ECON3510 Tutorial 5 Answers

2019

Exercise 1

1.1 Question 1

$MPL_F = 200 - L_f, MPL_C = 100 - 2L_C, L = 200$

From our equilibrium condition:

$$P_C MPL_C = w = P_F MPL_F$$
$$2(100 - 2L_C) = 1(200 - L_F)$$
$$200 - 4L_C = 200 - L_F$$
$$4L_C = L_F \quad (*)$$

Since total labor is $L_C + L_F = 200$ we can substitute (*) in for L_F to solve for L_C:

$$L_C + 4L_C = 200$$
$$L_C = 40$$

By (*) since $L_C = 40$ it must be that:

$$L_F = 4(40) = 160$$

Since we know L_C and L_F we know wages from using the wage formula:

$$w = 2(100 - 2(40))$$
$$w = 40$$

1.2 Question 2

If prices double nothing change in terms of employment between sectors, however wages will double to 80. You can see this in the wage equation where:

$$W = 2 \times P_1 MPL_1$$

However in the equilibrium equation we have:

$$2\ times P_1 MPL_1 = 2 \times P_2 MPL_2$$

So the multiplication by 2 cancels out and the equilibrium equation for employment does not change

1.3 Question 3

The price for one good in our equilibrium condition changes so we have

$$2(100 - 2L_C) = 2(200 - L_F)$$
$$200 - 4L_C = 400 - 2L_F$$
$$2L_F = 200 + 4L_C$$
$$L_F = 100 + 2L_C \quad (**)$$

Since $L_C + L_F = 200$, using (**) we have:

$$L_C + L_F = 200$$
$$L_C + 100 + 2L_C = 200$$
$$3L_C = 100$$
$$L_C = 100/3$$

Using (**) again we have:

$$L_F = 100 + 2(100/3) = 500/3 = 166.666$$

Again using the wage formula:

$$W = P_C MPL_C$$
$$W = 2(100 - 2L_C)$$
$$W = 2(100 - 2(100/3)$$

Exercise 2

2.1 Question 1

Using that $MPL_L = \frac{\partial Q_C}{\partial L_C}$ we have:

$$Q_C = 4L_C^{0.5}$$
$$MPL_C = 2L_C^{-0.5}$$
$$MPL_C = \frac{2}{\sqrt{L_C}} = \frac{2}{L_C^{0.5}}$$

Similarly for food:

$$Q_F = 2L_F^{0.5}$$
$$MPL_F = 1LF^{-0.5}$$
$$MPL_F = \frac{1}{\sqrt{L_F}} = \frac{1}{L_F^{0.5}}$$

2.2 Question 2

From our OC Formula:

$$OC_C = \frac{1}{LF^{0.5}} / \frac{2}{L_C^{0.5}}$$
$$OC_C = \frac{1}{L_F^{0.5}} \times \frac{L_C^{0.5}}{2}$$
$$OC_C = \frac{L_C^{0.5}}{2L_F^{0.5}}$$

Our production function for quantity equations were:

$$Q_C = 4L_C^{0.5} \Rightarrow L_C^{0.5} = \frac{Q_C}{4}$$
$$Q_F = 2L_F^{0.5} \Rightarrow L_F^{0.5} = \frac{Q_F}{2}$$

Now we can substitute in L for Q in the OC equation to get:

$$OC_C = \frac{Q_C}{4} / 2(\frac{Q_F}{2})$$
$$OC_C = \frac{Q_C}{4Q_F}$$

2.3 Question 3

Assuming $L_C = 125$ or $L_F = 125$ we get the following maximum quantities:

$$Q_C = 4(125)^{0.5} = 44.72$$
$$Q_F = 2(125)^{0.5} = 22.36$$

2.4 Question 4

We use the following PFQ_C, PFQ_F, and L equations to solve for the PPF:

$$(1) \quad Q_C = 4L_C^{0.5} \Rightarrow L_C = \frac{Q_C^2}{16}$$
$$(2) \quad Q_F = 2L_F^{0.5} \Rightarrow L_F = \frac{Q_F^2}{4}$$
$$(3) \quad L_F + L_C = 125$$

Substituting equations (1) and (2) into (3) yields:

$$\frac{Q_F^2}{4} + \frac{Q_C^2}{16} = 125$$
$$Q_F^2 + \frac{Q_C^2}{4} = 500$$
$$4Q_F^2 + Q_C^2 = 2000$$

2.5 Question 5

We have that $OC_C^* = \frac{4Q_C^*}{Q_F^*}, PPF^* = Q_F^{*2} + 4Q_C^{*2} = 2000$. Our equilibrium conditions under free trade are:

$$(1) \quad \frac{P_C^W}{P_F^W} = \frac{Q_C}{4Q_F}$$
$$(2) \quad \frac{P_C^W}{P_F^W} = (Q_F - X_F)/(Q_C - X_C)$$
$$(3) \quad 4Q_F^2 + Q_C^2 = 2000$$
$$(4) \quad \frac{P_C^W}{P_F^W} = 4\frac{Q_C^*}{Q_F^*}$$
$$(5) \quad \frac{P_C^W}{P_F^W} = (Q_F^* + X_F)/(Q_C^* + X_C)$$
$$(6) \quad Q_F^{*2} + 4Q_C^{*2} = 2000$$
$$(7) \quad P_C X_C + P_F X_F = 0$$

<u>Quantities</u>

From conditions (1) and (4) we have:

$$\frac{Q_C}{4Q_F} = \frac{4Q_C^*}{Q_F^*}$$

Since $Q_C^* = Q_F$ (and vice versa) we have:

$$\frac{Q_C}{4Q_F} = \frac{4Q_F}{Q_C}$$

$$\therefore Q_C^2 = 16Q_F^2 \quad (*)$$

$$\therefore Q_C = 4Q_F \quad (**)$$

Solving for quantities by using (*) in (3) we have:

$$4Q_F^2 + Q_C^2 = 2000$$

$$4Q_F^2 + 16Q_F^2 = 2000$$

$$20Q_F^2 = 2000$$

$$Q_F^2 = 100$$

$$\therefore Q_F = 10 \Rightarrow Q_C^* = 10$$

Using $Q_F = 10$ in condition (3) we have the following:

$$4(10)^2 + Q_C^2 = 2000$$

$$Q_C^2 = 1600$$

$$\therefore Q_C = 40 \Rightarrow Q_F^* = 40$$

<u>World Relative Price</u>

Substituting (**) into condition (1) gives us the world relative price:

$$\frac{P_C^W}{P_F^W} = \frac{4Q_F}{4Q_F} = 1$$

Since $\frac{P_C^W}{P_F^W} = 1$ we know from (7):

$$X_F = -X_C \quad (***)$$

<u>Exports</u>

Plugging equation (***) into condition (2) we can solve for exports:

$$\frac{P_C^W}{P_F^W} = (Q_F - X_F)/(Q_C - X_C)$$

$$1 = (Q_F - X_F)/(4Q_F + X_F)$$

$$4Q_F + X_F = Q_F - X_F$$

$$3Q_F = -2X_F$$

$$-X_F = (\tfrac{3}{2})Q_F$$

Since $Q_F = 10$ we have:

$$-X_F = (\tfrac{3}{2})Q_F = (\tfrac{3}{2})10 = 15$$

$$\therefore -X_F = 15$$

$$\therefore X_C = 15$$

Summary:$Q_F = 10, Q_C = 40, Q_F^* = 40, Q_C^* = 10, \frac{P_C^W}{P_F^W} = 1, -X_F = 15, X_C = 15$