ECON3510 Tutorial 3 Answers

2019

Exercise 1

1.1 Question 1

For autarky we have 3 Conditions:

$$\frac{P_C}{P_f} = \frac{Q_C}{4Q_F} \tag{1}$$

$$\frac{P_C}{P_F} = \frac{Q_F}{Q_C} \tag{2}$$

$$4Q_4^2 + Q_C^2 = 2000 \tag{3}$$

- For Home: from conditions (1) and (2) we have

$$\frac{Q_C}{4Q_F} = \frac{P_C}{P_F} = \frac{Q_f}{Q_C}$$

$$\frac{Q_C}{4Q_F} = \frac{Q_f}{Q_C}$$

$$\therefore Q_C^2 = 4Q_F^2 \tag{*}$$

Plugging equation $(*)$ into our PPF we obtain the following:

- For Q_C:

$$Q_C^2 + Q_C^2 = 2000$$

$$Q_C = \sqrt{2000/2}$$

$$Q_C = 10\sqrt{10} = 31.6$$

- For Q_F:

$$4Q_F^2 + 4Q_F^2 = 2000$$
$$Q_C = \sqrt{2000/8}$$
$$Q_C = 5\sqrt{10} = 15.8$$

For relative price we have from condition (2) that:

$$\frac{P_C}{P_F} = \frac{Q_F}{Q_C} = \frac{15.8}{31.6} = 0.5$$

- For Foreign: repeating the above for the Foreign country yields:

$$Q_F^* = 31.6$$
$$Q_C^* = 15.8$$
$$\frac{P_C}{P_F} = 2$$

1.2 Question 2

For free trade we have that the relative price must be the same across countries, yielding the following conditions:

$$\frac{P_C^W}{P_F^W} = \frac{Q_C}{4Q_F} \tag{1}$$
$$\frac{P_C^W}{P_F^W} = \frac{(Q_F - X_F)}{(Q_C - X_C)} \tag{2}$$
$$4Q_F^2 + Q_C^2 = 2000 \tag{3}$$
$$\frac{P_C^W}{P_F^W} = \frac{4Q_C^*}{Q_F^*} \tag{4}$$
$$\frac{P_C^W}{P_F^W} = \frac{(Q_F^* + X_F)}{Q_C^* + X_C} \tag{5}$$
$$Q_F^{2*} + 4Q_C^{*2} = 2000 \tag{6}$$
$$P_C X_C + P_F X_F = 0 \tag{7}$$

1.3 Question 3

From conditions (1) and (2) we have:

$$\frac{Q_C}{4Q_F} = \frac{4Q_C^*}{Q_F^*}$$

Substituting in the conjecture (contained in the question) yields the relationship between the goods for Home:

$$\frac{Q_C}{4Q_F} = \frac{4Q_F}{Q_C}$$
$$Q_C^2 = 16Q_F^2 \quad (*)$$
$$Q_C = 4Q_F \quad (**)$$

Substituting equation $(**)$ into (1) gives us the relative price:

$$\frac{P_{CW}}{P^FW} = \frac{4Q_F}{4Q_F} = 1$$

Substituting equation $(*)$ into our PPF conditiong (3) gives us the quantity of food prroduced:

$$4Q_F^2 + Q_C^2 = 2000$$
$$4Q_F^2 + 16Q_F^2 = 2000$$
$$Q_F^2 = 100$$
$$Q_F = 10$$

Since by equation $(**)$ we have that $Q_C = 4Q_F$, we know that:

$$Q_C = 4 \times 10 = 40$$

For the foreign quantities produced, since $Q_C = Q_F^*$ and $Q_F = Q_C^*$, it must be that:

$$Q_F^* = 40$$
$$Q_C^* = 10$$

Given our relative price we have that $P_C^W = P_F^W$, this allows us to take out prices from condition (7) by division, yielding:

$$X_C + X_F = 0$$
$$X_F = -X_C \quad (***)$$

Using that $(Q_C = 4Q_F)$, $(X_F = -X_C)$, and $(P_C^W/P_F^W = 1)$, we can use condition (2) to solve for X:

$$\begin{aligned}
\tfrac{P_C^W}{P_F^W} &= \tfrac{(Q_F - X_F)}{(Q_C - X_C)} \\
1 &= (Q_F - X_F)/(4Q_F + X_F) \\
4Q_F + X_F &= Q_F - X_F \\
2X_F &= -3Q_F \\
\therefore X_F &= -(\tfrac{3}{2})Q_F \\
\therefore X_C &= (\tfrac{3}{2})Q_F \qquad (****)
\end{aligned}$$

Using the quantities that we solved for, i.e. $Q_C = 40$ and $Q_F = 10$, by equation $(***)$ we have:

$$X_C = \tfrac{3}{2}Q_F = \tfrac{3}{2}(10) = 15$$

By equation $(***)$ we have:

$$\begin{aligned}
X_F &= -X_C \\
\therefore X_F &= -15
\end{aligned}$$

Exercise 2

2.1 Question 1

We have:

$$OC_C = Q_C/4Q_F$$
$$OC_C^* = 4Q_C^*/Q_F^*$$

Therefore, the opportunity cost depends on the quantities produced and so from the PPF it is not immediately clear who has a comparative advantage However, if we look at the case under autarky quantities we can calculate the opportunity cost as:

$$OC_C = 31.6/(4 * 15.8) = 1/2$$
$$OC_C^* = (4 * 15.8)/31.6 = 2$$

Therefore, since $OC_C < OC_C^*$, Home will export cloth

2.2 Question 2

See Claudio's answer sheet

2.3 Question 3

Yes, this is true since if opportunity cost under Autarky is $OC_C < OC_C^*$ and Home with free trade exports 15 cloth and imports 15 food

2.4 Question 4

See Claudio's answer sheet

2.5 Question 5

Prices will be equilized