CC Title

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Outline

- Intro and Motivation
- Model
- Recreate Dr. Phillips' Application
- Our Application

Introduction

Hypotheses:

- Family researchers comparing the attitudes, behaviors, and opinions of pairs
- Collect Dyadic data
  - Inter-individual reporting
  - Intra-individual reporting

Motivation

- Difference scores used to analyze dyadic data
- Difference scores allow to see how well they “fit” together
- Common types: algebraic, absolute, and squared difference

$$Z = \beta_0 + \beta_1(X - Y) + \epsilon \quad (1)$$

$$Z = \beta_0 + \beta_1|X - Y| + \epsilon \quad (2)$$

$$Z = \beta_0 + \beta_1(X - Y)^2 + \epsilon \quad (3)$$

Motivation

- Many methodological issues with Difference Scores
  1. Difficult to identify the underlying mechanism
  2. Problems with underlying assumptions

Motivation

- Any alternatives?
- Polynomial Regression
  e.g.

$$Z = \beta_0 + \beta_1 X + \beta_2 Y + \beta_3 X^2 + \beta_4 Y^2 + \beta_5 XY + \epsilon \tag{4}$$

- Take the Squared Difference and expand it

$$(X - Y)^2 = X^2 + Y^2 - 2XY \tag{5}$$

- Expand on the ideas from Simple Linear Regression

The Model

- Theoretical model:

$$Z = \beta_0 + \beta_1 X + \beta_2 Y + \beta_3 X^2 + \beta_4 Y^2 + \beta_5 XY + \epsilon \tag{6}$$

- Fitted model:

$$\hat{z} = b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_5 y^2 \tag{7}$$

- Fitted model in matrix notation:

$$\hat{z} = b_0 + \mathbf{d}^{'}\mathbf{b} + \mathbf{d}^{'}\mathbf{B}\mathbf{d} \tag{8}$$

where

$$\mathbf{d} = \begin{bmatrix} x \\ y \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} b_3 & b_4/2 \\ b_4/2 & b_5 \end{bmatrix}$$

Model

We can fit a model like this in R using the `lm()` function:

```
# Z - the response variable
# X - the first explanatory variable
# Y - the second explanatory variable
QuadFit <- lm(z ~ x + y + I(x^2) + I(x*y) + I(y^2), data=d)
```

Stationarity Points: How & Why?

- What are they?
  - Points where the slope is zero no matter which direction you take the derivative
- Values of our explanatory variables provide the "best" fit for the response
- How do you derive the stationary points?
  1. Take the derivatives of Equation 7 with respect to x and y
  2. Set the derivatives equal to zero
  3. Solve for x and y in terms of $\mathbf{b}$ and $\mathbf{B}$ to find the stationarity points
  4. Refer to these points as x_0 and y_0

Stationarity Points: How?

- Take the partial derivatives with respect to x and y

$$\begin{bmatrix} \frac{dz}{dx} = b_1 + 2b_3x + b_4y \\ \frac{dz}{dy} = b_2 + b_4x + 2b_5y \end{bmatrix} = \mathbf{b} + 2\mathbf{Bd}$$

- Set the derivatives to zero and solve for $\mathbf{d_0}$

$$\begin{aligned}
\mathbf{d_0} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} &= -\frac{\mathbf{B}^{-1}\mathbf{b}}{2} \\
&= -\frac{1}{2}\frac{1}{b_5b_3 - \frac{b_4^2}{2}} \begin{bmatrix} b_5 & -b_4/2 \\ -b_4/2 & b_3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \\
&= -\frac{2}{4b_5b_3 - b_4^2} \begin{bmatrix} b_5b_1 - b_2b_4/2 \\ -b_1b_4/2 + b_2b_3 \end{bmatrix} \\
&= \begin{bmatrix} \frac{b_2b_4 - 2b_5b_1}{4b_5b_3 - b_4^2} \\ \frac{b_1b_4 + 2b_2b_3}{4b_5b_3 - b_4^2} \end{bmatrix}
\end{aligned}$$