Laplace Xforms

$\delta(t) \to 1 \qquad u(t) \to \frac{1}{s}$ (signals)

$e^{-\sigma t} f(t) \to F(s+\sigma)$ (properties)

$\frac{d}{dt} \to s \qquad \int \to \frac{1}{s}$

$u(t) \times t^n \to \frac{n!}{s^{n+1}}$ (functions)

$u(t) \times e^{-\sigma t} \to \frac{1}{s+\sigma}$

$u(t) \times \sin(\omega t) \to \frac{\omega}{s^2+\omega^2}$

$u(t) \times \cos(\omega t) \to \frac{s}{s^2+\omega^2}$

2nd Order Xfer Funcs

$TF(s) = \frac{\sigma^2+\omega^2}{(s+\sigma)^2+\omega^2} = \frac{\sigma^2+\omega^2}{s^2+2\sigma s+\sigma^2+\omega^2}$

$TF(s) = \frac{{\omega_n}^2}{s^2+2\zeta\omega_n s+{\omega_n}^2}$

$\sigma = \zeta\omega_n$

$\omega = \beta\omega_n$

$\omega_n = \sqrt{\sigma^2+\omega^2}$

$\zeta = \frac{\sigma}{\sqrt{\sigma^2+\omega^2}}$

$\beta = \sqrt{1-\zeta^2}$

System ID

$T_s = \frac{4}{\sigma} = \frac{4}{\zeta\omega_n}$

$T_p = \frac{\pi}{\omega_n\beta}$

$OS = K_{DC} e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \qquad \%OS = \frac{OS}{K_{DC}} \times 100\%$

$FV = sFF(s)\Big|_{s=0} \qquad FF(s) = Force\ Func$

$T_r = \frac{1}{\omega_n\beta}\left(\pi - \tan^{-1}\frac{\beta}{\zeta}\right)$

$T_{r1} \approx \frac{4.65\zeta - 1.3}{\omega_n}, \qquad \zeta \geq 0.8$

2nd Order Approx.

$\omega_n = \frac{4}{T_s\zeta} \qquad \zeta = \frac{4}{T_s\omega_n}$

$\omega_n = \frac{\pi}{T_p\beta} \qquad \beta = \frac{\pi}{T_p\omega_n}$

$\omega_n = \frac{1}{T_r\beta}\left(\pi - \tan^{-1}\frac{\beta}{\zeta}\right)$

$\zeta = \sqrt{\frac{\ln^2\frac{OS}{K_{DC}}}{\pi^2+\ln^2\frac{OS}{K_{DC}}}}$

Electro-Mech Sys.

$F = Ma$ $F = Bv$ $F = Kx$

$\tau = J\dot{\omega}$ $\tau = B\omega$ $\tau = K\theta$

$R \rightarrow {}^{1}/_{B}$ $L \rightarrow {}^{1}/_{K}$ $C \rightarrow M$

$I \rightarrow F$ $V \rightarrow V$

Feedback Sys.

$$TF(s) = \frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$TF(s) = \frac{G_N H_D}{G_D H_D + G_N H_N}$$

$$CE: \quad G_D H_D + G_N H_N = 0$$

State-Space Rep.

$$\dot{\bar{x}} = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{u}$$

$$\bar{y} = \bar{\bar{C}}\bar{x} + \bar{\bar{D}}\bar{u}$$

$$\bar{\bar{\Phi}}(s) = \left[s\bar{\bar{I}} - \bar{\bar{A}}\right]^{-1}$$

$$\text{CE} = \det(s\bar{\bar{I}} - \bar{\bar{A}}) = 0$$

$$\overline{\overline{TF}}(s) = \frac{\bar{Y}(s)}{\bar{U}(s)} = \overline{\overline{C\Phi B}} + \bar{\bar{D}}$$

Root Locus

$$\sigma_A = \frac{\Sigma p - \Sigma z}{XS}$$

Lead (PD) Controller Dynamics

$K_p = 1$ $K_d = \dfrac{p - z}{pz}$

$$z = \frac{K_p p}{K_d p + K_p}$$

Lag (PI) Controller Dynamics

$K_p = \dfrac{1}{z}$ $K_i = 1$

$$z = \frac{1}{K_p}$$

DAQ Dynamics

$$H = \frac{2CF}{s + 2CF}$$

PID Controller Dynamics

$$K_p = \frac{z_1 + z_2}{z_1 z_2} - \frac{1}{p}$$

$$K_i = 1$$

$$K_d = \frac{1}{z_1 z_2} - \frac{K_p}{p} \quad = \quad \frac{1}{z_1 z_2} - \frac{z_1 + z_2}{p z_1 z_2} + \frac{1}{p^2}$$

$$z_1, z_2 = roots\left(s^2 + \frac{K_p p + K_i}{K_d p + K_p}s + \frac{K_i p}{K_d p + K_p}\right)$$

Physical Constants

$$1\,(RPM) = \frac{\pi}{30}\,({}^{rad}/_{s})$$

$$1\,(ft) = 0.3048\,(m)$$