Astrape: Anonymous Payment Channels with Boring Cryptography

ncreasing usage of blockchain-based cryptocurrencies like Bitcoin and Ethereum have run into inherent scalability limitations of blockchains. Payment channels networks, or PCNs, promise to greatly increase scalability by conducting the vast majority of transactions outside the blockchain while leveraging it as a final settlement protocol. Unfortunately, currently deployed PCNs have significant flaws in security and privacy. In particular, even though transactions are conducted off-chain, anonymity guarantees are very weak.

In this work, we present Astrape, a novel PCN construction that achieves strong security and anonymity guarantees with simple, black-box cryptography. Existing anonymous PCN constructions all integrate with specific, often custom-designed, cryptographic constructions, but Astrape can use any generic public-key signature scheme and any random-oracle hash function. This allows Astrape to achieve provable security without relying on any specific computational hardness assumption. Astrape's simple cryptography also lends itself to unusually straightforward security proofs compared to existing systems.

Furthermore, we evaluate Astrape's performance, including that of a concrete implementation on the Bitcoin Cash blockchain. We show that on the average case, Astrape operations require less than 20 milliseconds of computation and 1 KB of communication on commodity hardware. Astrape explores a novel avenue to secure and anonymous PCNs that achieves similar or better performance compared to all existing solutions.

1 Introduction

1.1 Payment channel networks

Blockchain-based cryptocurrencies, like Bitcoin, Lite-coin, and DASH, are gaining in popularity and becoming a significant alternative to traditional government-issued money. For instance, almost 300,000 Bitcoin transactions alone [5] are processed every day. Unfortunately, such high demand inevitably causes blockchain cryptocurrencies to run into well-known scalability bar-

riers [9]. In Bitcoin's case, less than 10 transactions per second in total can be processed [20], far less than the throughput required for a global payment system.

Payment channels [12] have become the most common way in practice of scaling cryptocurrency transactions. In a nutshell, Alice and Bob open a payment channel by submitting a single transaction to the blockchain, that locks up a sum of cryptocurrency from both of the parties. They can then perform payments to each other without using the blockchain, by simply mutually agreeing on which portion of the locked money belongs to Alice and which to Bob. Additional blockchain transactions are required only when the channel is closed, unlocking the latest balances of Alice and Bob on the blockchain. Payment channels solve blockchains' scalability problem by allowing the vast majority of transactions to be conducted outside the blockchain, yet retaining the blockchain for final settlement: as long as the blockchain is secure, nobody can steal funds.

More importantly, payment channels can be organized into payment-channel networks (PCNs) [20], where users without any open channels between them can pay each other through intermediaries that they do have open channels to. This allows horizontally scalable payment networks based on trustless off-chain settlement, vastly improving the usability of decentralized cryptocurrencies. Despite some disadvantages, most importantly the capital inefficiency of locking up funds in channels, PCNs are widely considered a promising avenue for scalable cryptocurrency payments.

1.2 Anonymity in PCNs

Unfortunately, "first-generation" PCNs based on the HTLC (hash time-locked contract), such as Lightning Network [12] and Raiden [4], have a significant problem — poor anonymity [18]. The very contracts that ensure blockchain-tied security also make payments in the worst case as transparently linkable as blockchain payments [18], since HTLC uses a unique hash preimage to identify each overall transaction. This threatens the improved privacy compared to on-chain transactions that is often cited as a benefit of PCNs. Furthermore, naive implementations fall victim to subtle fee-stealing attacks, like the "wormhole attack" [19], that threaten the viability of large PCNs.

A sizable body of existing work on fixing PCN security and privacy exists, mostly divided into two groups. On one hand, specialized constructions can achieve strong anonymity in specific settings, such as Bolt [15] for hub-based PCNs on the ZCash blockchain. These specialized solutions typically provide for indistinguishability of two concurrent transactions even when all intermediaries are malicious. On the other hand, general solutions for all PCN topologies, like AMHL [19] and Fulgor [18], achieve a somewhat weaker, topology-dependent notion of anonymity: relationship anonymity [7]. This property, common to onion-routing and other anonymous communication protocols, means that two concurrent transactions cannot be distnuished as long as they share at least one honest intermediary.

Unfortunately, all existing systems share one weakness. No existing anonymous PCN construction limits itself to the bare-bones cryptographic primitives used in HTLC — black-box access to a generic signature scheme and hash function. Instead, almost all existing anonymous payment channel networks are tightly coupled to the mathematics of specific cryptographic constructions. Tumblebit [16], for instance, uses a custom cryptosystem based on the RSA assumption. Systems that do use black-box primitives nevertheless require fairly exotic and difficult-to-implement primitives, such as the non-interactive zero-knowledge proofs used in Fulgor [18].

In the case of the first group of anonymous PCNs, this is understandable, as total unlinkability is a difficult goal to achieve, and likely impossible without some sort of zero-knowledge proof in settings like a hub-based PCN with one, untrusted hub. PCN constructions based on relationship anonymity, however, have no such excuse. First of all, relationship anonymity appears to be a relatively weak property for a cryptosystem to achieve. Standard and well-understood relationship-anonymous constructs, like onion routing and mix networks, exist for communication with no more than standard blackbox primitives used in secure communication (symmetric and asymmetric encryption). *Prima facie*, use of exotic cryptographic constructs to merely achieve relationship anonymity seems to be overkill.

Use of custom cryptography when not needed is not a merely academic problem. Black-box cryptography allows easy replacement of cryptographic primitives without wholesale protocol redesign, making it much easier to deal with possible cryptanalytic breakthroughs. For example, we can easily replace ECDSA in Bitcoin with any unforgeable signature scheme or AES in TLS with any secure block cipher, without compromising their security or requiring their wholesale redesign. This is simply not possible with existing anonymous PCNs.

Tumblebit [16], for instance, uses a custom cryptosystem based on the RSA assumption, and if RSA were to be broken, Tumblebit's entire design collapses. Furthermore, a black-box design allows us to seamlessly replace cryptographic primitives (like RSA) with more efficient but mathematically different primitives with the same properties (like elliptic curves). Finally, black-box cryptography makes provable security easier, as we can rely on high-level properties of the black-box primitives (such as unforgeability of a signature scheme) rather than only using low-level computational hardness assumptions.

Thus, we believe that any relationship-anonymous PCN construction should strive towards using "boring" cryptography — black-box, pervasively used cryptographic primitives.

1.3 Our contributions

In this paper, we present Astrape, the first PCN protocol that uses entirely "boring" cryptography yet achieves strong relationship anonymity. Despite achieving comparable security, privacy, and performance to other anonymous PCN constructions, Astrape does not introduce any cryptographic constructs other than those used in HTLC.

- We discuss existing payment channel networks, focusing on their security and privacy properties. In particular, we show that PCNs based on the hash time-lock contract, such as Lightning Network, have significant privacy and security problems. We discuss existing work on improving PCNs and their extensive reliance on either specialized environments or novel cryptographic constructions.
- We introduce a formal model of PCN constructions, "generalized multi-hop locks" (GMHL), based on existing work defining "anonymous multi-hop locks" [19]. that generalizes all sorts of PCN constructs. We model the security and privacy properties we desire from a PCN construction within this model.
- Within the GMHL framework, we introduce Astrape, a new PCN construction that solves the security and privacy issues of HTLC-based constructs with only black-box cryptographic primitives. Strong privacy without new cryptographic primitives solves an important open problem in the field. Astrape builds upon existing work [18] on private PCNs but uses a new technique that avoids the need for zero-knowledge proofs. We prove that Astrape fulfills all of our security and privacy goals, even with very powerful adversaries.

We implement Astrape and show that on average, Astrape requires less computation or communication than state-of-the-art private PCN constructions. We show that Astrape can easily be ported to different blockchains, due to its use of standard cryptographic primitives and the generality of the GMHL model.

2 Background and related work

2.1 Payment channels

The fundamental building block of payment channel networks is the bilateral payment channel [20]. A payment channel is a construct where two parties Alice and Bob deposit money into a blockchain-enforced "vault" that can be only opened with signatures from both Alice and Bob. Alice and Bob can then send each other money by privately agreeing on the distribution of funds between them by signing dated statements that split the money between them, without using the blockchain at all. When the funds locked in the vault are insufficient, or one of the parties need to access the funds, one of the parties can simply open the vault using any statement signed by both parties that was produced in the course of using the payment channel, subject to a short period of time where the other party may override the statement with a later-dated statement. This mechanism ensures security of funds even if one of the parties is malicious.

2.2 Multi-hop networks with HTLC

An extremely useful property of payment channels is that they can be used to construct payment channel networks (PCNs) [9, 11, 20]. A PCN is simply a directed graph of payment channels between users, allowing users without payment channels directly between each other to pay each other by routing the money over intermediaries. For example, the Lightning Network, one of the most prominent PCNs, has over 30,000 channels connecting more than 4,000 nodes [3]. Any of these 4,000 nodes can pay any other.

A crucial building block of secure PCNs is the secure multi-hop transaction (SMHT) — some way of Alice paying Bob to pay Carol without any trust in Bob. Most PCNs implement an SMHT using a smart contract known as the Hash Time-Lock Contract (HTLC). An HTLC is parameterized over a sender Alice, the recipient Bob, a deadline t, and a puzzle s. It locks up a

certain amount of money, unlocking it according to the following rules:

- The money goes to Bob if he produces π where $H(\pi) = s$ before time t, where H is a secure hash function.
- Otherwise, the money goes to Alice.

HTLCs are used to implement an SMHT in the following way. Consider a sender U_0 wishing to send money to a recipient U_n through untrusted intermediaries U_0,\ldots,U_n . At first, U_0 will generate a random π and $s=H(\pi)$, while sending the pair (π,s) to U_n over a secure channel (for instance, encrypted under U_n 's public key and transmitted over the Tor [13] network). U_0 can then lock money in a HTLC parameterized over U_0,U_1,s,t_1 , notifying U_1 . U_1 would send an HTLC over U_1,U_2,s,t_2 , notifying U_2 , and so on. The deadline must become earlier at each step $-t_1 > t_2 > \cdots > t_n$ — this ensures that in case of an uncooperative or malicious intermediary, funds always revert to the sender.

The payment eventually will be routed to U_n , who will receive an HTLC over U_{n-1}, U_n, s . He will claim the money by providing π ; this allows U_{n-1} to claim money from U_{n-2} using the same π , and so on, until all outstanding HTLC contracts are fulfilled. U_0 has successfully sent money to U_n , while the preimage resistance of H prevents any intermediary from stealing the funds.

2.3 Hub-based anonymous payment channels

An important property we may want from PCNs is strong anonymity. Intuitively, taking transactions away from the prying eyes of the public blockchain should enable two parties to transact without risking third-party surveillance. Unfortunately, HTLC has an inherent privacy problem — a common identifier $s = H(\pi)$ visible to all nodes in the payment path. Even when the communications between all PCN nodes are done over a robust anonymizing protocol, this property still severely hampers privacy [15, 18, 19]. Furthermore, when payment channels are closed and transactions end up settling on the blockchain, privacy degenerates to be similar to that of on-chain transactions.

Hub-based approaches form the earliest and most common kind of anonymous PCN design. In a hub-based anonymous PCN, the shape of the network is limited to a star topology with users communicating with a centralized (though typically untrusted) hub. Many of these networks are highly specialized, such as Green and Miers' BOLT [15], which relies on the Zcash blockchain's zero-knowledge cryptography, and

Lind and Eyal's Teechain design [17], which uses trusted execution environments. Other hub-based approaches, such as Tumblebit [16] and the more recent A2L [24], provide more general solutions that work on a wide variety of blockchains.

Hub-based PCN constructions tackle the inherently difficult problem of providing unlinkability between transactions despite the existence of only one, global, untrusted intermediary. It is therefore no surprise that highly specialized cryptography is needed to protect anonymity. Fortunately, actual payment channel networks often have intricate topologies with many participants and no huge hubs. General, topology-agnostic solutions, which may not need the kind of cryptography common in hub-based approaches, are thus more important to deploying private PCNs in practice.

2.4 Relationship-anonymous payment channels

Unlike hub-based approaches, where no intermediaries are trusted (since there is only one), general private PCN constructions target relationship anonymity. This concept, which these constructions share with onion routers and other anonymous communication protocols, assumes the existence of at least one honest intermediary between the counterparties. Thus intermediaries are in fact crucial to relationship-anonymous PCNs' privacy properties, since they achieve privacy by bouncing the payment between multiple non-colluding parties.

The earliest solution to PCN privacy in this family was probably Fulgor and Rayo [18], a closely related pair of constructions that can be ported to almost all HTLC-based PCNs. Fulgor/Rayo combines a "multihop HTLC" contract with out-of-band zero-knowledge proofs to remove the common identifier between different hops of the payment.

More recently, Malavolta et al. [19] introduced anonymous multi-hop locks (AMHL), a rigorous theoretical framework for analyzing private PCN contracts. The AMHL paper provided a concrete instantiation using linear homomorphic encryption, which provides significant performance benefits compared to systems based on zero-knowledge like BOLT or Fulgor/Rayo, while conjecturing that linear homomorphic encryption is necessary for implementing anonymous PCNs. They also presented a clever way of encoding homomorphic encryption in terms of ECDSA, allowing AMHL to be used in traditional blockchains like Bitcoin.

Another important contribution by the authors of AMHL was the discovery of "wormhole attacks" on HTLC-based PCNs. These attacks exploit a fundamen-

tal flaw in the HTLC construction to allow malicious intermediaries to steal transaction fees from honest ones. It turns out that anonymity techniques to remove the common HTLC identifier also take care of fixing this class of attack, making research into anonymous non-HTLC payment channel contracts worthwhile not only for privacy but also security improvements.

2.5 Why a new solution?

Ever since HTLCs were first proposed, anonymous PCNs solutions have progressed greatly. Systems like AMHL and A2L achieve rigorous anonymity guarantees without significantly sacrificing performance, while keeping compatibility with many existing blockchains. It is easy to conclude that anonymous payment channels are a mostly "solved problem".

Yet there remains a weakness common to all existing anonymous PCN constructions — reliance on custom cryptographic primitives. For example, AMHL uses linear homomorphic encryption, Fulgor uses non-interactive zero-knowledge proofs, and Tumblebit uses a custom cryptosystem based on the RSA assumption. Although these constructs are typically based on commonly accepted cryptographic assumptions, this situation is still not ideal.

First, non-standard cryptography poses barriers to adoption. Portability is reduced — AMHL's clever embedding of homomorphic encryption within ECDSA does not extend to other signature schemes, for example. More importantly, reliable and performant implementations of novel cryptographic functions are also difficult to obtain. Widely used, well-audited cryptographic libraries like libsodium [1] or OpenSSL [2] generally neither implement functions such as RSA-based homomorphic encryption nor provide tools to securely implement them.

Furthermore, tight coupling between a PCN protocol and the mathematics of a particular cryptographic construction makes swapping out cryptographic primitives impossible. For example, Tumblebit's cryptography crucially depends on RSA's algebraic structure [16]. In case RSA is broken through practical quantum computers or advances in factorization, Tumblebit must be completely redesigned.

Thus, we believe that finding a way of constructing efficient yet privacy-preserving PCNs by composing well-understood and easily replaced black-box cryptographic primitives is crucial to usable anonymous payment channels. In fact, AMHL's authors already posed finding the minimal set of cryptographic primitives as

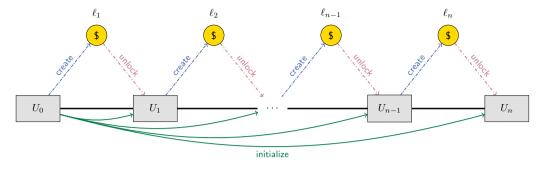


Fig. 1. An illustration of the GMHL model

an "interesting problem" [19] that they conjecturally answered with linear homomorphic encryption.

Astrape is our solution to the problem of cryptographically simple anonymous PCNs that refutes this conjecture. We show that anonymous and atomic multihop transactions can be constructed with nothing but the two building blocks of HTLCs — hashing and signatures. Unlike existing a work, no mathematical assumptions about the structure of the hash function or signature scheme are made, allowing Astrape to be easily ported to different concrete cryptographic primitives and its security properties to "fall out" from those of the primitives. This also allows Astrape to achieve high performance on commodity hardware using standard cryptographic libraries.

3 Problem statement

3.1 Generalized multi-hop locks

In our discussion of Astrape, we avoid describing the concrete details of a specific payment channel network and cryptocurrency. Instead, we introduce an abstract model — generalized multi-hop locks. This model readily generalizes to different families of payment channel networks and even other intermediarized payment technologies, such as on-chain "coin tumbling" services.

We model a sender, U_0 , sending money to a receiver, U_n , through intermediaries U_1, \ldots, U_{n-1} . We assume a "source routing" model, where the graph of all open payment channels in the network is publicly known and the sender can choose any valid path to the recipient. After an initialization phase where the sender may securely communicate parameters to each hop, each user U_i where i < n creates a coin and notifies U_{i+1} . This coin is simply a contract ℓ_{i+1} known as a lock, that essentially releases money to U_{i+1} given a certain key k_{i+1} . We call this lock the right lock of U_i and the left lock of U_{i+1} .

Finally, the payment completes once all coins created in the protocol have been unlocked and spent by fulfilling their lock scripts. Typically, this happens through a chain reaction where the recipient's left lock ℓ_n is spent, allowing U_{n-1} to unlock its left lock, etc.

Figure 1 illustrates the three GMHL components. Formally, we model a GMHL over a set of participants U_i as a tuple of four PPT algorithms $\mathbb{L} = (\text{Init}, \text{Create}, \text{Unlock}, \text{Vf})$, defined as follows:

Definition 3.1. A GMHL $\mathbb{L} = (Init, Create, Spend)$ consists of the following efficient protocols:

- 1. $\langle s_0^I, \ldots, (s_n^I, k_n) \rangle \leftarrow \langle \operatorname{Init}_{U_0}(1^{\lambda}, U_1, \ldots, U_n), \operatorname{Init}_{U_1}, \ldots, \operatorname{Init}_{U_n} \rangle$: the initialization protocol, started by the sender U_0 , that takes in a security parameter 1^{λ} and the identities of all hops U_i and returns an initial state s_i^I to all users U_i . Additionally, the recipient receives a key k_n .
- 2. $\langle (\ell_i, s_{i-1}^R), (\ell_i, s_i^L) \rangle \Leftarrow \langle \text{Create}_{U_{i-1}}(s_{i-1}^I), \text{Create}_{U_i}(s_i^I) \rangle$: the coin-creating protocol run between two adjacent hops U_{i-1} and U_i , creating the "coin sent from U_{i-1} to U_i ". This includes a lock representation ℓ_i as well as additional state on both ends unlocking the lock represented by ℓ_i releases the money.
- 3. $k_i \Leftarrow \operatorname{Unlock}_{U_i}(\ell_{i+1}, (s_i^I, s_i^L, s_i^R), k_{i+1})$: the coinspending protocol, run by each intermediary U_i where i < n, obtains a valid unlocking key k_i for the "left lock" ℓ_i given its "right lock" ℓ_{i+1} , its unlocking key k_{i+1} , and U_i 's internal state.
- 4. $\{0,1\} \leftarrow \mathsf{Vf}(\ell,k)$: given a lock representation ℓ and an unlocking key k, return 1 iff the k is a valid solution to the lock ℓ

As an example, in the classic HTLC protocol, Init consists of U_0 sending U_n the pair $k, s^I = U(k)$ and all other U_i simply s^I in the initialization phase. Then, in Create each pair of hops U_i, U_{i+1} creates the lock ℓ_{i+1} which can be unlocked by the preimage of s, saving no additional state ($s_i^L = s_i^R = \emptyset$ for all i). At the end,

all the coins are unlocked in Unlock through the propagation of the same preimage k "leftwards" — Unlock simply returns $k_i = k_{i+1}$. Finally, $\operatorname{Vf}(\ell,k)$ returns 1 iff $H(k) = \ell$.

Comparison to existing work

GMHL is an extension of anonymous multi-hop locks, the model used in the eponymous paper by Malavolta et al. [19]. In particular, AMHL defines an anonymous PCN construction in terms of the operations KGen,Setup, Lock, Rel, Vf, four of which correspond to GMHL functions. Table 1 compares the two models.

Table 1. A comparison of GMHL and AMHL

GMHL	AMHL	Notes				
_	KGen	A pre-setup key generation protocol be- tween each adjacent pair of nodes in AMHL. GMHL includes it in Init.				
Init	Setup	An initialization phase before coins are created.				
Create	Lock	Coins are created between each pair of users U_i and U_{i+1} .				
Spend	Rel	Coins are spent by satisfying their locks, releasing the money.				
Vf	Vf	A verification algorithm checking whether a key can unlock a representation of a lock.				

Although AMHL's model is very useful, we could not use it unmodified when presenting Astrape. This is because AMHL's PCN model is coupled with the homomorphic-encryption based system introduced by its authors — for example, the multi-party key setup indicated by KGen is absent from most PCN constructions. Furthermore, AMHL's definition includes its security properties, while we wish to be able to use the same framework to discuss PCNs with varying security and anonymity goals.

Nevertheless, GMHL can be considered as a minimally modified version of AMHL, generalized to encompass almost all existing PCN constructions. Thus, Astrape can be considered a solution to the problem posed by AMHL's authors — what are the cryptographic primitives needed to implement "anonymous multi-hop locks" [19].

3.2 Security model

Now that we have a model to discuss PCN constructions, we can discuss our threat model as well as our security and privacy objectives.

Active adversary

We use a similar adversary model to that of AMHL [19]. That is, we model an adversary \mathcal{A} with access to a functionality $\operatorname{corrupt}(U_i)$ that takes in the identifier of any user U_i and provides the attacker with the complete internal state of U_i . The adversary will also see all incoming and outgoing communication of U_i . It also sees all information that is posted onto any underlying blockchain. $\operatorname{corrupt}(U_i)$ will also give the adversary active control of U_i , allowing it to impersonate U_i when communicating with other participants.

Anonymous and authenticated communication

We assume there is a secure and anonymous message transmission functionality \mathcal{F}_{anon} that allows any participant to send messages to any other participant. Messages sent by an honest (non-corrupted) user with \mathcal{F}_{anon} hide the identity of the sender and cannot be read by the adversary, although the adversary may arbitrarily delay messages.

We also assume there is a secure and mutually authenticated message transmission functionality \mathcal{F}_{auth} . Like \mathcal{F}_{anon} , \mathcal{F}_{auth} lets any honest participant send a message to another honest participant without the adversary knowing. Unlike \mathcal{F}_{anon} , \mathcal{F}_{auth} does not hide the identity of the sender from the recipient, instead unforgeably indicating the sender. However, the adversary cannot know the sender or recipient of \mathcal{F}_{auth} messages between honest participants.

A good practical approximation of \mathcal{F}_{anon} and \mathcal{F}_{auth} recommended by existing work [18, 19] is an onion-routing circuit constructed over the same set of users U_i , constructed with a provably private protocol like SPHINX [10]. Public networks such as Tor may also be used to implement \mathcal{F}_{anon} and \mathcal{F}_{auth} .

Liveness and timeouts

We assume that every coin locks ℓ_i comes with an appropriate timeout that will return money to U_{i-1} if U_i does not take action. We also assume that each left lock ℓ_i 's deadline is at least δ later than that of the right lock ℓ_{i+1} , where δ is an upper bound on network latency between honest parties, even under disruption by the adversary. These assumptions allow us to omit time-

out handling from the description of the protocol, in line with related work (such as AMHL [19]).

Cryptographic assumptions

One of Astrape's main goals is to make minimal cryptographic assumptions. We assume only:

- Generic random oracle. We assume a random-oracle hash function H producing λ bits of output, where 1^{λ} is the security parameter. We use the random oracle both as a pseudorandom function and as a commitment scheme, which is well known [8] to be secure.
- Generic signature scheme. We assume a secure signature scheme that allows for authenticated communication between any two users U_i and U_j .

3.3 Security and privacy goals

Against the adversary we described above, we want to achieve the following security and privacy objectives:

Relationship anonymity

Given two simultaneous payments between different senders $S_{\{0,1\}}$ and receivers $R_{\{0,1\}}$ with payment paths of the same length intersecting at the same position at one honest intermediate user, an adversary corrupting all of the other intermediate users cannot determine, with probability better than 1/2 (guessing), whether S_0 paid R_0 and S_1 paid R_1 , or S_0 paid R_1 and S_1 paid R_0 . This is an established standard for anonymity in payment channels [18, 19] and is analogous to similar anonymity definitions for anonymous communication [7, 22]. It is important to note that the adversary is not allowed to corrupt the sender U_0 — senders always know who they are sending money to — and that anonymity of the sender against the recipient is not considered.

Balance security

For an honest user U_i , if its right coin ℓ_i is spent, U_i must always be able to spend its left coin ℓ_{i+1} even if all other users are corrupt. Combined with the timeouts mentioned in our security model, this guarantees that no intermediary node can lose money even if everybody else conspires against it.

Wormhole resistance

We need to be immune to the wormhole attack on PCNs, where malicious intermediaries steal fees from other intermediaries. The reason why is rather subtle [19], but for our purposes this means that given an honest sender and an honest intermediary U_{i+1} , ℓ_i cannot be spent by U_i until U_{i+1} spends ℓ_{i+1} . Intuitively, this prevents honest intermediaries from being "left out".

4 Construction

4.1 Core idea: broken + broken = working

Unlike existing systems that utilize the mathematical properties of some cryptographic construction to build a secure and anonymous primitive, Astrape is constructed out of two separate *broken* constructions, both of which use boring cryptography and are straightforward to describe:

- **XorCake**, which has relationship anonymity but lacks balance security if the sender U_0 is malicious
- HashOnion which has balance security, but loses relationship anonymity the Unlock phase. That is, an adversary limited only to observing Init and Create cannot break relationship anonymity, but an adversary observing Unlock can.

The key insight here is that if we can ensure that HashOnion's Unlock phase can only start when there is proof that the sender is malicious, we obtain a system that has both relationship anonymity and balance security. This is because the definition of relationship anonymity assumes an honest sender: if the sender is compromised, it can always simply tell the adversary the identity of its counterparty, breaking anonymity trivially.

We next describe XorCake and HashOnion, and their composition into Astrape.

4.2 XorCake: anonymous but insecure against bad senders

Let us first describe XorCake's construction. XorCake is an extremely simple construction borrowed from "multihop HTLC", a building block of Fulgor [18]. It has relationship anonymity, but not balance security against malicious senders.

Recall that in GMHL, the sender (U_0) wishes to send a sum of money to the recipient (U_n) through U_1, \ldots, U_{n-1} . At the beginning of the transaction, the

```
function Init_{U_0}^{XC}(1^{\lambda}, U_1, \dots, U_n)
Upon invocation by U_0:
    generate \lambda-bit random numbers \{r_1, \ldots, r_n\}
    for i in 1, \ldots, n do
         s_i \leftarrow H(r_i \oplus r_{i+1} \oplus \cdots \oplus r_n)
         if i = n then
              send (k_n = r_n, s_n) to U_n
         else
              send (r_i, s_i, s_{i+1}) to U_i
function Create_{U_i}^{XC}(s_i^I = (r_i, s_i, s_{i+1}))
Upon invocation by U_i, where i < n:
    return \ell_{i+1} = \mathsf{HTLC}[s_{i+1}]
function Unlock_{U_i}^{	ext{XC}}(\ell_{i+1}, s_i^I = (r_i, s_i, s_{i+1}), k_{i+1})
Upon invocation by U_i, where i < n:
    if Vf^{XC}(\ell_{i+1}, k_{i+1}) = 0 then
         abort
     k_i \leftarrow r_i \oplus k_{i+1}
    return k_i
function Vf^{XC}(\ell, k)
    parse \ell as \mathsf{HTLC}[s]
    if H(k) = s then
         return 1
    else
         return 0
```

Fig. 2. XorCake in the GMHL framework

sender samples n independent λ -bit random strings (r_1, \ldots, r_n) . Then, for all $i \in 1, \ldots, n$, she sets n values $s_i = H(r_i \oplus r_{i+1} \oplus \cdots \oplus r_n)$, where H is a secure hash function. That is, s_i is simply the hash of the XOR of all the values r_j for $j \geq i$.

 U_0 then uses the anonymous channel \mathcal{F}_{anon} to provide U_n the values (r_n, s_n) and the all other U_i with (r_i, s_i, s_{i+1}) .

Then, for each pair of neighboring nodes (U_i, U_{i+1}) , U_i sends U_{i+1} a coin encumbered by a regular HTLC ℓ_{i+1} asking for the preimage of s_{i+1} . U_n knows how to unlock ℓ_n , and the solution would let U_{n-1} unlock ℓ_{n-1} , and so on

Figure 2 formalizes XorCake in the GMHL framework.

XorCake by itself satisfies relationship anonymity. A full proof is available in the Fulgor paper from which XorCake was borrowed [19], but intuitively this is because r_i will be randomly distributed over the space of possible strings because H behaves like a random oracle.

This means that unlike in HTLC, no two nodes U_i and U_j can deduce that they are part of the same payment path unless they are adjacent.

Bad-state attack

Unfortunately, XorCake does not have balance security. Consider a malicious sender who follows the protocol correctly, except for sending an incorrect r_i to U_i . (Note that U_i cannot detect that r_i is incorrect assuming a random-oracle hash function) Then, when ℓ_{i+1} is unlocked, ℓ_i cannot be spent, since U_i cannot reconstruct the solution to its left lock from that of its right lock without the correct r_i . In an actual PCN such as the Lightning Network, all coins "left" of U_i will time out, letting the money go back to U_0 . U_0 paid U_n with U_i 's money instead of her own! We call this the "bad-state attack", and because of it, XorCake is not a viable PCN construction on its own. In Fulgor, XorCake was combined with out-of-band zero-knowledge proofs of the correctness of r_i , but as we will see shortly, Astrape can dispense with it.

4.3 HashOnion: secure but eventually non-anonymous

We now present HashOnion, a PCN construction that has balance security but not relationship anonymity. Note that unlike HTLC, HashOnion's non-anonymity stems entirely from information leaked in the Unlock phase, a property we will leverage to build a fully anonymous construction combining HashOnion and XorCake.

At the beginning of the transaction, U_0 generates random values s_i for $i \in \{1, ..., n\}$, then "onion-like" values x_i , recursively defined as:

$$x_i = H(s_i||x_{i+1})$$
$$x_n = H(s_n||0^{\lambda})$$

Essentially, x_i is a value that commits to all s_j where i > i.

The sender sends (x_i, s_i) to U_i . Then, each intermediary U_{i-1} sends to its successor U_i a lock ℓ_i , which can be only be unlocked by some $k_i = (s_i, \ldots, s_n)$ where $H(s_i||H(s_{i+1}||H(\ldots H(s_n||0^{\lambda})))) = x_{i-1}$. Finally, during the unlock phase, the recipient U_n solves ℓ_n with $k_n = (s_n)$. This allows each U_i to spend ℓ_i , completing the transaction.

For balance security, we need to show that with a solution $k_{i+1} = (s_{i+1}, \ldots, s_n)$ to ℓ_{i+1} and s_i , we can always construct a solution to ℓ_i . This is obvious: we just add s_i to the solution: $k_i = (s_i, s_{i+1}, \ldots, s_n)$.

One subtle problem is that U_i needs to make sure that its right coin is actually the correct ℓ_i and not some bad ℓ'_i parameterized over some $x_{i-1} \neq H(s_i||x'_i)$. Fortunately, this is easy: given x_{i-1} (which it always knows from ℓ_i), U_i can just check that $x_{i-1} = H(s_i||x_i)$ before sending out ℓ_{i+1} to the next hop. Thus, every user can make sure that if its right coin is spent, so can its left coin, so balance security holds.

We also see that although the unlocking procedure obviously breaks relationship anonymity by revealing all the s_i , before the unlock happens, HashOnion does have relationship anonymity. This is because the adversary cannot connect the different x_i as long as one s_i remains secret — that of the one honest intermediary we assume.

4.4 Securing XorCake+HashOnion with a fraud proof

We now move on to composing XorCake and HashOnion. We do so by creating a variant of HashOnion that embeds XorCake and recognizes *fraud proofs* that prove an attempt by the sender to execute a bad-state attack for XorCake.

After generating the XorCake parameters, U_0 creates $n \lambda$ -bit values x_i recursively:

XorCake parameters
$$x_i = H(r_i||s_i||s_{i+1}||o_i||x_{i+1})$$

$$x_n = o_n$$

where o_i is a random nonce sampled uniformly from all possible λ -bit values.¹ The intuition here is that x_i commits to all the information U_0 would give to all hops U_j where $j \geq i$.

Afterwards, the sender then uses an anonymous channel to send (o_i, x_i, x_{i+1}) to every hop i. Every hop U_i checks that all the parameters, including the Xor-Cake parameters (r_i, s_i, s_{i+1}) given earlier, are consistent with each other.

We next consider what will happen if the sender attempts to defraud an intermediate hop U_i with a bad-state attack. U_{i+1} would unlock its left lock ℓ_{i+1} by giving k_{i+1} where $H(k_{i+1}) = s_{i+1}$ but $H(r_i \oplus k_{i+1}) \neq s_i$. This then causes $\mathsf{Unlock}_{U_i}^{\mathsf{XC}}$ to fail.

But this attempt at fraud allows U_i to generate a cryptographic fraud proof verifiable to anybody know-

ing x_i : λ -bit values k_{i+1} , r_i , s_i , s_{i+1} , o_i , x_{i+1} where:

$$H(k_{i+1}) = s_{i+1}$$

$$H(r_i \oplus k_{i+1}) \neq s_i$$

$$H(r_i||s_i||s_{i+1}||o_i||x_{i+1}) = x_i$$

This fraud proof proves the preimage of s_{i+1} XORed with r_i does not equal the preimage of s_i , demonstrating that the values given to U_i are inconsistent and that U_0 is corrupt. Since U_{i-1} knows x_i , U_i can therefore prove that it was a victim of a bad-state attack to U_{i-1} .

Since x_i commits to all XorCake initialization states "rightwards" of U_i , U_i , in cooperation with U_{i-1} , can also produce a fraud proof that U_{i-2} can verify using x_{i-1} . This is a set of simply λ -bit values k_{i+1} , r_{i-1} , s_{i-1} , r_i , s_i , s_{i+1} , x_i , x_{i+1} where:

$$H(k_{i+1}) = s_{i+1}$$

$$H(r_i \oplus k_{i+1}) \neq s_i$$

$$H(r_i||s_i||s_{i+1}||o_i||x_{i+1}) = x_i$$

$$H(r_{i-1}||s_{i-1}||s_i||o_{i-1}||x_i) = x_{i-1}$$

We can clearly extend this idea all the way back to U_1 — given a fraud proof demonstrating a bad-state attack against U_i , U_{i-1} can verify the proof and generate a similar proof verifiable by U_{i-2} , and so on. This forms the core construction that Astrape uses to fix XorCake's lack of balance security.

4.5 Complete construction

We now present the complete construction of Astrape, as formalized in Figure 3 within the GMHL framework.

Initialization

In the first phase, represented as Init in GMHL, the sender U_0 first establishes communication to the n hops U_1, \ldots, U_n , the last one of which is the receiver. When talking to intermediaries, U_0 uses our abstract functionality $\mathcal{F}_{\text{anon}}$, but communication with the recipient is authenticated with $\mathcal{F}_{\text{auth}}$.

The sender then generates random λ -bit strings (r_1,\ldots,r_n) and (o_1,\ldots,o_n) . From these strings, she derives $s_i=H(r_i\oplus r_{i+1}\oplus\cdots\oplus r_n)$ and $x_i=H(r_i||s_i||s_{i+1}||o_i||x_{i+1}); x_n=H(o_n)$. She then sends to each intermediate hop U_i the tuple $s_i^I=(r_i,s_i,s_{i+1},x_i,x_{i+1},o_i)$ and gives the last hop U_n the initial state $s_i^I=(r_n,s_n)$ and the unlocking

^{1 ||} denotes concatenation. In our case, it's possible to unambiguously separate concatenated values, since we only ever concatenate λ -bit values.

```
function Unlock_{U_i}^{\mathrm{AS}}(\ell_{i+1}, s_i^I, k_{i+1})
function \operatorname{Init}^{\operatorname{AS}}_{U_0}(1^\lambda,U_1,\ldots,U_n)
                                                                                                            Upon invocation by U_i, where i < n:
Upon invocation by U_0:
                                                                                                                 \Gamma_i \leftarrow r_i ||s_i||s_{i+1}||o_i|
     generate \lambda-bit random numbers \{r_1, \ldots, r_n\}
                                                                                                                 parse s_i^I = (r_i, s_i, s_{i+1}, x_i, x_{i+1}, o_i)
     x_n \leftarrow \text{random } \lambda\text{-bit number}
                                                                                                                 if parse k_{i+1} = \mathsf{HSoln}[\kappa_{i+1}] then
     for i in n, \ldots, 1 do
                                                                                                                      if H(r_i \oplus \kappa_{i+1} \oplus s_{i+1}) = s_i then
          s_i \leftarrow H(r_i \oplus r_{i+1} \oplus \cdots \oplus r_n)
                                                                                                                            return k_i = \mathsf{HSoln}[r_i \oplus \kappa_{i+1}]
          o_i \leftarrow \text{random } \lambda\text{-bit number}
                                                                                                                      else
           if i < n then
                                                                                                                            return k_i = \mathsf{FSoln}[\kappa_{i+1}, x_{i+1}, \{\Gamma_i\}]
                x_i \leftarrow H(r_i||s_i||s_{i+1}||o_i||x_{i+1})
                                                                                                                 else
     for i in 1, \ldots, n do
                                                                                                                      parse k_{i+1} = \mathsf{FSoIn}[\kappa_i, x_i, \{\Gamma_{i+1}, \dots, \Gamma_i\}]
           if i = n then
                                                                                                                      return k_i = \mathsf{FSoln}[\kappa_j, x_j, \{\Gamma_i, \Gamma_{i+1}, \dots, \Gamma_j\}]
                send s_n^I = (k_n = \mathsf{HSoln}[r_n], s_n) to U_n
           else
                                                                                                           function Vf^{AS}(\ell, k)
                send s_i^I = (r_i, s_i, s_{i+1}, x_i, x_{i+1}, o_i) to U_i
                                                                                                                parse \ell = \mathsf{Astrape}[x, s]
                                                                                                                if parse k = \mathsf{HSoln}[\kappa] then
function Create_{U_i}^{AS}(s_i^I = (r_i, s_i, s_{i+1}, x_i, x_{i+1}, o_i))
                                                                                                                      return 1 iff H(\kappa) = s

ightharpoonup "normal" case
Upon invocation by U_i, where i < n:
                                                                                                                 else if parse k = \mathsf{FSoln}[\kappa, \chi, \{\Gamma_i, \dots, \Gamma_j\}] then
     if x_i \neq H(r_i||s_i||s_{i+1}||o_i||x_{i+1}) then
                                                                                                                      if \exists i \text{ s.t. } \Gamma_i.\text{length} \neq 4\lambda \text{ bits then}
           abort bad initial state
                                                                                                                            return 0
     if i > 0 then
                                                                                                                      parse \Gamma_j = r_j ||s_j||s_{j+1}||o_j|
           wait for \ell_i = \mathsf{Astrape}[\hat{x}_i, \hat{s}_i] to be created
                                                                                                                      if H(\kappa \oplus r_j \oplus s_{j+1}) = s_j then
           if \hat{x}_i \neq x_i or \hat{s}_i \neq s_i then
                                                                                                                                              ▷ state good, not fraud proof
                                                                                                                            return 0
                abort invalid left coin
                                                                                                                      \hat{x} \leftarrow H(\Gamma_i||H(\Gamma_{i+1}||\dots H(\Gamma_j||\chi)))
     return \ell_{i+1} = \mathsf{Astrape}[x_{i+1}, s_{i+1}]
                                                                                                                                                                         \triangleright "fraud" case
                                                                                                                      return 1 iff \hat{x} = x
```

Fig. 3. Astrape as a GMHL protocol

key $k_n = \mathsf{HSoln}[r_n]^2$. Every hop U_i checks that the values are self-consistent by verifying that $x_i = H(r_i||s_i||s_{i+1}||o_i||x_{i+1})$; if this check fails the hop must not proceed with the protocol. According to our liveness assumptions, an "abort" will lead to a cascading timeout of coins left of the aborting intermedary, restoring balance security.

Creating the coins

We now move on to Create, where all the coins are initially locked. U_0 then sends U_1 a coin encumbered with a lock represented as $\ell_1 = \mathsf{Astrape}[x_1, s_1]$. When each hop U_i receives a correctly formatted coin from its previous hop U_{i-1} , it sends the next hop U_{i+1} a coin with a lock $\ell_{i+1} = \mathsf{Astrape}[x_{i+1}, s_{i+1}]$.

Each of these locks ℓ_i can be spent either through solving a XorCake-type puzzle to find the preimage of s_i (the "normal" case) or by presenting a fraud proof with a HashOnion-type witness demonstrating x_i 's commitment to inconsistent data (the "fraud proof" case).

After all the transactions with Astrape-encumbered coins are sent, U_n can finally claim its money, triggering the next phase of the protocol.

Unlocking the coins

We finally arrive at the last step, Unlock. After receiving the final coin from U_{n-1} , the recipient unlocks its lock ℓ_n by providing the preimage of the HTLC puzzle: $k_n = \mathsf{HSoln}[r_n]$ — this is the only way an honest recipient can claim the money in a payment originating from an honest sender.

Each intermediate node U_i reacts when its right lock ℓ_{i+1} is unlocked with a key k_{i+1} :

- If U_{i+1} solved the HTLC puzzle with $k_{i+1} = \mathsf{HSoln}[\kappa_{i+1}]$, construct $\kappa_i = r_i \oplus \kappa_{i+1}$
 - If $H(\kappa_i \oplus s_{i+1}) = s_i$, this means that there is no bad-state attack happening. We unlock our left lock with $k_i = \mathsf{HSoln}[\kappa_i]$.
 - Otherwise, there must be a bad-state attack happening. We construct a fraud proof and create a key that embeds the fraud proof and a witness verifiable with x_i . This gives us $k_i = \mathsf{FSoln}[\kappa_{i+1}, x_{i+1}, \{\Gamma_i\}]$, where $\Gamma_i = r_i ||s_i|| |s_{i+1}||o_i$.

² We use the notation $\mathsf{Tag}[x_1, \ldots, x_n]$ to represent a *n*-tuple of values with an arbitrary "tag" that identifies the type of value.

- Otherwise, U_{i+1} unlocked ℓ_{i+1} with a fraud proof $k_{i+1} = \mathsf{FSoln}[\kappa_j, x_j, \{\Gamma_{i+1}, \Gamma_{i+2}, \dots, \Gamma_j\}]$ demonstrating that the sender attempted to defraud some U_j , where j > i.
 - We can simply construct $k_i = \mathsf{FSoln}[\kappa_j, x_j, \{\Gamma_i, \Gamma_{i+1}, \dots, \Gamma_j\}]$ where $\Gamma_i = r_i ||s_i||s_{i+1}||o_i|$. This transforms the fraud proof verifiable with x_{i+1} to a proof verifiable with x_i .

This process continues backwards towards the sender until all the coins created in the previous step are spent. It is clear that we have balance security — node U_i can unlock its left coin ℓ_i if and only if node U_{i+1} has unlocked ℓ_{i+1} , so no intermediaries can lose any money.

5 Security and privacy analysis

Let us see how Astrape accomplishes all of our security and privacy goals.

5.1 Relationship anonymity

Out of our three desired properties, relationship anonymity is the most important. Unlike plain XorCake, Astrape does not achieve relationship anonymity in a particularly obvious manner.

Thus, we present proof that Astrape does achieve relationship anonymity. Note that within this proof, we assume that the sender U_0 and one particular U_i is honest according to the definition of relationship anonymity, while everybody else can be compromised by the adversary A.

Lemma 5.1. The adversary cannot gain any information by corrupting \mathcal{F}_{anon} messages sent by U_0 to the honest intermediary U_i .

Proof. Because $\mathcal{F}_{\mathrm{anon}}$ hides the sender, all the messages sent by U_0 to U_i can trivially be forged. This can prove problematic, because in the rest of our analysis of relationship anonymity we want to treat messages that U_i obtained through $\mathcal{F}_{\mathrm{anon}}$ as authentic messages from U_0 . However, if we prove that the adversary cannot gain any information from corrupting these messages, corrupting these messages cannot lead to a break in relationship anonymity, so we can then assume the messages are not corrupted.

We first note that due to U_i 's self-consistency check of $x_i = H(r_i||s_i||s_{i+1}||o_i||x_{i+1})$, any forgery of one vari-

able requires the adversary either to know or forge every other variable. Therefore, after a corrupted lnit, the only values U_0 wished to send to U_i that the adversary doesn't know — r_i, o_i — are destroyed. There is thus nothing in s_i^I now that U_i could reveal to the adversary in later stages of the protocol to possibly break relationship anonymity.

TODO: make this argument rigorous

Lemma 5.2. Given an honest sender and uncorrupted communication between it and the honest intermediary U_i , the left and right locks of U_i , ℓ_i and ℓ_{i+1} , are always unlocked normally and never by fraud proof. That is, no malicious intermediary can "frame" an honest sender with a fraud proof.

Proof. There are only two cases where an honest U_i unlocks the left coin ℓ_i by fraud proof:

- 1. The right lock was unlocked normally, but with an XorCake solution κ where $H(\kappa) = s_{i+1}$ yet $H(r_i \oplus \kappa) \neq s_i$.
- 2. The right lock was unlocked with a fraud proof recognized by HashOnion.

The first case is impossible with an honest sender since our hash function is secure. Finding such a κ would imply finding two different values κ , κ' where $H(r_i \oplus \kappa) \neq s_i$ but $H(r_i \oplus \kappa') = s_i$.

In the second case, under the random oracle model, where we can assume that H acts as a binding commitment scheme, this means that x_{i+1} commits to inconsistent information that violates the protocol. But since we know that U_0 is honest, such an x_{i+1} could not have been sent to U_i , so this is impossible too.

Thus, there cannot be any fraud proofs used when unlocking ℓ_i and ℓ_{i+1} .

Lemma 5.3. Given two simultaneous payments with the same value and paths $\cdots \to U_{i-1} \to U_i \to U_{i+1} \to \cdots$ and $\cdots \to U'_{i-1} \to U_i \to U'_{i+1} \to \cdots$, the attacker cannot distinguish this situation from two payments with paths $\cdots \to U_{i-1} \to U_i \to U'_{i+1} \to \cdots$ and $\cdots \to U'_{i-1} \to U_i \to U_{i+1} \to \cdots$.

Proof. We assume that the attacker \mathcal{A} has corrupted all intermediate nodes before and after U_i . This means that in the lnit phase, \mathcal{A} learns all states s_j^I where $j \notin \{i,n\}$ for the first transaction, and also all states $s_j'^I$ where $j \notin \{i,n\}$ for the second transaction, but not the internal state of U_i , which is $s_i^{(')I}$. The only values revealed to \mathcal{A} that contain the internal state of U_i are commitments x_j where $j \leq i$ — but reconstructing U_i 's initial state

from x_j implies breaking the preimage resistance of H, which is impossible if H is a random oracle.

In the Create phase, we note that U_i obviously does not reveal any internal state except $s_{i+1}^{(\prime)}$ and $x_{i+1}^{(\prime)}$, which the attacker already knows from Init. Furthermore, by Lemma 5.2 we know that the two locks surrounding U_i can only be unlocked through the HTLC-based "normal" case, which again does not reveal more information about $s_i^{(\prime)}$.

Thus, the attacker's game reduces to guessing the predecessor and successor of U_i for both transactions, given all states $s_j^{(')I}$ for $j \notin \{i,n\}$, with probability nonnegligibly more than 1/2 (as a function of λ). We can further reduce this to \mathcal{A} guessing whether or not two states \hat{s}_{i-1}^I and \hat{s}_{i+1}^I from corrupt users adjacent to U_i come from the same transaction.

At this point, we can proceed with a proof by contradiction. Assume that \mathcal{A} can, in fact, guess whether or not \hat{s}_{i-1}^{I} and \hat{s}_{i+1}^{I} come from the same transaction. This implies that \mathcal{A} can distinguish between two possible values of Ω in

$$\hat{s}_{i-1}^{I} = (\hat{r}_{i-1}, \hat{s}_{i-1}, \hat{s}_{i}, \hat{x}_{i-1}, \hat{x}_{i}, o_{i-1}) \text{ where}$$

$$\hat{x}_{i} = H(\hat{r}_{i}||\hat{s}_{i}||\Omega||\hat{x}_{i+1})$$

with more than $1/2+\epsilon$ probability, given only the values of \hat{s}_{i-1}^I and $\hat{s}_{i+1}^I = (r_{i+1}, s_{i+1}, x_{i+1}, o_i)$. But this implies breaking preimage resistance of H, which is impossible for a random oracle.

Theorem 5.4. Astrape has relationship anonymity.

Proof. This follows immediately from 5.3.

5.2 Balance security

We now show that Astrape has balance security.

Lemma 5.5. An honest user U_i always receives enough information to unlock its left lock ℓ_i if its right lock ℓ_{i+1} is unlocked.

Proof. We have two cases to consider.

In the first case, ℓ_{i+1} is unlocked by solving the XorCake component — with $k_{i+1} = \mathsf{HSoln}[\kappa_{i+1}]$.

Either κ satisfies $H(r_i \oplus \kappa) = s_i$ or it does not. If it does, U_i can construct $\kappa' = H(r_i \oplus \kappa)$ and use $k_i = \mathsf{HSoln}[\kappa]$ to unlock $\ell_i = \mathsf{Astrape}[s_i, x_i]$. If it doesn't, we have a fraud proof, and we can construct $k_i = \mathsf{FSoln}[\kappa, x_{i+1}, \{\Gamma_i\}]$ where $\Gamma_i = r_i ||s_i||s_{i+1}||o_i$, and $H(\Gamma_i||x_{i+1}) = x_i$. This k_i is guaranteed to unlock ℓ_i .

In the second case, our right lock ℓ_{i+1} is spent by fraud proof using the HashOnion-like component. We

can construct our own k_i to unlock ℓ_i by simply extending the fraud proof with an additional Γ_i .

Theorem 5.6. Astrape has balance security.

Proof. For every intermediary U_i , its left lock has funds equal to its right lock (plus fees). Following 5.5, this means that any money lost by U_i 's right lock being spent can always be recovered by spending its left lock. Thus, intermediaries cannot lose any money.

Therefore, Astrape has balance security according to its definition in Section 3.3.

5.3 Wormhole immunity

The wormhole attack [19] is a generic attack on PCNs where given an honest sender and two corrupted nodes $U_i, U_j, j > i+1$ separated by at least one honest node, U_i can unlock its left coin once U_j 's right coin is unlocked. This "cuts out" all the participants in between, depriving them of fees from the sender.

Astrape is immune to the wormhole attack. This is because when U_{i+1} is honest, ℓ_{i+1} must be unlocked before U_i even has the information to unlock ℓ_i — information from "further down the line" at U_j where j > i+1 is not sufficient. We note that Astrape fits the model given in the AMHL paper [19] for wormhole-immune PCN constructions, since the sender must know the complete path before sending money and there are two rounds of communication.

6 Practical concerns

6.1 Blockchain implementation

So far we have presented Astrape in an abstract GMHL model. We now present how Astrape may be implemented in popular cryptocurrency blockchains.

Astrape is easy to implement on blockchains with Turing-complete scripting languages, like Ethereum, as well as layer-2 PCNs such as Raiden. One simply has to replace HTLC-based constructs with Astrape-based ones. Since Astrape does not use any novel cryptographic primitives, built-in hash and signature functions can be used, reducing "gas" costs significantly. Furthermore, in a Turing-complete scripting language, the Vf algorithm can be compactly represented, leading to less space usage.

Porting to a more traditional blockchain without Turing-complete scripts is not significantly harder since our contracts are very simple and do not involve unbounded loops. There are, however, two main issues that do not appear in Ethereum-like blockchains.

The first problem is that traditional blockchains typically do not allow any recursion in lock scripts. This means that we cannot directly implement the Vf function. Instead, we must "unroll" Vf to explicitly check for fraud proofs involving inconsistencies given to U_i , U_{i+1} , etc.

Thus, for an n-hop payment the size of every lock script will be O(n). In practice, the mean path length in Lightning Network is only 2.8 [23], and privacy-focused onion routing systems typically use 3 to 5 hops. We do not think exorbitant script sizes are a serious concern for Astrape deployment.

The second issue is more serious: some blockchains have so little scripting functionality that Astrape cannot be implemented. Astrape requires an "append-like" operation || that can take in two bytestrings and combine them in a collision-resistant manner. This functionality is present in many Turing-incomplete scripting languages, including that of Bitcoin Cash and its derivatives. Unfortunately, the most popular cryptocurrency Bitcoin has disabled all of its string-manipulation opcodes. In fact, the only operations possible for strings longer than 32 bits in Bitcoin are comparing or hashing them. This means that Astrape, which requires heavily on much longer cryptographic values, cannot be directly implemented on Bitcoin. Whether an implementation based solely on the 32-bit integer arithmetic that Bitcoin uses is practical is an interesting open question.

6.2 Side-channel deanonymization

We follow existing literature on anonymous PCNs [18, 19] in pursuing relationship anonymity as our privacy goal. This means that a powerful adversary that cannot distinguish between two payments sharing one honest intermediary that happen at exactly the same time with exactly the same values. However, real-life scenarios are rarely so clean, and violations of the assumptions of relationship anonymity can easily break privacy.

For example, the value of a transaction (say \$3.1415) may be fairly unique, allowing adversaries to distinguish it from other transactions. In other cases, timing patterns may deanonymize users — perhaps only AliceCorp pays employees large lump sums on the 3rd and 14th every month, letting the adversary detect AliceCorp employees.

Why, then, do we use relationship anonymity as our privacy goal? The main reason is that timing and value anonymity are largely *orthogonal* to relationship anonymity. Once we achieve relationship anonymity, we can adapt a large body of existing research in defeating side-channel attacks against anonymity systems like onion routing. Timing attack defenses can be directly lifted from works like Feigenbaum et al. [14], while the information leaked by cryptocurrency values can be vastly diminished simply by using fixed denominations. Restricting the possible "denominations" of coins in the network in a system with timing and relationship privacy would lead to cash-like anonymity — every quarter changing hands is indistinguishable from other quarters, every dime from other dimes, etc.

We further contrast relationship anonymity with the stronger privacy guarantees provided by hub-based approaches, where the sender and receiver are the only trusted parties. Hub topologies necessarily require a stronger privacy model, as there is only one intermediary, and we believe that for general PCNs that is too strong and constrains us to exotic cryptographic constructs. Furthermore, even these approaches often do not cover side channels based on timing and value [15].

Thus, we believe that relationship anonymity is the best way to evaluate a PCN construction's privacy.

6.3 Griefing attacks

An unusual property of Astrape is that its worst-case cost (the HashOnion backup mechanism) diverges sharply from its "average"-case cost (XorCake). A naive implementation of Astrape would then open the door to griefing attacks, where an adversary sends coins to itself while intentionally triggering HashOnion by giving intermediaries corrupt values. Linear lock sizes and verification costs would then be borne by intermediaries.

Fortunately, this is easy to mitigate by charging the sender a penalty P for every hop the HashOnion case is used. For each hop ℓ_i , in addition to the main payment coin encumbered by Astrape's composite Xor-Cake/HashOnion lock, we add a coin of value P(n-i) which is unlockable by the same parameters that unlock the HashOnion case of the main coin. This way, if HashOnion is activated, every hop burdened by verifying HashOnion proofs can collect an extra P from its predecessor, which ultimately penalizes U_0 .

7 Comparison with existing work

In this section, we compare Astrape with existing PCN constructions. First, we compare Astrape's design choices and features with that of other systems, showing that it explores a novel design space. Then, we evaluate Astrape's concrete performance. We compare Astrape's performance with that of other PCN constructions, both anonymous and non-anonymous. Finally, we explore how Astrape's performance on a real-world network graph from the Lightning Network.

7.1 Design comparison

Table 2. Comparison of different PCNs

	Topology	Anon	Efficient	Crypto
HTLC	Mesh	No	Yes	Sig. + hash
Tumblebit Bolt Teechain	Hub Hub Hub	Yes Yes Yes	No Yes Yes	Custom RSA NIZKP Trusted comp.
Fulgor/Rayo AMHL	Mesh Mesh	Yes Yes	No Yes	ZKP Homom. enc.
Astrape	Mesh	Yes	Yes	Sig. + hash

In Table 2, we compares Astrape's properties with those of existing payment channel networks. We see that except for HTLC, which does not achieve anonymity, all previous PCN networks use cryptographic constructions specialized for their use case. Furthermore, only more recent constructions achieve efficiency comparable to HTLC. It is thus clear that Astrape is the first and only PCN construction that works on all PCN topologies, achieves strong anonymity, and performs at high efficiency, while using the same simple cryptography as HTLC.

7.2 Implementation and benchmark setup

To demonstrate the feasibility and performance of our construction, we developed a prototype implementation in the Go programming language. We implemented all the cryptographic constructions of Astrape inside a simulated coin-passing model. We used the libsodium library's implementation of the ed25519 [21] signature scheme and blake2b [6] hash function. In addition, we generated script locks in Bitcoin Cash's scripting languages to illustrate script sizes for scripting languages with no loops.

All tests were done on a machine with a 3.2 GHz Intel Core i7 and 16 GB RAM. Network latency is not simulated, as this is highly application dependent. These conditions are designed to be maximally similar to those

under which Fulgor [18] and AMHL [19] were evaluated, allowing us to compare the results directly.

7.3 Resource usage

Our first set of tests compares Astrape's resource usage to that of other PCN constructions. We compare both a simulation of Astrape and a concrete implementation using Bitcoin Cash's scripting language to traditional HTLC, Fulgor/Rayo, and the ECDSA-embedded version of AMHL. Note that due to an unoptimized, naive Bitcoin Cash implementation, the lock sizes are not linear but quadratic in the number of hops. Nevertheless, we see that this does not create undue overhead.

We summarize the results in Table 3, where n refers to the number of hops, c to the size of an HTLC contract, and d to the size of an Astrape contract. We copy results for Fulgor/Rayo [18] and AMHL [19] from their original sources, which uses an essentially idendical setup.

Computation time

We measure computation time, with communication and other overhead ignored. The time measured is the sum of the CPU time taken by each hop, for all steps of the algorithm. We note that by eschewing non-standard cryptographic primitives, Astrape achieves lower computation times across the board compared to Fulgor/Rayo and AMHL, even when the fairly large overhead of our inefficent Bitcoin Cash implementation is included.

Communication overhead

We also measure the communication overhead of each system. This is defined as all the data that needs to be communicated other than the locks and their opening solutions. For example, in Astrape, this includes all the setup information sent from U_0 , while in AMHL this includes everything exchanged during the Setup, Lock, and Rel [19] phases. We see that Astrape has by far the least communication overhead of all the anonymous PCN constructions. Note especially the extreme overhead of the zero-knowledge proofs used in Fulgor/Rayo.

Lock overhead

The last measure is *per-coin* lock overhead — the size of each lock script and the information required to unlock it. This is a very important component of a system's resource usage, since lock sizes directly translate

Table 3. Resource usage of different PCN systems

	Plain HTLC	Fulgor/Rayo	AMHL (ECDSA)	Astrape	Astrape (BCH)
Computation time (ms) Communication size (bytes)	$<0.001\\32n$	$\approx 200n$ $1650000n$	$\approx 3n$ $544n$	$\approx 0.25n$ $192n$	$\approx 1.5n$ $192n$
Lock size (bytes) Unlock size, normal case (bytes) Unlock size, worst case (bytes)	32 + c 32 32	32 + c 32 32	32 + c 64 64	$64 + d$ 32 $32 \cdot n$	$120 + 56n^2$ 32 $32 \cdot n$

into transaction fees in blockchain cryptocurrencies. Astrape's performance differs in two important ways.

First of all, due to its complex lock scripts and our non-optimal implementation, Astrape has much higher lock sizes for blockchains like Bitcoin Cash that do not support recursion in lock scripts. Secondly, the worst-case unlock size is larger for Astrape. When the sender is malicious and all coins have to be "collapsed", we need n parameters $(\Gamma_1, \ldots, \Gamma_n)$ to unlock each coin.

We believe that Astrape still offers competitive lock performance though, since in practice n is limited in size, and the worst-case unlock size only applies when the sender is malicious. In the usual case, Astrape has 32-byte unlocks, just like traditional HTLC.

7.4 Statistical simulation

Finally, we simulate the performance of Astrape on the network graph of Lightning Network.

Setup

TODO: update info

We set up a mainnet Lightning Network node using the lnd [3] reference implementation. We then used the lncli describegraph command to capture the network topology of Lightning Network in August 2019. This gives us a graph of 4767 nodes and 32219 edges. Finally, we randomly sample 1,000 pairs of nodes in the network and calculate the shortest paths between them. This gives us a randomly sampled set of real-life payment paths.

Path statistics

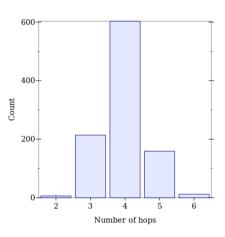


Fig. 5. Distribution of shortest path lengths

As we have previously shown, paths more than 10 hops long still have fairly small overhead even with non-recursive lock scripts, but much longer paths will cause unacceptably large unlock sizes. Will graph topology ever force payment to grow too long? Figure 5 illustrates the distribution of lengths for our 1,000 randomly selected payment paths. On average, a payment path was 3.92 hops long, while even the longest payment paths were only 6 hops long. This indicates that shortest payment paths long enough to pose seriously ballooning worst-case overhead are practically nonexistent. A privacy-optimizing routing algorithm thus has ample room to deviate from shortest-path routing.

Total scalability

One important attribute we wish to explore with the LN topology is the total scalability of the network — how fast can a PCN process transactions as a whole.

To do so, we keep track of how many times each node appears, or is "hit", in our 1,000 randomly selected payment routes. We see that the average number of hits is 1.97, but the vast majority of nodes are hit only once, while a few nodes are hit hundreds of times. This reflects that some nodes are unusually well-connected and act

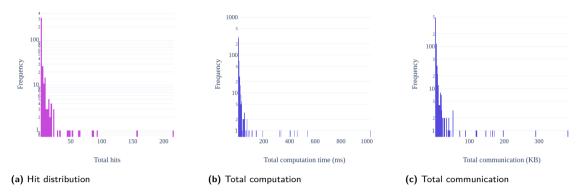


Fig. 4. Overhead distribution

as "hubs" for a large fraction of all transactions. The distribution of hits is plotted in Figure 4a as a log-linear histogram.

We then look at the distribution of overhead in the network for both computation and communication. This is by calculating the total computation and communication cost for each node "hit" by the 1,000 random payments, using values from the Bitcoin Cash implementation. This allows us to calculate the resources required for the largest "hubs" in the network whose throughput limits that of the entire system.

Computation cost is plotted in Figure 4b. We see that the most heavily loaded node in the entire network did around 1,000 ms of computation to process 1,000 transactions. This indicates that a PCN with the current Lightning Network topology will be able to process around 1,000 transactions a second with the largest hubs using commodity hardware. Such a throughput is orders of magnitude higher than that of typical blockchains and is within reach of many traditional payment systems. We note that the Lightning Network is still experimental; with a larger network this throughput will increase dramatically.

Communication cost is plotted in Figure 4c. We used the quadratically increasing lock sizes of the unoptimized Bitcoin Cash implementation of Astrape, and assumed that all payments are settled through a worst-case "collapse". Even so, the total network load averages to only about 3.92 KB per node. The largest hubs' total load still do not exceed 400 KB. This illustrates that even though communication cost is worst-case quadratic in the number of payment hops, the bottleneck is actually computation, not communication — with LN's topology major hubs will have saturated their processing power at 1,000 transactions per second, but would only have to transmit a meager 400 KB/s of data.

In summary, we see that Astrape's rather "scary" worst-case asymptotic performance poses no barriers to the total throughput of an Astrape-powered payment channel network. PCNs can enjoy the superb scalability associated with them just as easily with Astrape-powered privacy and security.

8 Conclusion

Payment channel networks and other trust-minimizing intermediarized cryptocurrency payment systems currently deployed lack strong privacy and security guarantees. Existing research, although solving the privacy and security problems, introduce either stringent requirements on underlying blockchain semantics or non-standard, difficult-to-port cryptographic primitives.

We presented Astrape, a novel PCN construction that breaks this conundrum. Astrape is the first PCN that achieves relationship anonymity and balance security with only black-box access to generic conventional cryptography. It relies on a general idea of using non-anonymous post-hoc fraud proofs to achieve balance security, while avoiding any information leaks when senders are not corrupt. This allows Astrape to avoid the zero-knowledge proofs used to achieve balance security in almost all other private PCNs, while having easily provable security poroperties.

Furthermore, we demonstrate that Astrape is practical to deploy in the real world. Performance is superior on average to existing private PCNs, even on blockchains that are unsuitable for free-form smart contracts. We also showed that Astrape achieves high scalability on a real-world payment channel network graph.

References

- [1] libsodium. Accessed on 2020-07-15.
- [2] Openssl. Accessed on 2020-07-15.
- [3] Lightning network daemon, 2019.
- [4] Raiden network high speed asset transfers for ethereum, 2019. Accessed on 2019-08-13.
- [5] Blockchain.com, 2020. Accessed on 2020-06-02.
- [6] J.-P. Aumasson, S. Neves, Z. Wilcox-O'Hearn, and C. Winnerlein. Blake2: simpler, smaller, fast as md5. In *International Conference on Applied Cryptography and Network Security*, pages 119–135. Springer, 2013.
- [7] M. Backes, A. Kate, P. Manoharan, S. Meiser, and E. Mohammadi. Anoa: A framework for analyzing anonymous communication protocols. In 2013 IEEE 26th Computer Security Foundations Symposium, pages 163–178. IEEE, 2013.
- [8] J. Camenisch, M. Drijvers, T. Gagliardoni, A. Lehmann, and G. Neven. The wonderful world of global random oracles. In Annual International Conference on the Theory and Applications of Cryptographic Techniques, pages 280–312. Springer, 2018
- [9] K. Croman, C. Decker, I. Eyal, A. E. Gencer, A. Juels, A. Kosba, A. Miller, P. Saxena, E. Shi, E. G. Sirer, et al. On scaling decentralized blockchains. In *International Conference on Financial Cryptography and Data Security*, pages 106–125. Springer, 2016.
- [10] G. Danezis and I. Goldberg. Sphinx: A compact and provably secure mix format. In 2009 30th IEEE Symposium on Security and Privacy, pages 269–282. IEEE, 2009.
- [11] C. Decker, R. Russell, and O. Osuntokun. eltoo: A simple layer2 protocol for bitcoin. White paper: https://blockstream.com/eltoo.pdf, 2018.
- [12] C. Decker and R. Wattenhofer. A fast and scalable payment network with bitcoin duplex micropayment channels. In Symposium on Self-Stabilizing Systems, pages 3–18. Springer, 2015.
- [13] R. Dingledine, N. Mathewson, and P. Syverson. Tor: The second-generation onion router. Technical report, Naval Research Lab Washington DC, 2004.
- [14] J. Feigenbaum, A. Johnson, and P. Syverson. Preventing active timing attacks in low-latency anonymous communication. In *International Symposium on Privacy Enhancing Technologies Symposium*, pages 166–183. Springer, 2010.
- [15] M. Green and I. Miers. Bolt: Anonymous payment channels for decentralized currencies. In *Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communica*tions Security, pages 473–489. ACM, 2017.
- [16] E. Heilman, L. Alshenibr, F. Baldimtsi, A. Scafuro, and S. Goldberg. Tumblebit: An untrusted bitcoin-compatible anonymous payment hub. In Network and Distributed System Security Symposium, 2017.
- [17] J. Lind, I. Eyal, P. Pietzuch, and E. G. Sirer. Teechan: Payment channels using trusted execution environments. arXiv preprint arXiv:1612.07766, 2016.
- [18] G. Malavolta, P. Moreno-Sanchez, A. Kate, M. Maffei, and S. Ravi. Concurrency and privacy with payment-channel networks. In *Proceedings of the 2017 ACM SIGSAC Conference* on Computer and Communications Security, pages 455–471. ACM, 2017.

- [19] G. Malavolta, P. Moreno-Sanchez, C. Schneidewind, A. Kate, and M. Maffei. Anonymous multi-hop locks for blockchain scalability and interoperability. In NDSS, 2019.
- [20] P. McCorry, M. Möser, S. F. Shahandasti, and F. Hao. Towards bitcoin payment networks. In *Australasian Conference* on *Information Security and Privacy*, pages 57–76. Springer, 2016.
- [21] N. Moeller and S. Josefsson. letf draft: Eddsa and ed25519, 2015.
- [22] A. Pfitzmann and M. Hansen. A terminology for talking about privacy by data minimization: Anonymity, unlinkability, undetectability, unobservability, pseudonymity, and identity management. 2010.
- [23] I. A. Seres, L. Gulyás, D. A. Nagy, and P. Burcsi. Topological analysis of bitcoin's lightning network. arXiv preprint arXiv:1901.04972, 2019.
- [24] E. Tairi, P. Moreno-Sanchez, and M. Maffei. A2L: Anonymous atomic locks for scalability and interoperability in payment channel hubs. Technical report, Cryptology ePrint Archive, Report 2019/589, 2019.