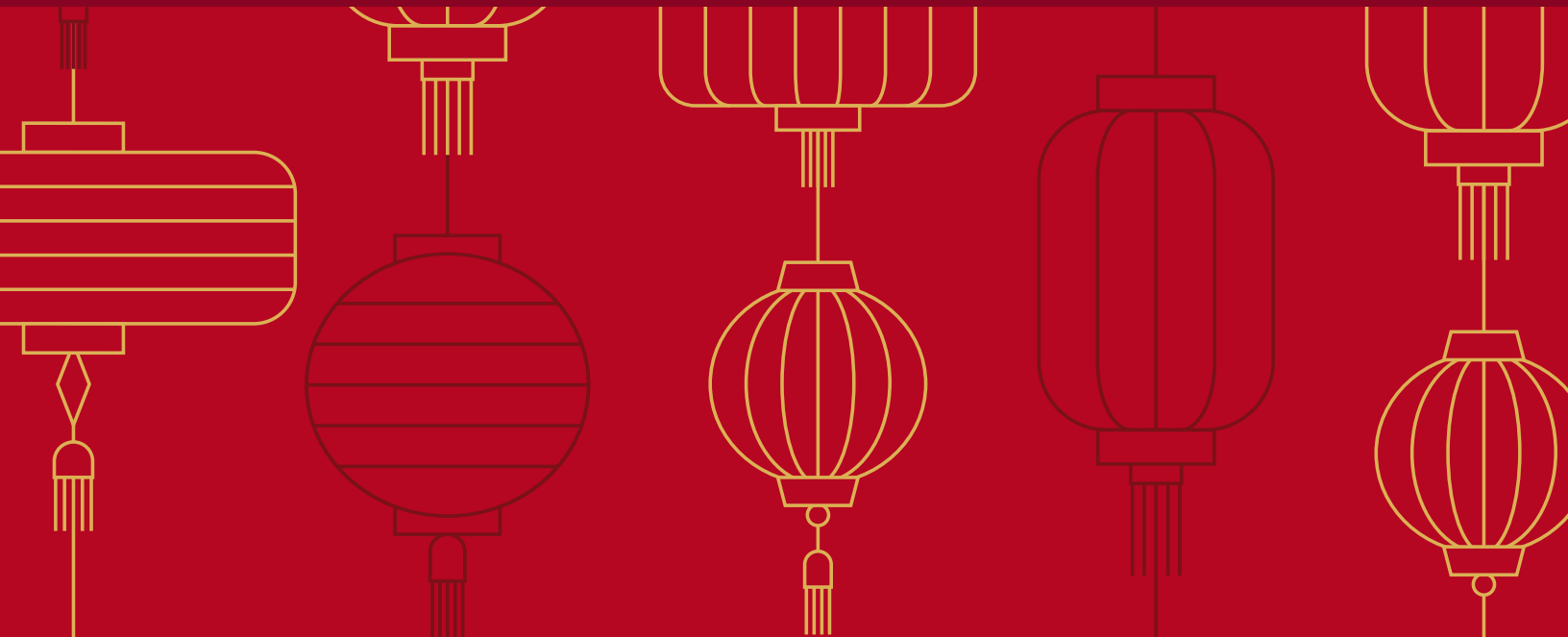
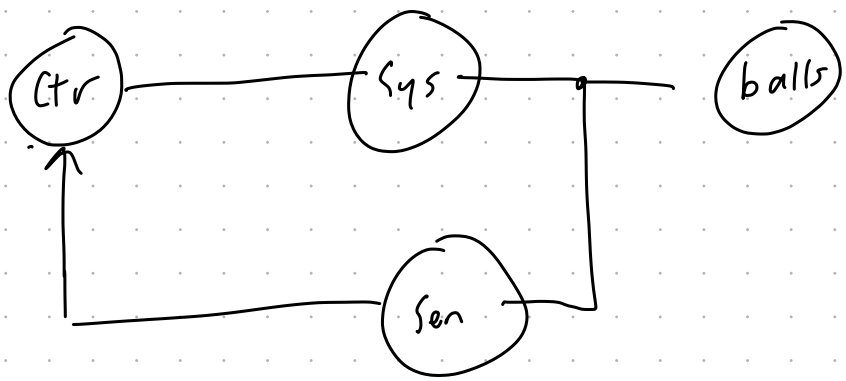


Balls

need a controller, a system, and a sensor to observe the system



$s = \sigma + j\omega \rightarrow$ all laplace xforms have σ instead of a.

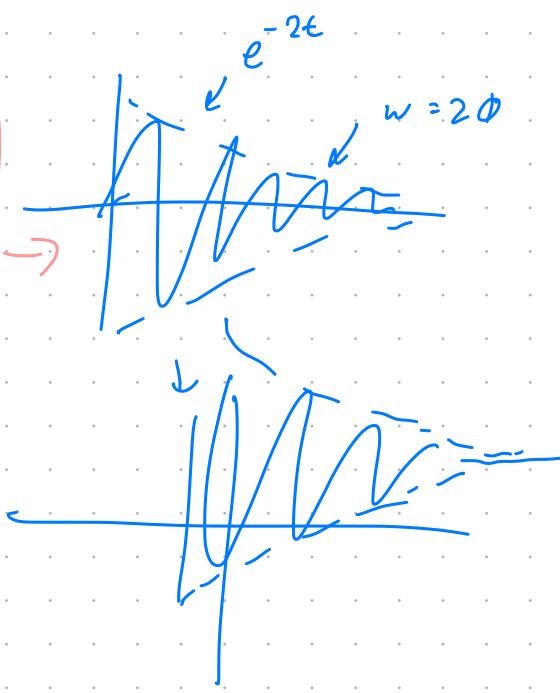
heaviside expansion

$$\frac{\sim}{(s+a)(s+b)} = \frac{\sim}{s+a} + \frac{\sim}{s+b}$$

$F(s)$

$$\frac{100}{s((s+2)^2+400)} = \frac{a}{s} + \boxed{\frac{b}{(s+2)^2+20^2}}$$

σ ω

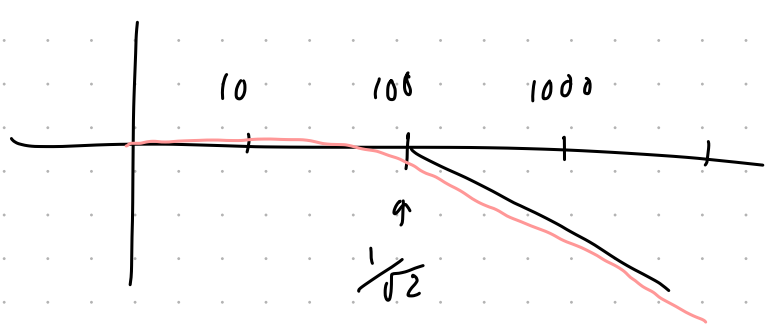


Bode Plots

$$\frac{100}{s+100} = a + jb \quad \text{when } s = \cancel{\sigma}^{0} + j\omega$$

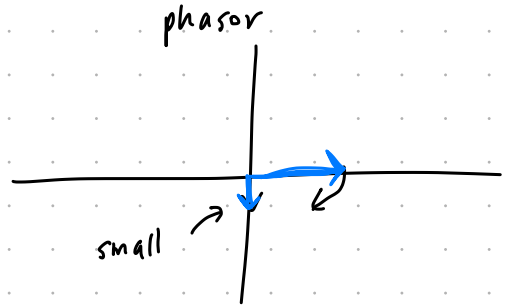
$$= c\angle d^\circ$$

denom	mag	$\frac{100}{\text{mag}}$
$100 + j0$	100	1
$100 + j1000$	$100\sqrt{2}$	$\frac{1}{\sqrt{2}}$
$100 + j\text{Big}$	big	small

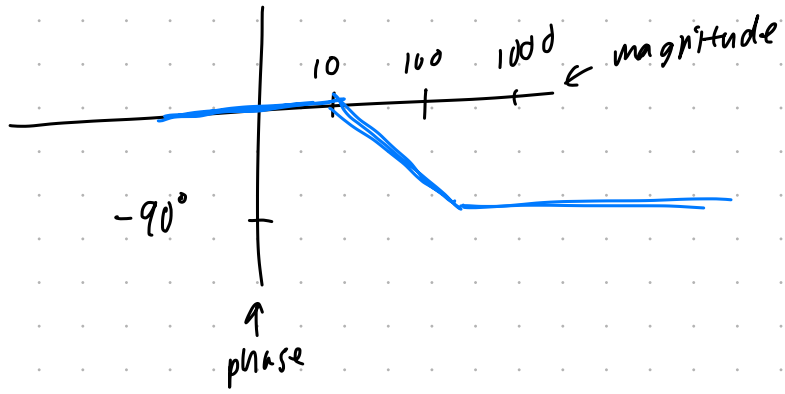


if $w = 0 = 1\angle 0$

$w = \text{big} = \frac{100}{j\,\text{big}} = -j\,\text{small}$

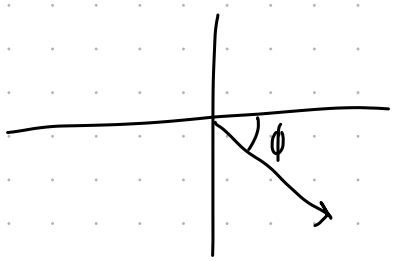


bode:



$\text{gain} = \sqrt{\text{real}^2 + \text{Im}^2}$

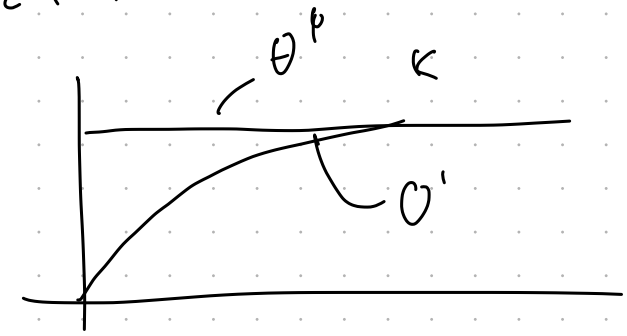
$\phi = \text{atan2}(\text{Imag}, \text{Real}) = \arg(H(j\omega))$



missed 1

missed!

24-1



step response

$$\frac{1}{s}\,\frac{k\sigma}{s+\sigma}$$

$$\mathcal{L}^{-1} \rightarrow k(u(t)-e^{-\sigma t})$$

$\downarrow$

first order system can't model and overshoot as $0 < e^{-\sigma t} < 1$ and $u(t) = 1, 0$

Transfer Function

$TF = \frac{num}{denom}$ $\leftarrow$ zeros, $\leftarrow$ poles Heaviside Terms

Poles $\rightarrow$ solve for s $den(s) = 0$ $\rightarrow$ characteristic eqn CE

0 order

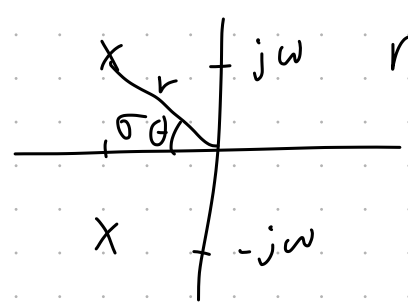
θ^0 : constant $\rightarrow$ no poles

θ^1 : 1st order $\therefore$ 1 real pole. $\rightarrow$ if you have imaginary poles then you get imaginary time after an inverse laplace transform

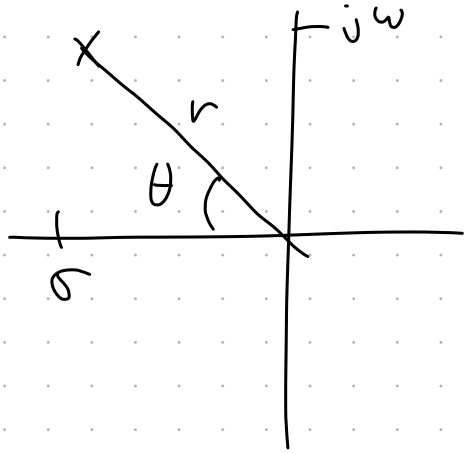
θ^2 : 2 real or a complex conjugate poles $\rightarrow$ characteristic equation

$(s+\sigma+j\omega)(s+\sigma-j\omega)$

$\hookrightarrow e^{-\sigma t}\sin(\omega t)$



$r = \sqrt{}$



$r = \sqrt{\sigma^2 + \omega^2}$

$\sigma = r\cos\theta$

$\omega = r\sin\theta$

$\theta = 90°$

$\hookrightarrow \sigma = 0 \quad \omega = r$

s-C Natural Frequency : what does this mean?

$\omega_n = \sqrt{\sigma^2 + \omega^2}$

when $\theta = 90°$ → your frequency becomes the natural frequency ($\sigma \to 0$, $\omega_n \to r = \sqrt{\sigma^2 + \omega^2}$

New Qtys (Def)

$\zeta = \cos\theta \qquad \zeta = \frac{\sigma}{\omega_n}$

$\beta = \sin\theta \qquad \beta = \frac{\omega}{\omega_n}$

$c^2 + s^2 = 1$

$\zeta^2 + \beta^2 = 1 \qquad \beta = \sqrt{1 - \zeta^2}$

$\zeta = 0$ → pure oscillation

$\zeta = 1$ → critical damping → similar to first order system

$\theta = 0$

$\zeta \propto$ rate of decay

CE

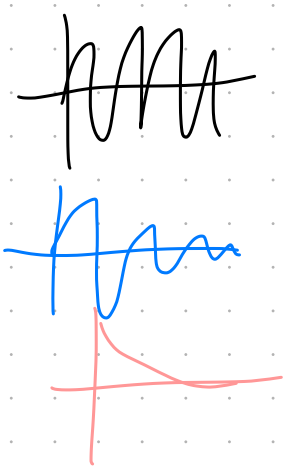
a) $s^2 + \overbrace{100}^{\text{radius}}$

b) $s^2 + 10s + 100$

c) $s^2 + 20s + 100$

d) $s^2 + 25s + 100$

ζ increasing $(\sigma \uparrow)$

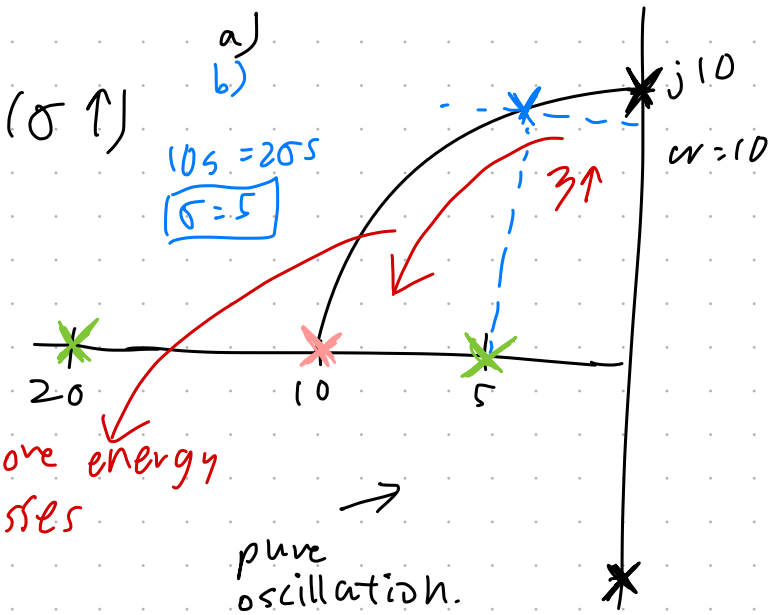


remember

$s^2 + 2\sigma s + \sigma^2 + \omega^2$

a)

b)



$10s = 2\sigma s$

$\boxed{\sigma = 5}$

$A\sin(\omega t)$ type beat

c) $20s = 2\sigma s$

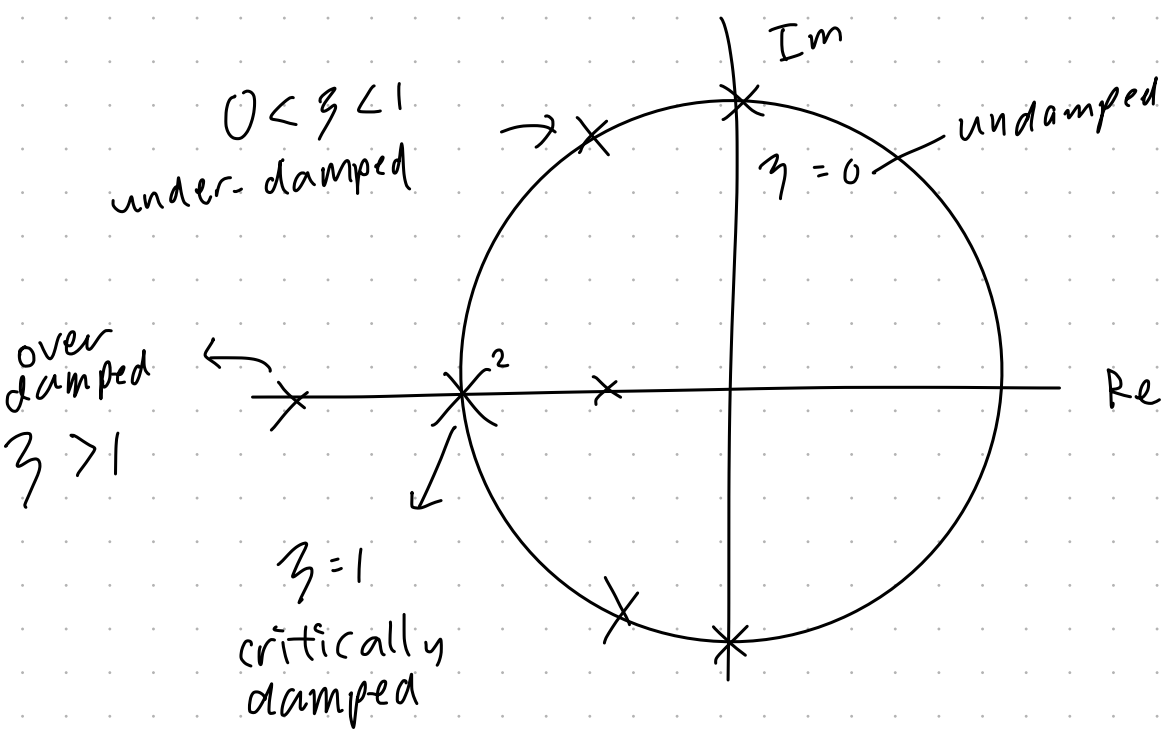
$\sigma = 10$

$\sigma^2 = 100 \quad \therefore \quad \omega = 0$

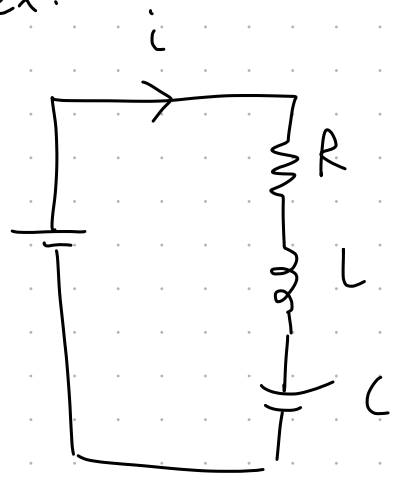
d) is going to be overdamped

$(s+5)(s+20)$

summary



ex.



$V = iZ \qquad \frac{i}{V} = \frac{1}{Z} = Y$ (attmitance)

$$\frac{i}{V} = \frac{1}{sL + R + \frac{1}{sC}} = \frac{sC}{s^2LC + sRC + 1}$$

pull out constant

$$= \frac{1}{L}\left(\frac{1}{s^2 + \frac{sR}{L} + \frac{1}{LC}}\right) \qquad (E = s^2 + \frac{sR}{L} + \frac{1}{LC}$$

$W_n = \sqrt{\frac{1}{LC}} \rightarrow$ $\underline{\underline{\text{if}}}$ $R = 0$ (not possible tho)

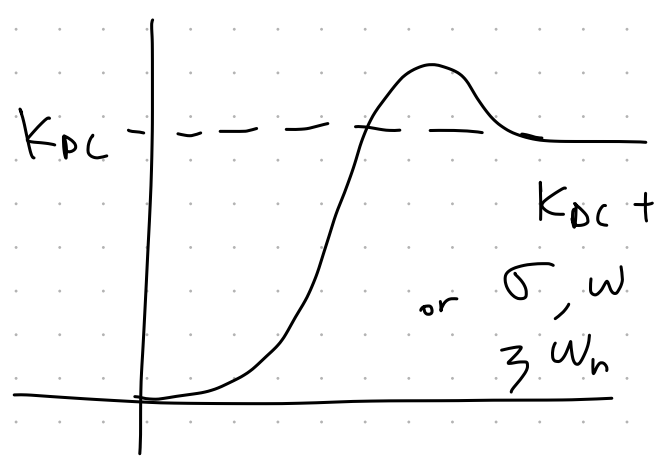
$2\sigma = \frac{R}{L} \qquad \sigma = \frac{R}{2L} \qquad \sigma^2 + w^2 = \frac{1}{LC}$ then $w = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

$\frac{1}{s^2 + 2\sigma s + \sigma^2 + w^2} \rightarrow$ DC gain $\rightarrow$ set $s = 0$ and set numerator to get unity

$w_n^2 = \sigma^2 + w^2 \qquad \sigma = w_n \underbrace{\cos\theta}_{\zeta}$

$$K_{DC} = \frac{\sigma^2 + w^2}{s^2 + 2\sigma s + \sigma^2 + w^2}$$

$$= \frac{w_n^2}{s^2 + 2w_n s + w_n^2}$$



Heaviside expansion

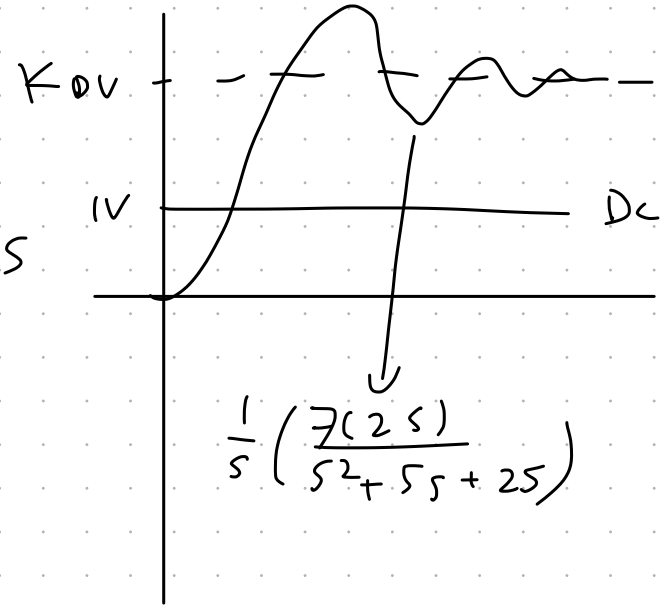
K_{DC} / Final Value

$$G(s) = \frac{25}{s^2+5s+25}$$

$2\zeta 5 = 5 \quad \omega_n = 5$

$\zeta = \frac{5}{10} = \frac{1}{2}$

$$\frac{V_o}{V_{in}} = G(s)$$

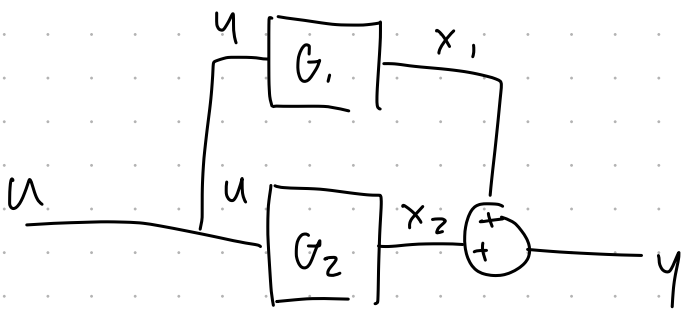


DC gain is value measured after $t=\infty$ with input of 1V (step)

Final value is the same but for <u>any</u> input

combining a network of subsystems:

Block Diagrams - simulink



4 types of elements

- system

signal

pick off point

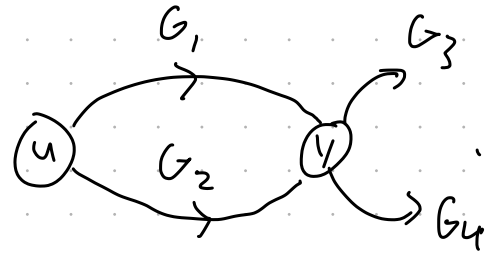
sum junction

$x_1 + x_2 = y$

$G_1 u + G_2 u = y$

$(G_1 + G_2) u = y$

signal Flow Graphs



1. sum
2. sig
3. P-O

y "makes" a new signal

u: input

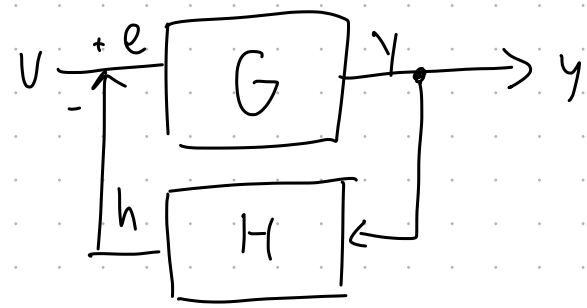
x: signals → doesn't really matter

y: output

Feedback:

Forward Path : G

Feedback Path : H



– negative feedback

$e = u - h$

$y = Ge$

$h = Hy$

$$y = G(u-h) = G(u - Hy)$$
$$= Gu - Ghy$$

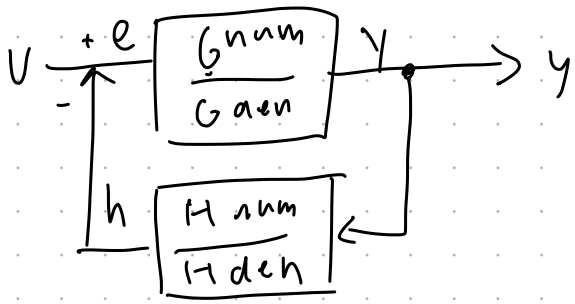
$$y(1+Gh) = Gu$$

$$\boxed{\frac{y}{u} = \frac{G}{1+Gh}}$$

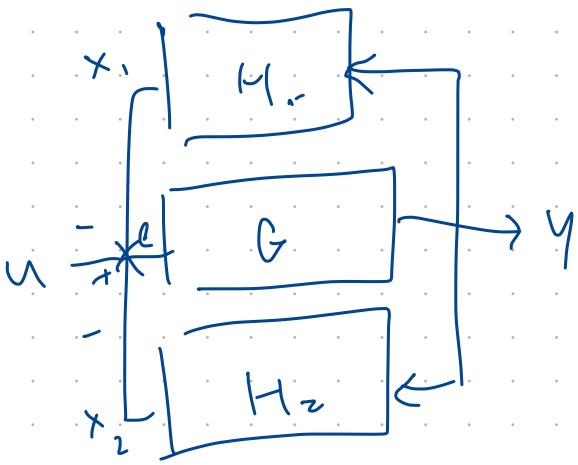
$$G = \frac{3s}{s+7} \qquad H = \frac{5}{s+5} \qquad \frac{y}{u} = \frac{\frac{3s}{s+7}}{1+\frac{3s \cdot 5}{(s+7)(s+5)}}$$

$$= \frac{3s(s+5)}{s^2+27s+35}$$

$$u \longrightarrow \boxed{\frac{3s^2+15s}{s^2+27s+35}} \longrightarrow$$



$$\frac{y}{u} = \frac{\frac{G_n}{G_d}}{1 + \frac{G_n H_n}{G_d H_d}} = \frac{G_n H_d}{G_n H_n + G_d H_d}$$



$$e = u - x_1 - x_2$$
$$= u - H_1 y - H_2 y$$
$$-(H_1 + H_2) y$$

$$y = Ge$$

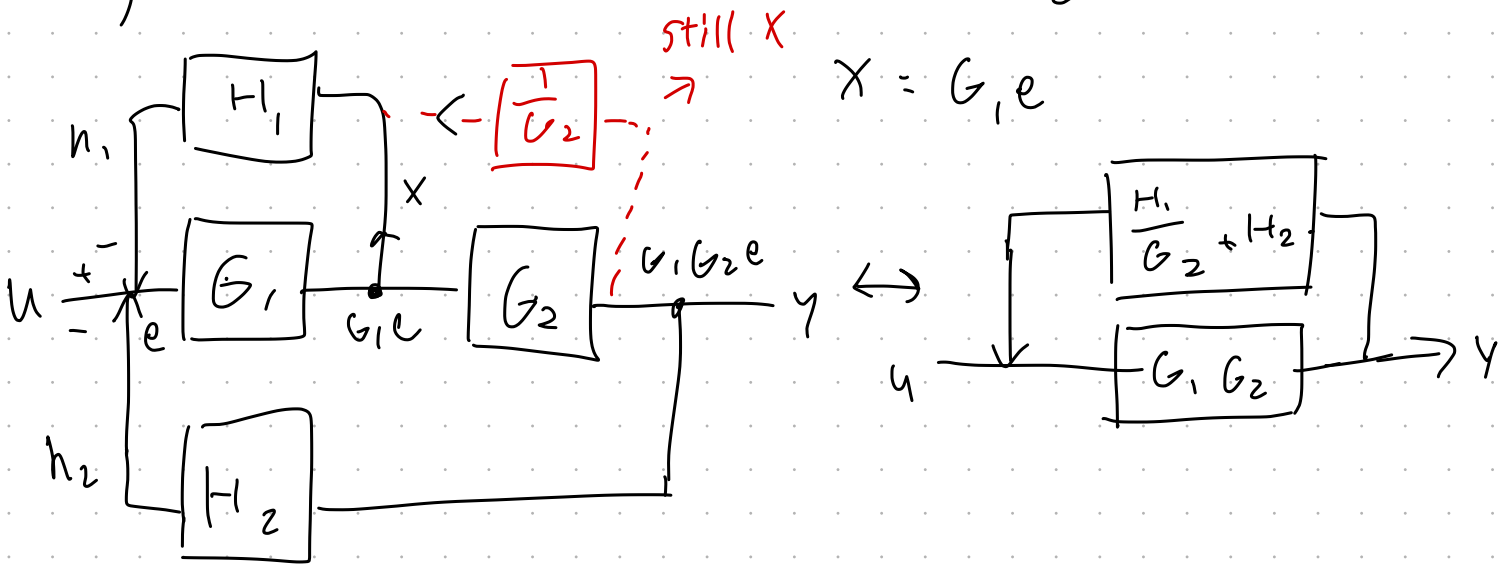
$$y = Gu - G(H_1 + H_2) y$$

$$y(1 + G(H_1 + H_2)) = Gu$$

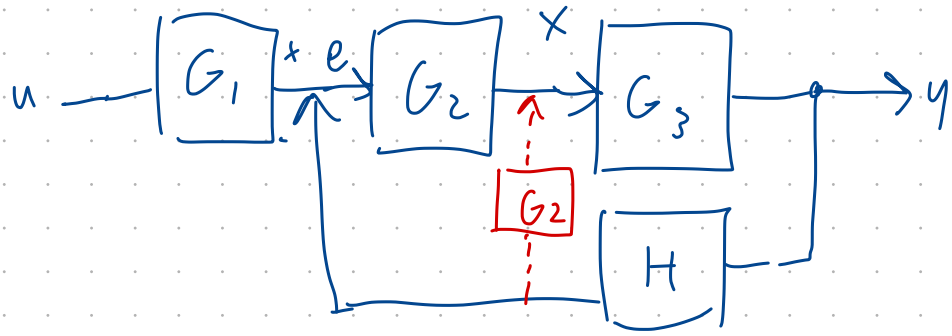
$$\frac{y}{u} = \frac{G}{1 + G(H_1 + H_2)}$$

Moving P-O

$e\, G_1\, G_2$



$$X = G_1 e$$

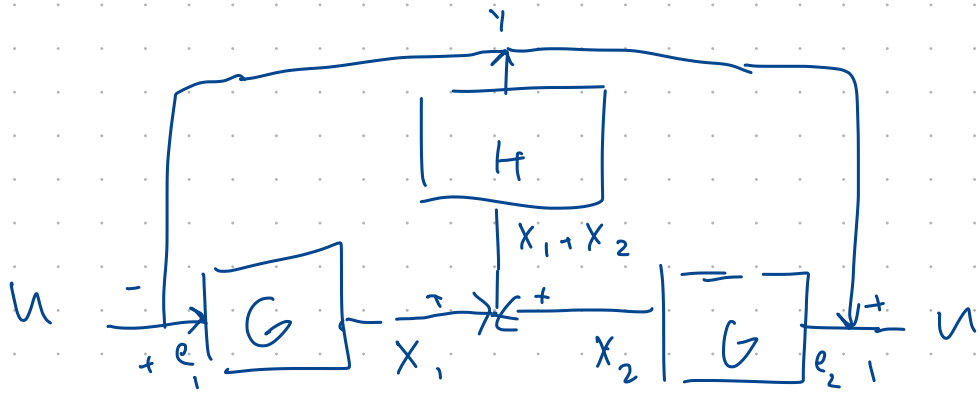


$$x = (G_1 u - Hy) G_2$$

$$= G_2 G_1 u - H G_2 y$$

PO ←

Σ →



$$y = H(x_1 + x_2)$$

$$x_1 = G e_1$$

$$e_1 = u - y$$

$$x_1 = G(u - y)$$

$$x_2 = G(u + y)$$

$$y = HG(u - y) + HG(u + y)$$

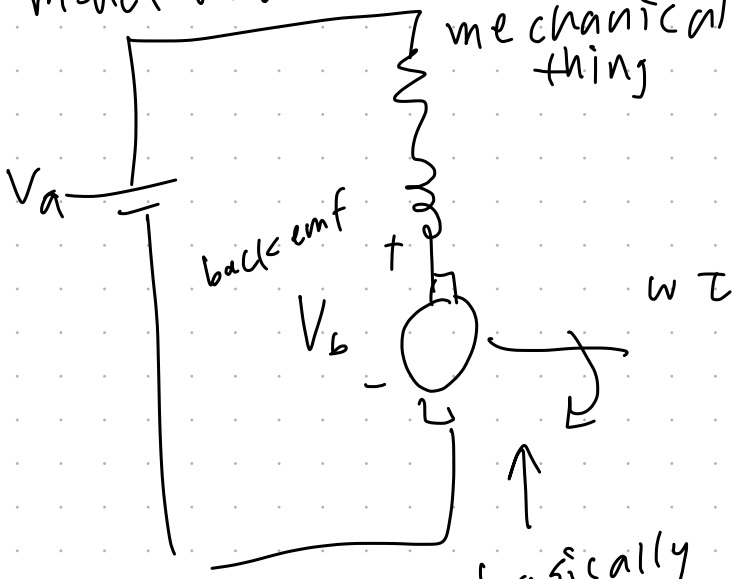
$$y = HG(u - \cancel{y} + u + \cancel{y})$$

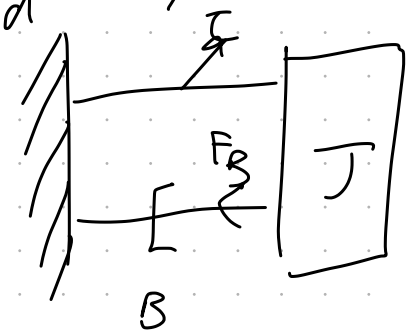
$$y = 2HGu$$

$$\frac{y}{u} = 2HG$$

missed half a lecture, something about motors

model of a motor - circuit and mechanical thing





$\tau = K_m i$ $\quad K_m =$ motor constant maybe

$V_b = K_m \omega$

basically a dependent voltage source

elec:

$$i = \frac{V_a - V_b}{Z_e} = Y_e (V_a - V_b)$$

$$= \frac{1}{sL+R}(V_a + V_b)$$

mech:

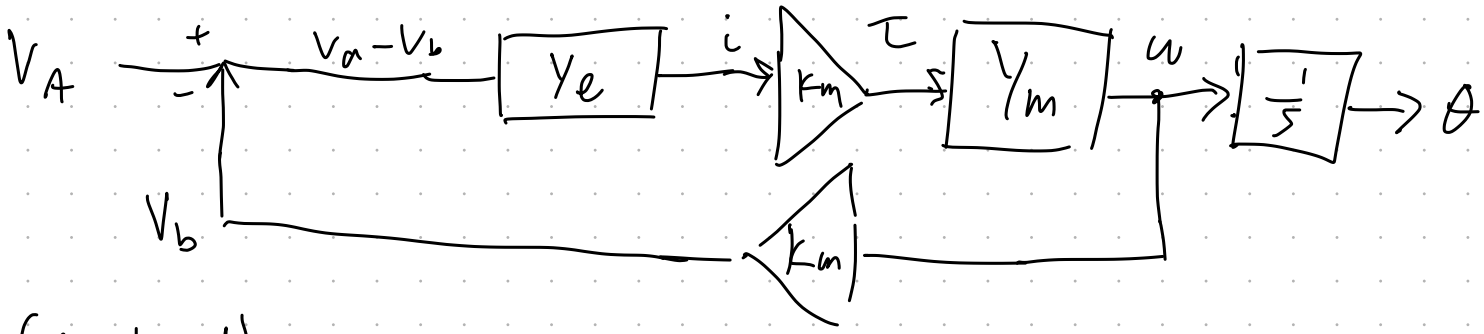
$$\tau - B\omega = J\dot{\omega}$$

$$\tau = \underbrace{(Js + B)}_{Z_m} \omega \quad \text{(in Laplace)}$$

$$\omega = Y_m \tau = \frac{1}{Js + B}\tau$$

and $\tau = K_m i$ so we can find stuff

$$\omega = \left(\frac{1}{Js+B}\right)\left(K_m\left(\frac{1}{sL+R}(V_a - V_b)\right)\right)$$



(no load)

If you add load (more J), then you just need to change your mechanical admittance

↳ same idea with the electrical side, Y_e needs to change

$\frac{\text{min}}{\text{rot}} \cdot \frac{60\,s}{\text{min}} \cdot \frac{1\ \text{rot}}{2\pi\ \text{rad}}$

Motor Xfer Func

$$\frac{\omega}{V} = \frac{G_N H_D}{G_N H_N + G_D H_D} = \frac{K_m \cdot 1}{K_m \cdot K_m + 1 \cdot (sL + R)(J_s + B)}$$

$$= \frac{K_m}{s^2 LJ + s(LB + JR) + RB + K_m^2}$$

$K_B = 1620\ \frac{RPM}{V} = \frac{1}{1620\ \cancel{RPM}} \cdot \frac{\cancel{V}\ \cancel{min}}{\cancel{rev}} \cdot \frac{60\,\cancel{s}}{\cancel{min}} \cdot \frac{\cancel{1\ rev}}{2\pi\ rad}$

↳ from DS

$$= \frac{60}{1620(2\pi)} \cdot \frac{V \cdot s}{rad} = 0.0059\ Vs \text{ or } \frac{Nm}{A}$$

$u \rightarrow \boxed{\frac{G_n}{G_D}} \rightarrow \boxed{s} \rightarrow$ not real, do:

$\rightarrow \boxed{\frac{1}{s}}\ G_n$

$u \rightarrow \boxed{\frac{s G_n}{G_D}}$ → derivative (loaded motor?)

$$\frac{rad}{s} \cdot \frac{60s}{min} \cdot \frac{1rev}{2\pi}$$

see latexid notes about state space

State Eqn

$$\dot{\bar{x}} = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{u}$$

Mechanical Space State

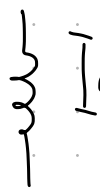
- id independent states
- state equations
- combine

→ - output equations
" ↓
"

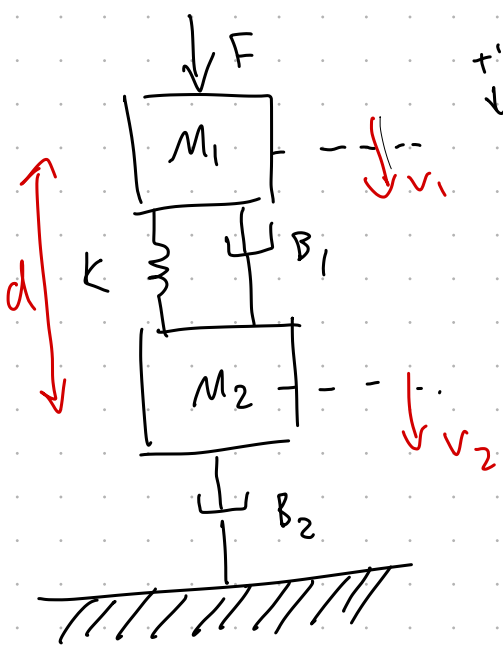
→ $\overline{\overline{T}}$

state variable

$v \} \bar{x}$ → $\dot{v}$ will be in state eqn → get from FBD



$d \} \bar{x}$ $\dot{d} = f(v)$



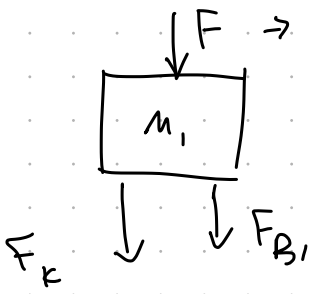
+'ve ↓

1) nothing dependent

$$\dot{\bar{x}} = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{u}$$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{d} \end{bmatrix} = \underset{3\times3}{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}} \begin{bmatrix} v_1 \\ v_2 \\ d \end{bmatrix} + \underset{3\times1}{\begin{bmatrix} \\ \\ \end{bmatrix}} \underset{(x)}{[F]}$$

FBDs



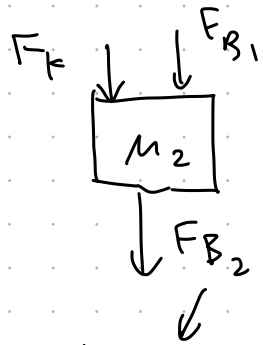
→ dynamic ∴ $\Sigma \vec{F} \neq 0 = m\vec{a} = m\dot{v}$

$$F + F_K + F_B = M\dot{v}_1$$

$$F + kd + B(v_2 - v_1) = M\dot{v}_1$$

$$\dot{v}_1 = \frac{-B_1}{M_1}v_1 + \frac{B_1}{M_1}v_2 + \frac{kd}{M_1} + \frac{F}{M_1}$$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{d} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{B_1}{M_1} & \frac{B_1}{M_1} & \frac{K_1}{M_1} \\ & & \\ & & \end{bmatrix}}_{3\times 3} \begin{bmatrix} v_1 \\ v_2 \\ d \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{M_1} \\ \\ \\ \end{bmatrix}}_{3\times 1} \underbrace{[F]}_{(\times)}$$



spring must be compressed to produce this force

$$F_K + F_{B_1} - F_{B_2} = M_2 \dot{v}_2$$

$$-kd + B_1(v_1 - v_2) + B_2 v_2 = M_2 \dot{v}_2$$

v_1 must be going faster for this force

don't assume v_2 is true

$$\dot{v}_2 = \frac{-kd}{M_2} - \frac{B_1 v_2}{M_2} + \frac{B_1 v_1}{M_2} - \frac{B_2 v_2}{M_2}$$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{d} \end{bmatrix} = \begin{bmatrix} -\frac{B_1}{M_1} & \frac{B_1}{M_1} & \frac{K_1}{M_1} \\ \frac{B_1}{M_2} & \frac{-(B_1+B_2)}{M_2} & \frac{-K}{M_2} \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ d \end{bmatrix} + \begin{bmatrix} \frac{1}{M_1} \\ 0 \\ 0 \end{bmatrix} [F]$$

$$\bar{Y} = \bar{\bar{C}}\bar{x} + \bar{\bar{D}}\bar{u}$$

what we want

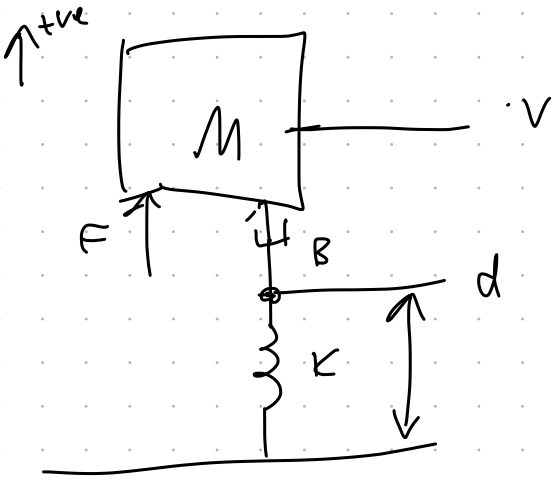
$$\begin{bmatrix} v_1 - v_2 \\ \text{spring force} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & K \end{bmatrix}}_{2\times 3} \begin{bmatrix} v_1 \\ v_2 \\ d \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{2\times 1} [F]$$

$$\bar{\bar{\Phi}} = [sI - \bar{\bar{A}}]^{-1}$$

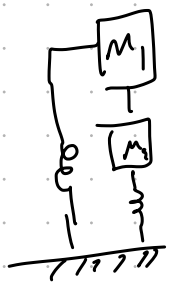
$$V_w = -K_m \omega M \perp$$

$$\bar{\bar{T}} = \bar{\bar{C}}\,\bar{\bar{\Phi}}\,\bar{\bar{B}} + \bar{\bar{D}} \rightarrow \begin{bmatrix} \frac{V_r}{F} \\ \frac{F_K}{F} \end{bmatrix}$$

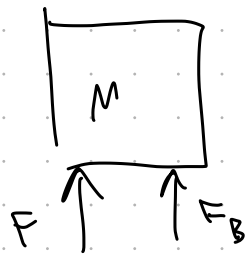
Messy example



$$\begin{bmatrix} \dot{v} \\ \dot{d} \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \begin{bmatrix} v \\ d \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \end{bmatrix} [F]$$



example of dependent mech system



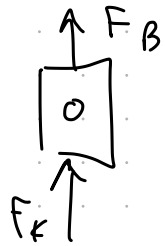
$$F + F_B = M\dot{v}$$

we will find $\dot{d}$ later, and it will be defined in terms of v and d

$$\frac{F}{M} + \frac{B(\dot{d} - v)}{M} = \dot{v}$$

fix later

$$\begin{bmatrix} \dot{v} \\ \dot{d} \end{bmatrix} = \begin{bmatrix} -\frac{B}{M} + \frac{B\dot{d}}{M} & 0 \\ \quad & \quad \end{bmatrix} + \begin{bmatrix} \frac{1}{M} \\ \quad \end{bmatrix} [F]$$



$$F_B + F_K = 0$$

$$B(V - \dot{d}) - Kd = 0$$

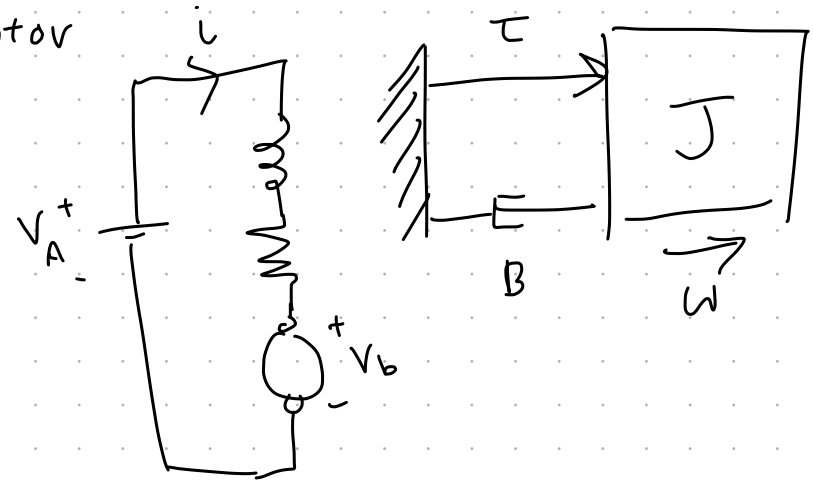
$$BV - Kd = B\dot{d}$$

$$\dot{d} = V - \frac{Kd}{B}$$

plug back in

$$\begin{bmatrix} \dot{v} \\ \dot{d} \end{bmatrix} = \begin{bmatrix} \cancel{-\frac{B}{M} + \frac{B}{M}}^{\,0} & -\frac{K}{M} \\ 1 & -\frac{K}{B} \end{bmatrix} \begin{bmatrix} v \\ d \end{bmatrix} = \begin{bmatrix} \frac{1}{M} \\ 0 \end{bmatrix} [F]$$

Motor



$$\begin{bmatrix} \ddot{i} \\ \ddot{\omega} \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \\ \end{bmatrix} \begin{bmatrix} V_A \end{bmatrix}$$

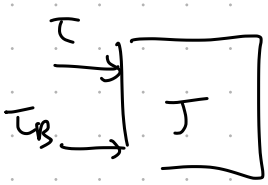
i: $L\ddot{i} = V$

$$-V_A + L\ddot{i} + Ri + V_b = 0 \qquad V_b =$$

$$\ddot{i} = \frac{V_A}{L} - \frac{Ri}{L} - \frac{V_b}{L} = \frac{V_A}{L} - \frac{Ri}{L} - \frac{k_m \omega}{L}$$

$$\begin{bmatrix} \ddot{i} \\ \ddot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{k_m}{L} \\ & \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ \end{bmatrix} \begin{bmatrix} V_A \end{bmatrix}$$

find ω



$$\tau + \tau_B = J\dot{\omega}$$

$$\tau - B\omega = J\dot{\omega} = \frac{k_m i}{J} - \frac{B\omega}{J} = \dot{\omega}$$

$$\begin{bmatrix} \ddot{i} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{k_m}{L} \\ \frac{k_m}{J} & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \begin{bmatrix} V_A \end{bmatrix}$$

How to get angle

$y = \omega$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \cancel{\begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} V_A \end{bmatrix}}^{\,0}$$

$$\frac{\omega}{V} = \bar{\bar{C}}\,\bar{\bar{\phi}}\,\bar{\bar{B}} \qquad \frac{\theta}{V} = \frac{\bar{\bar{C}}\,\bar{\bar{\phi}}\,\bar{\bar{B}}}{s} \leftarrow \text{integrated}$$

$$\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} i \end{bmatrix}$$

$$\dot{i} = -\frac{Ri}{L} - \frac{K_m}{L} i + \frac{1}{L} V_{in}$$

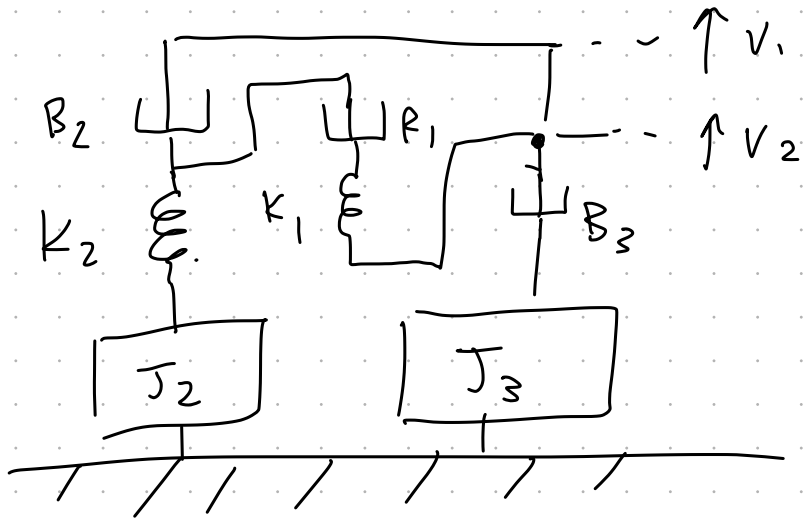
$$L\dot{i} = -Ri - K_m i + V_{in}$$

$$V_L = -V_R - V_b$$

$$V_{in} = L\dot{i} + Ri + K_m \omega_m$$

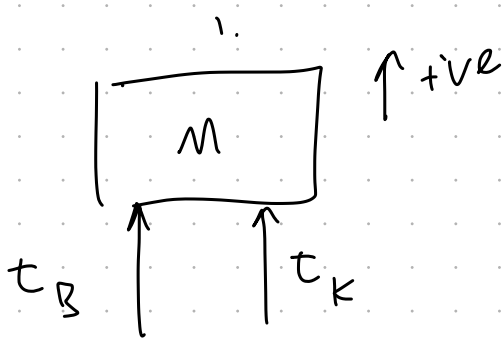
ELEC 341 Midterm
Andrew Chen
38113544

Q1.



Q2 below

Q4.



1. $M\dot{v} = \tau_B + \tau_K$

$M\dot{v} = -Bv + \tau_K$

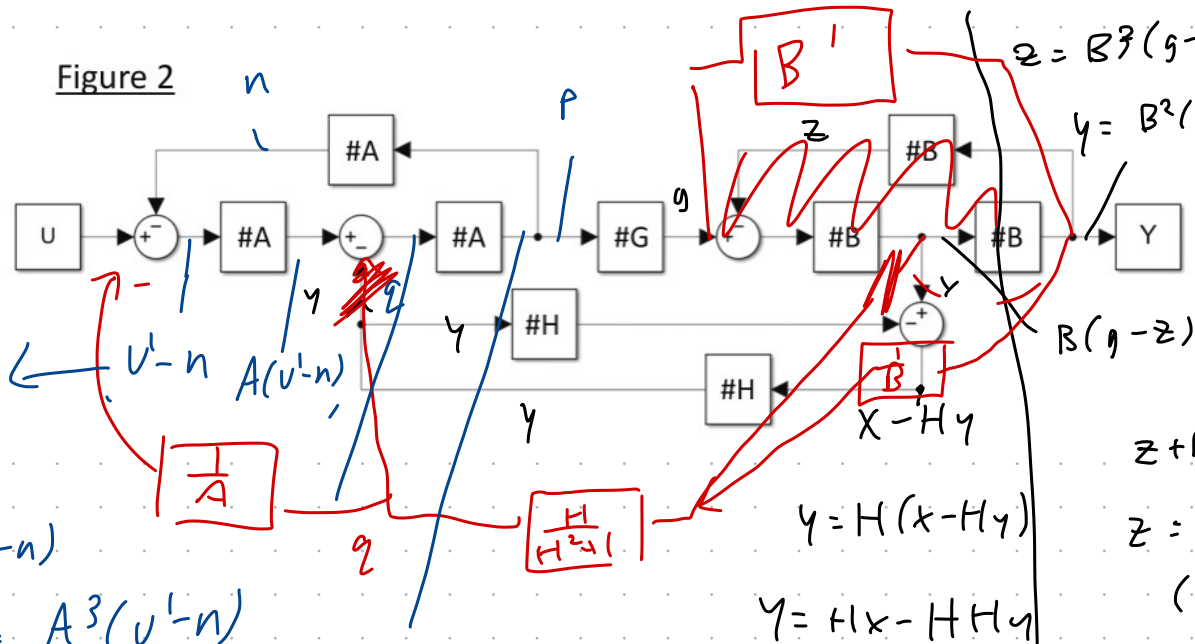
2. $0 = \tau_K + F$

$\tau_K = -F$

$$\begin{bmatrix} \dot{v} \\ \dot{d} \end{bmatrix} = \begin{bmatrix} \frac{-B}{M} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ d \end{bmatrix} + \begin{bmatrix} \frac{-1}{M} \\ 0 \end{bmatrix} [F]$$

Q3.

Figure 2



$z = B^3(g - z)$

$y = B^2(g - z)$

$B(g - z)$

$z + B^3 z = B^3 g$

$z = \frac{B^3}{(1+B^3)} g$

$y = B^2\left(g - \frac{B^3}{(1+B^3)} g\right)$

$y = g\left(B^2 - \frac{B^5}{(1+B^3)}\right)$

$y = g(B')$

$v' = v - \frac{q}{A}$

$v' - n$

$A(v' - n)$

$\frac{1}{A}$

$\frac{H}{H^2+1}$

$y = H(x - Hy)$

$y = Hx - HHy$

$(H^2+1)y = Hx$

$y = \frac{H}{(H^2+1)} x$

$p = A^2(v' - n)$

$n = Ap = A^3(v' - n)$

$n = A^3 v' - A^3 n$

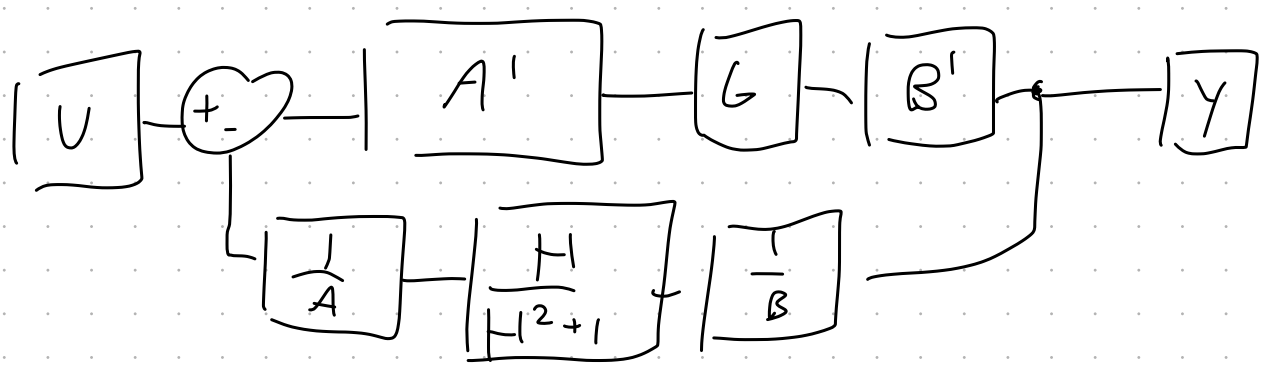
$(1 + A^3) n = A^3 v'$

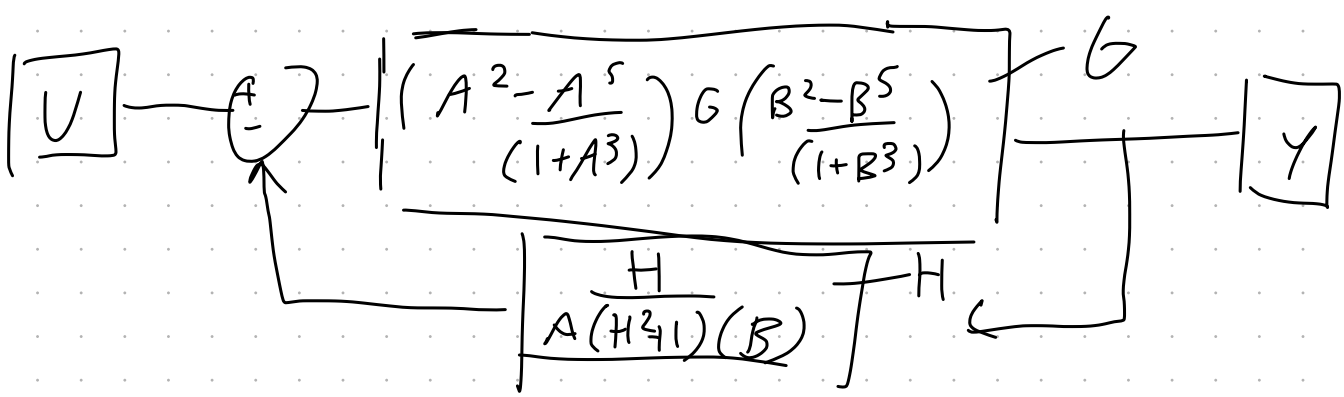
$n = \frac{A^3 v'}{(1+A^3)} \rightarrow p = A^2\left(v' - \frac{A^3 v'}{(1+A^3)}\right)$

$p = \left(A^2 - \frac{A^5}{(1+A^3)}\right) v'$

$A' = A^2 - \frac{A^5}{(1+A^3)}$

redraw





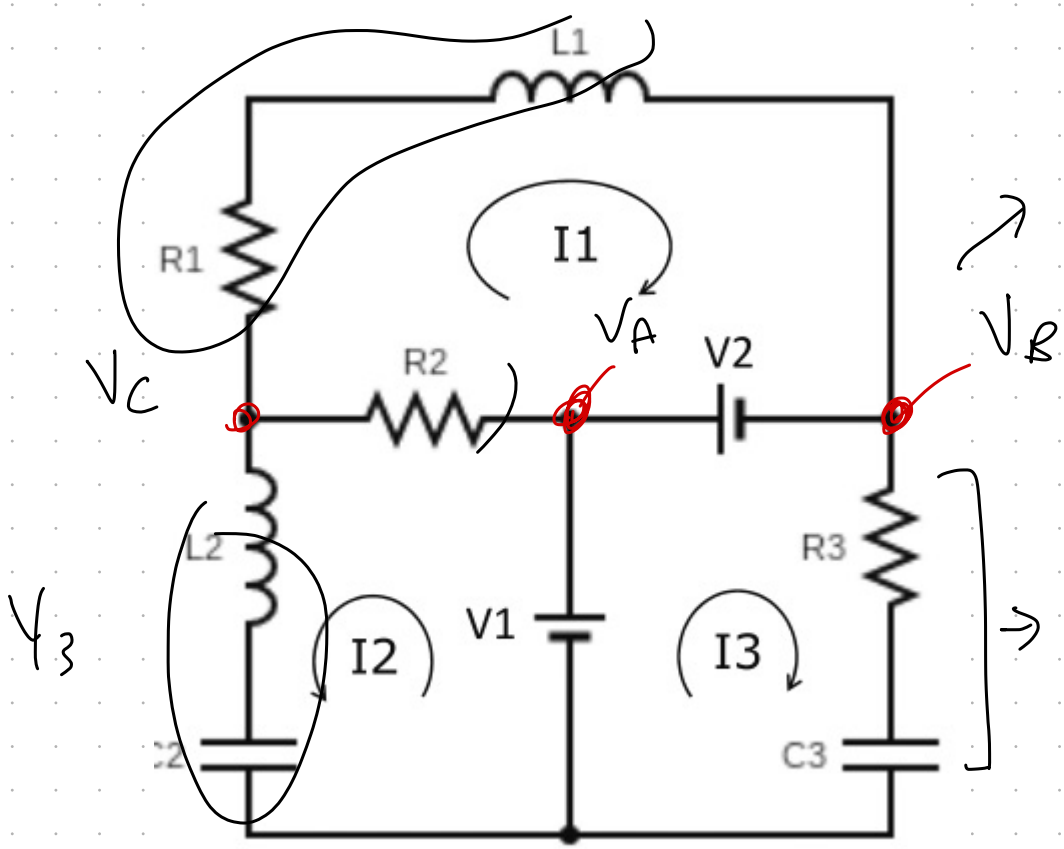
Gain: $\frac{V}{U} = \frac{G}{1+GH}$

Q_2:

$$\begin{bmatrix} 0 \\ 0 \\ \end{bmatrix} = \begin{bmatrix} \frac{1}{R_2} & 0 & -\frac{1}{R_2} \\ 0 & Y_1 + Y_2 & -Y_2 \\ -\frac{1}{R_2} & -Y_2 & Y_3 + Y_2 + \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

no time to solve for Ys.

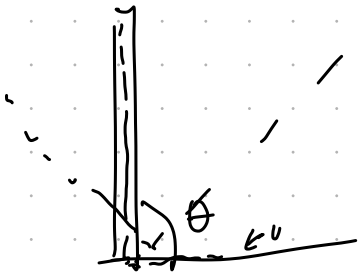
Y_2 = whatever



$V_B = V_A - V_2$

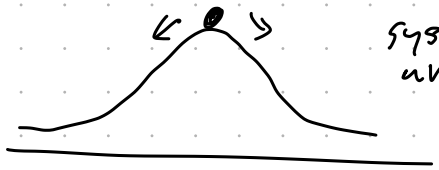
$V_A = V_1$

$$\frac{R_3 + \frac{1}{C_3 s}}{\left(R_3^2 + \left(\frac{1}{C_3 s}\right)^2\right)} = Y_1$$



$\theta_{desired} = 0$ nature wants the ruler to fall

marginal stability



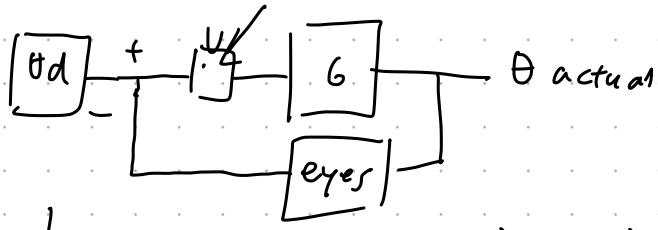
system is unstable and changes off small perturbations

open-loop control

(A)

converting θ to force

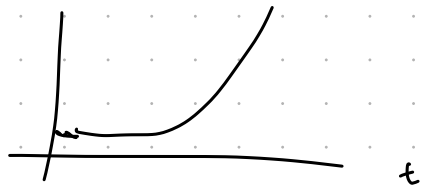
closed-loop/feedback control

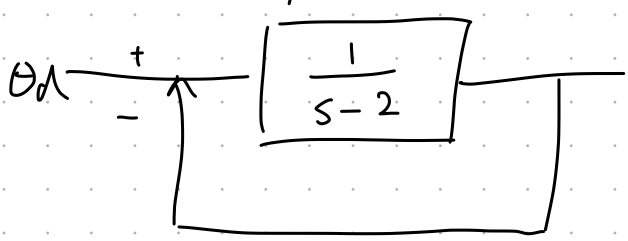


↳ your eyes compare θ_a and θ_d and adjust your "control" signal to try and match θ_a and θ_d

Trying to control system

let $G = \frac{1}{s-2}$ ← unstable $\xrightarrow{\mathcal{L}^{-1}}$ e^{2t}





is this stable?

$TF = \frac{G}{1+GH}$

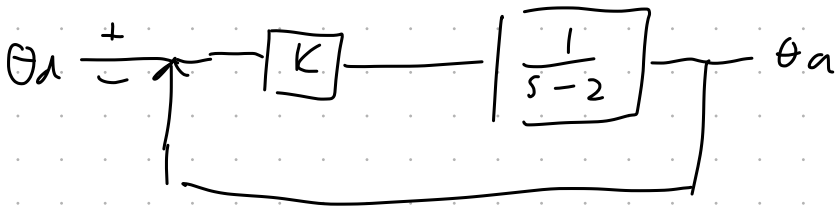
$= \frac{\frac{1}{s-2}}{1+\frac{1}{s-2}} = \frac{1}{s-2+1}$

$= \frac{1}{s-1}$

no

only doing a 1X feedback is not sufficient to reach stability

Try



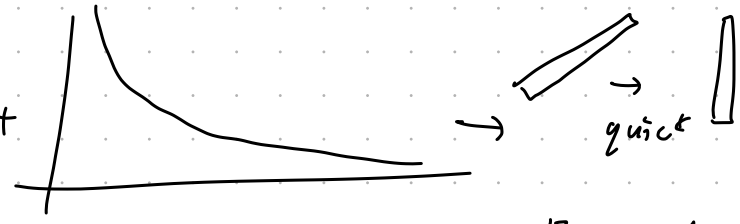
$TF = \frac{KG}{1+KGH} = \frac{K}{s-2+K}$

↳ set $K = 2$

This is marginally stable, but this doesn't let us get $\theta_d = 0$, it just makes θ_a a stable number as long as our input doesn't change

$K = 3 \rightarrow TF = \frac{3}{s+1} \xrightarrow{\mathcal{L}^{-1}}$

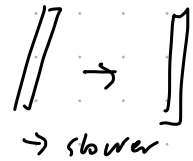
↳ eventually we will find that we need



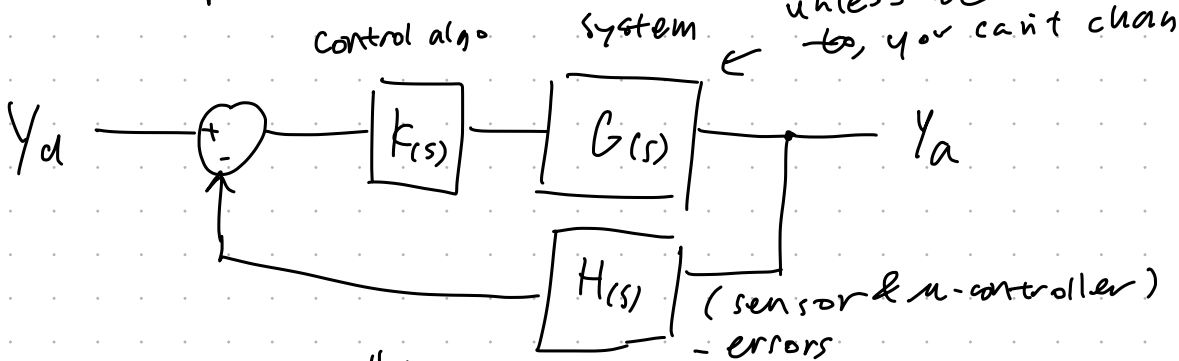
$? < K < K_{ultimate}$

↳ fucks up other stuff if you go to hard

(proportional control)



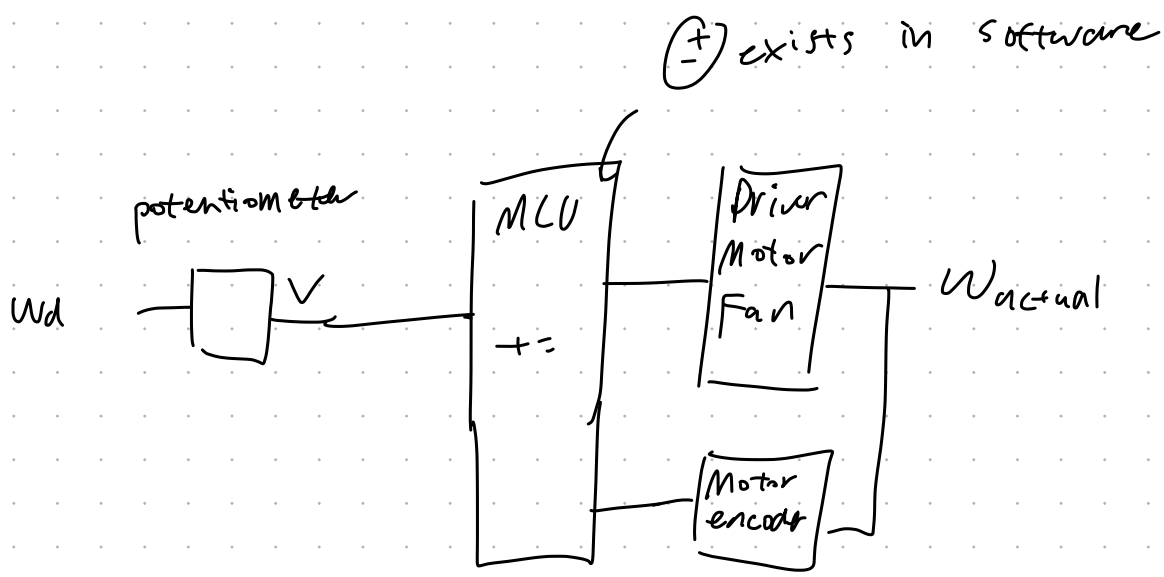
complete picture of proportional controller

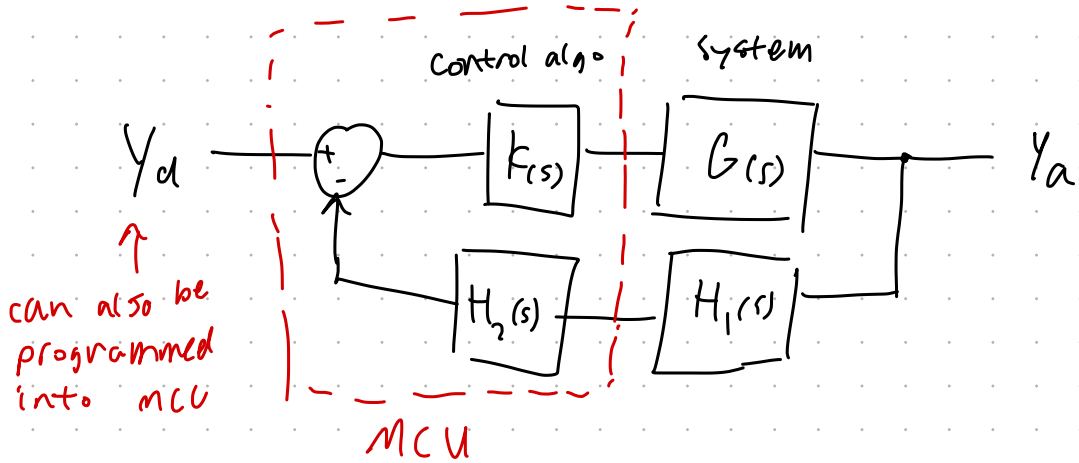


(sensor & µ-controller)
- errors
↳ - delay
- aliasing
- low sampling

we don't usually design this, but you can buy a component with suitable $H(s)$ to achieve your goals

Real life example





Controller.cpp

```
void pControl (){
    #define K 3 // constant
    double wd = readDAQ(potPin);
    double wa = readDAQ(senPin);
    double err = wd - wa;
    "      u = K * err;
    "      writeDAQ(u);
```

this is the difficult part

introduces delay, which affects K

Driver : O^2 or O^1 or O^n (if we design)

Motor : O^2

Mech : O^{2+}

Sens : O^{1+}

systems will be O^{6+}

Order(Sys) ≥ 6

poles ≥ 6

$$\frac{Y_a}{Y_d} = \frac{K G_N H_D}{\underbrace{K G_N H_N + G_D H_D}_{CE}}$$

↳ from open loop TF

$GH = \frac{\bigcirc}{\square}$ $CE = \bigcirc + \square$

→ can expand and find poles

↳ if you see $(s-a)$, you know something will be unstable

ex:

$s^6 + (K+3)s^5 + (K-1) \cdots$

↓

$(s+K+2)(s+K-7) \cdots$ ← check stability from here

$(s+7-K) \leftarrow K < K_u = 7$

Routhe-Hutwitz method can do this for us, but MATLAB works open loop

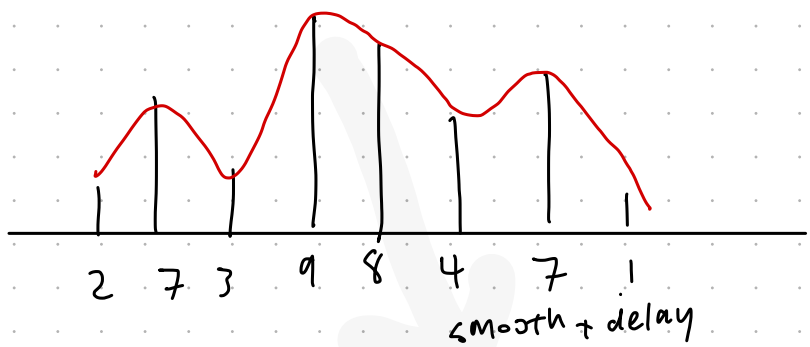
↳ K_v = margin (GH) TF

↓ tells you how much gain is stable

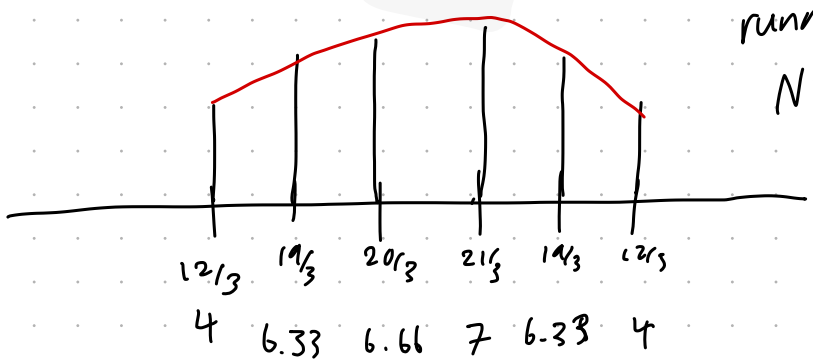
Filter can either be caused by things out of our control or we can add them

Filters:
- smooth things out
- add delay

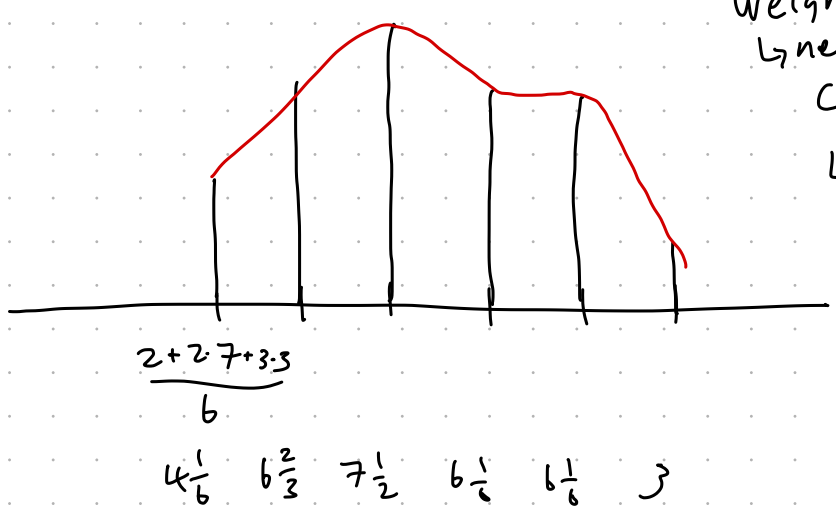
↳ running average (total average is too much)
(over N samples)



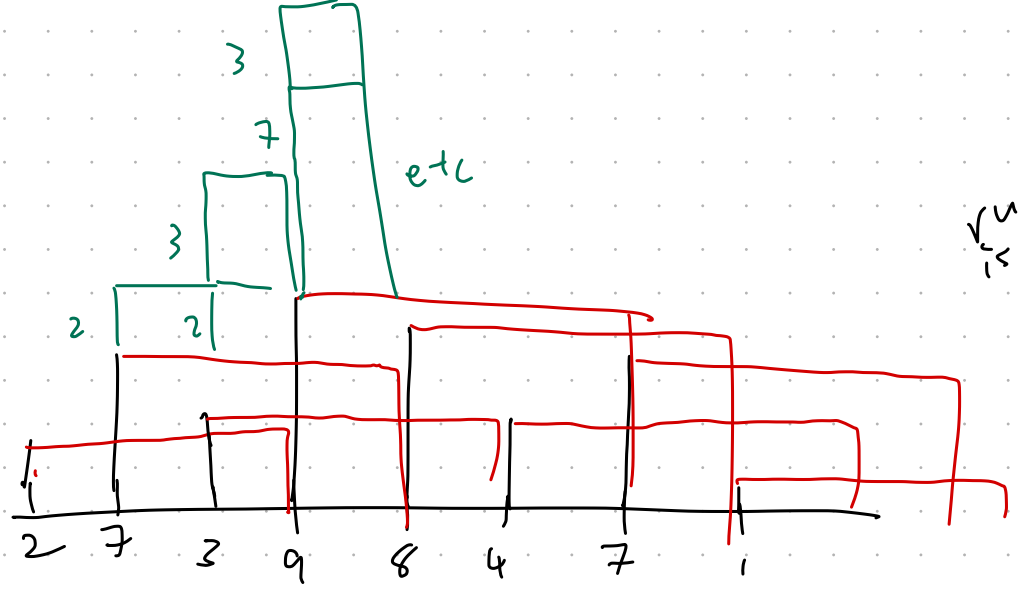
running average
N = 3



← dividing by 3 achieves unity gain



weighted average
↳ need weighting coefficients
C = [3 2 1] ← older samples less important
↳ divide by 6 for unity gain



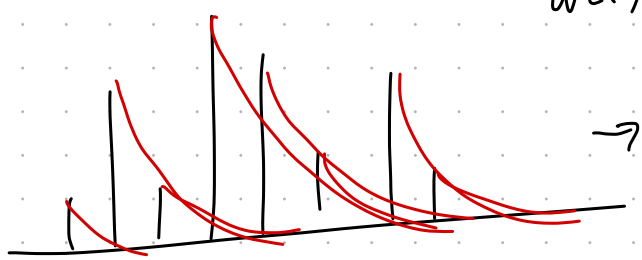
running average is like holding each sample for longer (N)

Filter $\mathcal{L}^{-1}$

$$\frac{1}{s+\sigma} \rightarrow e^{-\sigma t}$$

applying this to each sample weighted sum

↳



→ can be ignored after 4τ

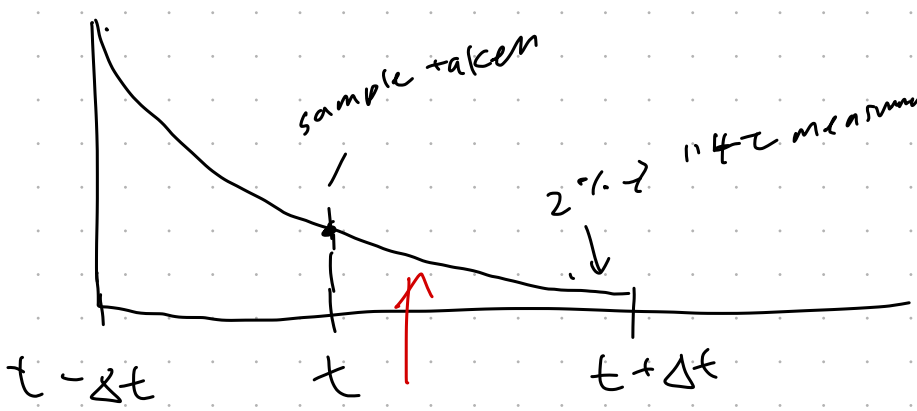
$$\tau = \frac{1}{\sigma}$$

Can create filters to replicate the effect of sampling delay

Continuous Approximation of sampling (Zero Order Hold)

Model for "doing nothing"

↳ use value you just read and assume no delay between



we want to model this tail by choosing where to put σ

$$\sigma = \frac{1}{\tau}$$

$$\Delta t = 4\tau = 2\Delta t$$

$\Delta t = \frac{1}{CF}$ ← control frequency (sample frequency)

$$\frac{4}{\sigma} = \frac{2}{CF} \qquad \sigma = 2CF \qquad \rightarrow \frac{2CF}{s + 2CF}$$

EX: CF = 1 Hz

$x = 7$ (at $t=1$)

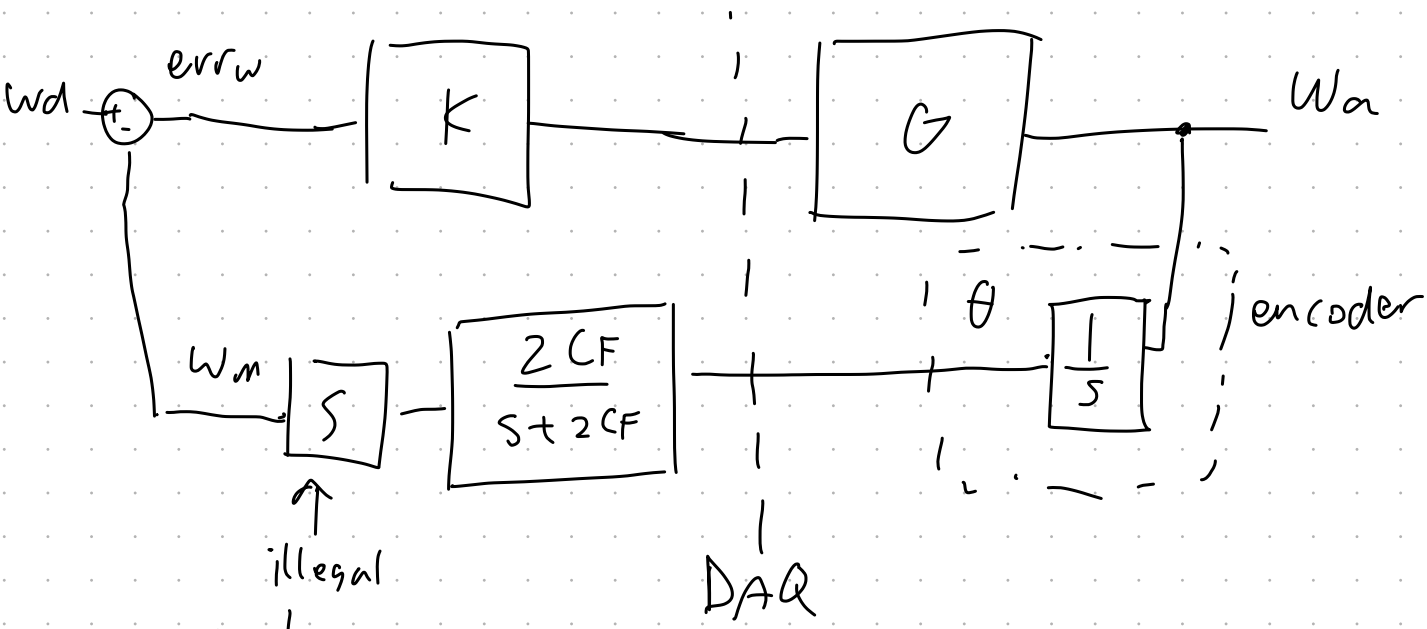
computer:

$$7e^{-(\frac{1}{1})t} = 7(0.37)$$ ← weighting coeff divide by 0.37 as you measure 1 sample at a time

computer reads $\frac{7(0.37)}{(0.37)} = 7$

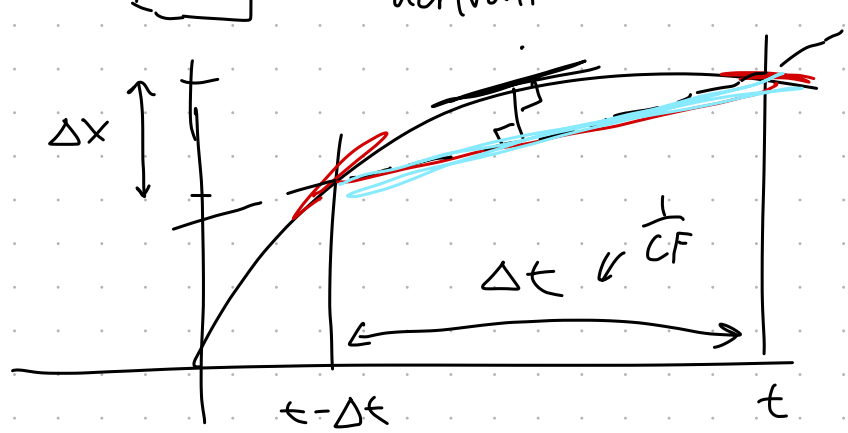
model → $\frac{2}{s+2}$

Takeaway: If you are running a MCU at 1 Hz, you need to account for a delay filter of $\frac{2}{s+2}$



use ↓

$\boxed{\frac{d}{dt}}$ "finite difference" derivative (rise/run for derivative)



$$FD = \frac{\Delta x}{\Delta t}$$

FD matches actual slope at some point, but this is delayed

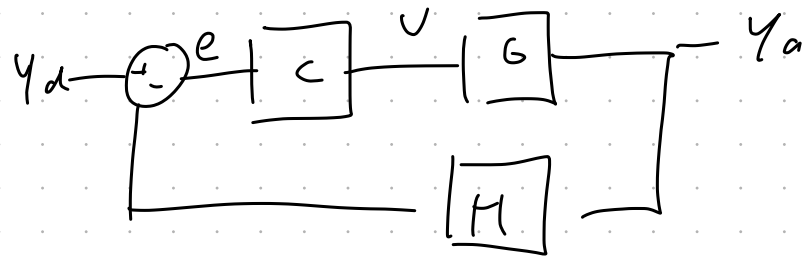
↓
this can be approximated as a filter

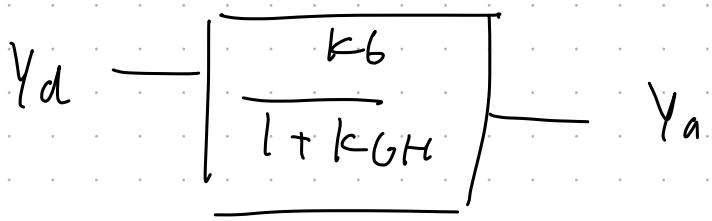
$\frac{2CF_s}{s+2CF_s}$, there's probably a good reason but it doesn't matter

Optimal Control - Proportional Only

RCG - requirements, contraints, goals

Steady-state error





FV - step response → set s to 0

$$K_{DC} = \frac{KG}{1+KGH}\Big|_{s=0}$$

e_{ss} = DC gain of error signal

↳ if we only care about error we can redraw

$$\frac{e}{y_d} = \frac{1}{1+KGH}$$

$$e_{ss} = \frac{1}{1+KGH}\Big|_{s=0}$$

TF $G = \dfrac{1}{(s+1)(s+2)(s+3)}$

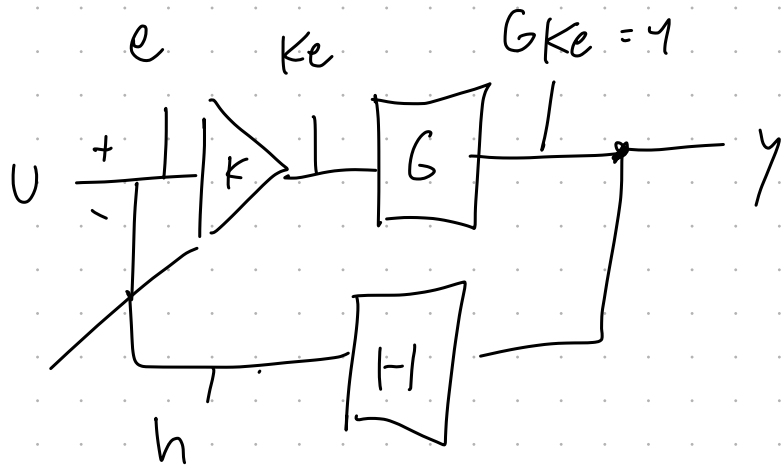
$H = \dfrac{200}{s+200}$

set $s \to 0$

$KGH|_{s=0} = \dfrac{\cancel{200}K}{1 \cdot 2 \cdot 3 \cdot \cancel{200}}$

$= \dfrac{K}{6}$

$e_{ss} = \dfrac{1}{1+\frac{K}{6}} = \dfrac{6}{K+6}$



CLTF:

$h = Hy = KGHe$

$e = u - h$

$GK(u - Hy) = y$

$KG\,u - KGHy = y$

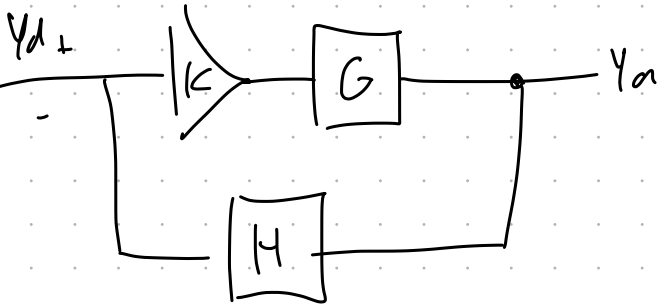
$KGu = y(1+KGH)$

$\dfrac{y}{u} = \dfrac{KG}{(1+KGH)}$

OLTF: $\dfrac{u}{h}$

$h = Hy = KGHu$

$\dfrac{u}{h} = \dfrac{1}{KGH}$



$$\frac{K G_N H_D}{K G_N H_N + G_D H_D}$$

K big $\rightarrow$ Den $\approx K G_N H_N$ $\leftarrow$ open loop zeros

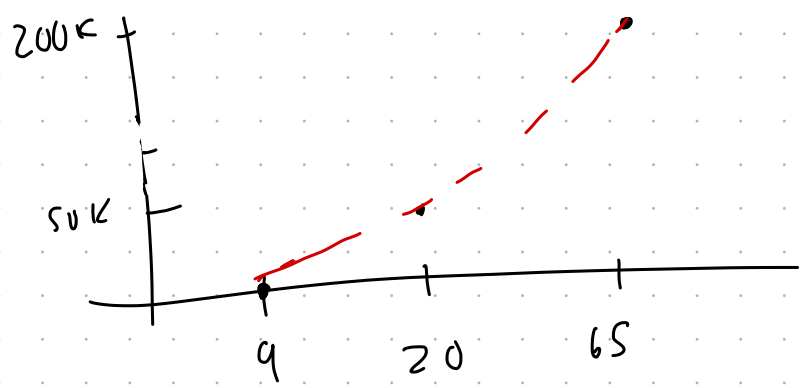
$K = 0 \rightarrow$ Den $= G_D H_D$ $\leftarrow$ open loop poles

↳ if poles are in right half plane we have an unstable system, and we use K to move them,

OL : $\dfrac{K G_N H_N}{\underbrace{G_D H_D}}$

↳ making this have a DC gain of 1, means K becomes your DC gain

Root Locus



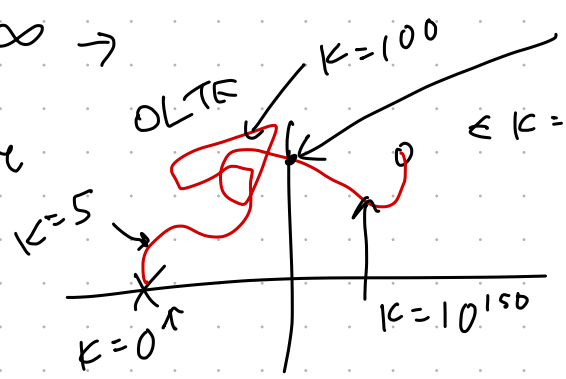
← money

Locus → collection of points

- A Root Locus plot is just all possible roots of a TF from $K = 0 \rightarrow \infty \rightarrow$

$K = K_U \rightarrow$ system is marginally stable

conceptual example



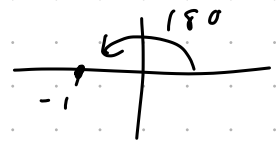
← $K = \infty$

→ this curve is the root locus

$CL = \frac{KG}{1+KGH}$ & CE: $1+KGH=0 \rightarrow KGH=-1$

$KGH = 1\angle 180^\circ$

magnitude criteria angle criteria



$|KGH|=1 \quad \angle KGH = \pm 180^\circ = \angle GH$

What does this tell us about s?

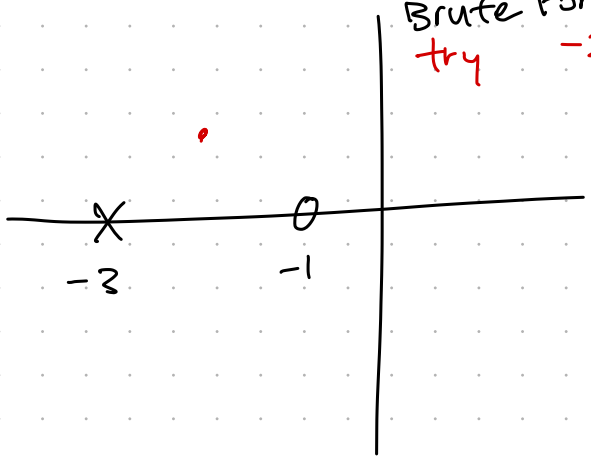
↳ all points in root locus satisfy $\angle G(s)H(s) = 180^\circ$

↳ that equation tells us it exists, plugging into $|KGH|=1$ gives you the actual value of K

Ex. $GH = \frac{s+1}{s+3}$

Brute Force

try $-2+j$



↳ $GH = \frac{-1+j}{1+j}$

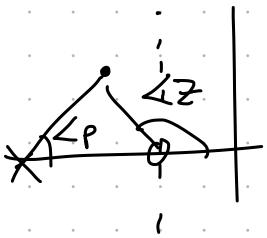
$= \frac{\sqrt{2}\angle 135}{\sqrt{2}\angle 45}$

$= 1\angle 90^\circ$

↳ not 180°

Geometric Trick:

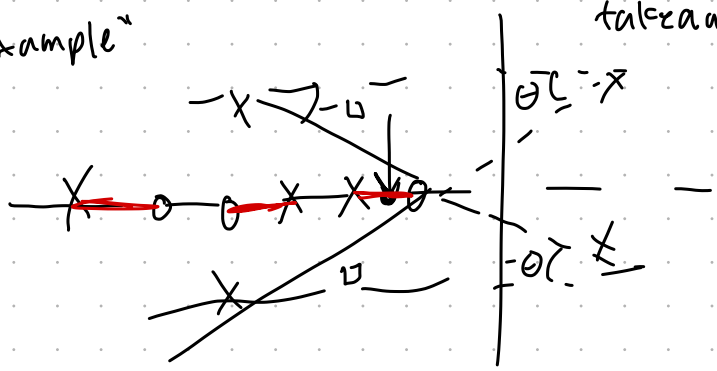
$\sum \angle \text{zeros} - \sum \angle \text{poles}$



$135 - 45 = 90^\circ$

Using the real axis

"hard example"



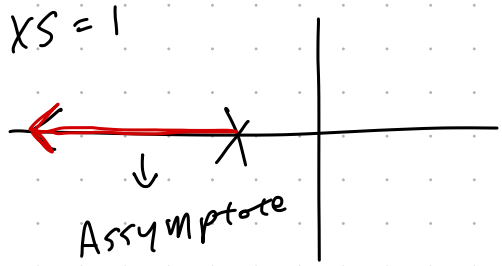
takeaways: anything to the left of your Point of interest will only contribute 0s, and those off the real axis also contribute only zeros

when on real axis ↑

Root Locus exists to the left of all odd # Real Poles & Zeros

$K_U = \infty$ if no parts of the root-locus cross imaginary axis

$XS = 1$

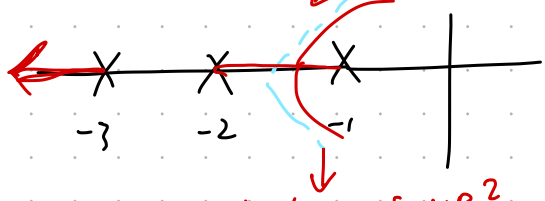


$\rightarrow \frac{K}{s+1} \quad \frac{s+\infty}{\infty} \approx 1$

$XS = \#\text{ poles} - \text{zeros} \rightarrow$ ignore $XS < 0$ as we hate derivatives

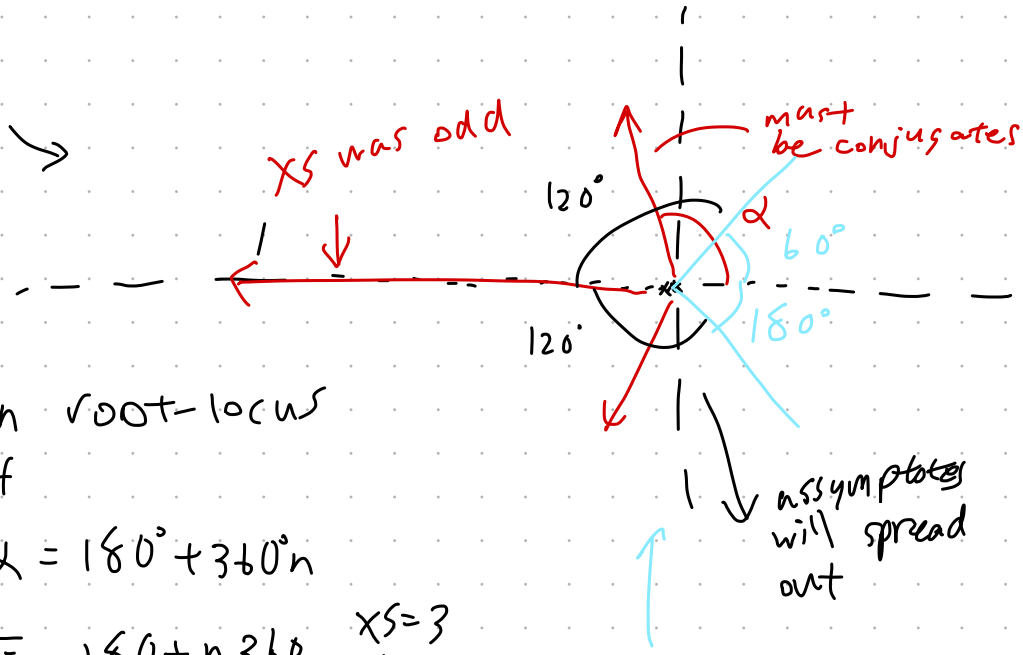
$XS = 3 \rightarrow 3$ assymptotes

no zeros to land in (squirrels hate each other)



which goes up?

↳ both, who cares



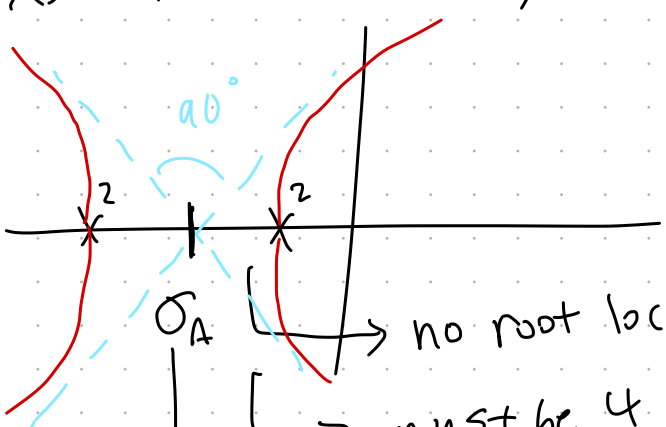
α:

$\Sigma \angle P = 180° + 360°$ on root-locus if

$XS \cdot \alpha = 180° + 360° n$

$\alpha = \frac{180 + n360}{3} \xrightarrow{XS=3} \alpha = 60° + n120°$

$XS = 4$ (even example)



↳ no root locus can go always ever to right

↳ must be 4, must be separated by $\frac{360°}{XS}$, must be complex conjugates

↳ ∴ must be big X

where do you put this?

↳ kinda like an average of OL P, Z

$$\sigma_A = \frac{\Sigma P - \Sigma Z}{XS}$$

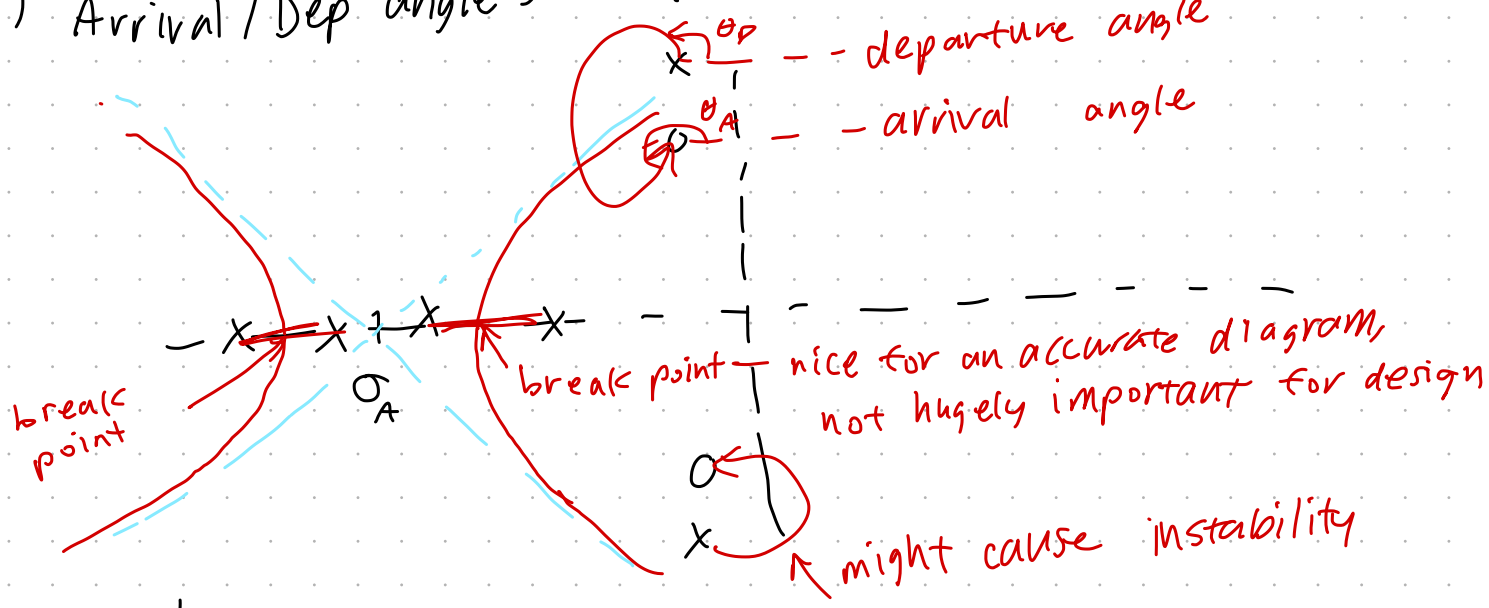
R-L (root locus) steps

1) Real Axis
2) Assymp
3) Assymp Cen

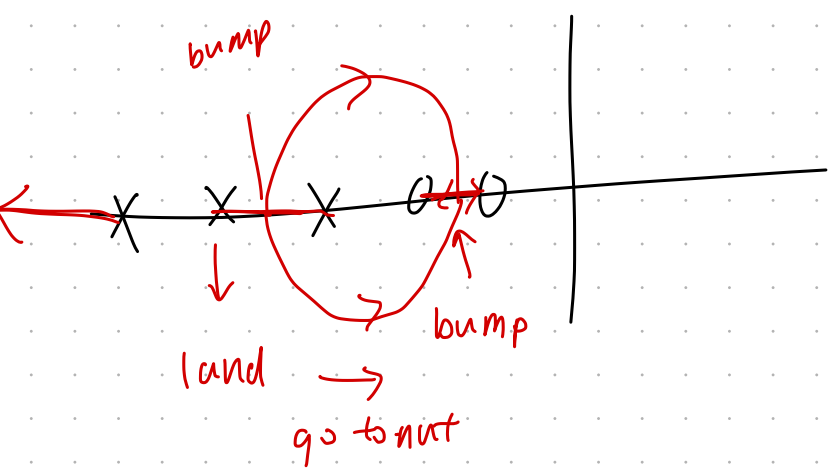
} eyeball

4) Break Pt
5) Arrival / Dep angle

} not covered above
↳ do in matlab



Cool example

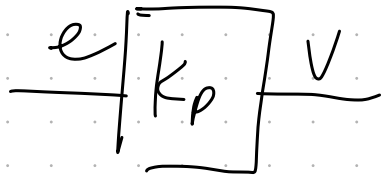


$OL = \frac{OL\ zeros}{OL\ poles}$ $\quad CL = \frac{CL\ zeros}{CL\ poles}$ ← characteristic equation

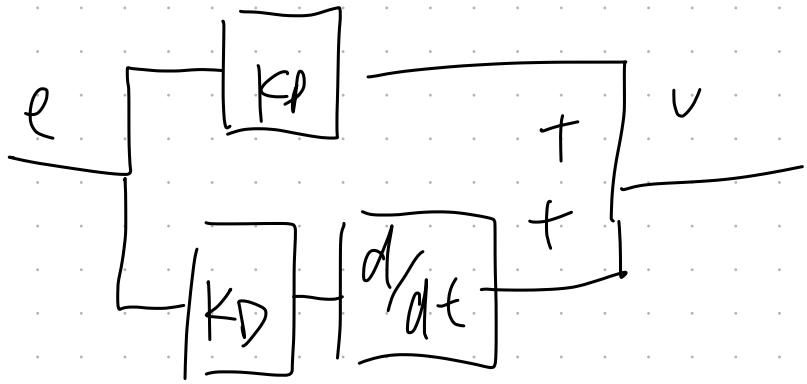
↳ all we care about really

In General

P-Contr



PD-Contr



PD-control not possible in digital world

LEAD control is the closest irl equivalent

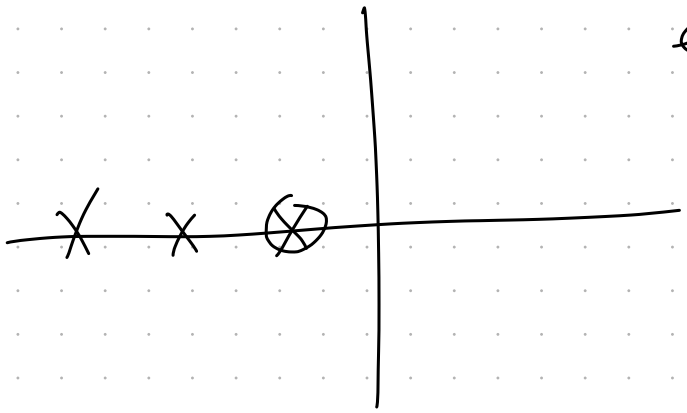
$\frac{d}{dt} \sim s$ → Finite Difference Derivative

$$C = K_p + K_D s$$

$$= K_D(s + K_p/K_D) = \underbrace{K_p}_{\text{Gain}} \underbrace{\frac{K_D}{K_p}\left(s + \frac{K_p}{K_D}\right)}_{\text{Unity gain}}$$

move your 0 wherever

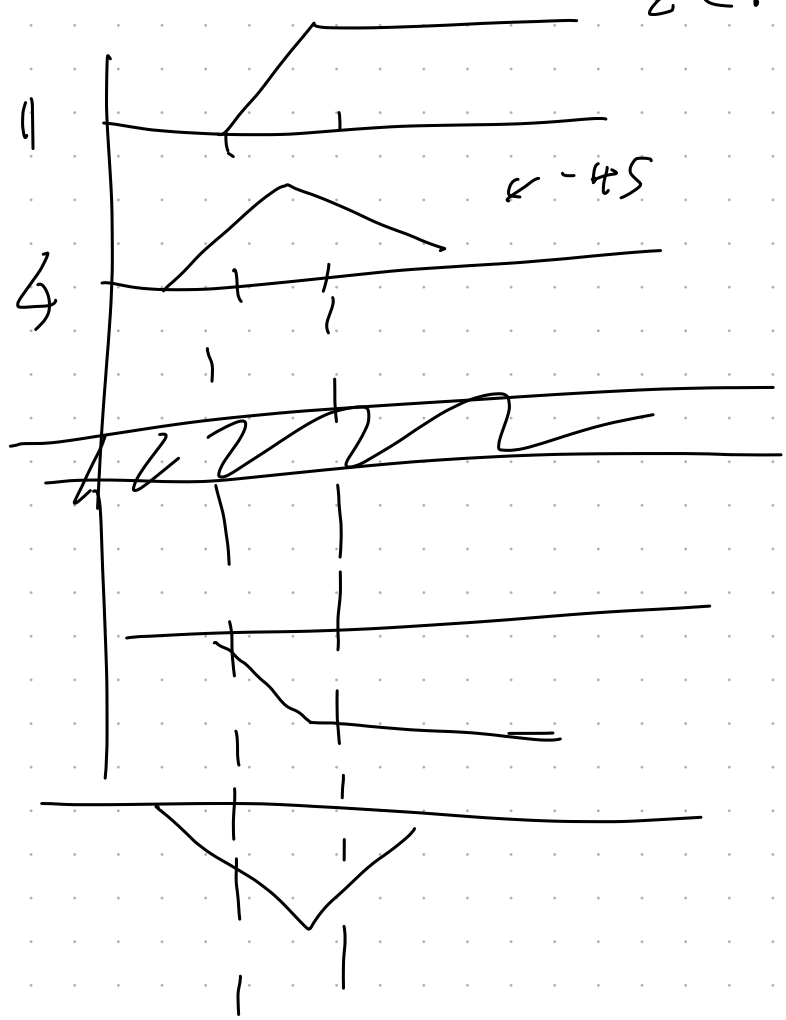
zero e $+K_p/K_D$



← PD control lets you add a zero, and kill a pole

but its impossible

$z < p$



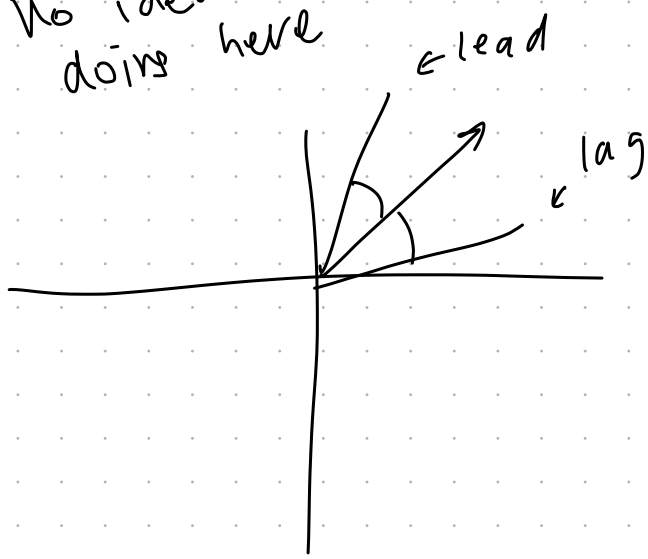
Lead/Lag control

$$C = KD = K\frac{p}{z}\frac{s+z}{s+p}$$

↓ dynamics only

↑ for unity gain

No idea with he was doing here



adding lead when $z < p$, lag when $p > z$

<u>Lead control</u>

$$C = K_P = K\frac{p}{z}\frac{s+z}{s+p} \approx \text{PD control} \rightarrow z < p$$

- must compute FPD $\frac{2CF}{s+2CF}\cdot s$ $\quad p = 2\cdot CF$

$z < 2CF$

$$D = \frac{p}{z}\left(\frac{s+z}{s+p}\right) = \frac{ps + pz}{zs + pz} = \frac{zs + pz - zs + ps}{zs + pz}$$

$$= 1 + \frac{ps - zs}{zs + pz} = 1 + \frac{(p-z)s}{zs + pz} = p - z\left(\frac{s}{z(s+p)}\right)$$

$$= \frac{p-z}{z} \cdot \frac{s}{(s+p)} = \underbrace{1}_{K_P} + \underbrace{\left(\frac{p-z}{zp}\right)}_{K_D}\left(\frac{sp}{s+p}\right)$$

↳ FPD (possible to implement)

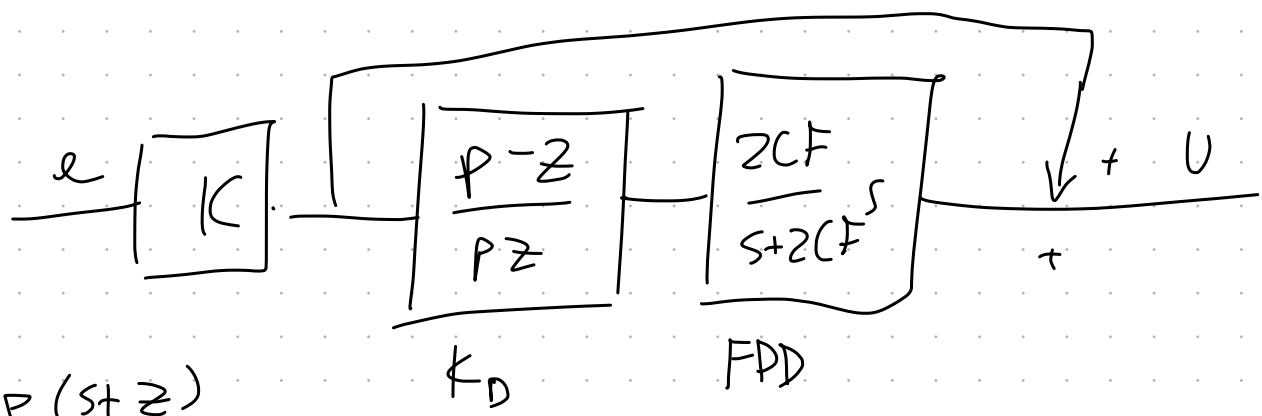
↳ FPD was s in the previous PD example

Ex: $p = 2CF = 200$

want $z = 3$

↳ $K_P = 1$ $K_D = 197/600$

Lead Control Diagram



$$K\frac{p(s+z)}{z(s+p)}$$

Steps:

$p = 2CF$

z = choose (design parameter (based on G)

calculate gains

$K_p = 1$ ← always true or you mess up everything.

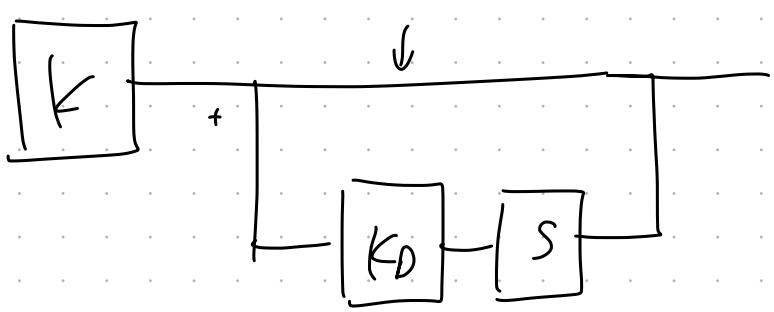
$K_D = \frac{p-z}{pz}$

K_p itself will end up being scaled by K

↓

K = choose using root locus

"K_p"



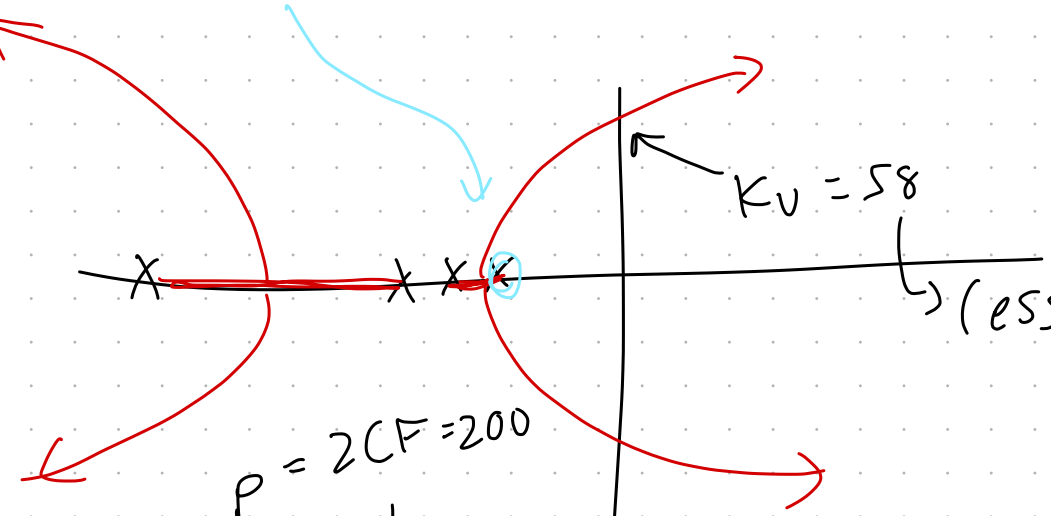
Ex:

$G = \frac{1}{(s+1)(s+2)(s+3)}$ $\quad CF = 100 \quad H = \frac{200}{s+200}$

$z = 1$ $\quad K_p = K_D$

$K_p = 1$

$K_D = \frac{199}{200} = 0.995$

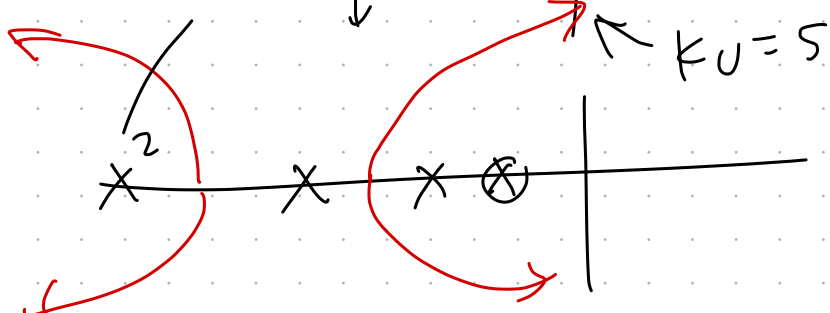


$K_U = 58$

(ess = 9 %)

$p = 2CF = 200$

↓



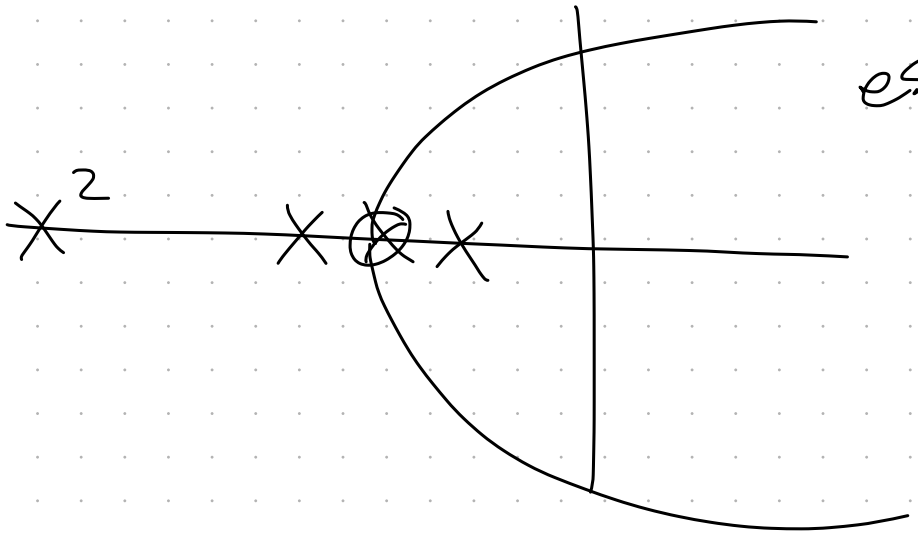
$K_U = 510$ (ess = 1.8%)

→ decreases settle time too

→ just better

Above was all $z = 1$, can try other zs

$z = 2$



$e_{ss} = .7\%$

↳ better error,

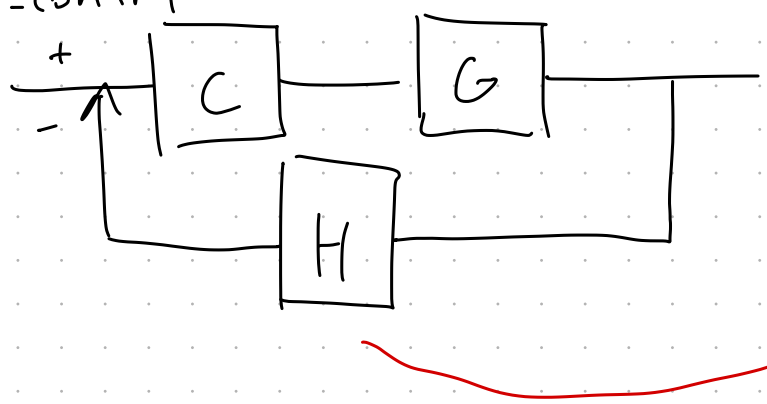
↳ lower k_v

↳ trade off

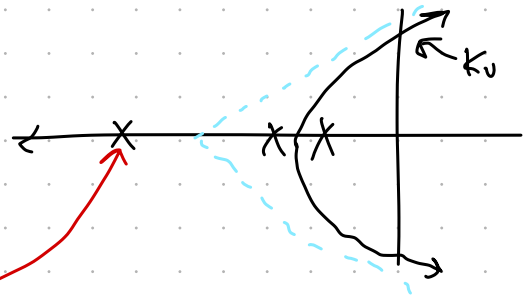
We need a tool to choose our zeros too, similar to how RL helps use choose gain

March 21st

P-control



For P-control, $C = K$ $K \leq K_u$



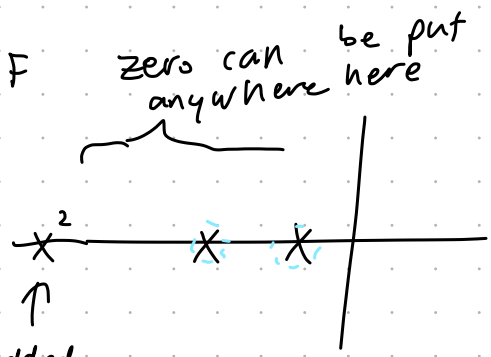
↳ P only results in bad e_{ss}

$e_{ss} \propto \frac{1}{K}$

Lead Control (Review)

$C = KD$ ← unity gain → place 2 poles at 2CF

↑ From root-locus



- adds 1 pole 1 zero → XS stays the same

↳ assymptotes stay the same, but gets shifted

↳ increases K_u → ↓ e_{ss}

· $z < p$ → z to the right

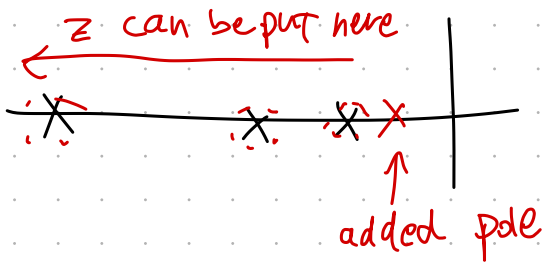
↳ the optimal choice of the zero is not clear

Lag Control

- same as lead control, but the order of which we put our p and z are swapped
- FDD not required
- no extra pole at 2CF (we put it there in lead control)

↳ p and z can be placed anywhere as long as $z > p$

↳ placing the p to the right of the rightmost system pole lets you use the zero to cancel any of the system poles



$$D = \frac{p}{z}\frac{s+z}{s+p}$$

$$e_{ss} = \frac{1}{1+KDGH\big|_{s=0}}$$

$$= \frac{1}{1+\frac{p(s+z)}{z(s+p)}KGH\big|_{s=0}}$$

sorta not 0, but in practice this is a tiny numerator with a giant denom

$$= \frac{z(s+p)}{z(s+p)+p(s+z)KGH}\Bigg|_{s=0} = \frac{zp}{zp+zpKG_0H_0}$$

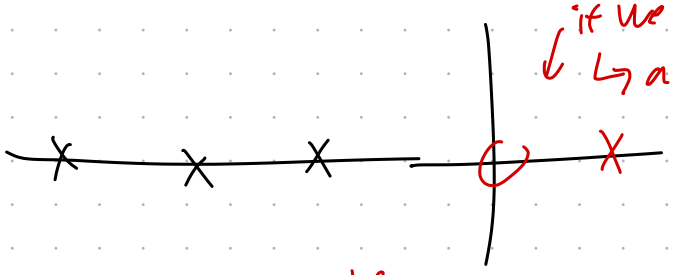
→ want to minimize

↳ set $p=0$

$\therefore z>0$

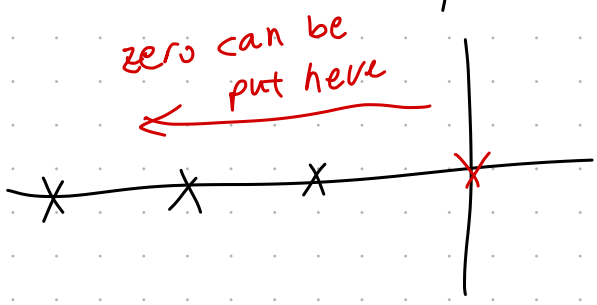
↳ PI control (proportional-Integrator)

$$\lim_{p\to 0}\frac{zp}{zp+zp}$$



if we put $p<0$
↳ always unstable

de-stabilizing



$$D = \frac{1}{z}\frac{s+z}{s}$$

$K_{DC} \neq 1$

Gain @ $\omega = 1 \approx 1$ (frequency)

↳ "1Hz is close enough to DC lol"

$$D = \frac{s+z}{s} = \frac{\cancel{s}}{z\cancel{s}} + \frac{\cancel{z}}{s\cancel{z}} = \frac{1}{z} + \frac{1}{s}$$

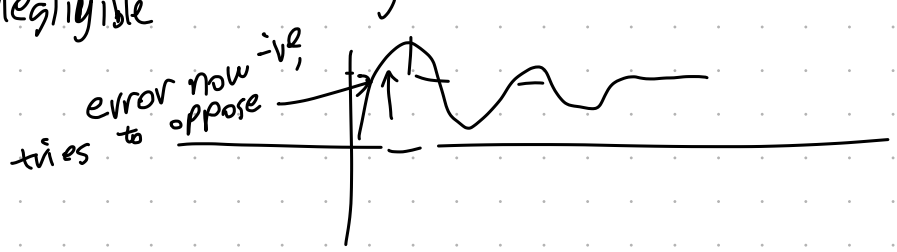
↓ inherently a filter

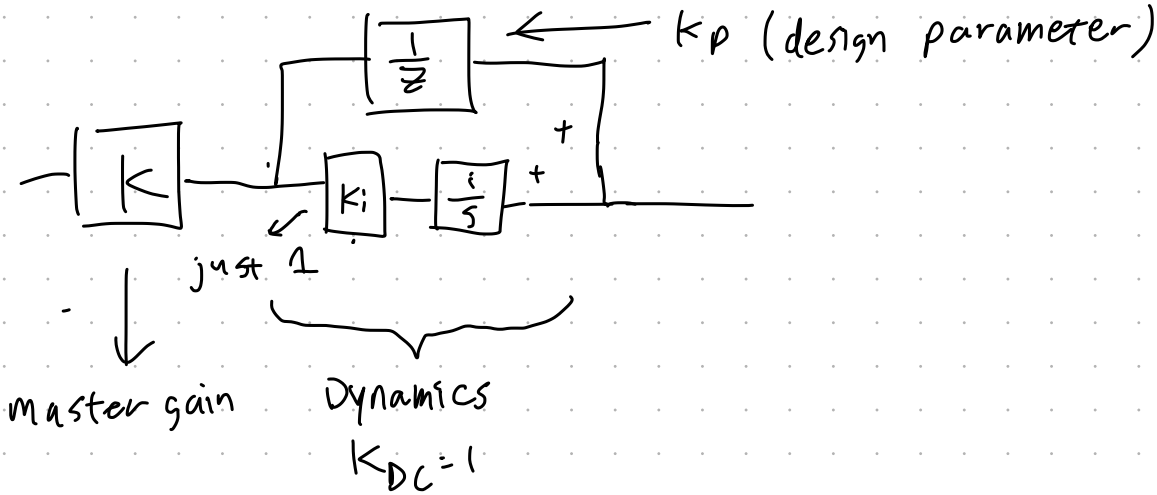
↳ adding small values (noise) to the bigger value "collected" by the integral is negligible

$K_p = \frac{1}{z}$ $K_i = 1$

in MCU
static double integ = 0;
integ += e · dt;

↳ Integration also almost guarentees overshoot





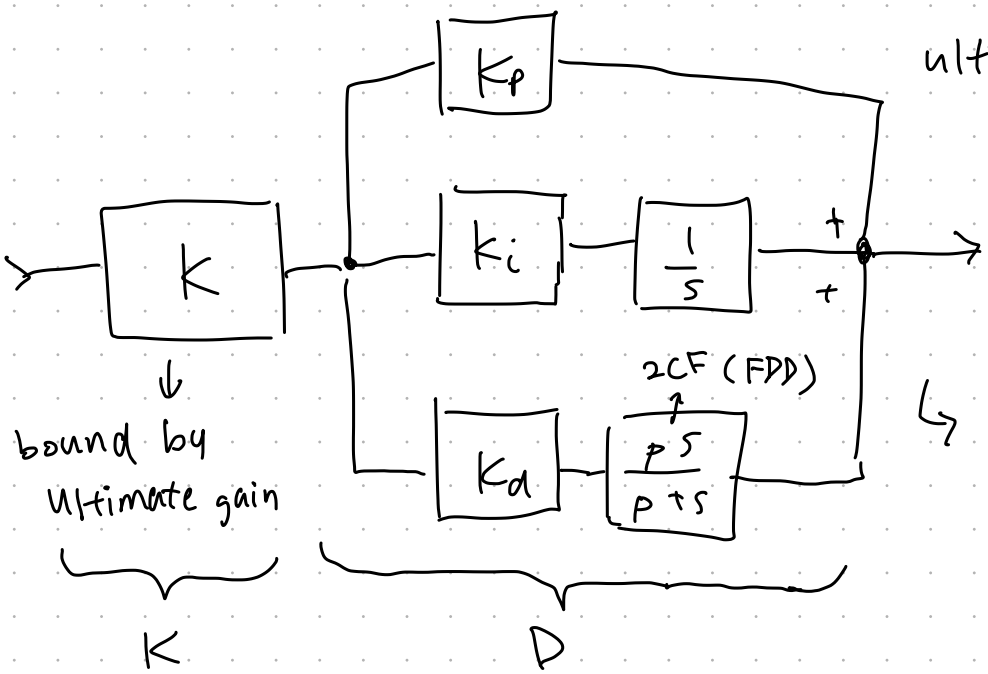
PID (Lead-Lag) Control

↳ just both

↳ 1 pole at 2CF → lead

1 pole at 0 → lag

↳ we have two zeros to work with ∴ complex zeros are possible



ultimately this is

$$\frac{(s+z_1)(s+z_2)}{s\cdot(s+p)}\cdot\frac{p}{z_1 z_2}$$

still Gain = 1 at $\omega = 1$

↳ $K\left[K_p + K_i\frac{1}{s} + K_d\left(\frac{ps}{s+p}\right)\right]$

Find K_p, K_i, K_d such that

$$\left[K_p + K_i\frac{1}{s} + K_d\left(\frac{ps}{s+p}\right)\right] = \frac{p}{z_1 z_2}\,\frac{(s+z_1)(s+z_2)}{s(s+p)}$$

$$\frac{K_p s(s+p) + K_i(s+p) + K_d p s^2}{\cancel{s(s+p)}} = \frac{p}{z_1 z_2}\left(\frac{s^2 + (z_1+z_2)s + z_1 z_2}{\cancel{s(s+p)}}\right)$$

collect powers of s

$$K_p(s^2+ps) + K_i(s+p) + K_d p s^2$$

$$s^2(K_p + pK_d) + s(pK_p + K_i) + pK_i = \frac{p}{z_1 z_2}s^2 + \frac{p(z_1+z_2)}{z_1 z_2}s + p$$

$\therefore$

$$K_p + pK_d = \frac{p}{z_1 z_2} \quad (1) \qquad pK_p + K_i = \frac{p(z_1+z_2)}{z_1 z_2} \quad (2) \qquad \boxed{K_i = 1} \quad (3)$$

(2) $$\boxed{K_p = \frac{(z_1+z_2)}{z_1 z_2} - \frac{1}{p}}$$

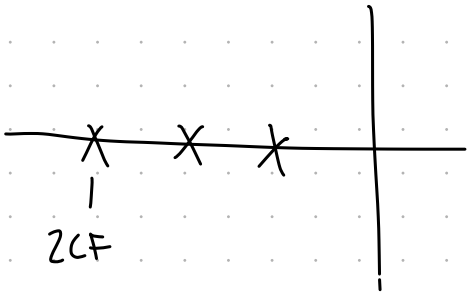
(1) $$\frac{(z_1+z_2)}{z_1 z_2} - \frac{1}{p} + pK_d = \frac{p}{z_1 z_2} \rightarrow \boxed{K_d = \frac{1}{z_1 z_2} - \frac{(z_1+z_2)}{p(z_1 z_2)} + \frac{1}{p^2}}$$

$$K_d = \frac{1}{z_1 z_2} - \frac{K_p}{p}$$

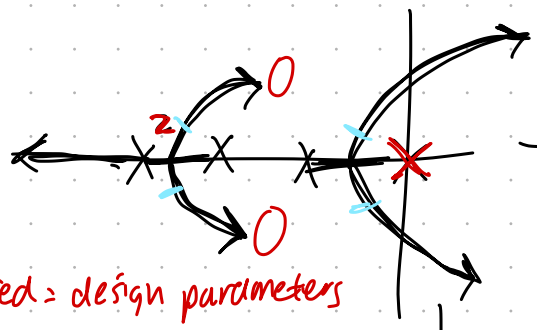
again, $p = 2CF$

Design Process

GH



DGH



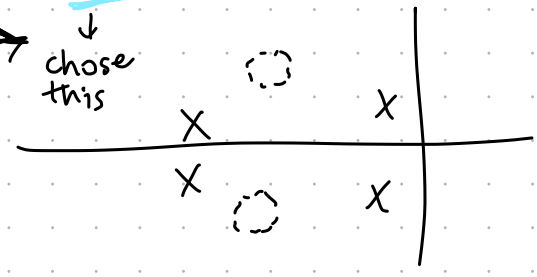
red = design parameters

↳ these come from K_p and K_d

$K = 0 \to \infty$

$K = 7$ C-L pole plot

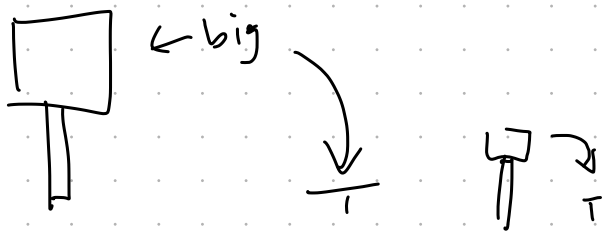
↓ chose this



March 23rd 2022

See supplementary Notes

weird hammer analogy



- amplitude of swings is proportional to how fast you can swing (ω)

$$\frac{V}{F} = \frac{1}{Js+B} = \frac{1/J}{s+B/J} \approx \frac{\sim}{s\big|_{s=j\omega} + a}$$

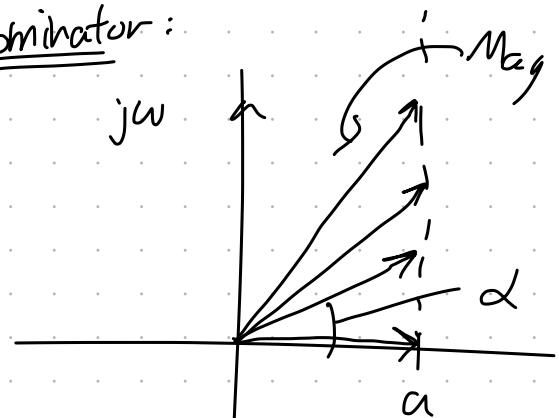
↑ hammer

$$\frac{1}{s+a}\Big|_{s=j\omega} = M\angle\theta$$

magnitude

$$M = \frac{1}{m} \qquad \theta = -\alpha$$

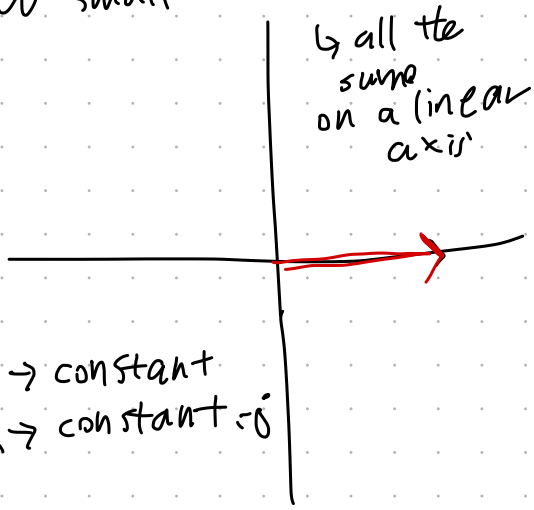
<u>denominator</u>:



$$m = \sqrt{a^2 + \omega^2}$$

ω small : $10^{-100} \to 10^{0}$

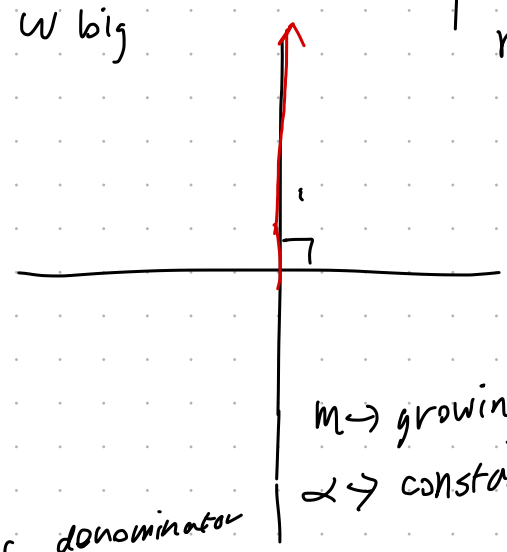
↳ all the same on a linear axis



$m \to$ constant

$\alpha \to$ constant $= 0$

ω big



$m \to$ growing with ω still

$\alpha \to$ constant $= 90°$

↳ pole does nothing to the magnitude of denominator until ω grows beyond a

$M \downarrow$ @ $\omega = a$ $\qquad \theta \downarrow 90°$ around $\omega = a$

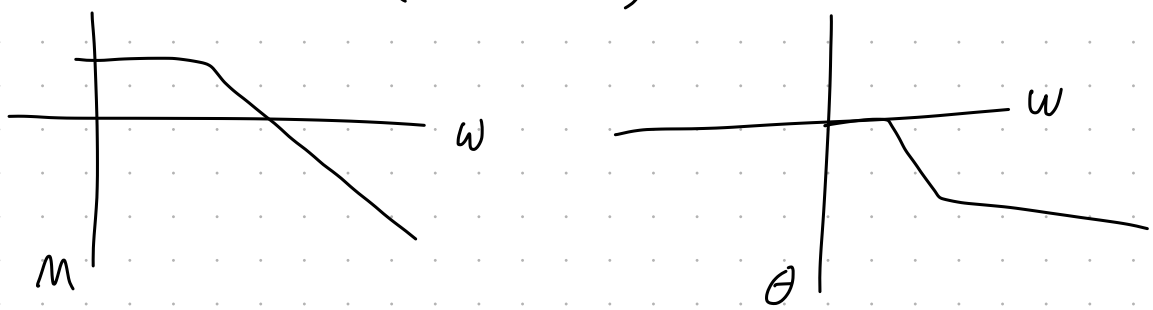
can make some general rules from this

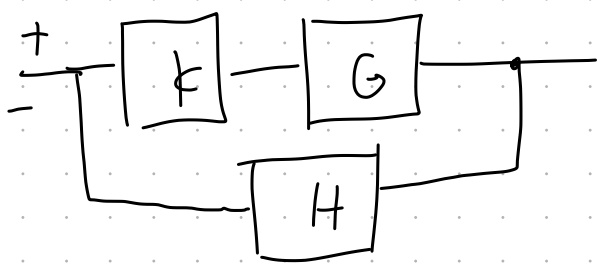
$\pm 90°$ over 2 decades

± 20 dB/dec starting @ a pole or zero

max θ change
M starts really changing } at $\omega \approx p$ or z

↳ this gives us a way to visualize magnitude and angle better
(Bode Plot)





$$T = \frac{KG}{1+KGH}\Big|_{s=j\omega} = M\angle\theta$$

↳ when is this unstable (without using matlab?)

↳ when denom $\to 0 \Leftrightarrow KGH \to -1$

$KGH = -1$

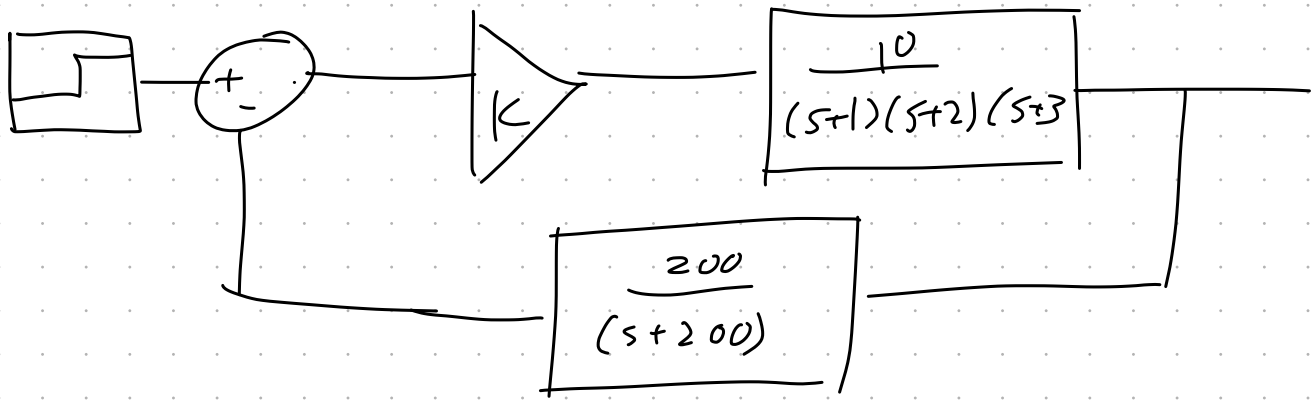
$= M\angle\theta$

$= 1\angle -180$

more unstable as:

$M \to 1$: Gain Margin

$\theta \to -180$: Phase Margin

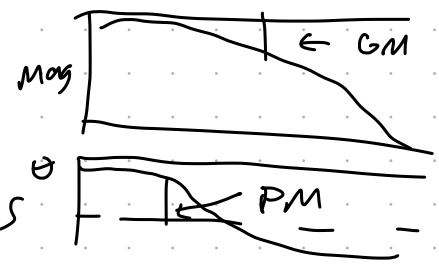


$$KGH = K\frac{2000}{(s+1)(s+2)(s+3)(s+200)}$$

see supplementary notes

↳ lines on bode plot show you gain margin and phase margin.

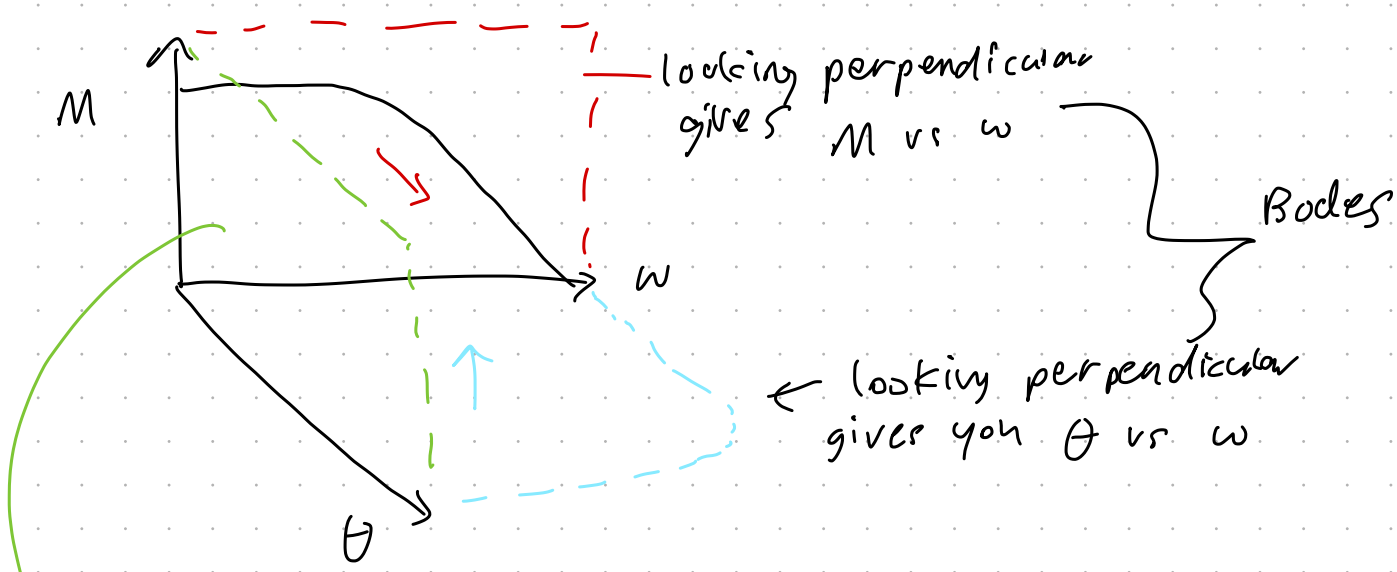
↳ can't get these from root-locus



Alternative

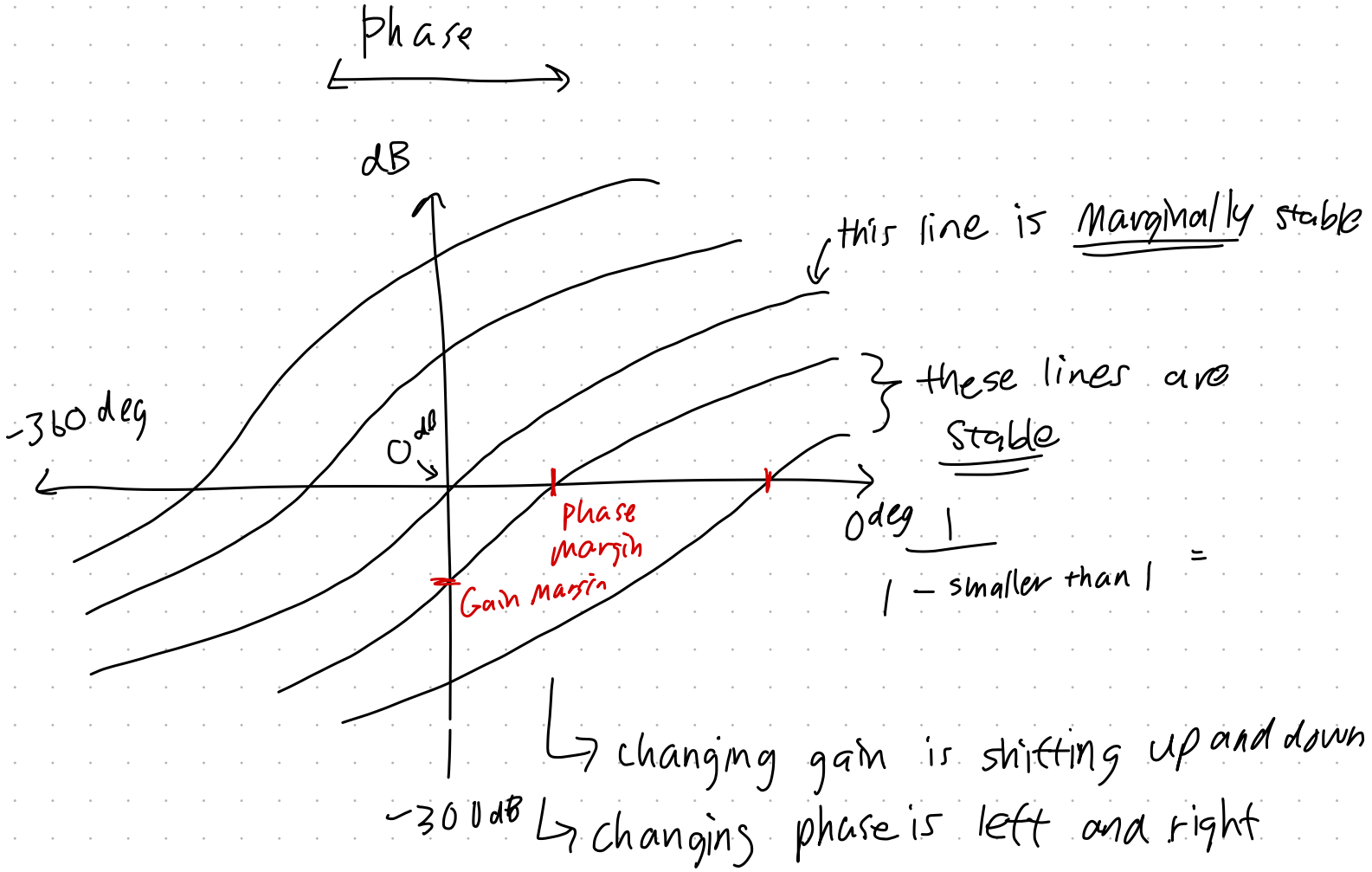
↳ Bode does M vs. ω θ vs. ω

↳ what if we did all 3.



Looking at this is new, you get M vs. θ

↓

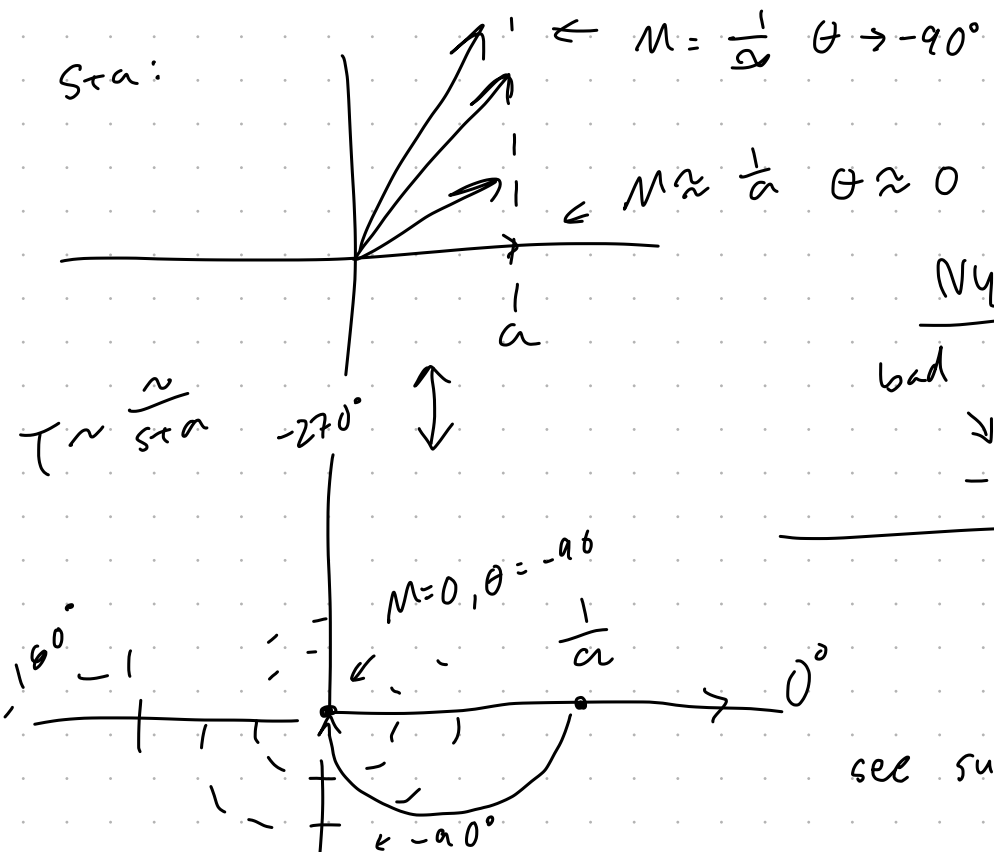


↳ changing gain is shifting up and down

↳ changing phase is left and right

↳ this is a Nichols Chart

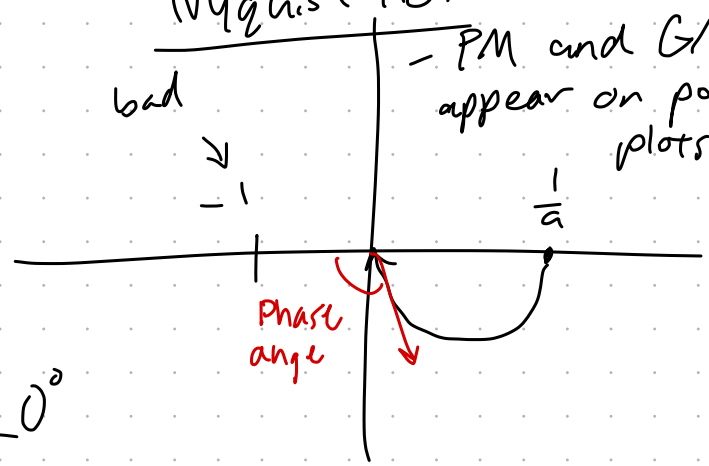
↳ why's the plot -360 → 0? surely there's a better way



→ why don't we do this in a polar plot?

Nyquist Plot

- PM and GM appear on polar plots too



see supplementary notes

OL $KGH = \dfrac{KG_N H_N}{G_{DD}}$

CL $= \dfrac{KG_N H_D}{KG_N H_N + G_D H_D}$

Plot:

$1 + KGH = \dfrac{G_D H_D + KG_N H_N}{G_D H_D}$ } CL Poles } OL Poles

Cauchy showed

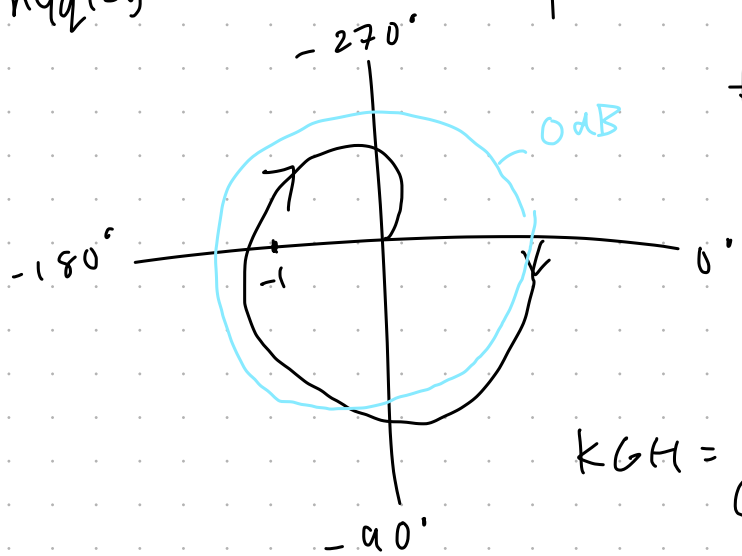
↳ $\boxed{\#\text{ CL Poles}}$ − # OL Poles

only thing we care about

= # Clockwise encircle of −1 on Nyquist → ω↑

(in Right Hand Plane)

nyqlog(100KGH)



#CL = # OL + # CW

if #CL > 0

#OL + #CW > 0 → we have instability

$KGH = \dfrac{200^2}{(s+1)(s+2)(s+3)(s+200)}$

↳ 0 poles on RHP

↳ 1 encirclement

↓

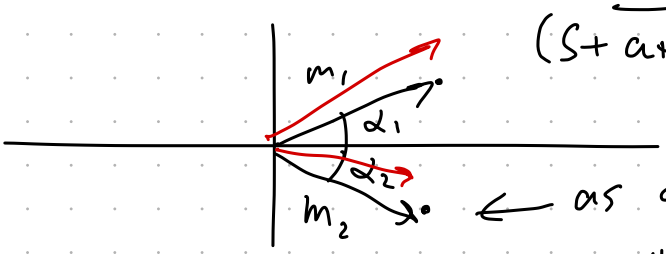
#CL = 0 + 1 → unstable gg

see supplementary notes

for $G = \dfrac{1}{s-2}$

Complex Poles

$\dfrac{1}{(s+a+jb)(s+a-jb)}$



← as α_1, α_2↑, m_1↑ m_2↓, kinda balances

m_2 still eventually grows

PD : $K \frac{2CF}{Z} \frac{(s+z)}{(s+2CF)}$

PI : $K \frac{1}{2} \frac{(s+2)}{s}$

PID : $K \frac{2CF}{Z_1 Z_2} \frac{(s+z_1)(s+z_2)}{s(s+2CF)}$

z_1, z_2, K, are design parameters

↓ certain choices of z_1, z_2 just yield PD or PI controllint

↳ deriv pole @ $-2CF$

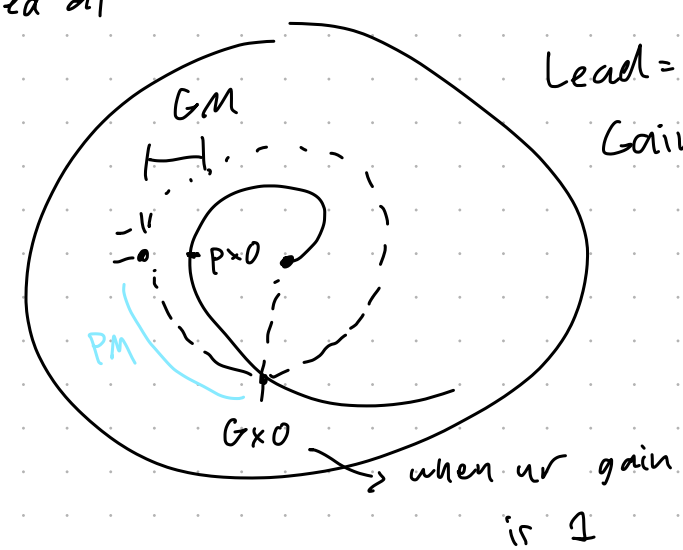
pole @ 0

Design Process:

- Calculate poles → just math
- Calculate zeros → position not clear
  - ↳ changes crossover point → changes stability
    - ↳ adjusts Px0, Gx0 (phase crossover, magnitude crossover)
      - ↓ Gives Gain Margin
      - ↓ Phase Margin

GM measured at Px0

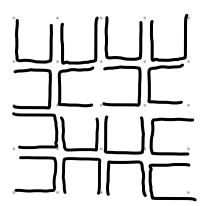
$\omega \to -180°$



→ when ur gain is 1

Lead = CCW

Gain = radial stretch



K (Gain) : Trade off

Z: affects gain/phase

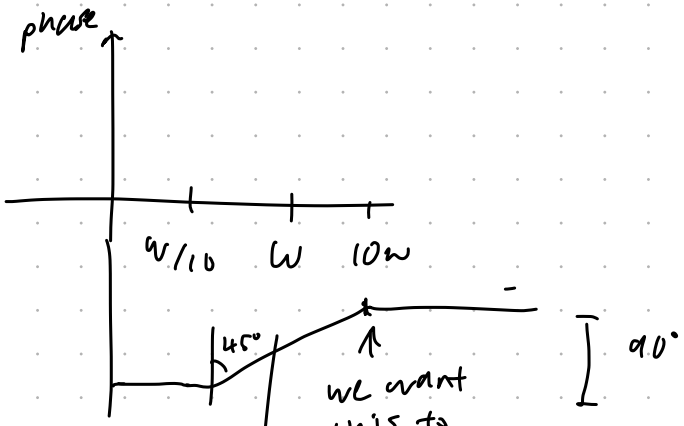
K: affects gain only

↳ do Z first

consider phase first and have shit gain

↳ adjust shit gain using K

Z must be < P×O as otherwise it doesn't do anything



∴

complex conjugat with no imaginary

→ put zero here

$P\times O = 10w$

↑ Z

$$\boxed{Z = \frac{P\times O}{10}}$$ (rule of thumb)

Now we have a zero → D

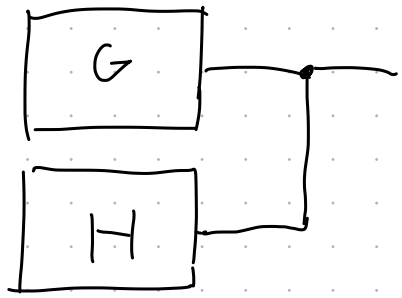
(Nyq (GDH) → zeros @ $\frac{P\times O}{10}$) ← ideal (adding zero in practice alters the plot)

↳ not perfect → need to iterate until close enough

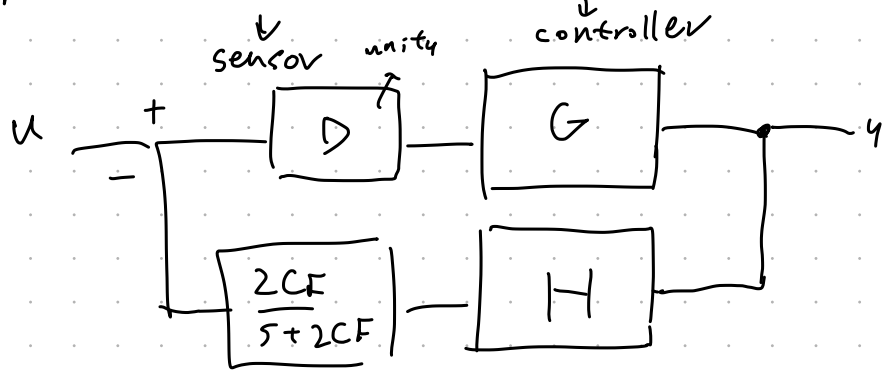
Now we have an "optimal" zero

Problem simplifiest to a gain choice

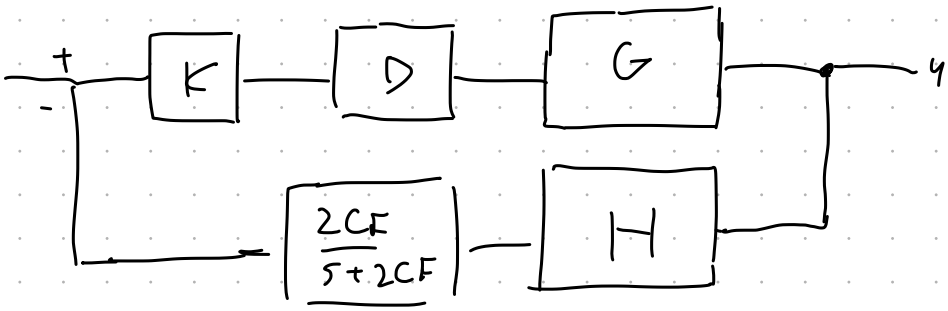
Step 1: System ID



Step 2: Add feedback and Dynamics



Step 3: Add Master gain



Step 4: Iterate

March 30

	P	PD	PI	PID
T_r	---	✓	X	----
T_s	----	✓	----	✓
e_{ss}	----	✓	✓✓	✓✓✓

Choose $C = KD$

find $z_0 \to D_0$

find K_0 "science"

Tune K_0, z_0 "art"

Tune K_p, K_i, K_d, K

Non-Lin

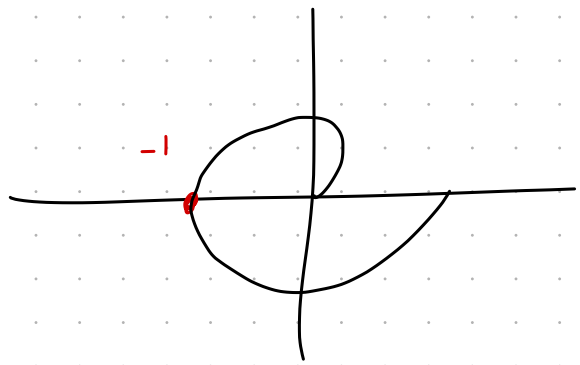
optional:

Choose $\mathcal{K} = \text{margin}(GH)$

$= K_u$ for P controller

nyqlog $(\mathcal{K} GH)$

↳ will touch -1



↳ $GM = 0$

$PM = 0$

↳ nice baseline

Calc D_0

↳ $\text{margin}(\mathcal{K} GH) \to z_1 \to D_1$ gives $P \times 0 \to z = \frac{P \times 0}{10}$

$\text{margin}(\mathcal{K} DGH) \to z_n \to D_N \to$ iterate until z stops changing

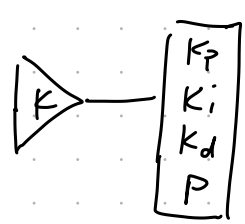
↳ $z_0 \to D_0$

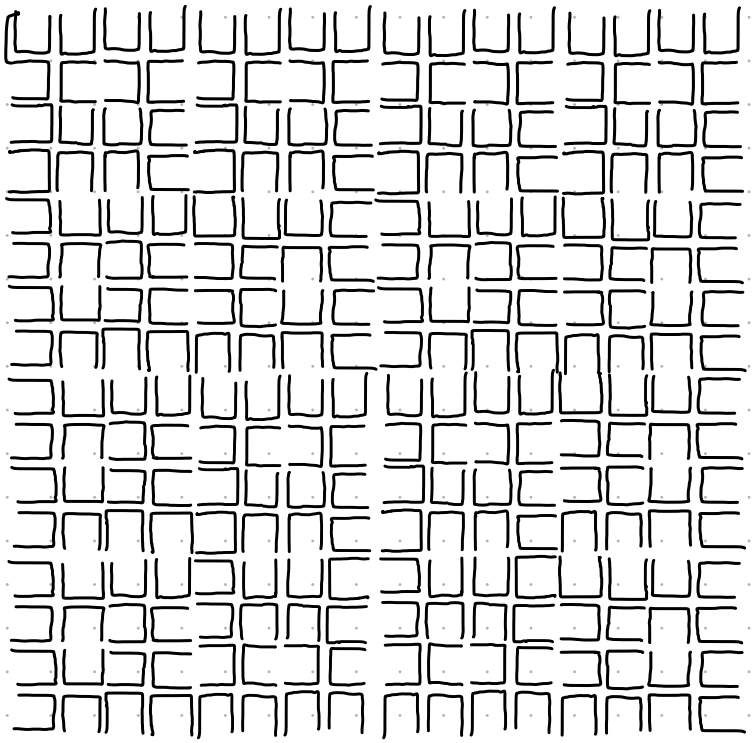
↳ compare with $\mathcal{K}$

$K, z \to K, K_p, K_i, K_d$

$\to K_i, K_p/K_i, 1, K_d/K_i \to$

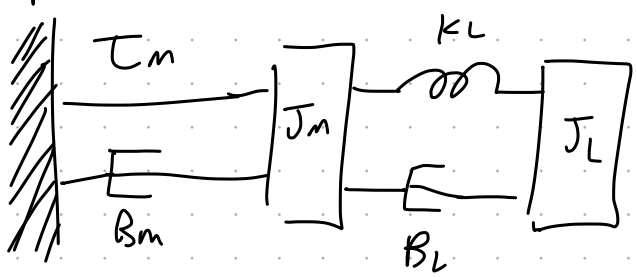
simulink



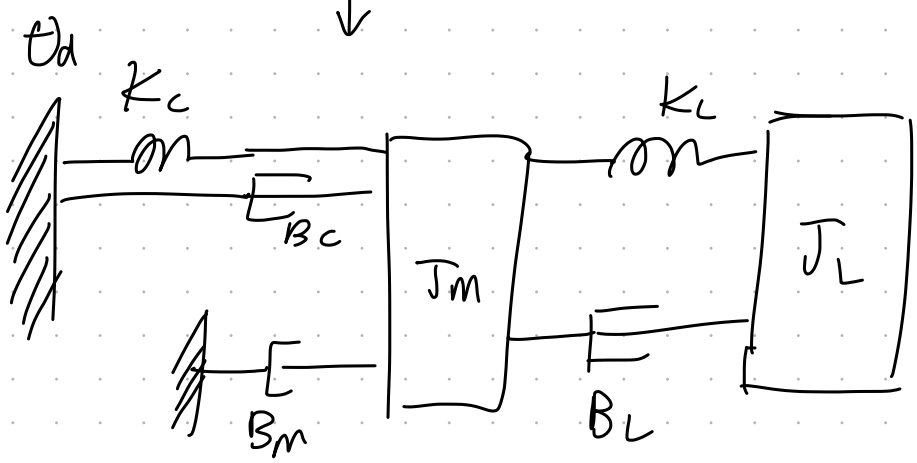


Impedance Control

Impedance control



- only torque is adjusted by controller



force is equivalent to controlling the position of the motor through a set of impedences

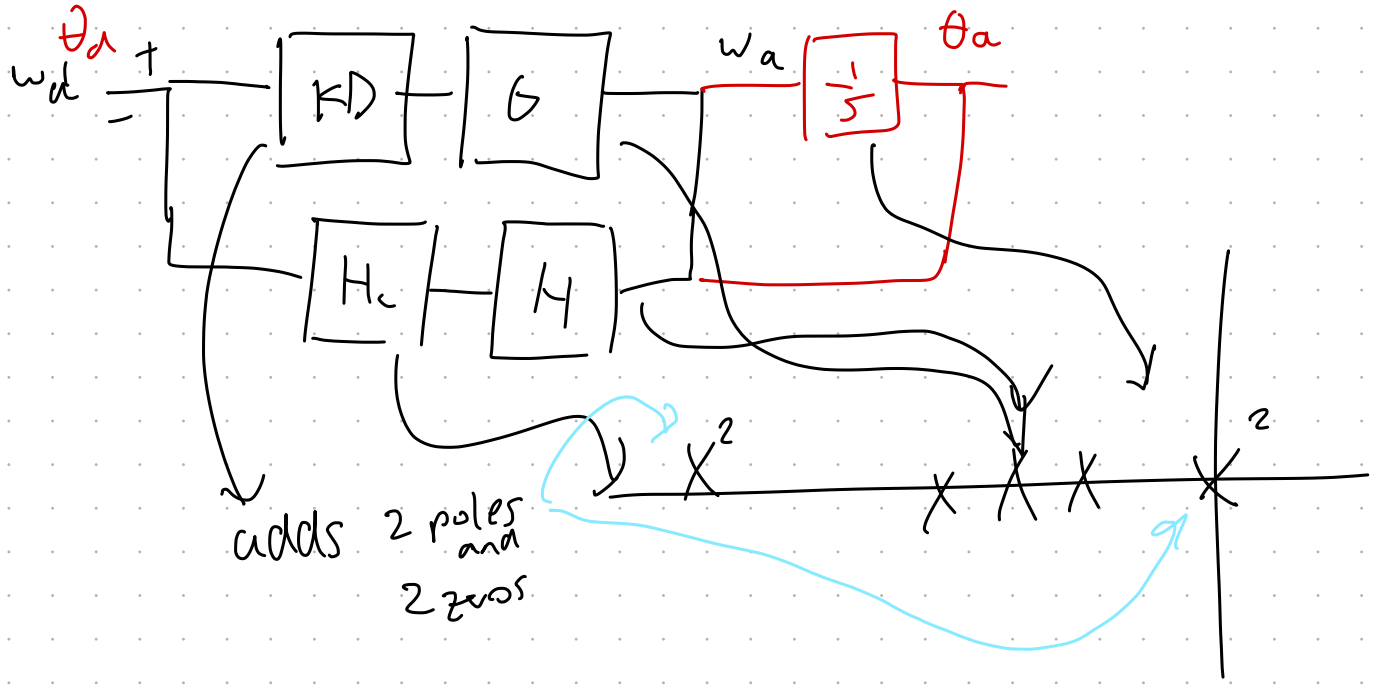
$$F = K_p(\theta_d - \theta_a) + K_d(\dot{\theta}_d - \dot{\theta}_a) = K_p e + K_d \dot{e}$$

$z = ?$

$$(s+z_1)(s+z_2) = s^2 + \frac{K_p p + K_i}{K_p + K_d p} s + \frac{K_i p}{K_p + K_d p}$$

$a = 1$ $\quad b = \downarrow$ $\quad c = \downarrow$

```
zeros = roots([1, b, c]);
```



April 4th

Heuristic Tuning

Z_0, K_0 : starting pt.

Z, K : Nyq, RL, SR (step response)

K, K_p, K_i, K_d : SR

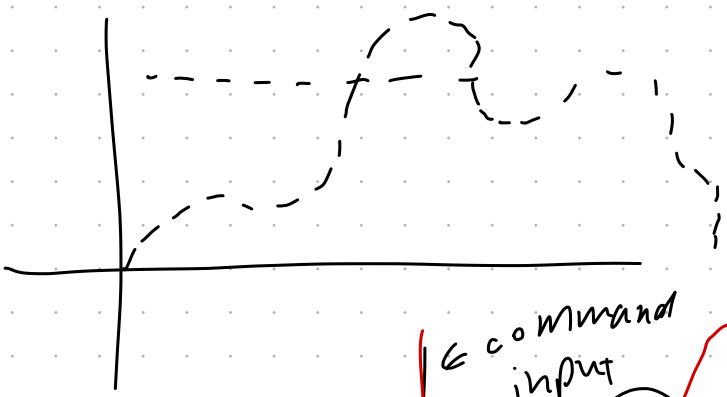
iterate

Final steps

specific requirements

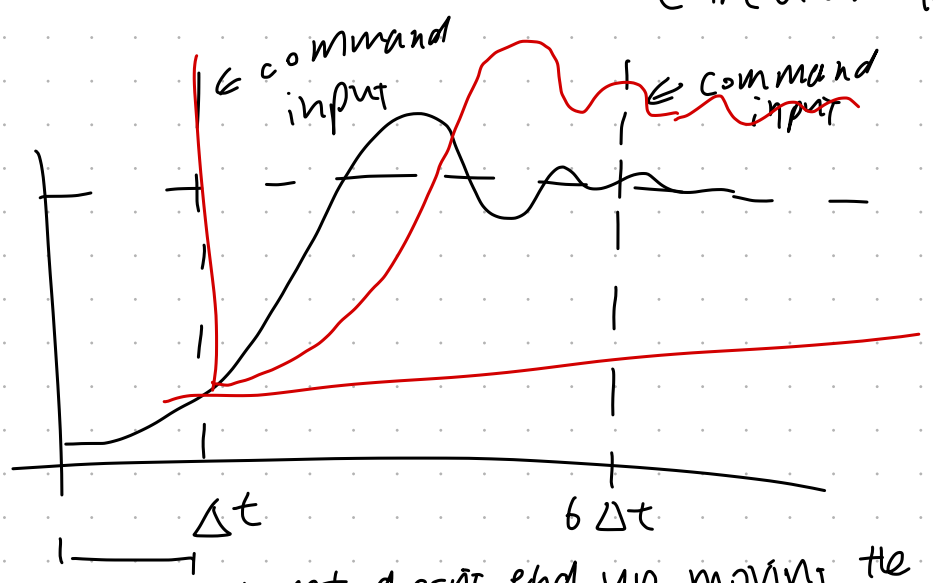
Non-Lin (Real)

Moving Targets



- Need T/E
- Controller depends on Path speed
- Path speed depends on C

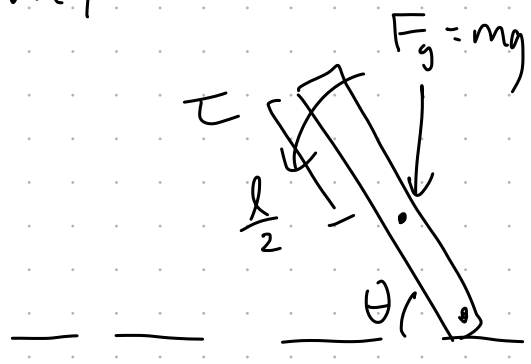
Circular Ref



fast movement doesn't end up moving the system in the correct place before moving to the next point

Non-Linearities ↓

Gravity



$\tau = \frac{l\,mg}{2}$

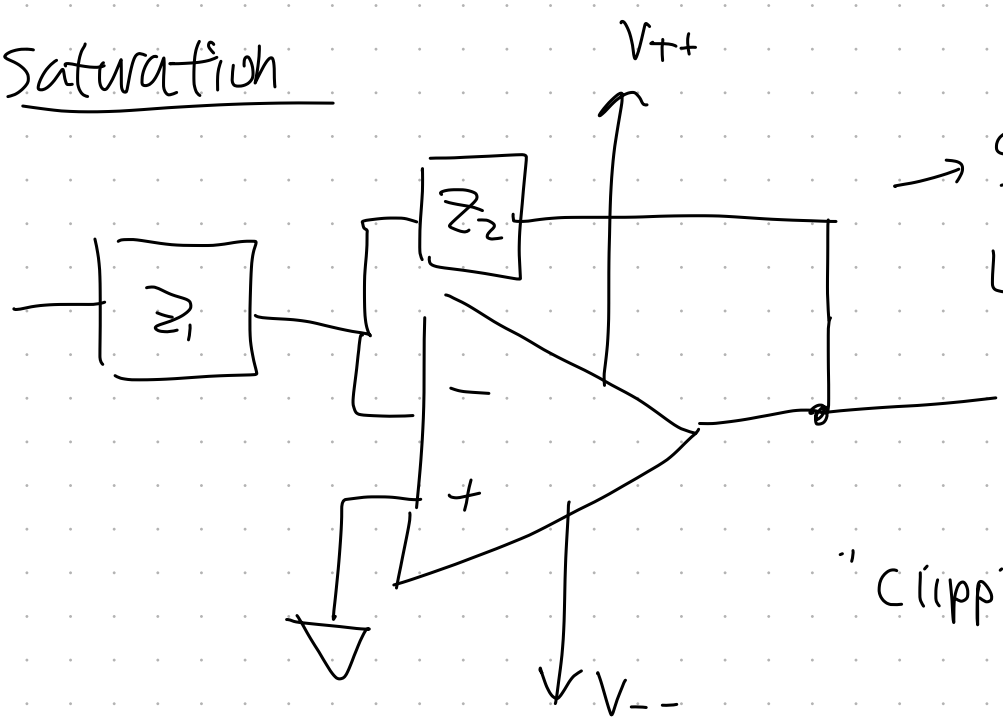
↳ torque depends on time
∴ system is not LTI
(not time invariant)

$\tau < 0 \quad \theta < 90°$
$\quad > 0 \quad \theta > 90°$

see supplemental notes

↳ K_i compensates for gravity cause gravity introduces steady-state error

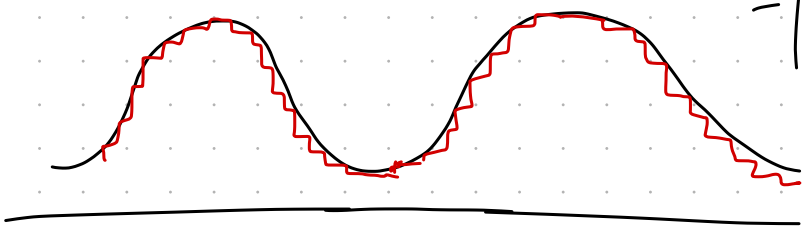
Saturation

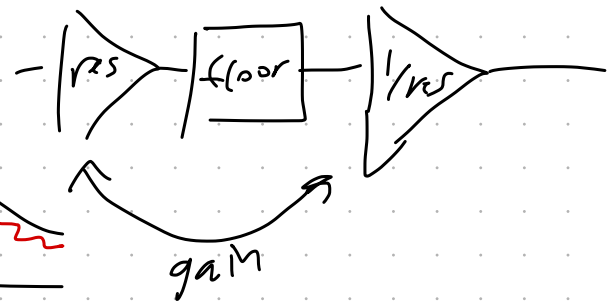


→ can't always get the gain you want
↳ controller ends up only "on" or "off" (bang-bang)
↓
"clipping" in output

↳ K (everything) is too high.
↳ not really tunable → becomes a design param.

Resolution





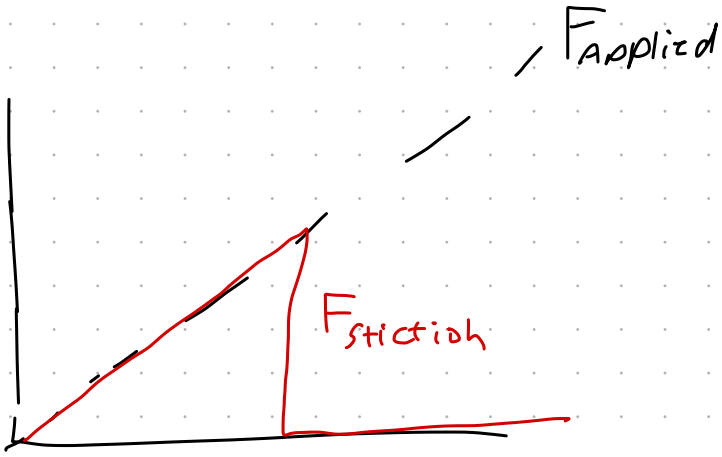
↳ add filter $\boxed{\frac{p}{s+p}}$ $p < 2CF$

↳ this is why bad motor drivers buzz so much

↳ good ones have better resolution

↑ just "some" filtering

Static Friction (stiction)



↳ not really something fixed in controls, need stuff like better components

SimulationX for modelling stuff

ELEC 341 – 2021 Winter Term

Final Exam
50 Marks

Each **BULLET POINT** is a **DELIVARABLE**

Figure 1

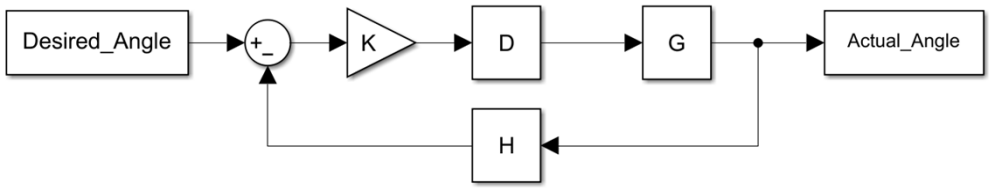


Figure 2

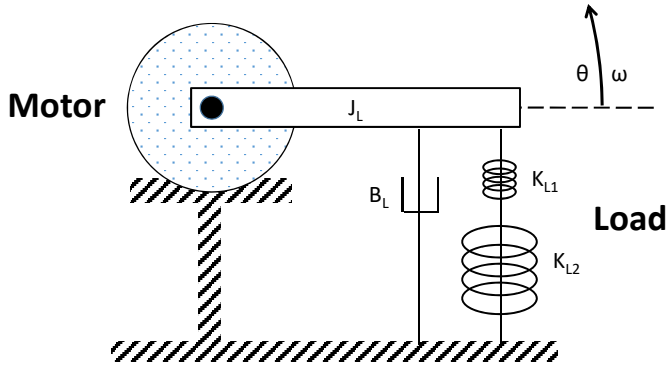


Table 1: Motor Specifications

Physical Parameter	Value	Physical Units
Nominal Voltage	12	V
No Load Current	250 + (10 x #A)	mA
Terminal Resistance	0.2	Ω
Terminal Inductance	0.5	mH
Rotor Inertia	100 + #B	gcm^2
Speed Constant	600 + (10 x #C)	RPM/V

Table 2: Micro-Controller Specifications

Physical Parameter	Value	Physical Units
Control Frequency	100 + (10 x #A)	Hz

Table 3: Load Specifications

Physical Parameter	Value	Physical Units
JL	#A + #B + #C	**μ**Nms^2/rad
BL	#D + #E + #F	**μ**Nms/rad
KL1	2	**m**Nm/rad
KL2	3 x #G	**m**Nm/rad

Table 4: Amplifier Specifications

Physical Parameter	Value	Physical Units
DC gain	#B	V/V
ωn	#A x #B x #C x #D	rad/s
zeta	#D / 10	pure

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A servo-motor is a closed-loop control system that tracks a desired angle, as shown in Figure 1.
The system G consists of a voltage amplifier, motor and load.
A SCHEMATIC of the motor and load is shown in Figure 2.
The motor, load, controller, and amplifier specifications are listed in Tables 1-4.
All load impedances are ROTATIONAL. Note the physical units in Table 3.

Use any method you choose to compute the following transfer functions.

- Voltage Amplifier (V/V)
- Motor and Load (rad/V)
- Micro-Controller DAQ (rad/rad)

*Use **checkGH()** to verify GH before continuing.*

Hint: It is always wise to annotate the signals in your block diagram with physical units.

Caution: There are no part marks for this question.
It must be correct before completing the remainder of this exam.
Once verified, you know you have 20/50 marks and don't have to check it again.

1. 20 mark(s) Step 1: System Identification

Compute the **OPEN-LOOP** transfer function GH and plot the Root Locus.

- GH (rad/V)
- Root Locus (figure)

Design a **PID** Controller.

2. 3 mark(s) Steps 2-4: Phase X-Over Frequency (PXO)

Compute your starting double-zero using PXO for a controller with no dynamics (D=1) and then iterate the controller dynamics until a stable double-zero is found.
The double-zero is stable when it stops changing to **2 SIGNIFICANT DIGITS**.
Combine **ALL DOUBLE-ZERO VALUES** into a vector **zvec**.
For double-zeros in the **Left-Half-Plane**, zvec should contain **NEGATIVE** numbers.

- zvec = vector of double-zeros (rad/s)

3. 3 mark(s) Step 5: Initial Gain (K1)

Find the initial gain K1 that places the double-zero at the Gain X-Over Frequency (GXO).

- K1 = initial gain of controller (V/rad)
- GXO = Gain X-Over Freq (rad/s)
- PXO = Phase X-Over Freq (rad/s)

4. 2 mark(s) Step 6: Check Result

Plot the **Root Locus** and **identify K1** on the roots that are heading toward the Right-Half-Plane.
Generate the **Nyquist Contour after applying K1**.

- Root Locus (figure)
- Nyquist Contour (figure)

5. 3 mark(s) Step 7: Step Response

Find the Optimal Gain **Kopt** that minimizes **Rise Time** with minimal increase in **Overshoot**, and **Settle Time**.
There is no single right answer to this question so make sure all of the above Goals are satisfied.
Plot on the same graph, the Closed-Loop Step Responses of **K1** and **Kopt**.

- Step Response (K1 & Kopt) (figure)

6. 2 mark(s) Step 7: PID Gains

Compute the PID gains you would use in a micro-controller that correspond to **Kopt**.

- K = Primary Gain
- Kp = Proportional Gain
- Ki = Integral Gain
- Kd = Derivative Gain

7. 5 mark(s) Step 8: Heuristic Tune

Perform Heuristic Tuning to improve your **Closed-Loop Step Response**.
Minimize **Rise Time**, **Overshoot** and **Settle Time** as much as possible.
Your curve **MUST** overshoot at least **2%** . Recall that over-damped curves have an INFINITE Rise Time.
Provide your gains after Heuristic Tuning.

- K = Primary Gain
- Kp = Proportional Gain
- Ki = Integral Gain
- Kd = Derivative Gain

8. 5 mark(s) Step 8: Heuristic Tune

Plot on the same graph, the Closed-Loop Step Responses, before and after Heuristic Tuning.
Include a **Legend**.

- Step Response (BEFORE & AFTER) (figure)

9. 2 mark(s) Effective Zeros

Compute the two zeros resulting from Heuristic Tuning.
For zeros in the Left-Half-Plane, zvec should contain **NEGATIVE** numbers.
For a double-zero, zvec should contain two identical real numbers.
For complex zeros, zvec should contain complex conjugates.

- zvec = vector of two zeros (rad/s)

10. 5 mark(s) 2nd Order Approximation

Use **Overshoot** & **Rise Time T_R** (not T_{R1}) to compute a 2nd Order Approximation of your Heuristic Tuned response.
Plot both (Heuristic Tuned & 2nd Order Approximation) on the same graph.
Show 0.5 seconds of data.

- Step Response (TUNED & APPROX) (figure)

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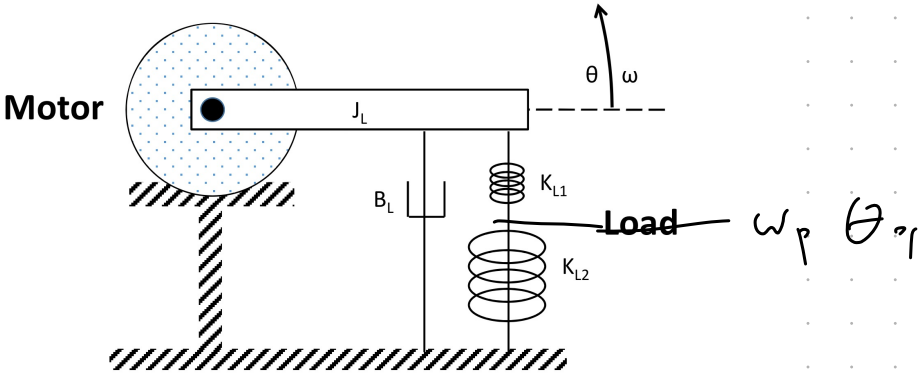

$$\dot{\bar{X}} = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{U}$$

inputs

$$\begin{bmatrix} \dot{\omega} \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} \omega \\ \theta \end{bmatrix} + \begin{bmatrix} & \\ & \end{bmatrix} \tau_{motor}$$

τ_{L1}

τ_{L2}

$$\tau_{L1} + \tau_{L2} = 0$$

$$K_1(\theta - \theta_1) - K_2\theta_1 = 0$$

$$\tau_{motor} - \tau_{B_L} - \tau_{K_{L1}} = J_L\dot{\omega}$$

$$\tau_{motor} - B(\theta) - K(\theta - \theta_1)$$