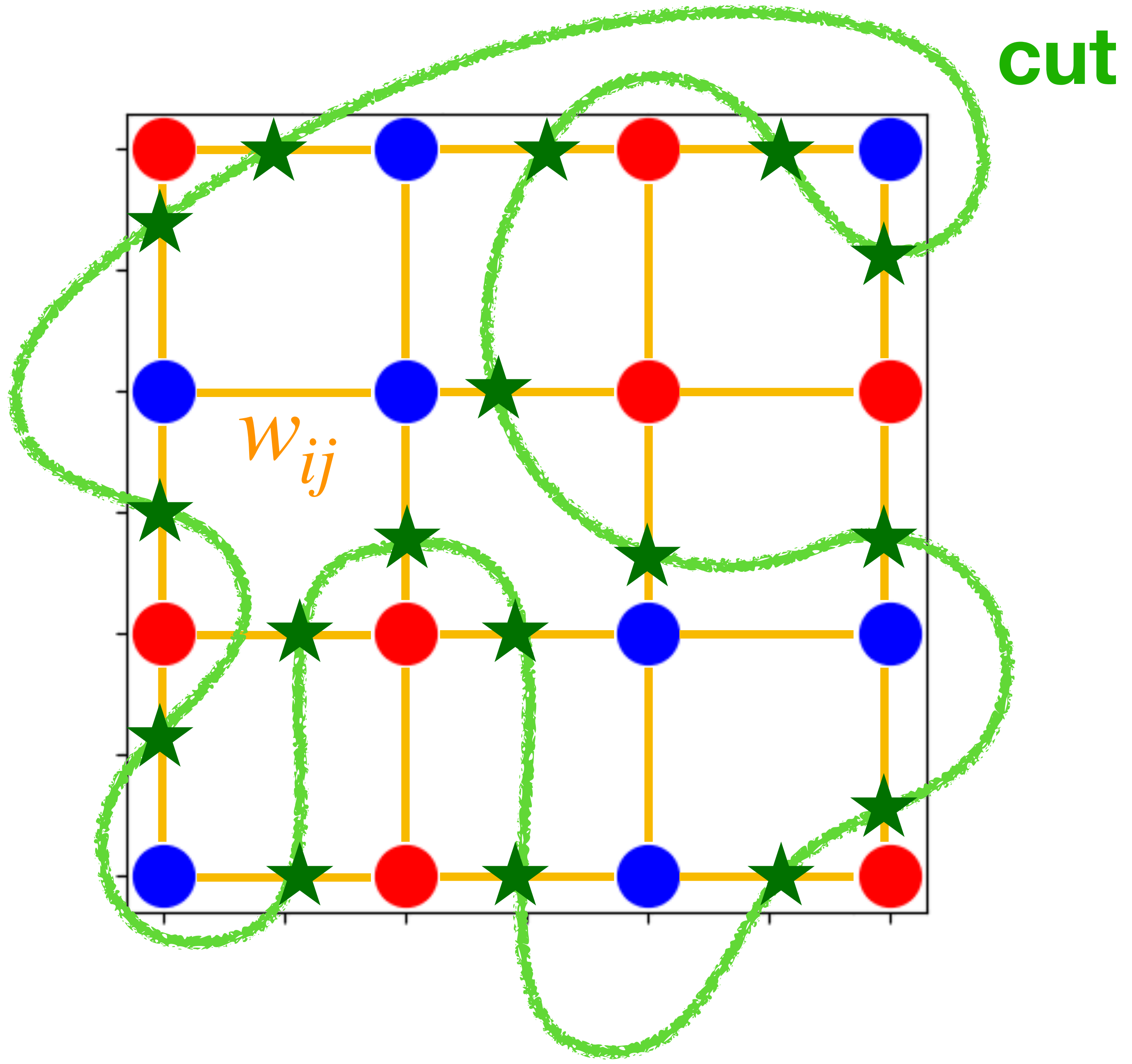
Solving the Max-Cut Problem with Classical & Quantum Algorithms

Final Presentation - 08/28/2020

Max-Cut Problem & Antiferromagnetic Ising Model

Max-Cut Problem *(NP-hard)*



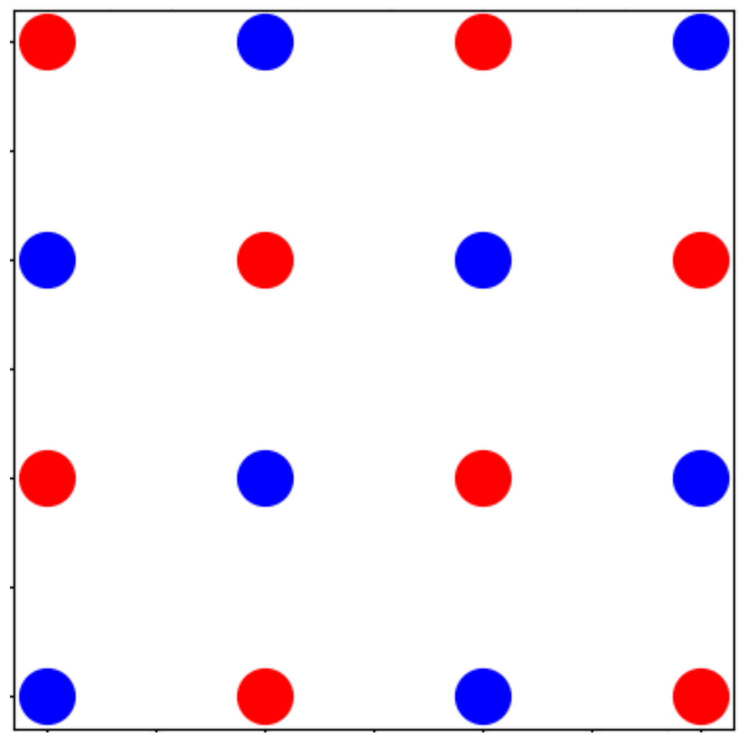
- Objective: find cut that maximizes the total weight of edges in the cut

Antiferromagnetic Ising Model *(Mother nature!)*

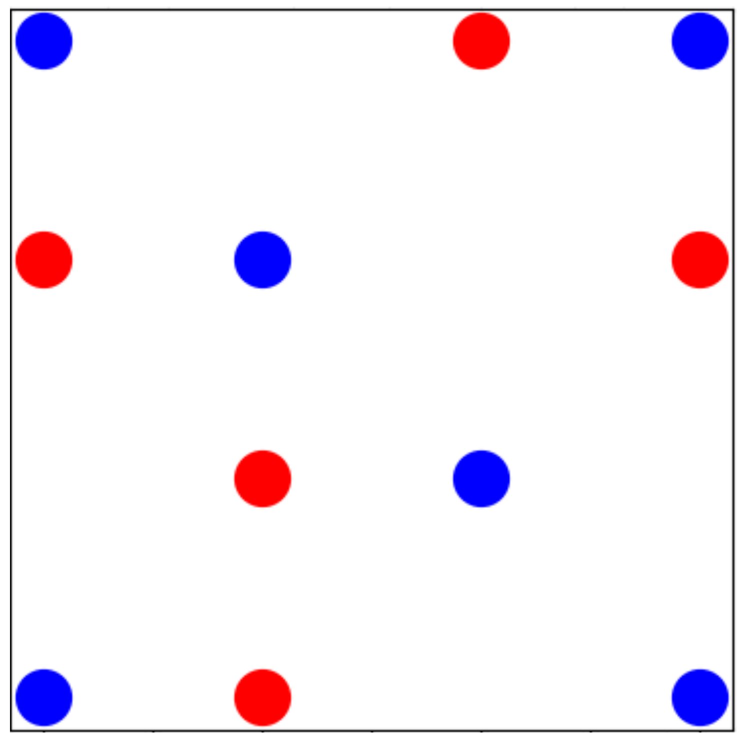
- Atoms i and interaction strengths $-J_{ij}$, $J_{ij} < 0$
- Spins σ_i^z: $(+1 \uparrow, -1 \downarrow)$
- Spins tend to anti-align
- Interactions between anti-aligned spins $\sigma_i^z \neq \sigma_j^z$
- Ground state: configuration that minimizes energy (maximizes negative energy)

Different System Configurations

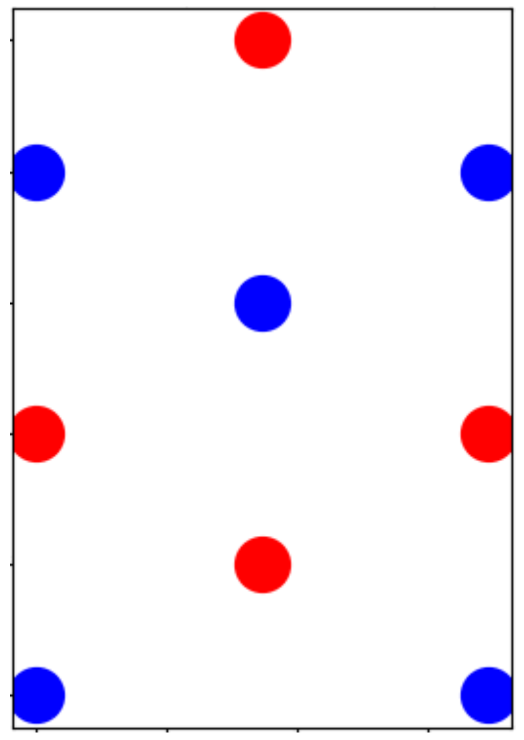
- **System structure**



Square lattice



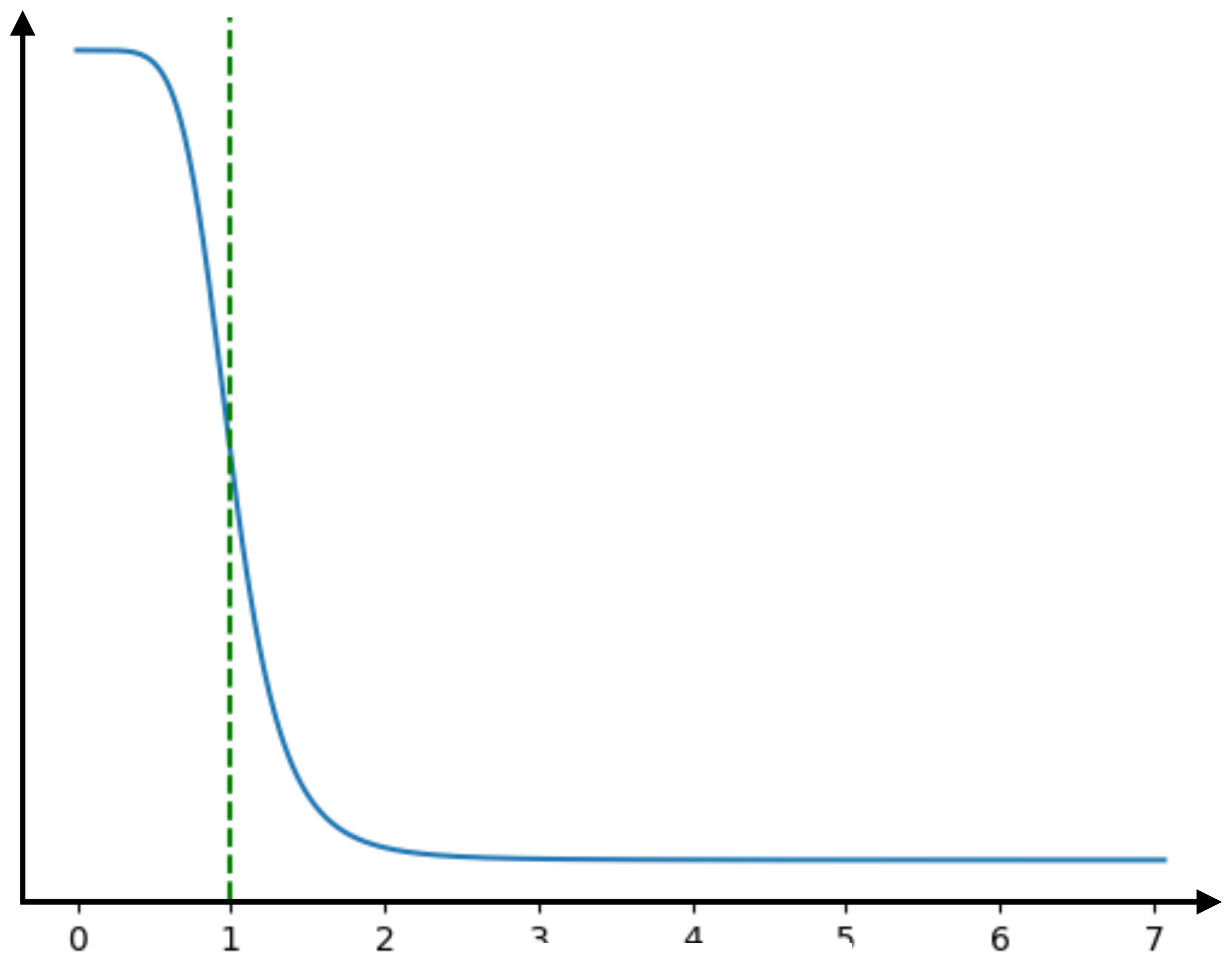
Lattice with empty slots



Triangular lattice

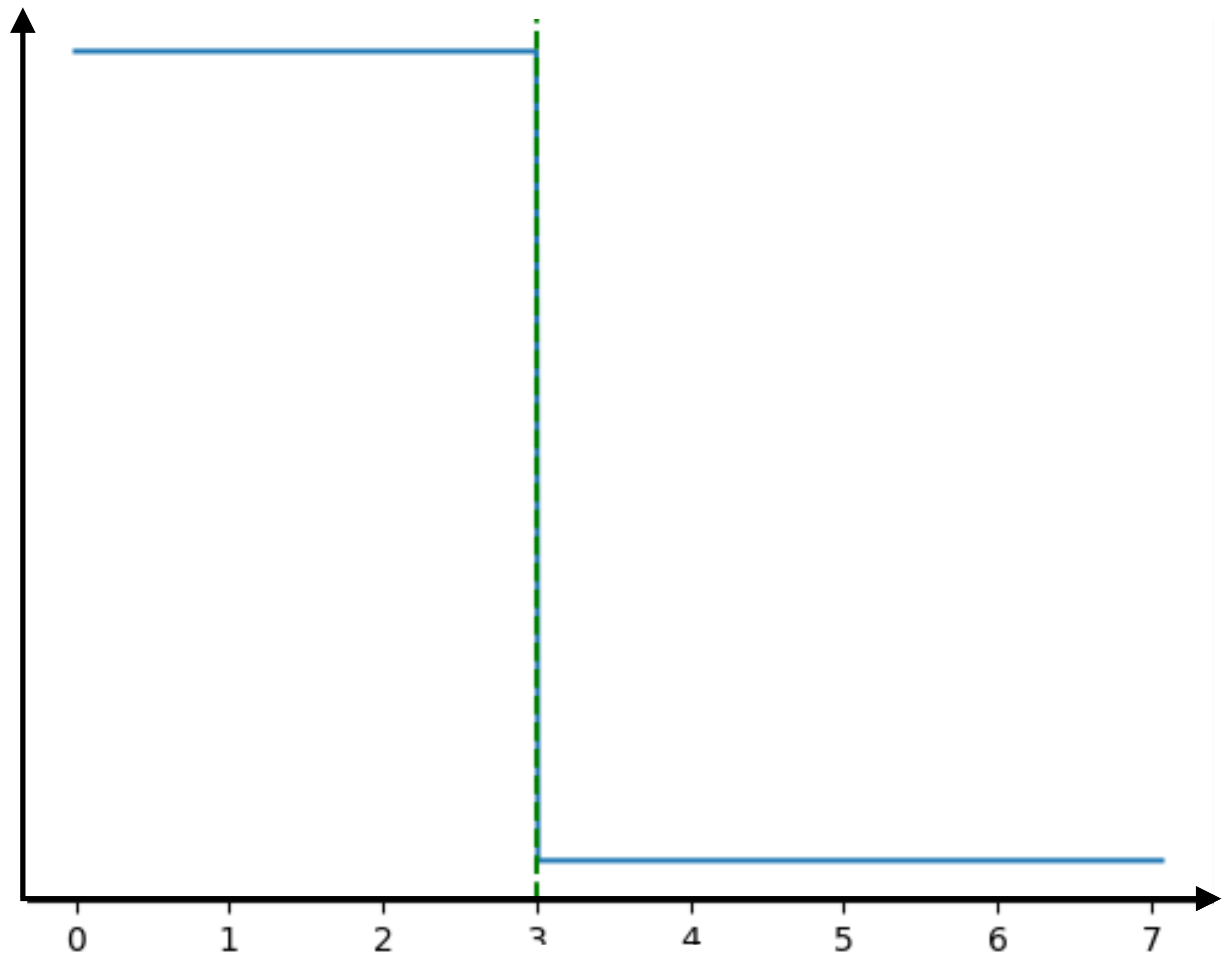
Disordered lattice

- **Interaction shape & range (radius, R)**



Distance, r

Step function $1 \cdot (d \leq R)$



Distance, r

Soft-core potential, $\dfrac{1}{1 + \left(\frac{d}{R}\right)^6}$

Random $R[0,1]$

- **System size**

Classical Algorithm

Simulated Annealing

Simulated Annealing Algorithm Parameters:

- Initial temperature T_0
- Cooling rate r

Repeat until convergence,
$T := r \cdot T$ (geometric cooling)

Initialize random partition p, set initial temperature $T := T_0$

Pick random vertex v

Proposal: switch assignment $p(v) := -p(v)$

Is $\Delta E \leq 0$?

Yes

No

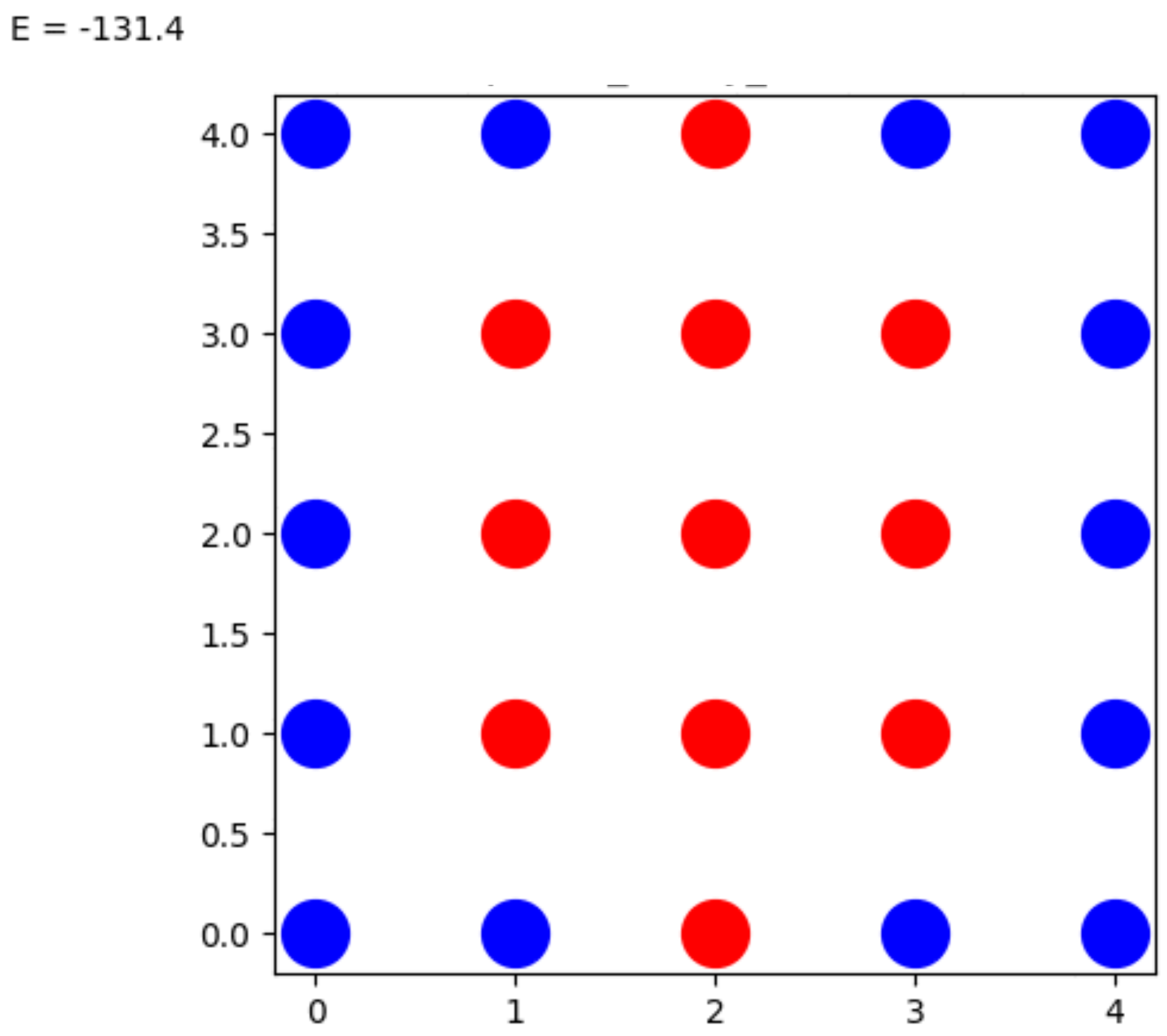
Accept proposal

Within probability $e^{-\Delta E/T}$?

Yes

No

Reject proposal



Metropolis-Hastings Algorithm
As temperature decreases, this probability gets smaller (“quenching” the process)

Quantum Algorithm

Quantum Approximate Optimization Algorithm (QAOA)

Classical optimization loop over angles $\overrightarrow{\beta}, \vec{\gamma}$:

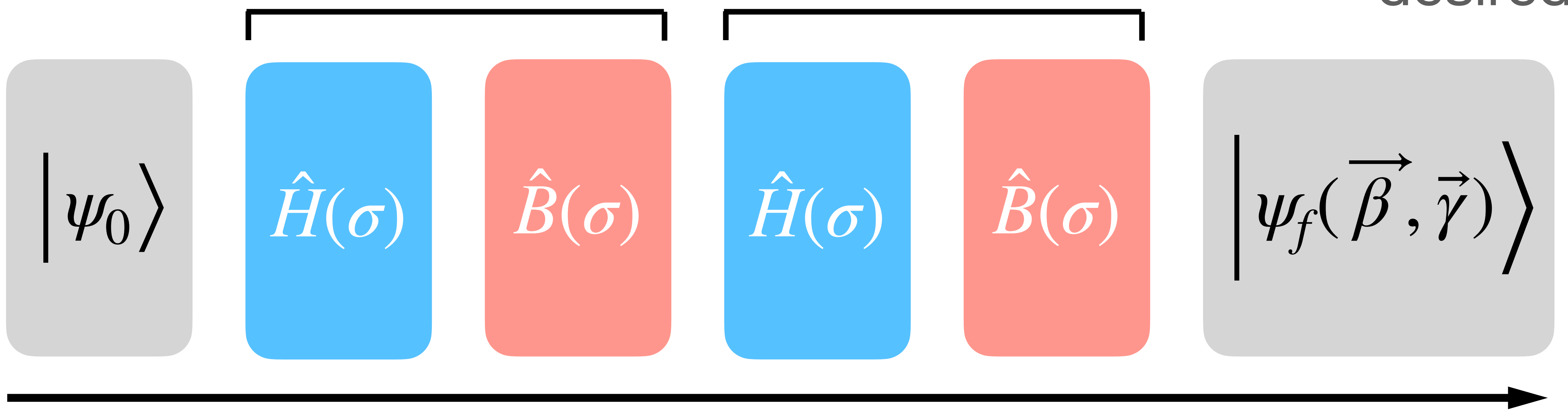
Quantum sub-routine:

Variational Quantum Eigensolver (VQE) minimizes total steps needed to reach a desired ground state fidelity

QAOA Algorithm Parameters:

- Circuit depth α
- Angles $\overrightarrow{\beta}, \vec{\gamma}$

sequence of length circuit depth α



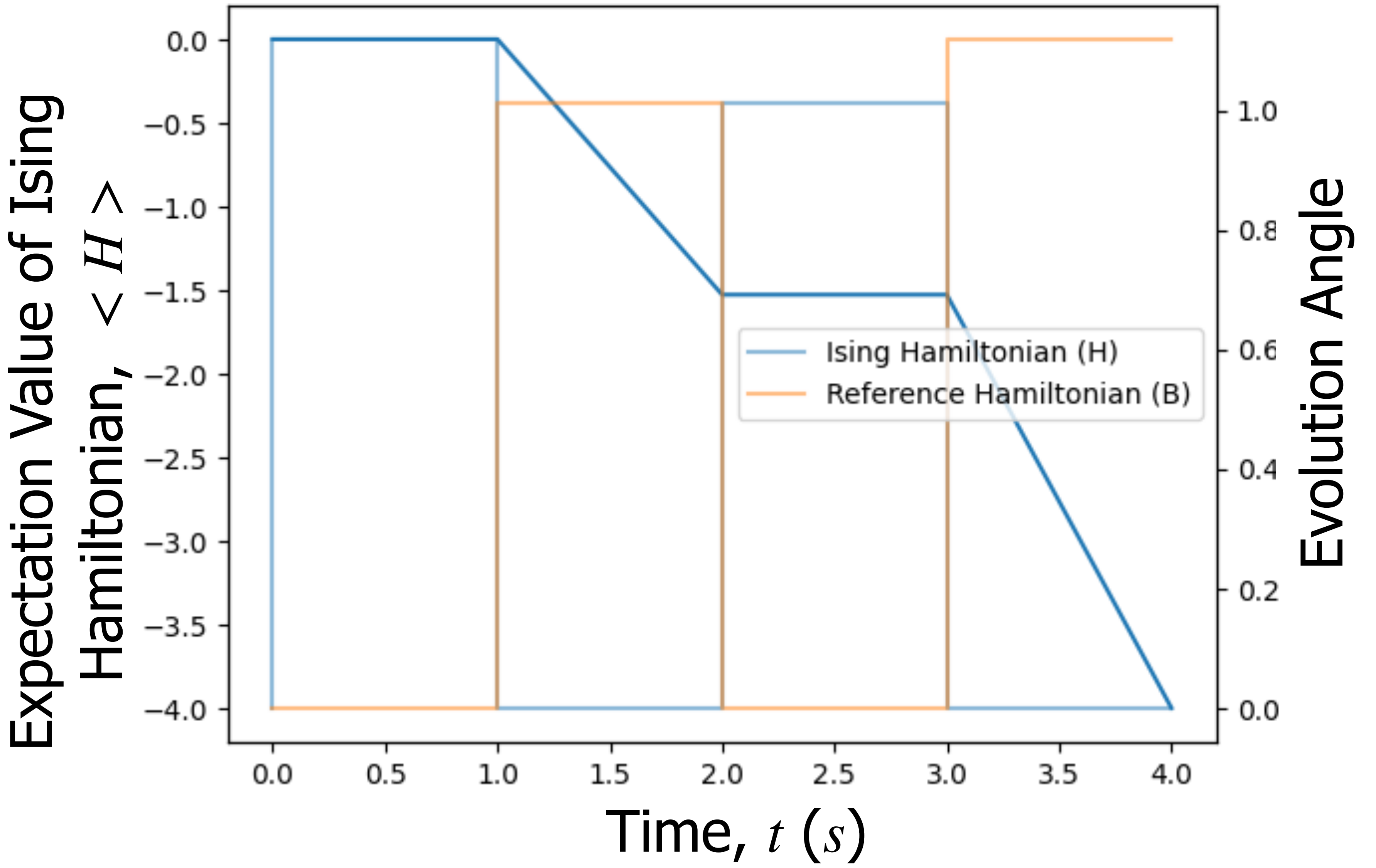
Unitary time evolution: $\hat{U}(\overrightarrow{\beta}, \vec{\gamma}) = e^{-i\hat{B}\beta_{(\alpha-1)}} e^{-i\hat{H}\gamma_{(\alpha-1)}} \dots e^{-i\hat{B}\beta_0} e^{-i\hat{H}\gamma_0}$

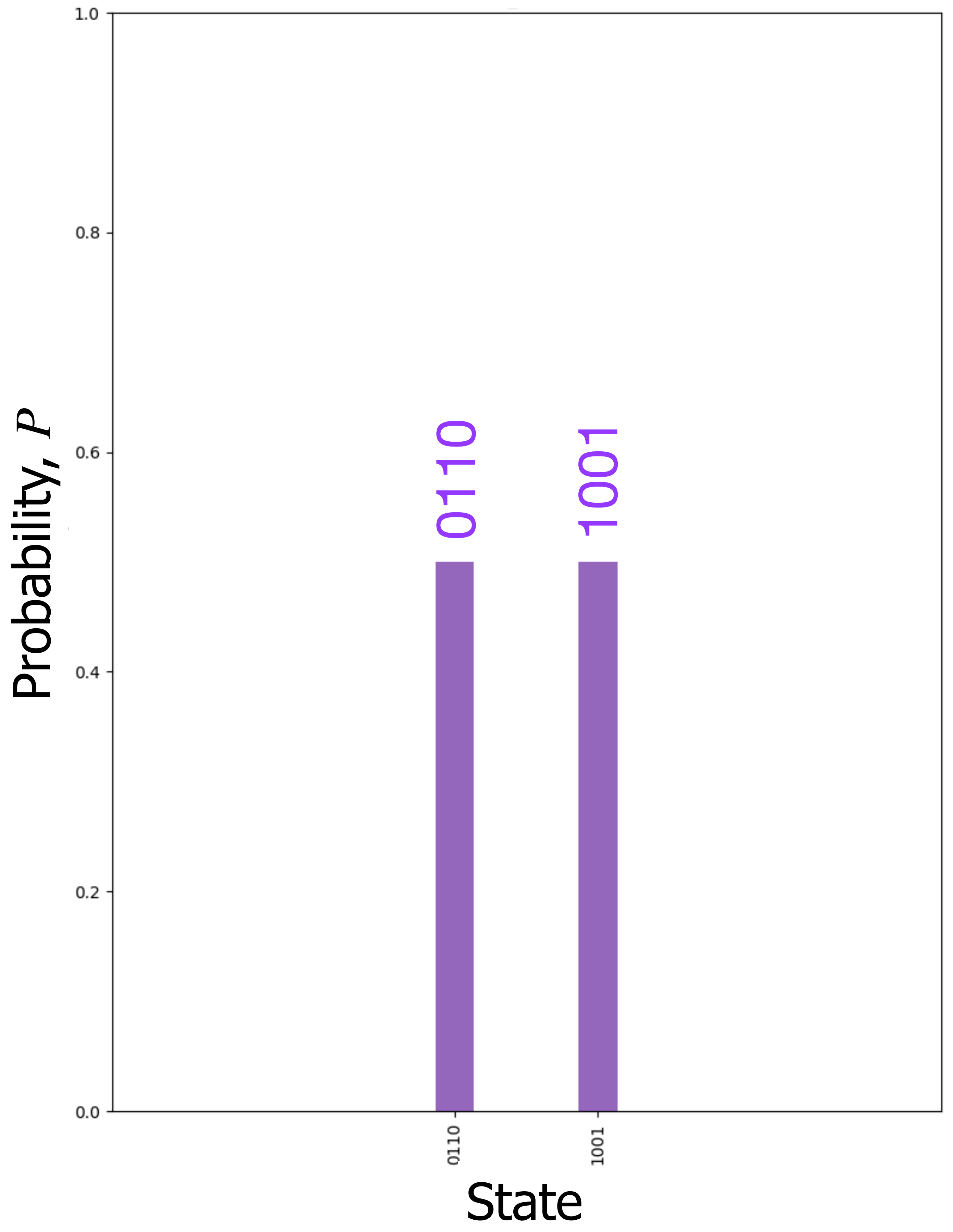
Ising Interaction Hamiltonian
(acting on spins in z direction)

$$\hat{H}(\sigma) = -\sum_{ij} J_{ij} \sigma_i^z \sigma_j^z$$

Reference Hamiltonian
(transverse field in $-x$ direction)

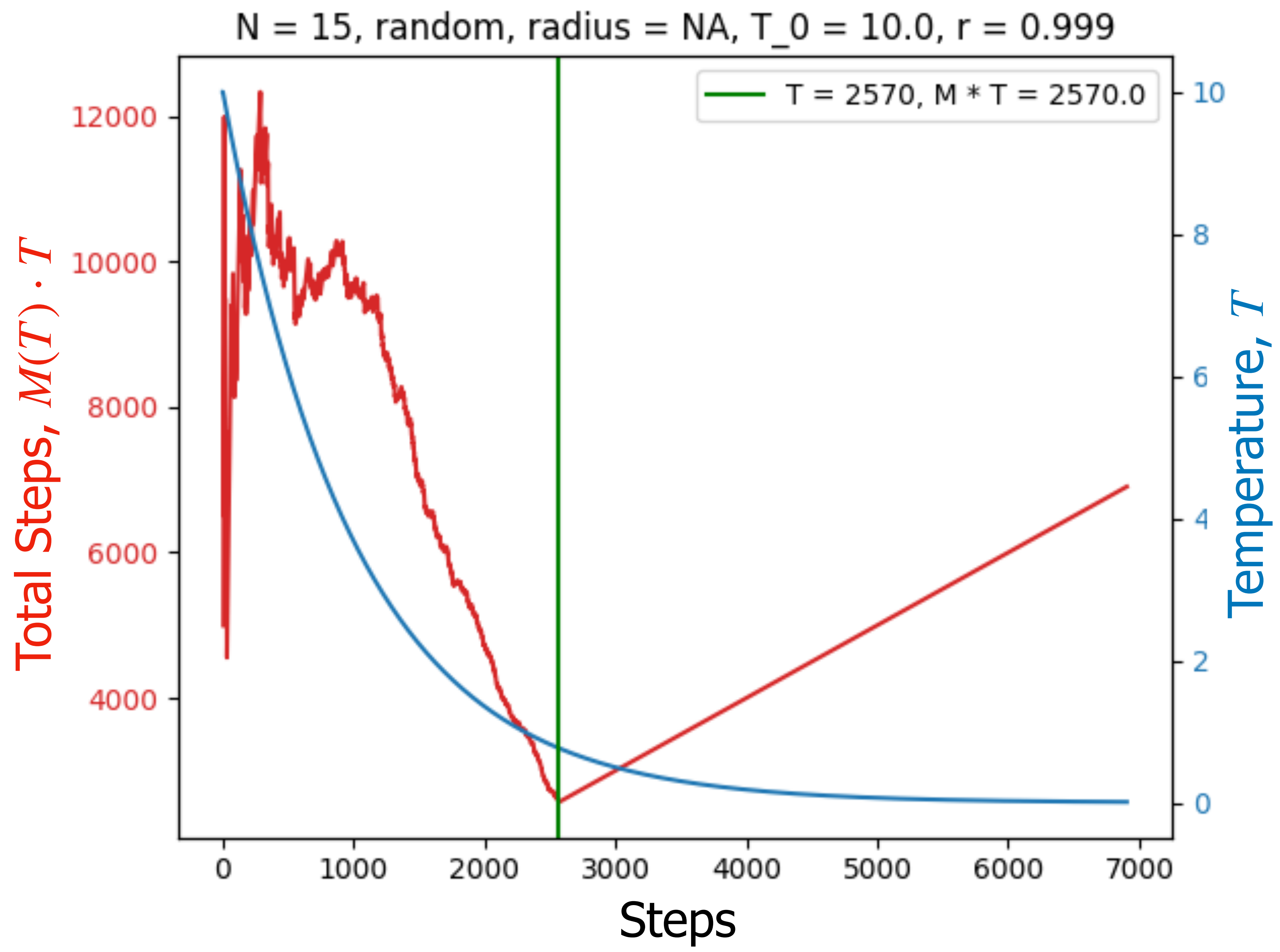
$$\hat{B}(\sigma) = -\sum_{i} \sigma_i^x$$





Algorithm Evaluation Metric

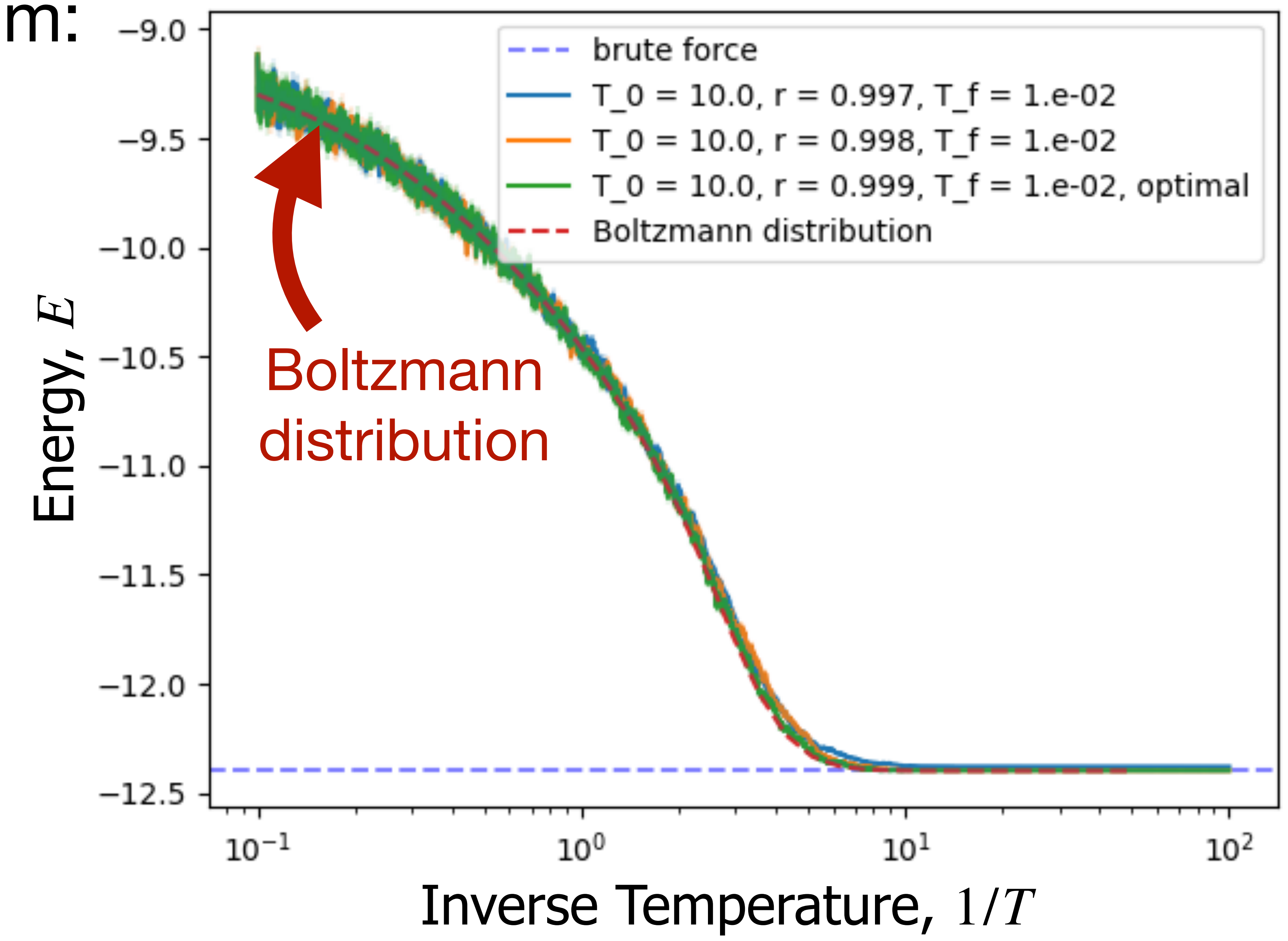
- Evaluation metric: **minimize** total # of steps $M(T) \cdot T$ to reach desired ground state probability P_*
- Optimization parameters: M runs each for a time T
- What is T ?
  - Simulated Annealing: number of iterations
  - QAOA: length of Hamiltonian sequence (less obvious)
- Key idea: willing to have multiple shorter runs to reach desired ground state probability more quickly

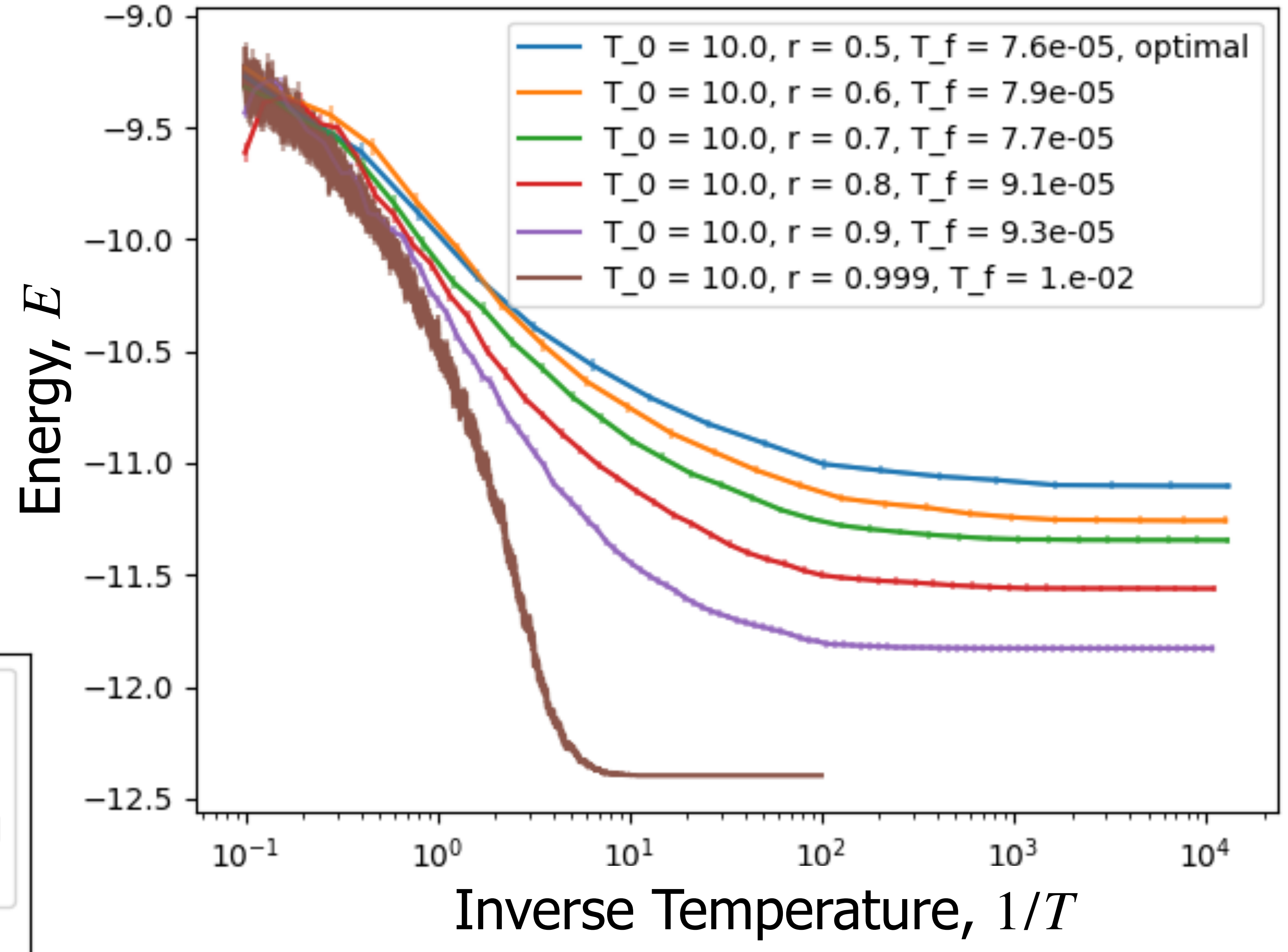


Sample Case: Simulated Annealing for All-to-all Random Interactions, N = 9

- Simulated Annealing
  - Evaluation metric: minimize $M(T) \cdot T$
- Compare with: brute force solution
  - Boltzmann distribution plotted using all states explored by brute force algorithm:

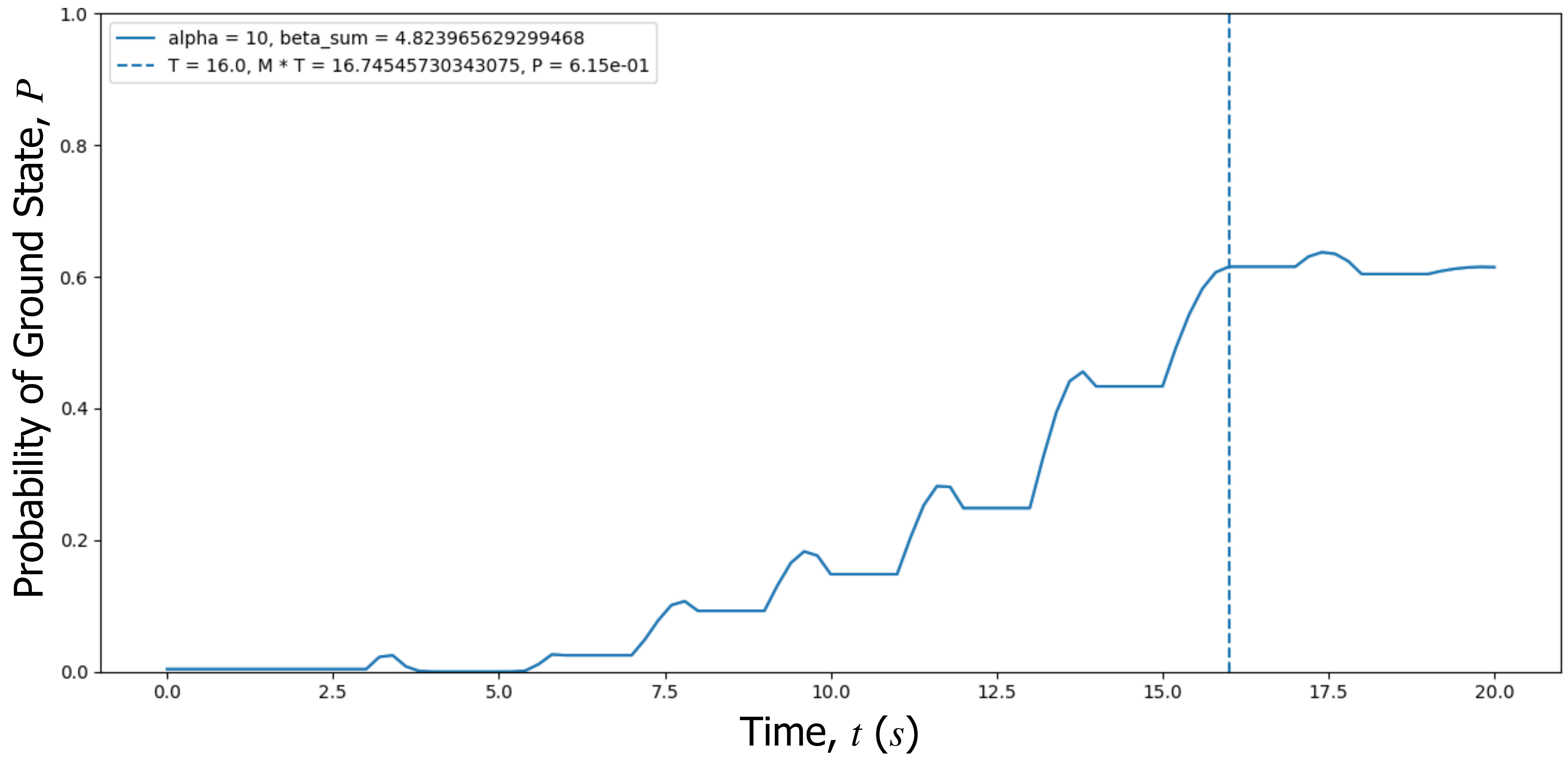
$$\frac{\sum_i E_i e^{-E_i/T}}{\sum_i e^{-E_i/T}}$$

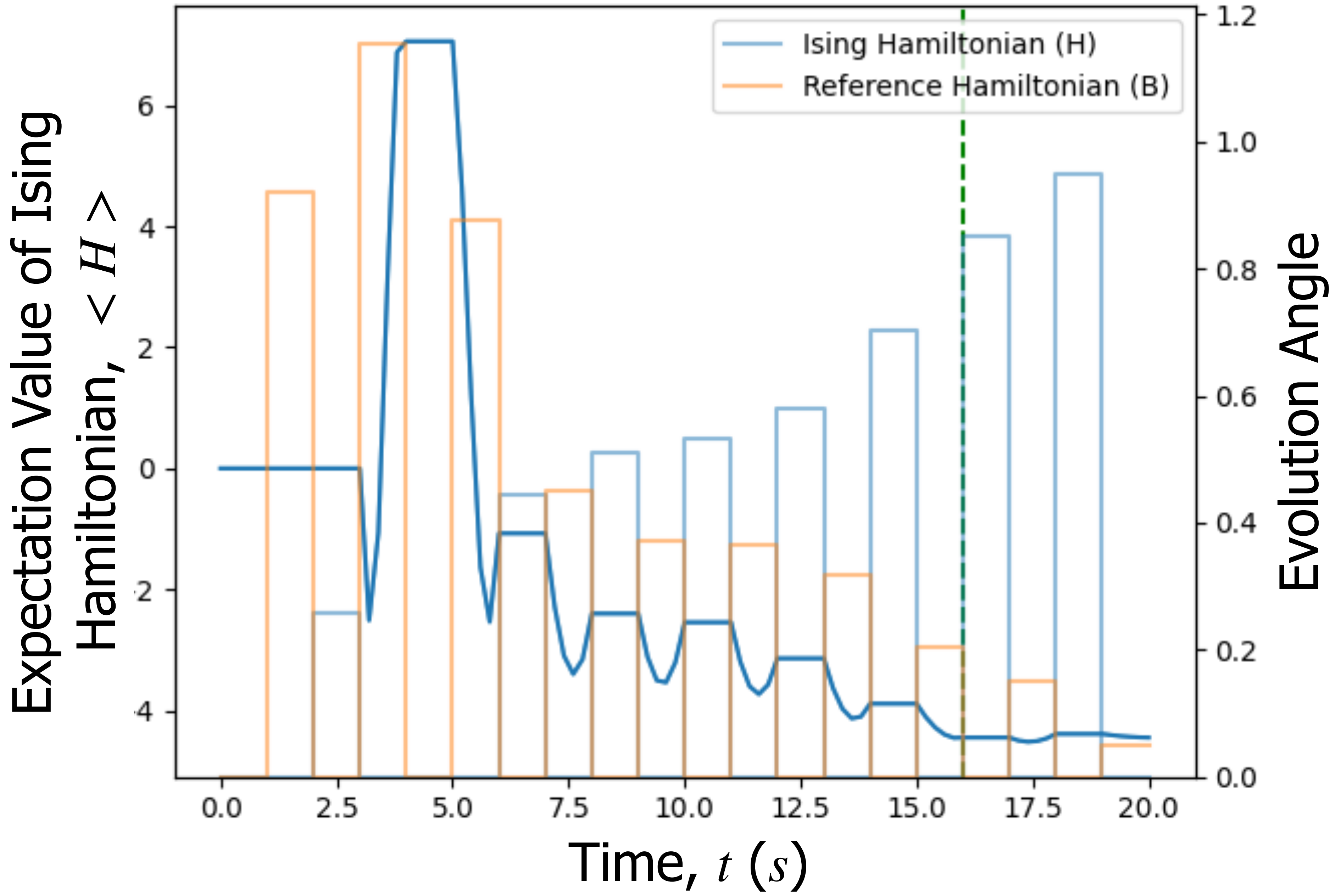




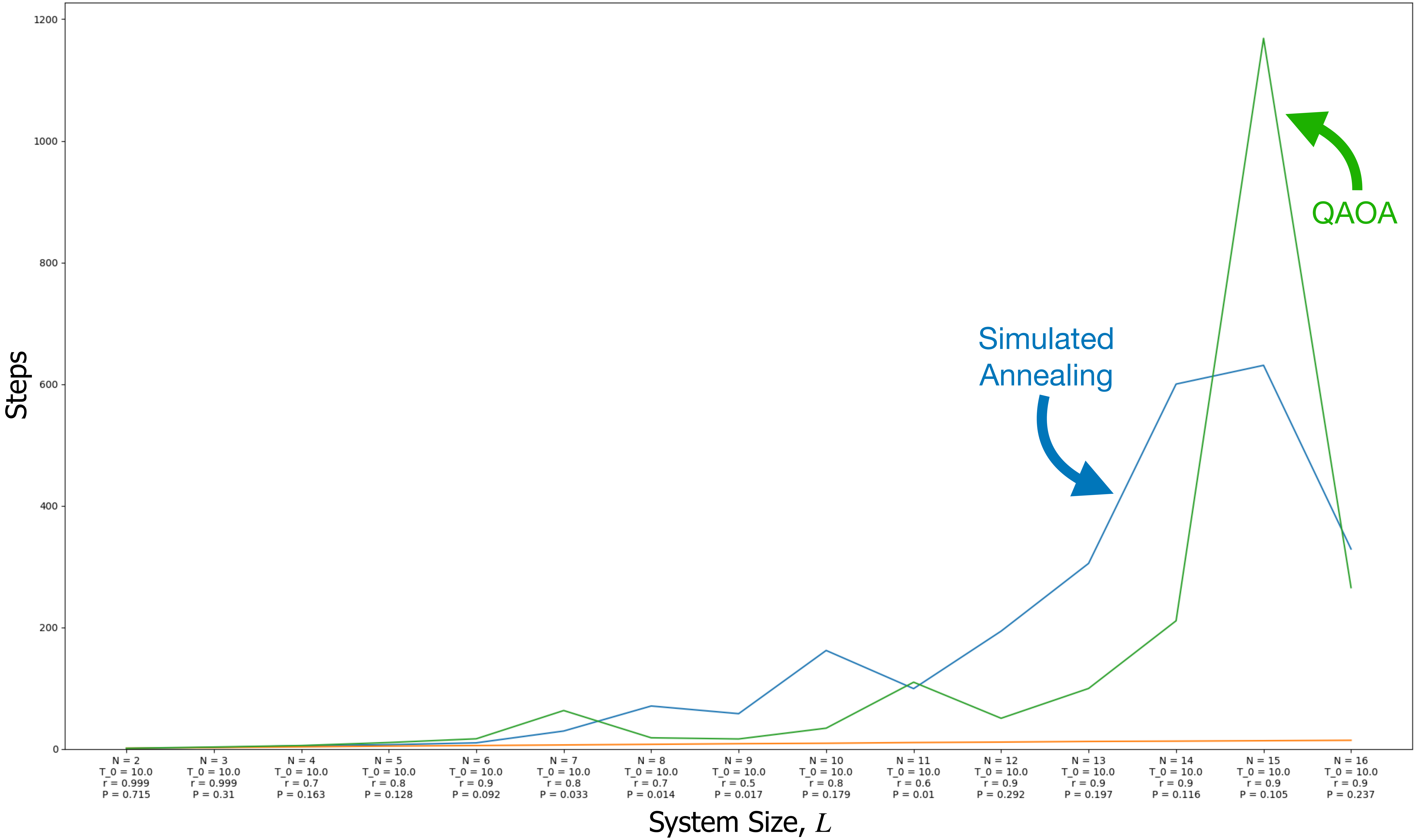
Sample Case: QAOA for All-to-all Random Interactions, N = 9

- Variational Quantum Eigensolver
    - Evaluation metric: minimize $M(T) \cdot T$, where $T \in [1,\ 2\alpha]$
- Quantum Approximate Optimization Algorithm





Sample Case: Scaling with System Size in All-to-all Random Interactions, N = 9



Next Steps

- Solidify grounds for comparing classical & quantum algorithms
  - Common optimization: minimize $M(T) \cdot T$, i.e. number of trials x number of “steps” per trial - what is T?
- Compare our findings for random all-to-all interactions with existing literature
- Simulate and compare classical & quantum algorithms for system configurations attainable in our experimental setup
- Run QAOA physically on our experiment!