Kinematic Equations Solver (KES) Reference

Nathan Ullberg

Welcome to the reference sheet concerning the Kinematic Equations Solver (KES) application! In this document I describe what this app is useful for, how to operate it, and how it works. The first section encompasses the general relevant points to the app, and the second section goes into more detail.

General information

Let's jump right in. Concerning linear translational motion in one dimension under constant acceleration, the 5 kinematic equations are

$$(1) \quad \mathrm{v_f} = \mathrm{v_i} + at$$

$$(2) \quad d = \mathrm{v_i}\, t + \frac{1}{2}at^2$$

$$(3) \quad d = \mathrm{v_f}\, t - \frac{1}{2}at^2$$

$$(4) \quad d = \frac{1}{2}(\mathrm{v_f} + \mathrm{v_i})t$$

$$(5) \quad \mathrm{v_f}^2 = \mathrm{v_i}^2 + 2ad$$

Now, we recall that there are 5 variables involved in kinematics, including d, $\mathrm{v_f}$, $\mathrm{v_i}$, a, t (displacement, final velocity, initial velocity, acceleration, and time). However only 3 of these variables are necessary, for describing any kind of kinematic motion. Thus, what we are doing in some arbitrary kinematics problem is, finding the value of an additional variable from the set of 5, which is not one of 3 the givens. Solving for this variable does not change anything about the motion, of course, rather it simply gives us more information that we may want to know, such as how far some projectile travelled.

This means that since 3 givens specify the motion, there are 2 more variables which we can solve for. Now, for each equation, there is one of the kinematic variables "missing". That means that since for a given problem, we are solving for one variable, then there will always be one of the equations that contain the 3 givens, and the one unknown that we are solving for. So the first step is to find which of the 5 equations that one is, based on the "missing" variable, and then isolate the variable we are solving for.

The KES app dynamically takes into account all of these processes, and solves the correct equation, based on a user-specified "Find" and 3 provided values in the "Given" pane. Notice that the selected "Find" becomes disabled in the "Given" pane, and that once you fill in 3 given values, the remaining field is automatically disabled, to prevent redundancy and contradictions. Click "Solve" when ready or "Reset" to start over.

Details

Now, for clarity, I have repeated the 5 kinematic equations from above, indicating which variable is "missing" in each.

$$
\begin{array}{lll}
(1) & \mathrm{v_f} = \mathrm{v_i} + at & \text{missing } d \\
(2) & d = \mathrm{v_i}\, t + \frac{1}{2}at^2 & \text{missing } \mathrm{v_f} \\
(3) & d = \mathrm{v_f}\, t - \frac{1}{2}at^2 & \text{missing } \mathrm{v_i} \\
(4) & d = \frac{1}{2}(\mathrm{v_f} + \mathrm{v_i})t & \text{missing } a \\
(5) & \mathrm{v_f}^2 = \mathrm{v_i}^2 + 2ad & \text{missing } t
\end{array}
$$

Now, let us first consider the case that we want to find the displacement. In that case, the first equation above is not relevant, since we cannot find the displacement value from an equation that does not have the displacement variable in it... Thus we are faced with the following possibilities:

$$
\begin{array}{lll}
(2) & d = \mathrm{v_i}\, t + \frac{1}{2}at^2 & \text{missing } \mathrm{v_f} \\
(3) & d = \mathrm{v_f}\, t - \frac{1}{2}at^2 & \text{missing } \mathrm{v_i} \\
(4) & d = \frac{1}{2}(\mathrm{v_f} + \mathrm{v_i})t & \text{missing } a \\
(5) & d = \frac{1}{2a}(\mathrm{v_f}^2 - \mathrm{v_i}^2) & \text{missing } t
\end{array}
$$

Notice that this is of course the same equations 2,3,4,5 from the top of this page, except rearranged for the displacement value. We take the same approaches for the other values. Thus for final velocity:

$$
\begin{array}{lll}
(1) & \mathrm{v_f} = \mathrm{v_i} + at & \text{missing } d \\
(3) & \mathrm{v_f} = \frac{1}{t}\left(d + \frac{1}{2}at^2\right) & \text{missing } \mathrm{v_i} \\
(4) & \mathrm{v_f} = \frac{2d}{t} - \mathrm{v_i} & \text{missing } a \\
(5) & \mathrm{v_f} = \sqrt{\mathrm{v_i}^2 + 2ad} & \text{missing } t
\end{array}
$$

And, similarly, this is what we get for initial velocity:

$$
\begin{array}{lll}
(1) & v_i = v_f - at & \text{missing } d \\
(2) & v_i = \frac{1}{t}\left(d - \frac{1}{2}at^2\right) & \text{missing } v_f \\
(4) & v_i = \frac{2d}{t} - v_f & \text{missing } a \\
(5) & v_i = \sqrt{v_f^2 - 2ad} & \text{missing } t
\end{array}
$$

Now for acceleration we have the following:

$$
\begin{array}{lll}
(1) & a = \frac{1}{t}(v_f - v_i) & \text{missing } d \\
(2) & a = \frac{2}{t^2}(d - v_i t) & \text{missing } v_f \\
(3) & a = \frac{-2}{t^2}(d - v_f t) & \text{missing } v_i \\
(5) & a = \frac{1}{2d}(v_f^2 - v_i^2) & \text{missing } t
\end{array}
$$

And now finally for time we have the following:

$$
\begin{array}{lll}
(1) & t = \frac{1}{a}(v_f - v_i) & \text{missing } d \\
(2) & t = \frac{1}{a}(-v_i \pm \sqrt{v_i^2 + 2ad}) & \text{missing } v_f \\
(3) & t = \frac{1}{a}(v_f \pm \sqrt{v_f^2 - 2ad}) & \text{missing } v_i \\
(4) & t = \frac{2d}{v_f + v_i} & \text{missing } a
\end{array}
$$