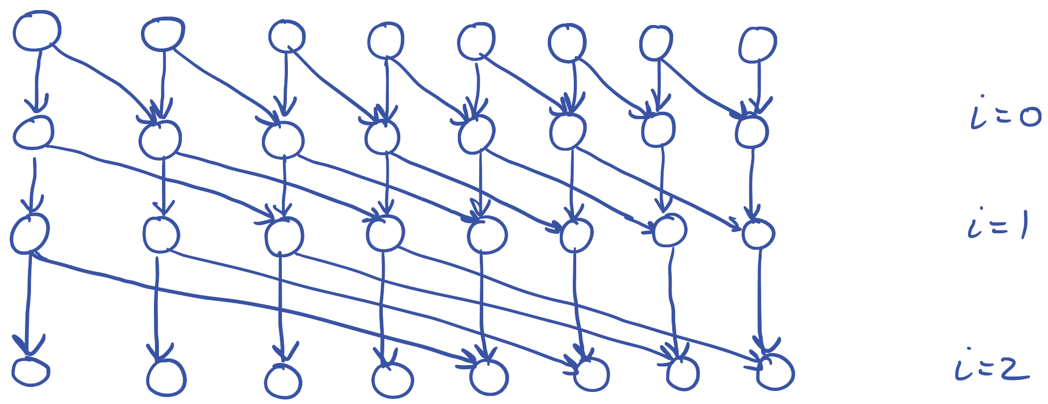
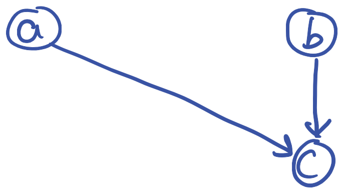
Aakash 2018B4A70887P PC- Assignment 1

Ans 1 (a) Task graph for Hillis algorithm:
(n=8)

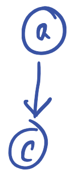


where



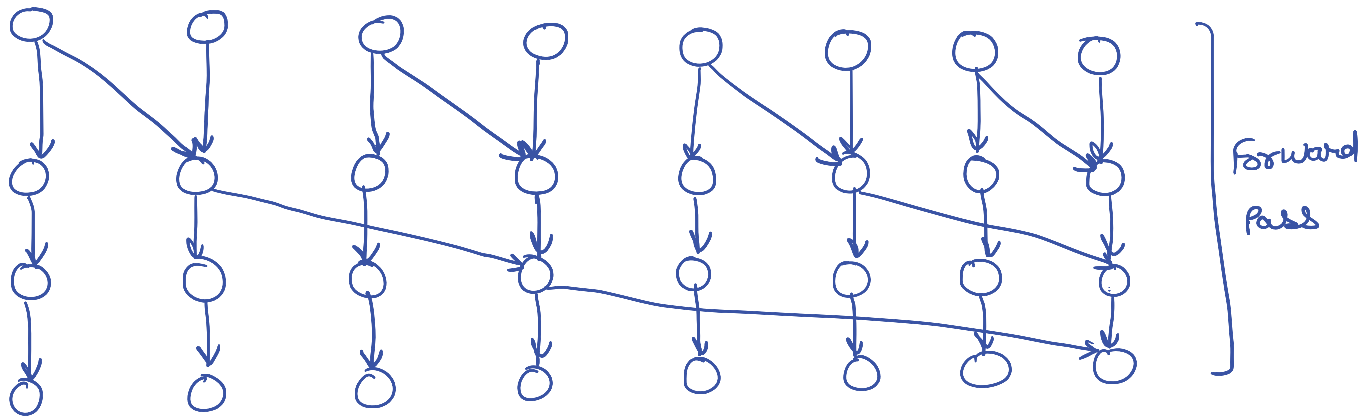
means $c = a + b$

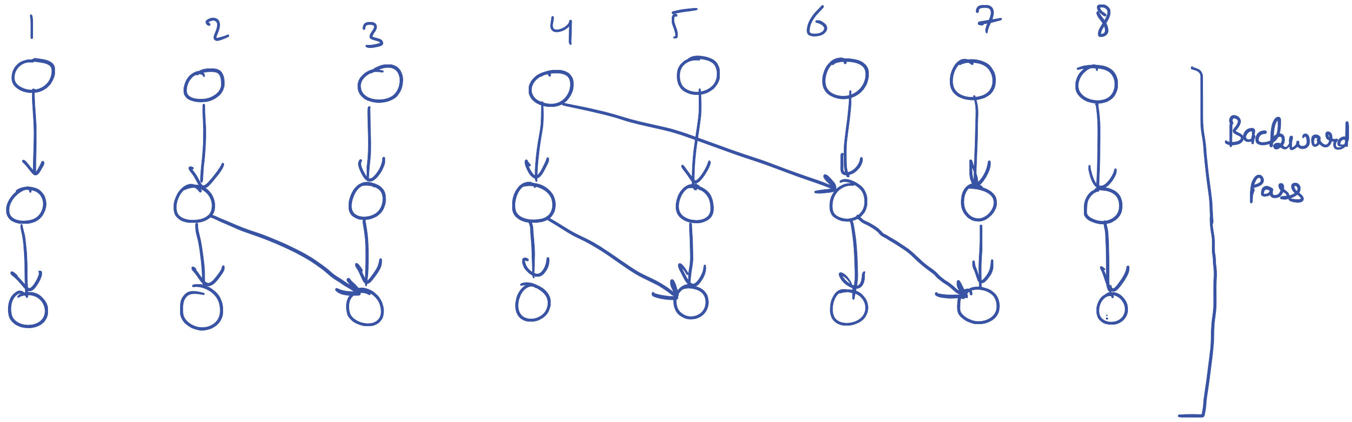
and



means $c = a$

b) Task graph for Blelloch algorithm:
(n=8)





Degree of concurrency: max no. of threads running in parallel
→ specified by user

In practice, array size $= n$ means max n threads in parallel.

Speedup is calculated later.

Ans 2

Algorithm (a) is not speed optimal.

Proof: no. of parallel iterations: $\lceil \log_2 n \rceil$

In each iteration $i = 0$ to $\lceil \log_2 n \rceil - 1$, no. of working threads $= n - 2^i$

$$\Rightarrow \text{Total computations} = \sum_{i=0}^{\lceil \log_2 n \rceil - 1} n - 2^i$$

$$= n \lceil \log_2 n \rceil - (2^{\lceil \log_2 n \rceil} - 1)$$

$$= O(n \log_2 n)$$

whereas we can compute prefix sum in $O(n)$ in single thread.

Algorithm (b) is cost optimal.

Proof: There are two passes of algorithm and no. of computations in second pass are same as of first pass.

Therefore, cost of algo $= 2 \times$ cost of first pass.

For first pass, we have $i = 1$ to $\lceil \log_2 n \rceil$.

For each i, no. of computations $= \frac{n}{2^i}$

$$\Rightarrow \text{Total} = 2 \times \sum_{i=1}^{\lceil \log_2 n \rceil} \frac{n}{2^i} = 2n \left[\frac{1 - \frac{1}{2^{\lceil \log_2 n \rceil}}}{1 - \frac{1}{2}} \right]$$

$$= 2n \left[1 - \frac{1}{2^{\lceil \log_2 n \rceil}} \right]$$

$= O(n)$ → cost optimal.

Ans 3 According to Brent's theorem,

let T_1 = Time taken by single processor.

T_p = ——— p processors.

T_∞ = depth of computation of DAG.

Then $$\frac{T_1}{p} \le T_p \le \frac{T_1}{p} + T_\infty$$

For Hillis / Blelloch algo,

$T_1 = n$

T_p = ① no. of computations/thread $\sim \frac{n}{p}$

② depth of tree $\sim \lceil \log_2 n \rceil$

$\Rightarrow$ computation time $\sim \frac{n \lceil \log_2 n \rceil}{p}$

$T_\infty \sim \lceil \log_2 n \rceil$

$$\Rightarrow \underbrace{\frac{n}{p} \le \frac{n\lceil \log_2 n \rceil}{p}}_{\text{Trivial}} \le \frac{n}{p} + \lceil \log_2 n \rceil$$

$$\Downarrow$$

$$\frac{n \log_2 n}{p} \le \frac{n}{p} + \log_2 n$$

$$\Leftrightarrow \frac{n}{p}\left[\log n - 1\right] \le \log n.$$

$$\Rightarrow \boxed{p \ge \frac{n\left(\log n - 1\right)}{\log n}}$$

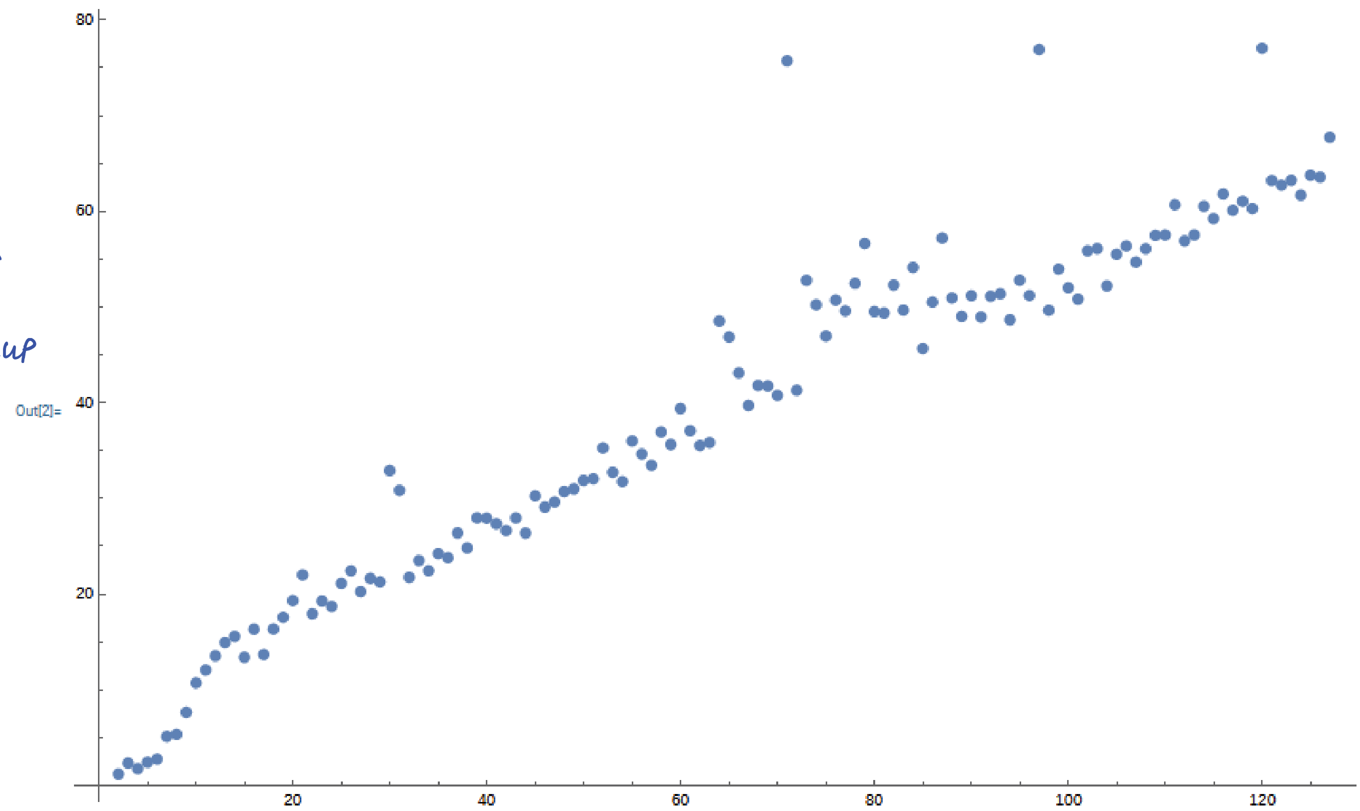
Ans 4 We know that maximum speedup from Amdahl's law is given by

$$\frac{T_1}{T_\infty}$$

$T_1 = n$
$T_\infty = \log n$ $\Rightarrow$ max speedup $= \dfrac{n}{\log n}$

Ans 5 Output of code for $n = 128$ and speedup:

speedup ↑



num_threads →

text file for speedup values is attached in submission.