1. The Horseshoe distribution can be described as follows:

$$\begin{aligned} y_{ij} &\overset{ind}{\sim} N(\theta_i, \sigma^2) \\ \theta_i &\overset{ind}{\sim} N(0, \lambda_i^2 \tau_i^2) \\ \lambda_i &\sim Ca^+(0,1) \\ \tau_i &\sim Ca^+(0,1) \\ \pi(\sigma) &\propto 1/\sigma \end{aligned}$$

   (a) The goal for this sampler is to make it in as few steps as possible. Here are the steps used:

      i. Sample $\theta = (\theta_1, \cdots, \theta_G) \sim p(\theta|\cdots)$

         A. For $g = 1, \cdots, G$, sample $\theta_g \sim N(\mu_g, \gamma_g^2)$

      ii. Sample $\sigma, \lambda_g^2, \tau$

         A. Sample $\sigma^2 \sim IG\Big(\frac{n}{2}, \frac{1}{2}\sum_{g=1}^{G}\sum_{i=1}^{n_g}(y_{ig} - \theta_g)^2\Big)$

         B. Sample λ_i^2 via a Metropolis-Hastings step the posterior from using the standard non-informative prior as its proposal

         C. Sample τ^2 via a Metropolis-Hastings step the posterior from using the standard non-informative prior as its proposal

   More specifically, we'll sample θ via:

$$\begin{aligned} \theta_g|\cdots &\overset{ind}{\sim} N(\mu_g, \gamma_g^2) \\ \gamma_g^2 &= \left[\frac{1}{\lambda_i^2\tau_i^2} + \frac{n_g}{\sigma^2}\right]^{-1} \\ \mu_g &= \gamma_g^2\left[0 * \lambda_i^{-2}\tau_i^{-2} + \bar{y}_g n_g \sigma^{-2}\right] \\ &= \gamma_g^2\left[\bar{y}_g n_g \sigma^{-2}\right] \end{aligned}$$

   σ via:

$$\sigma|\cdots \sim IG\left(\frac{n}{2}, \frac{1}{2}\sum_{g=1}^{G}\sum_{i=1}^{n_g}(y_{ig} - \theta_g)^2\right)$$

   λ_i^2 is sampled via a Metropolis-Hastings step since it's conditional posterior is unknown. For the proposal, we will use the posterior from using the standard non-informative prior($p(\lambda_i^2) \propto 1/\lambda_i^2$):

$$\lambda_i^2|\cdots \sim IG\left(\frac{n}{2}, \frac{1}{2\tau^2}\sum_{g=1}^{G}(\theta_g)^2\right)$$

   Similarly, τ^2 is sampled via a Metropolis-Hastings step since it's conditional posterior is unknown. For the proposal, we will use the posterior from using the standard non-informative prior($p(\tau^2) \propto 1/\tau^2$):

$$\tau^2|\cdots \sim IG\left(\frac{n}{2}, \sum_{g=1}^{G}\frac{1}{2\lambda_i^2}(\theta_g)^2\right)$$