ECON6080/8080 Assignment 3

Total marks: 10 marks. Marks will depend on accuracy and clarity. Speed is not an issue.

Use Python to answer all questions.

Q1 [3 marks]

Consider approximating

$$f(x) = (x+1)^{1/4}$$

over $[-1, 1]$.

Use

- Chebyshev interpolation (using 5 Chebyshev nodes and $T_0, T_1, ..., T_4$);
- Lagrange interpolation (using 5 equally-spaced grid points over $[-1, 1]$);
- Cubic spline (using 5 equally-spaced grid points over $[-1, 1]$) with the natural spline (in Judd's sense); and
- Cubic spline (using 5 equally-spaced grid points over $[-1, 1]$) with the secant Hermite spline.

Plot the four interpolants and the true function f, and discuss the performance of each of the interpolation methods. For methods that work poorly, try explaining potential reasons.

Q2 [3 marks]

Consider approximating

$$f(x) = \min\left[1, \max[-1, 4(x-0.2)]\right]$$

over $[-1, 1]$.

Use

- Chebyshev interpolation (using 5 Chebyshev nodes and $T_0, T_1, ..., T_4$);
- Lagrange interpolation (using 5 equally-spaced grid points over $[-1, 1]$);
- Cubic spline (using 5 equally-spaced grid points over $[-1, 1]$) with the natural spline (in Judd's sense); and
- Linear interpolation (using 5 equally-spaced grid points over $[-1, 1]$).

Plot the four interpolants and the true function f, and discuss the performance of each of the interpolation methods. For methods that work poorly, try explaining potential reasons.

Q3 [3 marks]

In lecture 6 notebook, we confirmed that Chebyshev regression can approximate $1/(1+x^2)$ very well on $[-1, 1]$. In lecture 5, we saw that the fit of the Lagrange interpolant was not very well for the same function on $[-5, 5]$ even when Chebyshev nodes are used.

Extend the Chebyshev regression code in the lecture 6 notebook so that it can handle any interval $[a, b]$, and evaluate the performance of Chebyshev regression for the same function $1/(1+x^2)$ on $[-5, 5]$. Use $m = 11$ (the number of Chebyshev nodes) as before, and vary n (the polynomial order) to see how the performance depends on n.

Q4 [1 marks]

Consider the following function:

$$f(x_1, x_2) = \frac{5\sin(x_1)}{x_1} \times \{\max(20 - |x_2|, 0)\}^{1.2}.$$

Find the global minimum of this function. We will go in steps.

- Plot this function in 3 dimesnions. Use the mplot3d from matplotlib for example. `https://jakevdp.github.io/PythonDataScienceHandbook/04.12-three-dimensional-plotting.html`

- Now start from $(x_1^0, x_2^0) = (-25, 25)$, try both BFGS and Nelder-Mead methods and report the outcomes of each method.

- Now think about how you can improve your algorithm. One hint could be adding some randomness into your search for the global minimizer. Think about how you can incorporate the Sobol equence (`https://en.wikipedia.org/wiki/Sobol_sequence`) into your algorithm for example. Again, start from the initial value $(x_1^0, x_2^0) = (-25, 25)$, and implement your new algorithm. Marks of this question depend on how successfully your new algorithm finds the global minimizer.