Macroeconomics B Notes

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1 Dynamic Optimisation in Continuous Time

Continuous vs Discrete Time: in discrete time the intervals between periods are $\Delta > 0$ (e.g. the interval between t and t+1 is 1), in continuous time the intervals between periods are $\Delta \to 0$

<u>Continuous Variables Issue</u>: while state variables are often naturally continuous, when using computational methods we can only evaluation the value function on a finite number of points

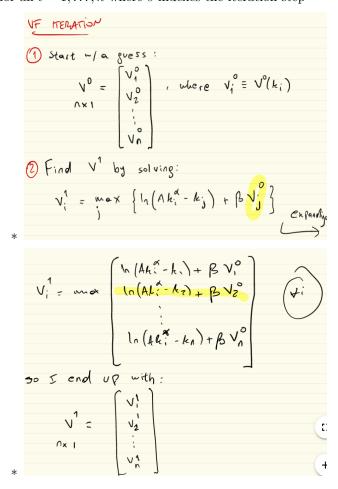
• <u>Solution</u>: we can address this problem via using (1) discretization of the state space, or, (2) polynomial approximation to the value function

<u>Discretization Method</u>: consider a problem with continuous state and control variables $\mathfrak{x} \in \mathbb{R}$, discretization just replaces \mathfrak{x} and \mathfrak{u} by the finite grids $\hat{\mathfrak{x}} = \{x^1, \dots, x^n\}$ and $\hat{\mathfrak{u}} = \{u^1, \dots, u^n\}$

- <u>Value Function Impact</u>: now the value function becomes a finite list of numbers, $V = [V^1, \dots, V^n]^T$
- Advantage: the maximization step is much simpler than under the original bellman equation, which is a key advantage of discretization methods
- Disadvantage: there is a "curse of dimensionality" in muiltidimensional state spaces where we must decides whether to have N points for a one-dimensional state space vs N^k points for a k-dimensional state space
 - <u>Grid Decision</u>: requires some a-priori information about the state space, which is sometimes difficult to obtain (e.g. upper and lower bounds)
- Optimal Growth Illustration: suppose $V(k) = \max_{k'} \{ \ln(Ak^{\alpha} k') + \beta V(k') \}$ where in this case $\mathfrak{x} = \mathfrak{u}$

^{*}Please see google notes

- <u>Discretization</u>: if we discretize \mathfrak{x} then the Bellman equation becomes $V_i = \max_j \{\ln(Ak_i^{\alpha} k_j) + \beta V_j\}$ for all $i = 1, \ldots, n$
- <u>Value Function Iteration Solution</u>: suppose we applied value function iteration and therefore iterate on the mapping $V_i^s = \max \left\{ \ln(Ak_i^{\alpha} k_1) + \beta V_1^{s-1}, \dots, \ln(Ak_i^{\alpha} k_n) + \beta V_n^{s-1} \right\}$ for all $i = 1, \dots, n$ where s indexes the iteration step



Ordinary Differential Equations (ODEs): a "differential equation" is one where the unknown is a function (instead of a variable) and the equation includes one or more of the derivatives of the function, an "ordinary differential equation" equation is one for which the unknown is a function of only one variable (typically time)

- Partial ODEs: where the unknown is a function of more than one variable
- First-Order ODE Form: $\dot{x}(t) = F(t, x(t))$ where $\dot{x}(t) \equiv dx(t)/dt$ and $t \in [t_a, t_b]$
 - <u>Unknown</u>: here the unknown is a function x(t) with $x:[t_a,t_b]\to\mathbb{R}$
 - <u>Uniqueness</u>: the solution is not unique with the form having infinitely many solutions indexed by an integrating constant C. However, generally the constant C can be uniquely determined by requiring the solution to pass through a given point on the tx-plane

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