Assignment 3

QUESTIONS:

Q1) 1 Point for part a, 1 point for part b; total 2 points

In a research survey in a certain city, a sample of 400 voters are asked if they favor an additional 4% gasoline sales tax to provide badly needed revenues for street repairs. If more than 220 but fewer than 260 favor the sales tax, we shall conclude that 60% of the voters are for it.

a) Find the probability of committing a type I error if 60% of the voters favor the increased tax.
b) What is the probability of committing a type II error using this test procedure if actually only 48% of the voters are in favor of the additional gasoline tax?

(a) $n = 400$, $p = 0.6$, $\mu = np = 240$, and $\sigma = \sqrt{npq} = 9.798$, with

$$z_1 = \frac{259.5 - 240}{9.978} = 1.990, \quad \text{and} \quad z_2 = \frac{220.5 - 240}{9.978} = -1.990.$$

Hence,

$$\alpha = 2P(Z < -1.990) = (2)(0.0233) = 0.0466.$$

(b) When $p = 0.48$, then $\mu = 192$ and $\sigma = 9.992$, with

$$z_1 = \frac{220.5 - 192}{9.992} = 2.852, \quad \text{and} \quad z_2 = \frac{259.5 - 192}{9.992} = 6.755.$$

Therefore,

$$\beta = P(2.852 < Z < 6.755) = 1 - 0.9978 = 0.0022.$$

Q2) 1 Point for part a, 1 point for part b; total 2 points

A company has developed a new fishing line, which the company claims has a mean breaking strength of 15 kilograms with a standard deviation of 0.5 kilogram. To test the hypothesis that $\mu = 15$ kilograms against the alternative that $\mu < 15$ kilograms, a random sample of 50 lines will be tested. The critical region is defined to be $\bar{x} < 14.9$.

(a) Find the probability of committing a type I error when H_0 is true.

(b) Evaluate β for the alternatives $\mu = 14.8$ and $\mu = 14.9$ kilograms.

$n = 50, \ \mu = 15, \ \sigma = 0.5,$ and $\sigma_{\bar{X}} = 0.5/\sqrt{50} = 0.071,$

$$\begin{aligned} \alpha &= P(Reject \ \ H_0 \mid H_0 \ \ TRUE) \\ &= P(\bar{X} < 14.5 \mid \mu = 15) \\ &= P(Z < \frac{14.5 - 15}{0.071}) \\ &= P(Z < -7.07) \\ &\approx 0.00 \end{aligned}$$

$n = 50, \ \mu = 15, \ \sigma = 0.5,$ and $\sigma_{\bar{X}} = 0.5/\sqrt{50} = 0.071,$

$$\begin{aligned} \beta &= P(NOT \ Reject \ \ H_0 \mid H_0 \ \ FALSE) \\ &= P(\bar{X} > 14.5 \mid \mu = 14.8) \\ &= P(Z > \frac{14.5 - 14.8}{0.071}) \\ &= P(Z > -4.243) \\ &\approx 1.00 \end{aligned}$$

Q3) 2 points

The average height of females in the freshman class of a certain college has historically been 162.5 centimeters with a standard deviation of 6.9 centimeters. Is there reason to believe that there has been a change in the average height if a random sample of 50 females in the present freshman class has an average height of 165.2 centimeters? Use a P-value in your conclusion. Assume the standard deviation remains the same.

The hypotheses are

$$H_0 : \mu = 162.5 \text{ centimeters},$$
$$H_1 : \mu \neq 162.5 \text{ centimeters}.$$

Now, $z = \frac{165.2-162.5}{6.9/\sqrt{50}} = 2.77$, and P-value$= 2P(Z > 2.77) = (2)(0.0028) = 0.0056$. Decision: reject H_0 and conclude that $\mu \neq 162.5$.

Q4) 2 points

A researcher in the University of Toronto claims that the average life span of mice can be extended by as much as 8 months when the calories in their diet are reduced by approximately 40% from the time they are weaned. The restricted diets are enriched to normal levels by vitamins and protein. Suppose that a random sample of 10 mice is fed a normal diet and has an average life span of 32.1 months with a standard deviation of 3.2 months, while a random sample of 15 mice is fed the restricted diet and has an average life span of 37.6 months with a standard deviation of 2.8 months. Test the hypothesis, at the 0.05 level of significance, that the average life span of mice on this restricted diet is increased by 8 months against the alternative that the increase is less than 8 months. Assume the distributions of life spans for the regular and restricted diets are approximately normal with equal variances.

The hypotheses are

$$H_0 : \mu_1 - \mu_2 = 8,$$
$$H_1 : \mu_1 - \mu_2 < 8.$$

$\alpha = 0.05$ and the critical region is $t < -1.714$ with 23 degrees of freedom.

Computation: $s_p = \sqrt{\frac{(9)(3.2)^2+(14)(2.8)^2}{23}} = 2.963$, and $t = \frac{5.5-8}{2.963\sqrt{1/10+1/15}} = -2.07$.

Decision: Reject H_0 and conclude that $\mu_1 - \mu_2 < 8$ months.

Q5) 2 points

Engineers at a large automobile manufacturing company are trying to decide whether to purchase brand A or brand B tires for the company's new models. To help them arrive at a decision, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are as follows:

Brand A: $\bar{x}_1 = 37{,}900$ kilometers, $s_1 = 5100$ kilometers.

Brand B: $\bar{x}_2$ = 39,800 kilometers, $s_2 = 5900$ kilometers.

Test the hypothesis that there is no difference in the average wear of the two brands of tires. Assume the populations to be approximately normally distributed with equal variances. Use a P-value.

The hypotheses are

$$H_0 : \mu_1 = \mu_2,$$
$$H_1 : \mu_1 \neq \mu_2.$$

Computation: $s_p = \sqrt{\frac{5100^2+5900^2}{2}} = 5515$, and $t = \frac{37{,}900-39{,}800}{5515\sqrt{1/12+1/12}} = -0.84$.

Using 22 degrees of freedom and since $0.20 < P(T < -0.84) < 0.3$, we obtain $0.4 < P\text{-value} < 0.6$. Decision: Do not reject H_0.

Q7) 2 points

At engineering faculty, it is estimated that at most 25% of the students ride bicycles to class. Does this seem to be a valid estimate if, in a random sample of 90 college students, 28 are found to ride bicycles to class? Use a 0.05 level of significance.

The hypotheses are

$$H_0: p = 0.25,$$
$$H_1: p > 0.25.$$

$\alpha = 0.05$.

Computation:

$$P\text{-value} \approx P\left(Z > \frac{28 - (90)(0.25)}{\sqrt{(90)(0.25)(0.75)}}\right) = P(Z > 1.34) = 0.091.$$

Decision: Fail to reject H_0; No sufficient evidence to conclude that $p > 0.25$.