Physics – Higher Level

Internal Assessment

Topic – Investigating the effect of concentration of sucrose solution on the critical angle at which Total Internal Reflection occurs

Research Question: How does varying the concentration of sucrose solution affect the critical angle at which total internal reflection occurs to trap light within a circular region?

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1. Introduction

I was inspired to perform this investigation regarding Total Internal Reflection (TIR) due to a phenomenon I studied in one of my other subjects. As a Computer Science student, I came across optic fibers in one of our class discussions where we discussed the working of optic fibers and TIR was a guiding principle for the function of these cables. Being curious about this, I researched more on optic fibers and TIR and I found that TIR in the working of optic fibers can occur regardless of the shape and size of the object it is passing through and this intrigued me. This led me to think about the differences in light traveling through optic fibers because different wires have different densities. This prompted me to formulate my research question, on which this internal assessment is based.

1.1 Research Question

How does varying the concentration of sucrose solution affect the critical angle at which total internal reflection occurs to trap light within a circular region?

2. Background Information

The critical angle of light entering a substance can be defined as the angle of incidence beyond which TIR occurs. When all incident light is reflected off the boundary, this is referred to as total internal reflection. There must be two conditions in place for TIR to occur:

1. A light ray is entering a less dense medium (rarer medium) from a denser medium.
2. The angle of incidence is greater than or equal to the critical angle.

When the critical angle is reached, at that instance, the refracted light will travel 90º to the normal alongside the boundary (assuming a flat boundary). Any angle greater than the critical angle will result in TIR.

The critical angle is essentially defined as the angle of incidence for which the angle of refraction is 90º. As a result, Snell's Law can be used to determine the relationship between refractive indices, angle of incidence (**critical angle**), and angle of refraction in order to calculate the critical angle.

The critical angle will vary depending on the combination of materials used as the rarer and denser medium. Since light travels at different speeds in different mediums, it is fair to assume that a change in the medium will result in different refractive indices, changing the critical angle required for a 90º refraction.

Considering two mediums: medium A and medium B of refractive indices n_1 and n_2 respectively, and a critical angle $_i$ for which the angle of refraction $_r$ is 90º, we can calculate the critical angle. If these values are substituted into Snell's Law, the following general relationship can be used to determine the critical angle for any two given mediums:

$$n_i \times \sin(\theta_{i\,(critical)}) = n_r \times \sin(\theta_r)$$

$$n_i \times \sin(\theta_{critical}) = n_r \times \sin(90°)$$

$$\sin(\theta_{critical}) = \frac{n_r}{n_i}$$

$$\theta_{critical} = \sin^{-1}\frac{n_r}{n_i}$$

For the mentioned equation to be used, n_r/n_i needs to have a value of less than one, otherwise, the sine inverse will not exist. Therefore, the light must travel from a denser medium of higher refractive index to a rarer medium of lower refractive index.

For this experiment, I considered opting for salt or other soluble substances instead of sugar in the solution between the Petri dishes. However, because sugar was more appropriate for the methodology, I resorted to using it. This is because, after doing some research, I found that the volume of sugar that would dissolve in 150ml of water is around 270g, whereas salt was only 52.5g. To create different concentration conditions while keeping the volume of water constant, the 52.5g of salt would have to be divided into different ratios. This would be difficult and time-consuming, as a small change in the mass of salt can cause a large difference in the concentration of the salt solution. Because a large amount of sugar is dissolved in 150ml of water, these problems are eliminated when sugar is used.

3. Hypothesis

The hypothesis for this experiment can be reasonably inferred to be that the concentration and the critical angle have a negative correlation. This is because the critical angle decreases with an increase in concentration.

4. Variables

4.1 Independent Variable

The **independent variable** in the experiment is the concentration of the sucrose solution which is measured in grams of granulated sugar per milliliter of distilled water.

4.2 Dependent Variable

The **dependent variable** in the experiment is the angle past which TIR; the critical angle. This is measured in degrees.

4.3 Controlled Variables

1. Volume of Water – the volume of water needs to be controlled as a change in those results in a change in the level of concentration of sugar. A standard level of 150ml was used as the solubility level depends on the volume of water and for the range of 0-125g of sugar used, a water volume level of 150ml is appropriate and balanced.
2. Temperature – the temperature of the room and water needs to be controlled as a change in temperature can have a cascading effect on the concentration and an increase in temperature could decrease the solution’s concentration which would affect
3. Laser Wavelength – wavelength can affect the angle of refraction, therefore, changing the wavelength will change the results obtained.

5. Apparatus

1. Granulated Sugar
2. A3 Sheet of Paper
3. Protractor
4. Red Light Laser
5. Spatula
6. Stirrer
7. Electronic Weighing Scale
8. Small Petri Dish
9. Big Petri Dish
10. Thermometer
11. Ring Stand and Utility Clamp
12. Stabilizing Wooden Block

6. Safety, Environmental and Ethical Issues

1. As a laser was used, the space used to experiment was directed towards the wall and cornered off to ensure the laser beam doesn’t get in a person’s eye causing possible eye damage.
2. The Petri dishes or the beaker may shatter when moving them around. To protect oneself from this, ensure to wear goggles to ensure that no glass splints hits our eyes in case the glass equipment breaks.
3. It is to be noted that there were no environmental or ethical issues faced during this experiment as no toxic chemicals or living organisms were used or involved.

7. Procedure

Firstly, before we start the experiment we have to make the modified setup using the big and small Petri dishes. For this, we need to mark and align the center of the dishes to make a cylindrical shape between them.

7.1 Setting up the sugar solution and Experiment

1. Fill 6 beakers with 150ml of water (measured on the electronic mass scale)
2. Add sugar in the 6 glasses starting from 0g and increased by 25g in every subsequent beaker (0g to 125g)
3. Stir each beaker with the stirrer till the sugar in each beaker is completely dissolved.
4. While the solution is being stirred setup the Petri dishes on the stabilizing blocks with the sheet of paper under for getting readings
5. Position the ring stand at its initial place which is perpendicular to the petri dish.

7.2 Experiment

1. Fill the petri dish setup with the sucrose solution from the beaker and turn on the laser.
2. Move the ring stand from the starting point till TIR can be observed.
3. Draw a line for the critical angle measured and calculate it using a protractor.
4. Empty the petri dish setup and repeat all the steps for the concentrations of sugar (25g, 50g, 75g, 100g, 125g)
5. Repeat steps 1 to 4, for 4 more trials to have 5 trials in total.

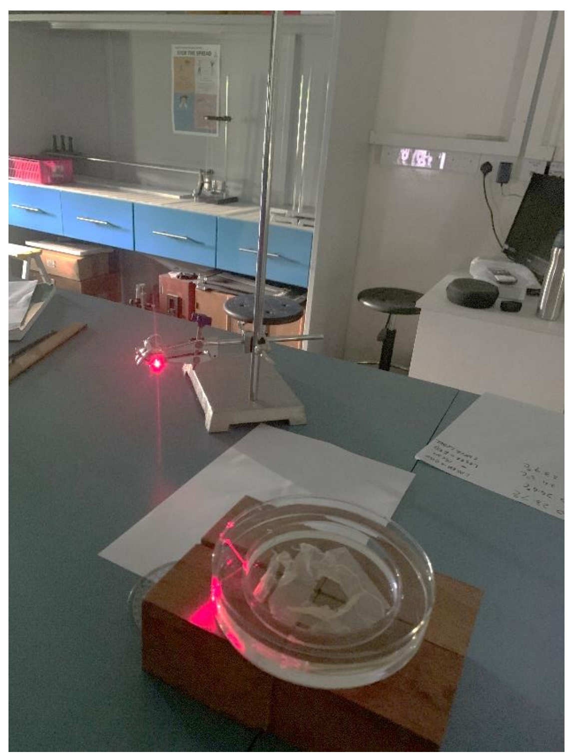


Figure 1 – Image of the experimental setup

8. Data Collection

Below is the table of raw data from all the trials in the experiment. The data will be processed to calculate the average critical angle and the concentration at each sugar level.

Amount of Sugar added in grams (±1g)	Critical Angle (±1°)					Average Critical Angle (°)
	1st Trial	2nd Trial	3rd Trial	4th Trial	5th Trial	
0.0	63	65	66	65	65	64.8 ≈ 65
25.0	61	60	58	62	60	60.2 ≈ 60
50.0	54	56	57	56	55	55.6 ≈ 56
75.0	51	51	52	50	52	51.2 ≈ 51
100.0	47	48	49	48	49	48.2 ≈ 48
125.0	45	46	45	45	47	45.6 ≈ 46

Table 1 – Raw Data collected from all the trials in the experiment

8.1 Absolute Uncertainties

- The uncertainty of the mass of sugar added is equivalent to the uncertainty of the electronic weighing scale. Since the minimum reading on the scale is 1g, the uncertainty of the mass of sugar is ±1g.
- The uncertainty of the volume of water is the same as the electronic weighing scale. The minimum reading for volume is accurate to 0.1ml which means the uncertainty of volume of water is ±0.1ml.
- The uncertainty of the critical angle is also the same as its instrumental uncertainty. Since the protractor's increments are by 1°, the uncertainty of the critical angle measured is ±1°.

9. Data Processing

9.1 Formulas Used

$$Percentage\ Uncertainty = \frac{Absolute\ Uncertainty}{Reading} \times 100\%$$

$$Concentration = \frac{Mass\ (Sugar)}{Volume\ (Water)}$$

$$Absoslute\ Uncertainty\ of\ Average = \frac{Maximum\ Reading - Minimum\ Reading}{2}$$

$$Percentage\ Uncertainty = \frac{Absolute\ Uncertainty_1}{Reading_1} + \frac{Absolute\ Uncertainty_2}{Reading_2} \times 100\%$$

9.2 Data Calculation and Error Propagation

At each stage, all the values were cross-referenced with theoretical calculations to check if the experiment was adhering to phenomena explained by classical physics.

Amount of Sugar added in grams (±1g)	Absolute Uncertainty (1s.f.)	Percentage Uncertainty (%)
0.0	1	0
25.0	1	4
50.0	1	2
75.0	1	1
100.0	1	1
125.0	1	1

Table 2 – Amount of Sugar added and its absolute and percentage uncertainties

Average Critical Angle (°)	Absolute Uncertainty (1s.f.)
65	2
60	2
56	2
51	1
48	1
46	1

Table 3 – Average Critical Angle and its absolute percentage

Amount of Sugar added in grams (±1g)	Concentration (g/ml)	Percentage Uncertainty (% to 1s.f.)	Absolute Uncertainty (1s.f.)
0.0	0.000	0.07	0
25.0	0.167	4	0.00668 ≈ 0.007
50.0	0.333	2	0.00666 ≈ 0.007
75.0	0.500	1	0.005
100.0	0.667	1	0.00667 ≈ 0.007
125.0	0.833	0.9	0.0075 ≈ 0.008

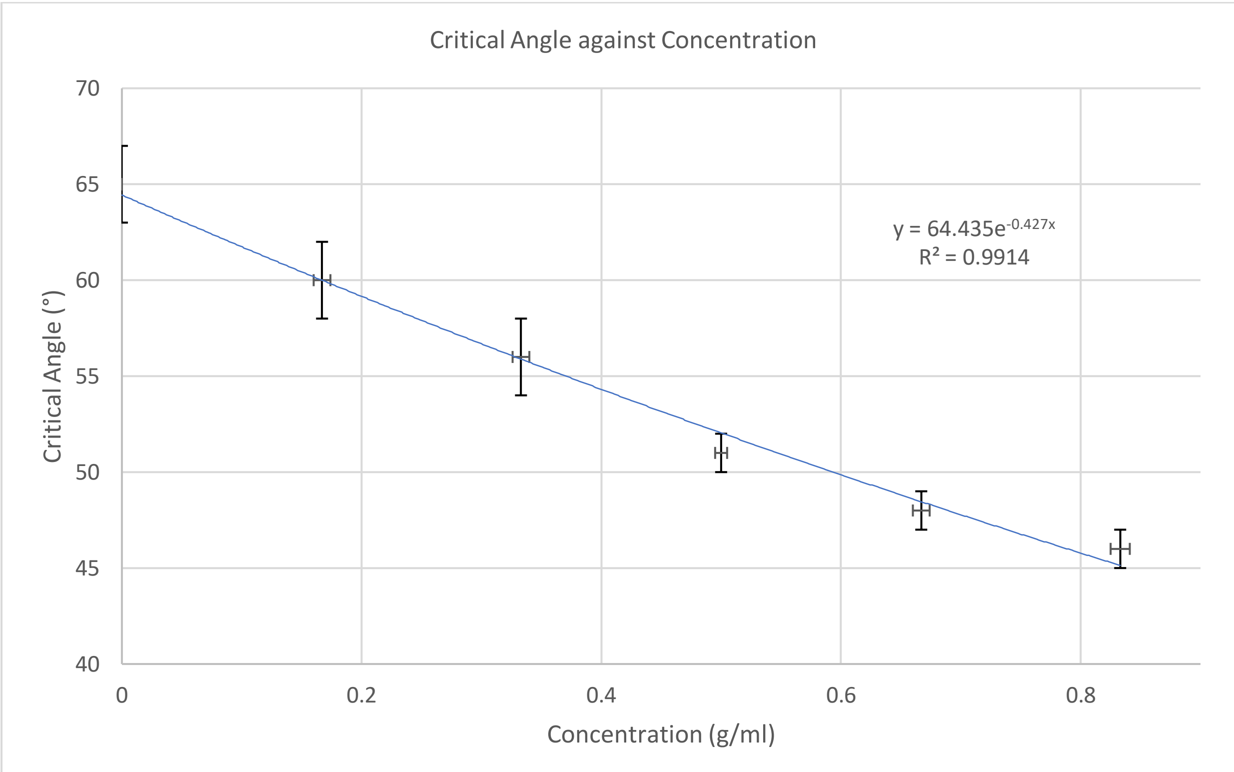
Table 4 – Calculating Concentration at each mass of sugar and its Percentage Uncertainty

9.3 Data Analysis

From the data collection, we can see that we have the data needed for graphing the independent and dependent variables; the critical angle against the concentration. Once the scatter plot was plotted, it was evident that there was a negative correlation between the critical angle and concentration, such that when the concentration increased the critical angle decreased. The next step was to find the best fit model that suited this relationship, and for this, we have to take the R^2 value of each type of model that fitted this relationship. The results are as follows:

Model	Exponential	Linear	Polynomial (Quadratic)
R^2	0.9914	0.9811	0.9971

From this we can see that out of the values, the quadratic model has the highest R^2 value (indicates it is the best model). However, this means that after reaching a certain point, the critical angle will begin to increase with the increase in concentration and as observed from the results we can see that this is untrue and not evident. Thus, the best model to graph the relationship is the exponential model.



Graph 1 – Critical Angle and Concentration

10. Conclusion

In conclusion, the experiment results support the previously proposed hypothesis that there is a negative non-linear relationship between the concentration of sucrose solution and the critical angle at which TIR occurs. The relationship also explains that the rate of decrease of the critical angle becomes lower as the concentration becomes higher. According to the findings, the relationship of the equation is exponential and can be modeled using the formula: $y = 64.435e^{-0.427x}$, where **y** is the critical angle and **x** is the concentration of the solution. As a result, I can state conclusively that the relationship is negative and non-linear, and that it is best represented by an exponential equation.

11. Evaluation

Most of the other variables besides the independent and the dependent variables were well-controlled and had little or no effect on the experiment. Since highly precise instruments were used, like an electronic weighing scale as opposed to a measuring cylinder for measuring the volume of water, the readings were highly precise.

Through the experiment, there were a few errors that were faced. Firstly, one systemic error that was faced was the saturation point of the sucrose solution. As large amounts of sugar were used, it was hard to dissolve it in the water even with the use of a stirrer. One possible solution to save time and dissolve the sugar is to heat the sugar and then cool it to ensure there are no impurities. Another solution is to use a magnetic stirrer which will speed the process of dissolving the sugar faster.

Secondly, a random error was a possible limitation was the human error that was faced in reading the protractor. This meant that if the reading was in between two values (1° increment in a protractor) then it would not give an accurate value, which affects the precision of the results. One solution that could work is by using a protractor that has increments in amounts smaller than 1°. Not only will this help in recording more accurate values but it will also help in reducing the absolute uncertainty.

Thirdly, another random error that is a limitation of the experiment which affects precision is the width of the laser. Using a laser with a larger width meant that it made the recording process of the critical angle harder. This would possibly lead to the results not being reproducible. To tackle this, one solution is to use a laser that has a smaller width. By doing so, the results will be more accurate while reading them using a protractor. This can help improve the precision of the results.

Apart from the errors, one strength of the experiment which helped give a different perspective and insight is the setup. For the experiment, I had designed the petri dish setup to create a torus-like shape where past one point the light would be trapped in the space between the two Petri dishes (TIR) where the sucrose solution is. This helped show how TIR occurs in objects regardless of the shape of the object. One modification that could be made is to use a more advanced glass like borosilicate glass instead of basic glass for the Petri dishes for the reflections to occur more naturally.

12. Future Investigation

After exploring whether the concentration of sugar in water influences the critical angle at which TIR, as a future extension we can look at the other factors that could have an effect on this relationship like the type of sugar or other soluble materials, temperature change.

13. Bibliography

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