Market-Neutral Volatility Harvesting (MNVH): A Discrete Self-Financing Construction Unifying AMMs, HFT Market Making, Options, and SPT

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Abstract

We propose *Market-Neutral Volatility Harvesting* (MNVH) as a unifying principle behind seemingly distinct mechanisms: constant-function automated market makers (AMMs), high-frequency market making (HFT/MM), delta-hedged options (gamma scalping), and Stochastic Portfolio Theory (SPT). The core is an exact discrete self-financing P&L identity that separates first-order (directional) exposure from a second-order volatility-driven component. Under market neutrality and contrarian rebalancing, quadratic price variation becomes a systematic source of expected P&L. We show how AMMs, HFT/MM, options hedging, and SPT arise as constrained or limiting cases of the same MNVH functional, and we discuss practical bounds due to transaction costs, adverse selection, inventory risk, and capacity limits.

1 Introduction

Volatility is often treated as risk to be minimized, yet many trading mechanisms systematically monetize it without directional prediction. This paper formalizes a discrete-time principle—*Market-Neutral Volatility Harvesting* (MNVH)—that explains how self-financing, contrarian rebalancing can transform quadratic variation into expected P&L under neutrality constraints. The same structural mechanism reappears across domains with different vocabularies: *fees* in AMMs, *spread capture* in market making, *gamma* in options, and *excess growth* in SPT. Our contribution is to place these mechanisms in a single discrete framework suitable for implementation.

2 Discrete Self-Financing Decomposition and Market-Neutral Volatility Harvesting

2.1 Discrete-time setup

We consider discrete time $t = 0, 1, 2, \ldots$. Let $p_t \in \mathbb{R}^n$ be the vector of tradable prices and $q_t \in \mathbb{R}^n$ the holdings *just before* rebalancing at time t. Define $\Delta q_t := q_{t+1} - q_t$ and $\Delta p_t := p_{t+1} - p_t$. The net asset value (NAV) is

$$\mathrm{NAV}_t := q_t \cdot p_t.$$

We assume self-financing trading, excluding explicit transaction costs for now.

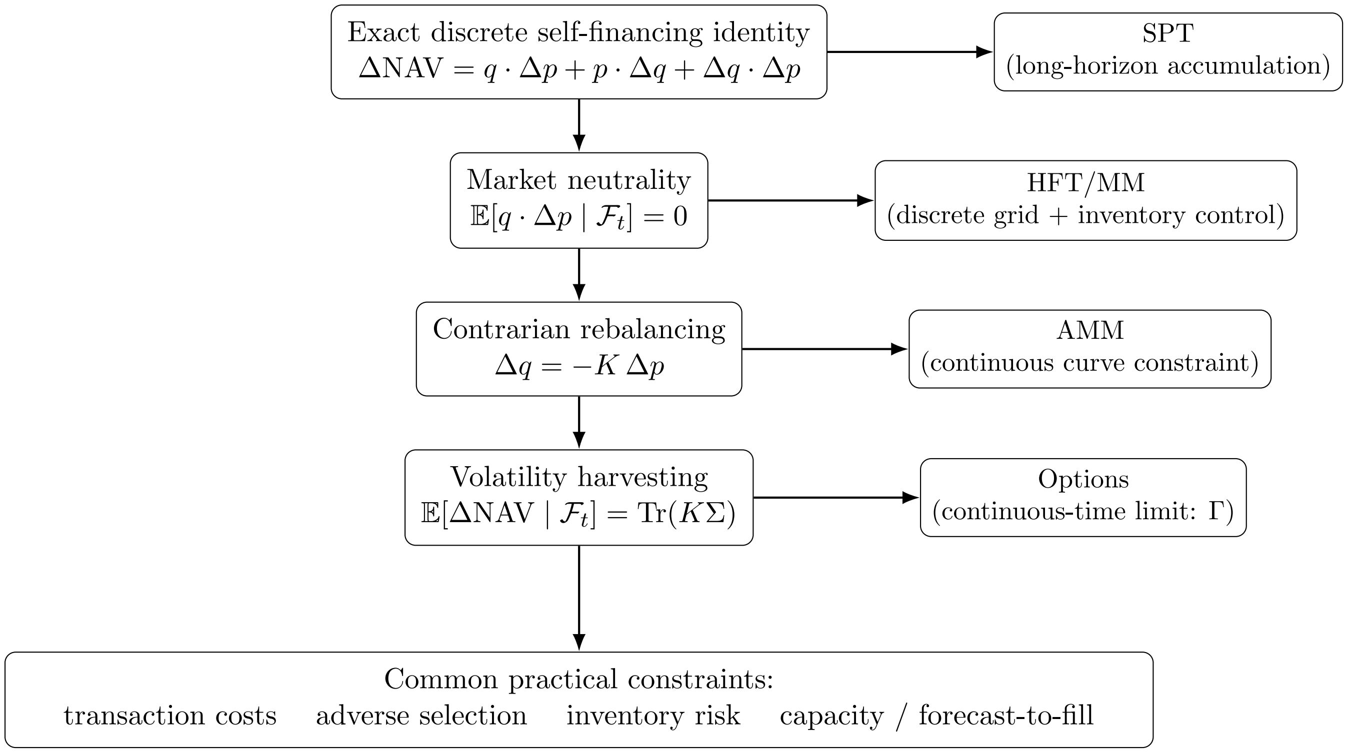

Figure 1: MNVH as a unifying principle across mechanisms and time scales. AMMs, HFT/MM, delta-hedged options, and SPT emerge as constrained or limiting cases of the same discrete volatility-harvesting functional. Practical frictions bound but do not negate the mechanism.

2.2 Exact discrete P&L identity

The one-step change in NAV admits the exact decomposition

$$\Delta\mathrm{NAV}_t := \mathrm{NAV}_{t+1} - \mathrm{NAV}_t = q_t \cdot \Delta p_t + p_t \cdot \Delta q_t + \Delta q_t \cdot \Delta p_t.$$

The first term is directional exposure, the second is rebalancing cashflow, and the third is a second-order interaction term that becomes central in discrete trading.

2.3 Market-neutral constraint

We impose neutrality via

$$Bq_t = 0,$$

where $B \in \mathbb{R}^{k \times n}$ encodes factor exposures (e.g. dollar-neutrality, beta-neutrality, multi-factor neutrality). Under standard assumptions,

$$\mathbb{E}[q_t \cdot \Delta p_t \mid \mathcal{F}_t] = 0,$$

so systematic growth must arise from non-directional components.

2.4 Self-financing contrarian rebalancing

We consider trading rules

$$\Delta q_t = -K_t\,\Delta p_t,$$

where $K_t \succeq 0$ is adapted to $\mathcal{F}_t$ and controls aggressiveness, risk, and capacity. This includes grid-based market making, AMM-style rebalancing, and gamma scalping limits.

2.5 Emergence of volatility harvesting

Substituting the rebalancing rule into the identity yields

$$\Delta\mathrm{NAV}_t = q_t \cdot \Delta p_t - p_t \cdot K_t \Delta p_t - \Delta p_t^\top K_t \Delta p_t.$$

Under neutrality, the expected directional term vanishes and the second-order term dominates:

$$\mathbb{E}[\Delta\mathrm{NAV}_t \mid \mathcal{F}_t] = \mathrm{Tr}(K_t \Sigma_t), \quad \Sigma_t := \mathbb{E}[\Delta p_t \, \Delta p_t^\top \mid \mathcal{F}_t].$$

Definition 1 (MNVH functional)**.** *We define the* MNVH functional *as*

$$\mathcal{H}_t(K) := \mathbb{E}[\Delta\mathrm{NAV}_t \mid \mathcal{F}_t] = \mathrm{Tr}(K_t \Sigma_t),$$

to be maximized subject to inventory, cost, and capacity constraints.

3 Automated Market Makers as Constrained MNVH Systems

3.1 AMM as a self-financing rebalancing mechanism

Consider a two-asset AMM with reserves (x_t, y_t) and invariant $\Phi(x_t, y_t) = k$ where Φ is strictly convex. Trades are self-financing moves along the constraint manifold $\Phi(x, y) = k$.

3.2 Constant-product AMM as contrarian rebalancing

For the constant-product AMM, $\Phi(x, y) = xy$, so $xy = k$ and the implied marginal price is $p = y/x$. When the external market price increases, arbitrage induces the AMM to sell the appreciating asset and buy the depreciating one. Thus AMMs implement continuous contrarian rebalancing under a geometric constraint.

3.3 Impermanent loss as directional component; fees as volatility harvesting

Impermanent loss corresponds to the directional cost of contrarian rebalancing when volatility is absent or insufficient. Trading fees scale with turnover induced by price variation and therefore with realized volatility. In MNVH terms, the AMM curvature induces an effective K_t under the constraint $\Phi(x, y) = k$.

4 High-Frequency Market Making as Discrete MNVH with Inventory Control

4.1 Abstract view: discrete MM as MNVH with inventory penalties

Market making executes piecewise-constant contrarian trades on an order book. A generic model is

$$\Delta q_t = -K_t \Delta p_t + u_t,$$

where $K_t \succeq 0$ captures passive contrarian fills (spread capture) and u_t is an inventory-control term.

4.2 Inventory-controlled objective

A canonical objective is

$$\max_{K_t,u_t} \operatorname{Tr}(K_t\Sigma_t) - \lambda\,\mathbb{E}[\|q_t\|^2] - (\text{costs}),$$

where $\lambda > 0$ controls inventory risk.

4.3 Concrete view: Avellaneda–Stoikov as MNVH

In a standard single-asset model with mid-price volatility σ, optimal quotes decompose into a symmetric spread component (volatility harvesting) and an asymmetric inventory component (mean reversion in q_t). This yields an effective rule

$$\Delta q_t \approx -K\,\Delta S_t - \beta q_t,$$

aligning with the MNVH form.

5 Delta-Hedged Options and Gamma Scalping as the Continuous Limit of MNVH

5.1 Delta-neutral hedging and quadratic variation

For an option value $V(t,S)$, the delta-hedged portfolio $\Pi_t = V - \Delta S$ with $\Delta = \partial_S V$ satisfies

$$d\Pi_t = \tfrac{1}{2}\Gamma_t\,d\langle S\rangle_t, \quad \Gamma_t = \partial_{SS}V.$$

Thus P&L is driven purely by quadratic variation. In the continuous limit, gamma plays the role of K.

5.2 Discrete hedging, costs, and capacity

With discrete hedging, costs create an interior optimal rehedge intensity and finite effective K, directly analogous to MM inventory and capacity constraints.

6 Stochastic Portfolio Theory as Long-Horizon MNVH

6.1 Excess growth as integrated volatility harvesting

In SPT, the log-relative performance admits a decomposition with an *excess growth rate* term depending only on covariance. This term is the long-horizon accumulation of the MNVH volatility-harvesting component.

6.2 Functionally generated portfolios as structured rebalancers

Functionally generated portfolios rebalance by selling relative winners and buying relative losers, implementing persistent contrarian rebalancing on market weights. MNVH provides a micro-to-macro bridge: short-horizon harvesting integrates into deterministic long-horizon growth.

7 Practical Constraints on MNVH

7.1 Transaction costs, adverse selection, and inventory risk

In practice, $\mathcal{H}_t(K)$ is opposed by costs, adverse selection, and inventory risk. Costs scale roughly linearly with turnover, while harvesting scales with quadratic variation, implying an interior optimal intensity.

7.2 Capacity and forecast-to-fill limitation

Let $F_t(K)$ be expected fill probability for aggressiveness K. Realized harvesting behaves like

$$\mathbb{E}[\Delta \mathrm{NAV}_t] \approx F_t(K)\,\mathrm{Tr}(K_t \Sigma_t) - \text{costs},$$

so diminishing fills and queue competition bound scalability (forecast-to-fill constraints).

7.3 Adaptive control via $R - r$ signals

The MNVH functional explains *where* returns come from; practical strategy design requires deciding *when* and *how much* to harvest. We introduce a control signal $R_t - r_t$ where:

- R_t is a realized variability measure (e.g. realized variance/covariance over a window),
- r_t is a reference level (e.g. implied variance, fee-implied "required" variance, or a long-run baseline).

We then adapt harvesting intensity by

$$K_t = g(R_t - r_t),$$

with $g(\cdot)$ increasing and bounded to ensure stability under costs and capacity. This unifies common control variables across domains: realized vs implied variance in options, spread adequacy in market making, fee-vs-IL adequacy in AMMs, and covariance-driven excess growth in SPT.

8 Illustrative Simulations (Minimal)

We simulate driftless price increments $\Delta p_t \sim \mathcal{N}(0, \Sigma)$ to isolate volatility-driven effects. Across discrete MNVH, AMM-style constrained harvesting, inventory-penalized MM-style harvesting, and discrete gamma-scalping limits, we observe: (i) near-zero directional exposure under neutrality; (ii) expected NAV growth proportional to realized variance for moderate intensity; (iii) diminishing returns under costs, adverse selection proxies, and fill saturation, producing an interior optimal intensity.

9 Discussion

MNVH was not previously unified due to fragmentation across communities, divergent terminology (fees/spread/gamma/excess growth), and continuous-time biases that obscure discrete interaction terms. MNVH adds a single discrete P&L identity, a design-oriented harvesting functional, and a micro-to-macro bridge from HFT to SPT. Limitations include estimation error in Σ_t, simplified cost models, and absence of strategic interaction; extensions include multi-factor neutrality, endogenous liquidity response, and adaptive covariance estimation.

10 Conclusion

We presented Market-Neutral Volatility Harvesting (MNVH) as a discrete self-financing principle that converts quadratic price variation into expected P&L under neutrality and contrarian rebalancing. AMMs, HFT market making, delta-hedged options, and SPT arise as constrained or limiting cases of the same functional. Practical frictions bound but do not negate the mechanism. MNVH reframes volatility as a tradable structural resource recurring across markets and time scales.