Definition 1 (Execution). *Let π be an error trace of length n. An execution of π is a sequence of states $s_0, s_1 ... s_n$ such that $s_i, s_{i+1} \models T$, where T is the transition formula of $\pi[i]$.*

Definition 2 (Blocked Execution). *An execution of a trace π of size n is called a blocked execution, if there exists a sequence of states $s_0, s_1 ... s_j$ where $i < j \leq n$ such that $s_i, s_{i+1} \models T$ where T is the transition formula of $\pi[i]$ and there exists an assume statement in the trace π at position j such that $s_j \not\Rightarrow guard(\pi[j])$*

Definition 3 (Relevance of a Statement). *Let $\pi = st_1, ..., st_n$ be an error trace of length n where st_i is an assignment or a havoc statement of the form $x := t$ or $havoc(x)$ respectively. The statement at position i is relevant if there exists an execution $s_1, ... s_{n+1}$ of π and some value v such that every execution of the trace $x := v; \pi[i + 1, n]$ starting in s_i is has a blocked execution.*

Lemma 1. *For a program statement st and predicates P and Q, where P is condition that is true before the execution of the statement and Q is a post condition, the following two implications are equivilant(also known as the duality of WP and SP):*

$$SP(P, st) \Rightarrow Q$$

$$P \Rightarrow WP(Q, st)$$

Lemma 2. *For a predicate Q and an assignment statement of the form $x := t$ where x is a variable and t is an expression, we have:*

$$WP(Q; havoc(x)) \subseteq WP(Q; x := t)$$

and

$$SP(P; x := t) \subseteq SP(P; havoc(x))$$

Lemma 3 (IGNORE FOR NOW). *For $P := WP(Q, x := t)$ and a set of states R, if $P \cap R \nsubseteq WP(Q, havoc(x))$ for some Q then $Q \subsetneq SP(P, havoc(x))$.*

Proof. We will show that $Q := SP(P; x := t) \subseteq SP(P; havoc(x)) \nsubseteq SP(P; x := t)$ from which it follows that the first inclusion is strict. The first inclusion is from Lemma 2. It is obvious that a state reachable after $x := t$ is also reachable after $havoc(x)$. Hence $SP(P; x := t) \subseteq SP(P; havoc(x))$.
By assumption $WP(Q; x := t) \cap R \nsubseteq WP(Q, havoc(x))$, which is equivalent to $WP(Q; x := t) \nsubseteq WP(Q; havoc(x))$ which by Lemma 1 is equivalent to.

$$SP(WP(Q; x := t); havoc(x)) \nsubseteq Q$$

or

$$SP(P; havoc(x)) \nsubseteq SP(P; x := t)$$

$\square$

Theorem 1 (Relevance of an assignment statement)**.** *Let π be an error trace of length n and $\pi[i]$ be an assignment statement at position i having the form $x := t$, where x is a variable and t is an expression. Let P and Q be two predicates where $P = \neg WP(False; \pi[i,n]) \cap SP(True; \pi[1, i-1])$ and $Q = \neg WP(False; \pi[i+1, n])$. The statement $\pi[i]$ is relevant iff:*

$$P \not\Rightarrow WP(Q, havoc(x))$$

Proof. Let $P' = WP(Q; havoc(x)) \cap SP(True; \pi[1, i-1])$ and $Q' = SP(P; havoc(x))$. It is obvious that P can also be written as $WP(Q; x := t) \cap SP(True; \pi[1, i-1])$.
"$\Rightarrow$"
If $\pi[i]$ is relevant, then

$$P \not\Rightarrow WP(Q; havoc(x))$$

Obviously all the transition from P' end up in Q. Relevancy of $x := t$ implies that there is a state in $s \in P$ such that there is a transition from s to $\neg Q$. That would mean:

$$P \not\Rightarrow P'$$

$$P \not\Rightarrow WP(Q; havoc(x))$$

"$\Leftarrow$"
$\pi[i]$ is relevant, if:

$$P \not\Rightarrow WP(Q; havoc(x))$$

From lemma 1, we can write:

$$SP(P; havoc(x)) \not\Rightarrow Q$$

$$Q' \not\Rightarrow Q$$

This shows the existence of a state s in Q' such that $s \in \neg Q$ and hence a value v for x such that if we replace $x := t$ with $x := v$, then every execution is becoming blocking. Also, from our assumption, it is clear that there exists an execution till P, since P is not empty.

$\square$

Theorem 2 (Relevance of a havoc statement)**.** *Let π be an error trace of length n and $\pi[i]$ be a havoc statement at position i having the form $havoc(x)$, where x is a variable. Let P and Q be two predicates where $P = \neg WP(False; \pi[i,n]) \cap SP(True; \pi[1,i-1])$ and $Q = \neg WP(False; \pi[i+1,n])$. The statement $\pi[i]$ is relevant iff:*

$$P \not\Rightarrow WP(Q, havoc(x))$$

Proof. Let $Q' = SP(P; havoc(x))$.
"$\Rightarrow$"
If $\pi[i]$ is relevant, then:

$$P \not\Rightarrow WP(Q; havoc(x))$$

Obviously $WP(Q; havoc(x))$ is a set of states from which all the transitions end up in Q. Relevancy of $havoc(x)$ implies that there is a transition from a state $s \in P$ which ends in $\neg Q$. That is:

$$P \not\Rightarrow WP(Q; havoc(x))$$

"$\Leftarrow$"
$\pi[i]$ is relevant, if:

$$P \not\Rightarrow WP(Q; havoc(x))$$

From lemma 1:

$$SP(P; havoc(x)) \not\Rightarrow Q$$

$$Q' \not\Rightarrow Q$$

This show the existence of a state s in Q' such that $s \in \neg Q$ and hence a value v for x such that if we replace $havoc(x)$ with $x := v$, then every execution is becoming blocking. Also, from our assumption, it is clear that there exits an execution till P, since P is not empty. □