig. 2b) and *C+D* (Fig. 2d). For the COPD game, all other patterns are also possible, i.e., *all A* (Fig. 2c), *C + A* (Fig. 2e) and *C + D + A* (Fig. 2f) phases can also be observed. Of course, the size of the clusters and the average link weight at the stationary state will depend on the parameter settings. However, for any scenario, it was observed that the population always evolves to one of these patterns. Further analysis on the effects of varying the parameter settings have been shown in previous studies [32–34].

Another interesting result is that, although previous research has claimed that "intermediate link weight amplitude can provide best environment for the evolution of cooperation" [34], our experiments reveal that there is no global optimal value of Δ/δ (defined by Huang et al. [34] as the link weight amplitude) nor δ for all environmental settings. Moreover, despite the fact that high δ usually leads to more cooperation, it does not mean that high δ is always the best option. Fig. 3, for example, illustrates a scenario in which high δ is actually a bad choice. In fact, as already expected (Section 2), in many cases it is possible to observe that the population evokes the existence of Griffiths-like phases, which makes it very difficult for the system to converge to a stationary state. For instance, the population evolves into a frozen pattern in the scenarios shown in Fig. 2d and Fig. 2e; moreover, the curves for $\{\Delta = 0.45, \delta = 0.5\}$, $\{\Delta = 0.63, \delta = 0.7\}$ and $\{\Delta = 0.81, \delta = 0.9\}$ in Fig. 3 are also evidence of the same technical difficulties in classifying the stationary state. Note that, in some scenarios, the presence of the cyclic dominance for the COPD (i.e., coexistence of the three strategies as observed in Fig. 2f) may eliminate the emergence of the frozen patterns when the population has only two strategies, i.e., $C + D$ or $C + A$. This phenomenon has been discussed in the literature of evolutionary games [3,40,47]. Furthermore, the dynamical behaviour observed in Fig. 3 illustrates the nature of enhanced network reciprocity [48] promoted when $\delta > 0.1$. In these scenarios, we can see that defectors are dominated by abstainers, allowing a few clusters of cooperators to survive; as a result of the absence of defectors, cooperators invade most (or all) of the abstainers in the population, which explains the initial drop, and the subsequent recovery, of the fraction of cooperators in Fig. 3. Similar behaviour has also been observed in previous work [42].

These results motivate the search for a better understanding of the evolutionary dynamics of the link weights. In the following sections, we will discuss how the link weights evolve over time.

(a) $b = 1.6, \Delta = 0.1, \delta = 0.1$.

(b) $b = 1.2, l = 0.3, \Delta = 0.08, \delta = 0.2$.

(c) $b = 1.7, l = 0.1, \Delta = 0.45, \delta = 0.5$.

(d) $b = 1.3, l = 0.4, \Delta = 0.18, \delta = 0.2$.

(e) $b = 1.9, l = 0.8, \Delta = 0.45, \delta = 0.5$.

(f) $b = 1.7, l = 0.4, \Delta = 0.45, \delta = 0.5$.

Fig. 2. Typical distribution of edges after a sufficiently long thermalization time (10^6 MCS). The opacity of each edge represents its current weight (w) in the network and is adjusted to be transparent at minimum (i.e., $w = 1 - \delta$) and bright at maximum (i.e., $w = 1 + \delta$). The pattern (a) is only possible for the CPD game, which can also exhibit the patterns (b) and (d). The patterns (b), (c), (d), (e) and (f) can be observed in the COPD game.
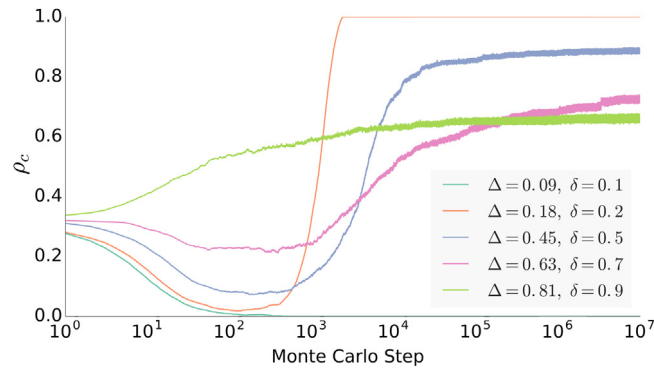

Fig. 3. Time course of the fraction of cooperation for different values of Δ and δ when $b = 1.9, l = 0.8$ and $\Delta/\delta = 0.9$. Contrary to what Fig. 1 may suggest, here we see that high δ is not always the best option to promote cooperation.

3.2. Understanding how the link weights evolve

In order to better understand how the link weights between agents evolve over time, we investigate the distribution of link weights for different values of b (temptation to defect), Δ and δ, for both the Coevolutionary Prisoner's Dilemma (CPD) [34] and the Coevolutionary Optional Prisoner's Dilemma (COPD) [32] games, where the latter also involves the variation of the loner's payoff (l).

Fig. 4 shows the distribution of link weights for each type of agent interaction when $b = 1.6, l = 0.2, \Delta = 0.2$ and $\delta = 0.8$ which is representative of the outcomes of other values as well. As discussed previously, we know that the ratio Δ/δ can be used to determine the number of link weights that an agent is allowed to have, which is actually evidenced when the link weight distribution is plotted over time. Despite the fact that the percentage of each type of link varies according to factors such as the total number of states and the value of b and l (for the COPD game), which will consequently affect the final outcome, it was observed (as shown in Fig. 4) that for any ratio Δ/δ the initial dynamics of all types of links is exactly the same for both games, that is:
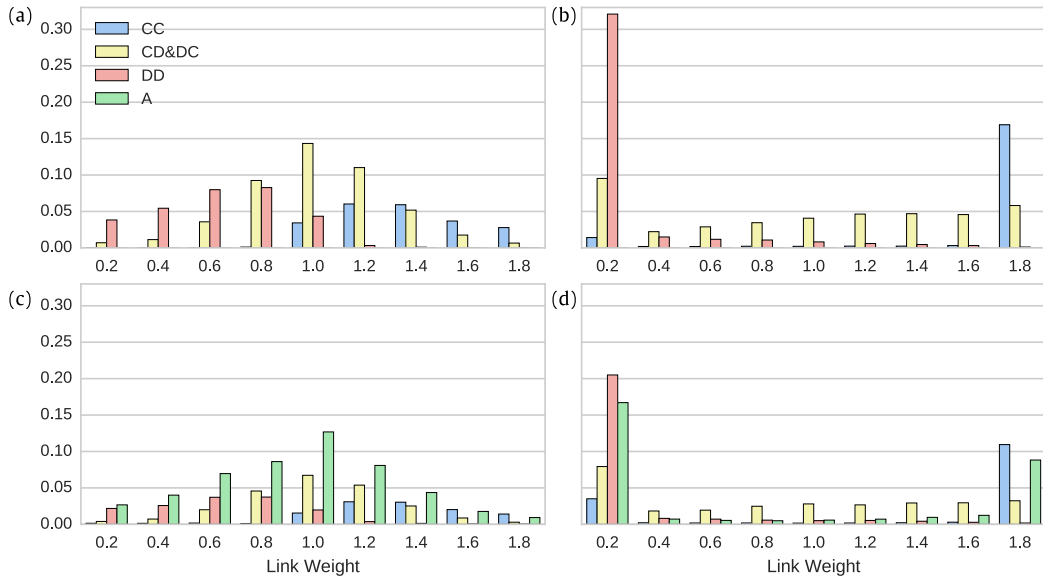
Fig. 4. Distribution of link weight at the Monte Carlo steps 1 (left) and 5 (right) for a population of 102×102 agents playing the Coevolutionary Prisoner's Dilemma game (top) and the Coevolutionary Optional Prisoner's Dilemma game (bottom) for $l = 0.2$, $b = 1.6$, $\Delta = 0.2$ and $\delta = 0.8$. It was observed that the initial behaviour of all types of links is the same for both games regardless of the ratio Δ/δ, i.e., the initial link weight distribution looks the same for all scenarios, where the only difference is in the proportion of each type, which depends on the values of all parameter settings.

- **Observation 1** Defector–Defector (DD) tends to move to states of lowest link weight.
- **Observation 2** Cooperator–Defector (CD) and Defector–Cooperator (DC) move to the extremes, keeping a small amount of intermediate states.
- **Observation 3** Cooperator–Cooperator (CC) tends to move to states of highest link weight, but will also occupy the state of lowest link weight as the DCs and ACs will eventually become CCs.
- **Observation 4** Abstainers (AC, AD, CA, DA or AA) move to the extremes.

Considering that utility is obtained by the product of link weight and payoff (Eq. (4)), and that the payoff of DD and CD is equal to zero (Eq. (2)), the utility (u_{xy}) associated with these link types will always be equal to zero, which is always the worst case as $u_{xy} \geq 0.0$. In this way, these agents will always be punished by Δ (Eq. (6)) and consequently, occupy states of lowest link weight (Observation 1).

Also, note that CD and DC are unstable configurations as the first will always get $u_{xy} = 0.0$ and the second is prone to get higher utilities as the temptation to defect (b) is always the highest payoff (Eq. (2)). Thus, these agents are constantly receiving $\pm\Delta$, which explains the phenomenon of having a small number of them along the intermediate states (Observation 2).

Although the payoff obtained by mutual cooperators (CC) is smaller than the one obtained by a defector–cooperator interaction (DC), i.e., $T > R$, the mutual cooperators are much more stable than DCs as both agents always get the same payoff (i.e., R). That is exactly the reason why these agents tend to a maximum link weight (Observation 3). Also, note that as the link weight is updated based on the comparison of the local utility of each connection with the average utility of the eight neighbours, when a cluster of nine cooperators is formed (i.e., one cooperator surrounded by eight cooperators), their links will remain in equilibrium, where the average link weight will tend to the value of R. In this way, for most scenarios of full dominance of cooperation, approximately half of the links will have a minimum weight and the other half will have a maximum weight.

It is noteworthy that we count all types of Abstainer's connections (AC, CA, AD, DA and AA) together because in the Optional Prisoner's Dilemma game, when a agent abstains, both agents receive the same payoff (l). In this way, the main reason why abstainers move to the extremes in Fig. 4 is that DD and CD agents ($u_{xy} = 0.0$) tend to abstain to increase their local utility ($u_{xy} > 0.0$) becoming ADs, which consequently are allocated in states of lowest link weight. For the same reason, abstention might be the best option in mixed clusters of C's and D's, where the chances of getting $u_{xy} = 0.0$ increases, then the agents may tend to abstain, eventually going to states of highest link weight (Observation 4).

Moreover, we point out that, as we force all link weights to be within the range $1 - \delta$ to $1 + \delta$, the phenomenon of having more agents occupying the maximum and minimum states is clearly expected. However, the observation of the initial dynamics of both games being the same for any combination of the parameters (i.e., b, l, Δ and δ) is a counter-intuitive result, which in turn shows that the observations discussed above are valid for both models.
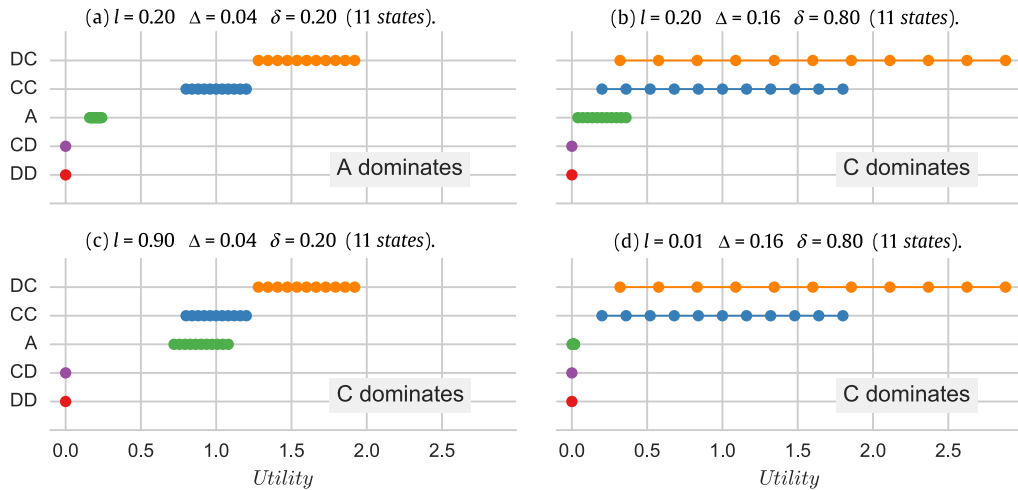
Fig. 5. Possible utilities (i.e., the product of payoff and link weight) for each type of edge, i.e., *Defector–Cooperator* (DC), *Cooperator–Cooperator* (CC), *Cooperator–Defector* (CD), *Defector–Defector* (DD) and the ones with one or two *Abstainers* (A). We observe that there is a connection between the overlap of possible utilities and the final outcome.

3.3. Investigating the role of heterogeneity

The reason why higher values of δ promote cooperation best remains one of the central open questions in this model. Based on the results discussed in previous sections, we know that the link weights usually evolve heterogeneously, which makes the effective payoff matrix unpredictable, adding a new layer of complexity to the model. For instance, in the traditional Prisoner's Dilemma game, any defector who plays with a cooperator will always get the value of the constant b; however in the coevolutionary model, this is unpredictable and heterogeneous as each defector–cooperator interaction might be in a different state.

Considering that the boundary states and the set of possible link weights is determined by the parameters Δ and δ, we can calculate all the possible utilities for each type of edge (i.e., CD, DC, DD, CC and A), which may allow us to better understand how the parameter settings affect the interplay between the evolution of strategies and their possible utilities. In this way, Fig. 5 shows the shape of all possible utilities for four different scenarios, all for the same temptation to defect ($b = 1.6$) and the same number of states ($\Delta/\delta = 0.2$, i.e., 11 states). Monte Carlo simulations revealed that cooperation is the dominant strategy in the scenarios of Fig. 5b-d; and that abstention dominates in Fig. 5a.

In fact, when we plot the possible utilities side by side (Fig. 5) we can see that the outcomes obtained through Monte Carlo simulations were actually expected. For instance, in Fig. 5a, the DC connections will always be the most profitable option in the initial steps, which in turn make the population of cooperators die off. After that, with the lack of cooperators in the population, DC is not possible anymore and abstention starts to be the best option as its payoff is always greater than the punishment for mutual defection (i.e., $lw_a > 0$).

However, notice that when the value of δ is increased (i.e., Fig. 5b), the overlap between the possible utilities for each type of connection also increases. In this case, we see that DC is the best option only in 5/11 of the cases, which enable cooperators to survive and as DC tends to minimum, CC will tend to maximum and abstention is sometimes better than DC (Section 3.2). Thus, the dominance of cooperators is also expected. We can also observe that due to the huge overlap of DCs and CCs, even if the loner's payoff is very low, i.e., Fig. 5d, or if abstention does not exist (i.e., CPD game), cooperation would still be expected.

Moreover, Fig. 5c illustrates interesting evidence of how abstention can support cooperation. The only difference between this scenario and Fig. 5a is the value of l. At a first glance, intuition may lead us to believe that if Fig. 5a with $l = 0.2$ resulted in full dominance of abstainers, increasing the value of l would just make the option to abstain more profitable, which consequently would not change the outcome. Surprisingly, this does not occur. Actually, abstainers only dominate the whole population when the population of cooperators is decimated. In this way, despite the fact that DC is still the best option and that the population of cooperators tend to decrease, they will not die off. Then, when the population of defectors become too high, they will prefer to turn into abstainers and with the increase of abstention, mutual cooperation will now be the best option, which allows abstainers to fully dominate the environment.

Thus, results show that if any of the possible DC, CC and A utilities do not overlap, then abstention will be the dominant strategy for the COPD game and defection will be the dominant strategy for the CPD game, except of course, when b is too low (i.e. $b < 1.1$), which usually promotes the coexistence of the available strategies. In general, we can observe that the greater the overlap between DC and CC utilities or/and CC and A utilities, then the more chances cooperators have to survive and dominate.

Notice that both the loner's payoff (l) and the link weights (which are controlled by the parameters Δ and δ) are actually mechanisms to weaken the benefits of defecting (i.e., effective utility of DC). As the parameter δ will act in the expansion of the utility boundaries, the greater the value of δ, the greater are the number of cases in which CC overlaps DC, which in turn promote cooperation best. The same scenario occurs when CC and A overlap, which will work as an extra mechanism to strengthen cooperators. That is, when the overlap of DC and CC is scarce or absent, overlapping CC and A can help cooperators to survive. This also explains why COPD is better than CPD in adverse scenarios [32].

The drawback of a large overlap of utilities is that the population may evolve into a frozen pattern in which a Griffiths-like phase can occur. In these scenarios, it might be very difficult to reach the full dominance of cooperative behaviour (Fig. 3). Thus, higher values of δ may promote cooperation best in a wider range of scenarios, but it might evoke the presence of frozen patterns of $C + A$. It is noteworthy that as the utility overlaps are also dependent of the values of b and l, all parameter settings may, in fact, influence the emergence of these frozen patterns.

4. Conclusions

This work investigates the role of heterogeneity in a population of agents playing the Prisoner's Dilemma (PD) game and the Optional Prisoner's Dilemma (OPD) game on a weighted square network with boundary conditions. Coevolutionary rules are adopted, enabling both the game strategies and the network to evolve over time, leading to the so-called Coevolutionary Prisoner's Dilemma (CPD) and the Coevolutionary Optional Prisoner's Dilemma (COPD) games respectively. A number of Monte Carlo simulations are performed in which each agent is initially assigned to a strategy with equal probability (i.e., random initial distribution of strategies). Echoing the findings of previous research [32], we show that independently of the link weight heterogeneity, the COPD game is still much more beneficial for the emergence of cooperation than the traditional OPD or the CPD games. Moreover, although previous research has claimed the opposite [34], we show that there is no global optimal value of the parameters Δ and δ for all environmental settings.

Experiments revealed that the correlation between the emergence of cooperation and heterogeneity does not hold for all scenarios, indicating that heterogeneity itself does not favour cooperation. Actually, it was observed that the higher the heterogeneity of states, the greater the chance of overlapping states, which is the actual mechanism for promoting cooperation. Namely, when considering the COPD game, if any of the possible *Defector–Cooperator* (DC), *Cooperator–Cooperator* (CC) and *Abstention* (A) utilities do not overlap, then abstainers dominate the environment; while for the CPD game, defection will be the dominant strategy. In general, we observed that the greater the overlap between DC and CC utilities or/and CC and A utilities, the more chances cooperators have to survive and dominate.

Finally, we highlight that both the loner's payoff and the link weights are actually mechanisms that weaken the benefits of defecting. In addition, abstention also works as an extra mechanism to strengthen cooperators, which explains why COPD is better than CPD in adverse scenarios. We believe that it might be possible to analytically define, through the analysis of utility overlap, which is the best value of δ for a given payoff matrix. Also, considering this model for regular graphs, it might be interesting to consider pair approximation techniques to describe the evolutionary dynamics of weighted networks [49,50]. To conclude, this paper provides a novel perspective for understanding cooperative behaviour in a dynamic network, which resembles a wide range of real-world scenarios. We hope this paper can serve as a basis for further research on the role of utility overlap to advance the understanding of the evolution of cooperation in coevolutionary spatial games.

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