Credit risk

Basics of credit risk management

0.1 Basic definitions

- **Liquidity risk**: This refers to the inability to raise funds at *normal cost*. This risk increases when the short term value of assets is lesser than the liabilities.
- **Interest rate risk**: This refers to the risk of decline in earnings due to movements in the interest rate.
- **Market risk**: Refers to the risk of adverse deviation in the *mark to market* value of trading portfolios during the period when are looking to liquidate the transaction.
- **Foreign exchange risk**: Risk of losses due to changes in exchange rates.
- **Solvency risk**: When there are multiple risk factors at play, this is the risk of not being able to cover losses by using *available capital*. It is basically the risk of a corporation defaulting. We note that the ultimate protection for such potential losses is *capital* - It is advisable to adjust levels of capital such that it is capable of absorbing the potential losses caused by various risk factors.
- **Operational risk**: This refers to the malfunctioning of information systems and their reporting.
- **Technical risk**: This refers to risk generated due to errors in the recording process of information. It increases in the absence of proper tools to measure risks.
- **Risk** is the volatility of unexpected returns.
- **Financial risk** refers to the possible losses owing to financial market activities.
- **Financial risk management** refers to the design and implementation of procedures for identifying, measuring and managing financial risk.

- **Credit risk**: This is the risk of losses owing to the fact that counterparties may be unwilling to or unable to fulfill their contractual obligations. Typically, its effect is measured by the cost of replacing the cash flows if the other party defualts.
- **Risk** might also refer to the uncertainty of outcomes, characterized in terms of probability distributions.
- **Risk** could also be denoted as the dispersion of possible outcomes.

0.2 Risk measures: Variance

- μ refers to the mean and $f(x)$ is the pdf of the random variable that represents percentage returns. Variance is given by:

$$Var = \int_{-\infty}^{\infty} (\mu - x)^2 f(x) dx \tag{1}$$

Units of measurement of the variance is $\%\%$ (percent percent). In case our random variable is discrete the variance would be given by:

$$Var = \sum_x (\mu - x)^2 P[X = x] \tag{2}$$

- **Example 1**: The pdf of a **continuous** random variable is given by:

$$f(x) = 0.00075[100 - (x - 5)^2], \text{ where } -5 \leq X \leq 15 \tag{3}$$

Now we are required to find the **variance**. Before we calculate variance, we must calculate the **mean** which is computed using the following general formula:

$$E[X] = \int x f(x) dx \tag{4}$$

Putting the values of our pdf function, along with the range of the random variable, we get the following:

$$E[X] = 0.00075 \int_{-5}^{15} x[100 - (x - 5)^2] dx \tag{5}$$

$$= 0.00075 \int_{-5}^{15} x[100 - x^2 - 25 + 10x] dx = 0.00075 \int_{-5}^{15} [75x + 10x^2 - x^3] dx \tag{6}$$

$$= 0.00075 \left[\frac{75x^2}{2} + \frac{10x^3}{3} - \frac{x^4}{4} \right]_{-5}^{15} = 5 \tag{7}$$

Now that we got the mean as 5, we will now subtitute this value into our variance formula given by equation 1 and get the following equation:

$$Var[X] = 0.00075 \int_{-5}^{15} (5 - x)^2 [100 - (x - 5)^2] dx \tag{8}$$

Now we note that $(5-x)^2$ is essentially the same thing as $(x-5)^2$ and hence we can simply rewrite the above equation as follows, by opening the inner brackets:

$$Var[X] = 0.00075 \int_{-5}^{15} [100(x-5)^2 - (x-5)^4] dx \tag{9}$$

$$= 0.00075 \left[\frac{100(x-5)^3}{3} - \frac{(x-5)^5}{5} \right]_{-5}^{15} = 20\%\% = 0.0020 \tag{10}$$

- **Example 2**: We now consider a **discrete** random variable. We consider a discrete distribution that has only two points, with associated probabilities as follows:

$$X = \begin{cases} -7, \ p(x) = 0.04 \\ 5.5, \ p(x) = 0.96 \end{cases} \tag{11}$$

Again we would attempt to compute the variance of this random variable, but before that we must compute its expected value first, given by:

$$E[X] = \sum_x xp(x) = -7(0.04) + 5.5(0.96) = 5\% \text{ per annum} \tag{12}$$

Note - the mean is written in 'percent' terms because our random variable represents *percentage return*. So a mean of 5% represents 5% returns on average. Now we find the variance using equation 2 as follows:

$$Var[x] = [(5-(-7))^2(0.04)] + [(5-5.5)^2(0.96)] = 6\%\% = 0.0006 \tag{13}$$

0.3 Risk measures: DSV, VaR and other terms

- **Downside semi variance (DSV)** represents the variance of percentage returns that fall below the mean value of returns. The definitions for continuous and discrete cases are given below, respectively:

$$DSV = \int_{-\infty}^{\mu} (\mu - x)^2 f(x) dx \tag{14}$$

$$DSV = \sum_{x<\mu} (\mu - x)^2 P[X = x] \tag{15}$$

- **Shortfall probability** is the probability that our actual percentage returns **fall short of** or are **lower than** a prespecified benchmark or **target** rate of return. So it is the probability of the random variable lying below the target L. The continuous and discrete cases are given below respectively:

$$SP = \int_{-\infty}^{L} f(x) dx \tag{16}$$

$$SP = \sum_{x<L} P[X = x] \tag{17}$$

Note that shortfall probability can often be misleading as a measure, since a small company and a big company can essentially have the same shortfall probability. However, even if they have the same shortfall probability, the intensity or the level of shortfall is often what matters more to companies as a risk measure.

- **Value ar Risk (VaR)**: VaR can be thought of as a statistical measure of potential loss. It summarizes the worst loss over a target horizon (time period) that will not be exceeded with a given level confidence. It is the **quantile** of the projected distribution of gains and losses over the target horizon. We can also say - *VaR is the worst loss over a target horizon such that there is a low, prespecified probability that the actual loss will be larger*.

$$VaR(X) = -t, \text{ where } P[X < t] = p \tag{18}$$

$$VaR(X) = -t, \text{ where } t = \max\{x : P[X < x] \leq p\} \tag{19}$$

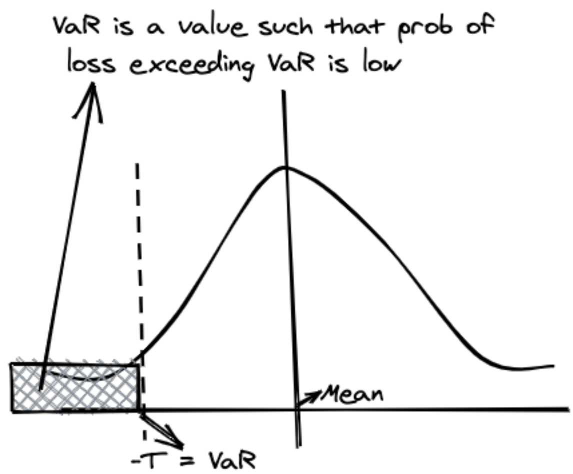

Figure 1: Value at Risk

- **Expected shortfall**: We saw earlier that shortfall probability is nothing but a simple probability measure. But expected shortfall gives us the actual amounbt (on average) by which we will fall short of our percentage returns as compared to target (benchmark) L. It embodies the absolute amount of risk that we are exposed to in our portfolio and is given, for continuous and discrete cases, by:

$$ES = \int_{-\infty}^{L} (L - x) f(x) dx \tag{20}$$

$$ES = \sum_{x<L} (L - x) P[X = x] \tag{21}$$

- **DSV with symmetrical data**: If the data distribution is symmetrical (as in the case of example 1 earlier), our DSV will simply be half the variance. Recall that in example 1 our variance came our to be $20\%\%$, so the DSV would be:
$$DSV = \frac{1}{2}Var = 10\%\% \tag{22}$$
Note that this rule might not necessarily hold for discrete distributions. Taking the values from example 2 earlier we can compute teh DSV as:
$$\sum_{x<5}(5-x)^2 P[X = x] \tag{23}$$
Since ours was a two point distribution, there is only one point less than the mean. Therefore the DSV would be:
$$DSV = [5-(-7)]^2 \times 0.04 = 5.76\%\% \tag{24}$$
Note the difference, the variance in example 2 was $6\%\%$ and its half is not equal to DSV.

- **Example 3**: We will now compute the **shortfall probability** under a benchmark rate of 0%. We take the same continuous pdf as in example 1.
$$SP = 0.00075\int_{-5}^{0}[100-(x-5)^2]dx \tag{25}$$
$$= 0.00075\left[100x - \frac{(x-3)^2}{3}\right]_{-5}^{0} = 0.15625 \tag{26}$$
Now if we wanted to compute the shortfall probability for the same 0% benchmark in the case of the discrete distribution in example 2, then we notice that since there is only one possible case in which our returns might be less than 0, the shortfall probability is simply:
$$P[X < 0] = P[X = -7] = 0.04 \tag{27}$$

0.4 Examples for VaR

- **Example 4**: We want to find the value of t such that $P[X < t] = 0.05$. Taking the same continuous distribution as in example 1 we have:
$$P[X < t] = 0.00075\int_{-5}^{t}[100-(x-5)^2]dx = 0.05 \tag{28}$$
$$= 0.00075\left[100x - \frac{(x-3)^3}{3}\right]_{-5}^{t} = 0.05 \tag{29}$$
Now to find t such that the LHS is 0.05, we use the method of **interpolation** wherein by **trial and error** we compute values of t such that the associated

LHS probability is **less than** 0.05 in one case and **greater than** 0.05 in another case. By trial and error we find the following:

$$\text{if } t = -3 \longrightarrow LHS = 0.028 \tag{30}$$

$$\text{if } t = -2 \longrightarrow LHS = 0.06075 \tag{31}$$

Thus, by interpolation, we can get the value of t associated with LHS 0.05 as follows:

$$t = -3 + \frac{0.05 - 0.028}{0.06075 - 0.028} = -2.293 \tag{32}$$

Suppose we had a portfolio worth $\$100$ million. Then now we can say that the 95% VaR over one year on a $\$100$ million portfolio is $100 \times 2.293\% = \$2.293$ million. *We are* 95% *certain that we will not lose more than* $\$2.293$ *million over the next year*.

- **Example 5**: We will now compute the VaR for the discrete distribution example taken in example 2 earlier. So we have to find t such that $t = \max\{x : P[X < x] \leq 0.05\}$. Since $P[X < -7] = 0$ is lesser than $P[X < 5.5] = 0.04$ and we are looking for the *max* value of t such that the probability is less than or equal to 0.05, we say that $t = 5.5$. **Note that the positive value of** t **indicates a profit. We say that we are** 95% **certain that we will not make profit less than** $\$5.5$ **million in the next year**.

- **Example 6**: The return on a portfolio follows a normal distribution with mean 8 and variance 8^2. Basically if we let X denote the returns as a random variable, then $X \sim N(8, 8^2)$. Find VaR at 97.5% confidence limit. Now, in order to find VaR, we need to find the value of t such that the following holds:

$$P[X < t] = 0.025 \tag{33}$$

Since we are given normal distribution, we can easily convert it to a standard normal distribution and rewrite the above as follows:

$$P\left[Z < \frac{t-8}{8}\right] = \phi\left(\frac{t-8}{8}\right) = 0.025 \tag{34}$$

Note further that ϕ is nothing but a representation of the CDF of the standard normal distribution. So we want the point at which the CDF takes a value of 0.025. We know that at 0.025 the associated value is -1.96.

$$\phi(-1.96) = 0.025 \tag{35}$$

Therefore now we can easily find the value of t by the following:

$$\frac{t-8}{8} = -1.96 \tag{36}$$

$$t = -7.68\% \tag{37}$$

Now, if we had a $\$200$ million portfolio, then we can say that **We are** 95% **certain that we will not lose more than** $200 \times 7.68\% = \$15.36$ **million over the next year**.