Bayesian Hierarchical Models and Influenza Modeling

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- Approx $4,000,000$ cases annually result in severe illness
- Approx $400,000$ annual deaths
- Causes a significant economic and resource burden on the healthcare system
- Vaccines are a simple and effective way of preventing the spread of influenza

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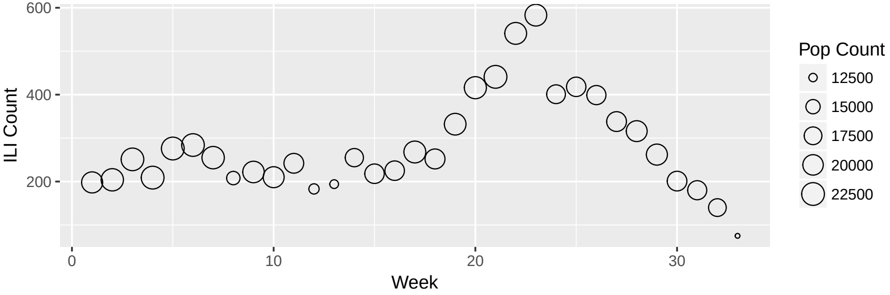

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- Change form of the scaling factor to something borrowed from Werker and Jaggard (1997)

$$ASG(w|\dots) = \begin{cases} \beta_1 + (\eta - \beta_1)\, exp[-(w-\mu)^2/2\sigma_1^2] & w < \mu \\ \beta_2 + (\eta - \beta_2)\, exp[-(w-\mu)^2/2\sigma_2^2] & w \geq \mu \end{cases}$$

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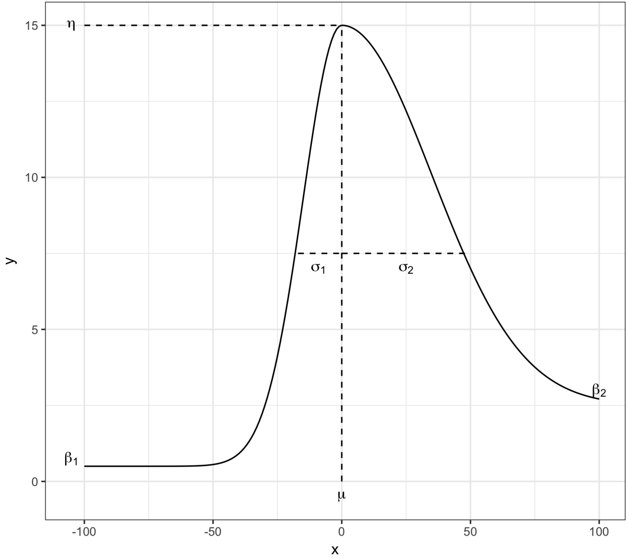

Full Model

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$$
\begin{aligned}
y_{wrs} &\overset{ind}{\sim} Bin(n_{wrs}, \phi_{wrs}) \\
logit(\phi_{wrs}) &= ASG(w|\theta_{rs}) \\
\theta_{rs} &\overset{ind}{\sim} N(\mu_r, \Delta_r \Omega \Delta_r)
\end{aligned}
$$

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\begin{aligned}
\mu_r &\overset{ind}{\sim} N(\mu, \Delta\Omega\Delta) & \mu &\overset{ind}{\sim} N(m_0, C_0) \\
\Delta_r &= diag(\sigma_{r,1}, \cdots) & \Delta &= diag(\sigma_1, \cdots) \\
\sigma_{r,i} &\overset{ind}{\sim} t_4^+(a, b) & \sigma_i &\overset{ind}{\sim} t_4^+(c, d) \\
& & \Omega &\overset{ind}{\sim} LKJ(n)
\end{aligned}
$$

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- Do you need the Asymmetrical Gaussian component?

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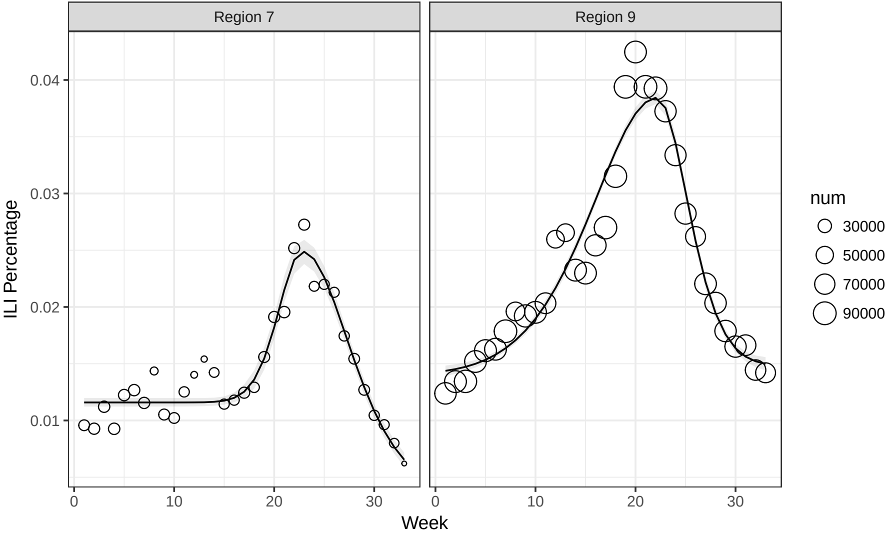
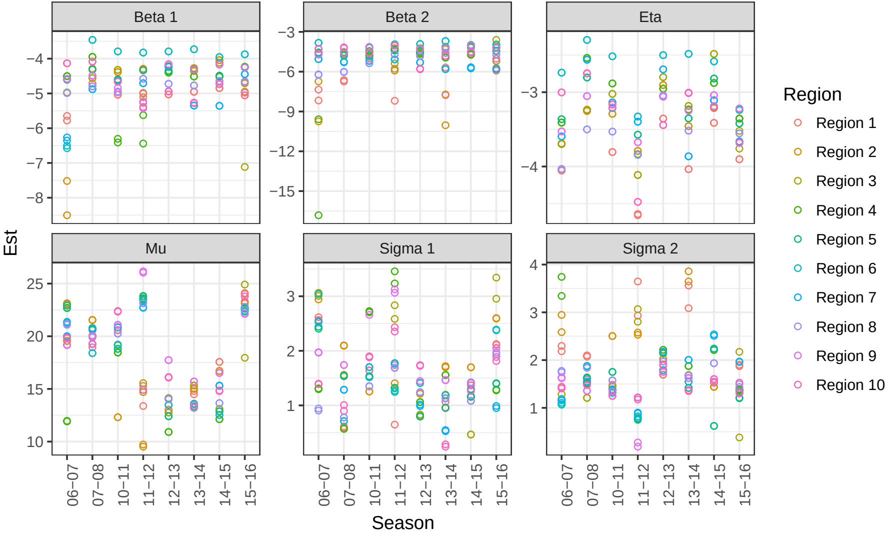
Results for Regions 7 and 9 in the $15 - 16$ Influenza Season:



Does it work?

What does the form tell us about the Influenza seasons:



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$$P(\beta_1 > \beta_2|y) \approx \frac{1}{n.iter}\sum_{i=1}^{n.iter} I\big(\beta_1^{(i)} > \beta_2^{(i))}\big)$$

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If β_1 and β_2, and σ_1 and σ_2, are really not different then the posterior probabilities will hang out around 0.5, but if they hang out around the boundaries, then we need the flexibility.

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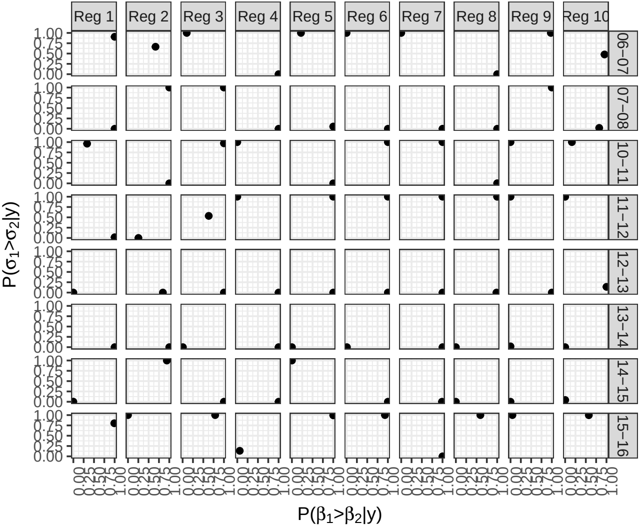

Final/Future Thoughts

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- Focus more on forecasting