MONASH University

MONASH BUSINESS SCHOOL

ETF3231/5231 Business forecasting

Week 8: ARIMA models

https://bf.numbat.space/

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Outline

ARIMA models

AR: autoregressive (lagged observations as inputs)
I: integrated (differencing to make series stationary)
MA: moving average (lagged errors as inputs)

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Make data stationary (variance & mean), fit model, reverse, forecast.

Outline

AR(1) model including a constant

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t, \quad |\phi_1| < 0 \quad (P.T.O)$$

- When $\phi_1 = 0$ and $c = 0$, y_t is equivalent to WN;
- When $\phi_1 = 1$ and $c = 0$, y_t is equivalent to a RW;
- When $\phi_1 = 1$ and $c \neq 0$, y_t is equivalent to a RW with drift;
- When $\phi_1 > 0$, y_t tends to hang below or above the mean of y_t.
- When $\phi_1 < 0$, y_t tends to oscillate below and above the mean of y_t.
- If $E(y_t) = \mu$, $\mu = \frac{c}{1-\phi_1}$, c is related to the mean of y_t.

For stationarity we require $-1 < \phi_1 < 1$

Let's set $\phi_1 = 2$ $\quad y_t = 2y_{t-1} + \varepsilon_t$ and see what happens:

$$y_1 = 2y_0 + \varepsilon_1$$

$$y_2 = 2y_1 + \varepsilon_2 = 4y_0 + 2\varepsilon_1 + \varepsilon_2$$

$$y_3 = 2y_2 + \varepsilon_3 = 8y_0 + 4\varepsilon_1 + 2\varepsilon_2 + \varepsilon_3 \quad \ldots \text{ and so on}$$

$$\vdots$$

- Getting larger & larger
- Hence, we only allow for the new obs. to be a fraction of the previous ones.

Autoregressive models

A multiple regression with lagged values of y_t as predictors.

$$\begin{aligned} y_t &= c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t \\ &= c + (\phi_1 B + \phi_2 B^2 + \cdots + \phi_p B^p) y_t + \varepsilon_t \end{aligned}$$

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&= c + (\phi_1 B + \phi_2 B^2 + \cdots + \phi_p B^p) y_t + \varepsilon_t
\end{aligned}$$

$$\begin{aligned}
(1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p) y_t &= c + \varepsilon_t \\
\phi(B) y_t &= c + \varepsilon_t
\end{aligned}$$

- ε_t is white noise.
- $\phi(B) = (1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p)$

Stationarity conditions

We normally restrict autoregressive models to stationary data, and then some constraints on the values of the parameters are required.

General condition for stationarity

Complex roots of $\phi(z) = 1 - \phi_1 z - \phi_2 z^2 - \dots - \phi_p z^p$ lie outside the unit circle on the complex plane.

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Complex roots of $\phi(z) = 1 - \phi_1 z - \phi_2 z^2 - \cdots - \phi_p z^p$ lie outside the unit circle on the complex plane.

- For $p = 1$: $-1 < \phi_1 < 1$.
- For $p = 2$:

  $$-1 < \phi_2 < 1 \qquad \phi_2 + \phi_1 < 1 \qquad \phi_2 - \phi_1 < 1.$$
- More complicated conditions hold for $p \geq 3$.
- fable takes care of this.

Moving Average (MA) models

A multiple regression with past *errors* as predictors.

$$\begin{aligned} y_t &= c + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \cdots + \theta_q\varepsilon_{t-q} \\ &= c + (1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q)\varepsilon_t \\ &= c + \theta(B)\varepsilon_t \end{aligned}$$

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y_t &= c + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \cdots + \theta_q\varepsilon_{t-q} \\
&= c + (1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q)\varepsilon_t \\
&= c + \theta(B)\varepsilon_t
\end{aligned}
$$

- ε_t is white noise.
- $\theta(B) = (1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q)$

Invertibility

General condition for invertibility

Complex roots of $\theta(z) = 1 + \theta_1 z + \theta_2 z^2 + \cdots + \theta_q z^q$ lie outside the unit circle on the complex plane.

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Complex roots of $\theta(z) = 1 + \theta_1 z + \theta_2 z^2 + \cdots + \theta_q z^q$ lie outside the unit circle on the complex plane.

- For $q = 1$: $-1 < \theta_1 < 1$.
- For $q = 2$:

  $-1 < \theta_2 < 1 \qquad \theta_2 + \theta_1 > -1 \qquad \theta_1 - \theta_2 < 1.$
- More complicated conditions hold for $q \geq 3$.
- fable takes care of this.

PROPERTIES OF AR & MA models

- AR(1) $\quad y_t = c + \phi\, y_{t-1} + \varepsilon_t\,;\quad \varepsilon_t \sim NID(0, \sigma^2)\,;\quad |\phi| < 1\,;\quad y_0$ (starting value).

$$y_t = c + \phi\, y_{t-1} + \varepsilon_t$$

$$= c + \phi\left(c + \phi\, y_{t-2} + \varepsilon_{t-1}\right) + \varepsilon_t$$

$$= c + \phi c + \phi^2 y_{t-2} + \phi\,\varepsilon_{t-1} + \varepsilon_t$$

$$= c + \phi c + \phi^2\left(c + \phi\, y_{t-3} + \varepsilon_{t-2}\right) + \phi\,\varepsilon_{t-1} + \varepsilon_t$$

$$= c + \phi c + \phi^2 c + \phi^3 y_{t-3} + \phi^2 \varepsilon_{t-2} + \phi\,\varepsilon_{t-1} + \varepsilon_t$$

$$\vdots$$

$$\Rightarrow y_t = c\left(1 + \phi + \phi^2 + \dots\right) + \phi^t y_0 + \varepsilon_t + \phi\,\varepsilon_{t-1} + \phi^2 \varepsilon_{t-2} + \dots$$

for $t \to \infty$ $\quad = \dfrac{c}{1-\phi} + \varepsilon_t + \phi\,\varepsilon_{t-1} + \phi^2 \varepsilon_{t-2} + \dots\quad$ MA(∞)

PROPERTIES OF AR & MA models

- AR(1) $\quad y_t = c + \phi\, y_{t-1} + \varepsilon_t$; $\varepsilon_t \sim NID(0, \sigma^2)$; $|\phi| < 1$; y_0 (starting value).

$$
\begin{aligned}
y_t &= c + \phi y_{t-1} + \varepsilon_t \\
&= c + \phi\,(c + \phi\, y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t \\
&= c + \phi c + \phi^2 y_{t-2} + \phi \varepsilon_{t-1} + \varepsilon_t \\
&= c + \phi c + \phi^2 (c + \phi\, y_{t-3} + \varepsilon_{t-2}) + \phi \varepsilon_{t-1} + \varepsilon_t \\
&= c + \phi c + \phi^2 c + \phi^3 y_{t-3} + \phi^2 \varepsilon_{t-2} + \phi \varepsilon_{t-1} + \varepsilon_t \\
&\ \ \vdots
\end{aligned}
$$

$$\Rightarrow y_t = c\,(1 + \phi + \phi^2 + \dots) + \phi^t y_0 + \varepsilon_t + \phi \varepsilon_{t-1} + \phi^2 \varepsilon_{t-2} + \dots$$

for $t \to \infty$ $\quad = \dfrac{c}{1-\phi} + \varepsilon_t + \phi \varepsilon_{t-1} + \phi^2 \varepsilon_{t-2} + \dots \qquad MA(\infty)$

- $E(y_t) = \dfrac{c}{1-\phi} = \mu$

- $var(y_t) = \gamma_0 = E(y_t - \mu)^2 = \dfrac{\sigma^2}{1-\phi^2}$ (stat)

$$
\begin{aligned}
\gamma_0 &= E\left(\frac{c}{1-\phi} + \varepsilon_t + \phi \varepsilon_{t-1} + \phi^2 \varepsilon_{t-2} + \dots - \frac{c}{1-\phi}\right)^2 \\
&= E(\varepsilon_t + \phi \varepsilon_{t-1} + \phi^2 \varepsilon_{t-2} + \dots)^2 \\
&= E(\varepsilon_t)^2 + \phi^2 E(\varepsilon_{t-1})^2 + \phi^4 E(\varepsilon_{t-2})^2 + \dots \quad \text{cross-products} \\
&= \sigma^2 + \phi^2 \sigma^2 + (\phi^2)^2 \sigma^2 + (\phi^2)^3 \sigma^2 + \dots \\
&= \sigma^2 \left(1 + \phi^2 + (\phi^2)^2 + (\phi^2)^3 + \dots\right) = \frac{\sigma^2}{1-\phi^2}
\end{aligned}
$$

$$\mathrm{Cov}(y_t, y_{t-1}) = \gamma_1 = \phi \frac{\sigma^2}{1-\phi^2} = \phi \gamma_0$$

$$\gamma_1 = E(y_t - \mu)(y_{t-1} - \mu)$$

$$= E(\varepsilon_t + \phi\varepsilon_{t-1} + \phi^2\varepsilon_{t-2} + \phi^3\varepsilon_{t-3} + \ldots)(\varepsilon_{t-1} + \phi\varepsilon_{t-2} + \phi^2\varepsilon_{t-3} + \phi^3\varepsilon_{t-4} + \ldots)$$

$$= \phi E(\varepsilon_{t-1})^2 + \phi^3 E(\varepsilon_{t-2})^2 + \phi^5 E(\varepsilon_{t-3})^2 + \ldots$$

$$= \phi\sigma^2 + \phi\phi^2\sigma^2 + \phi\phi^4\sigma^2 + \ldots$$

$$= \phi\sigma^2 \left(1 + \phi^2 + (\phi^2)^2 + (\phi^2)^3 + \ldots\right)$$

$$= \phi \frac{\sigma^2}{1-\phi^2}$$

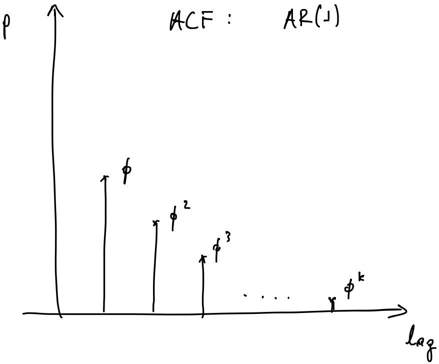
In general $\gamma_k = \phi^k \gamma_0$ (cov stat)

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{\phi\gamma_0}{\gamma_0} = \phi$$

$$\rho_2 = \frac{\gamma_2}{\gamma_0} = \frac{\phi^2\gamma_0}{\gamma_0} = \phi^2$$

$$\vdots$$

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{\phi^k\gamma_0}{\gamma_0} = \phi^k$$



Properties of an MA(1); $y_t = c + \varepsilon_t + \theta \varepsilon_{t-1}$, $\varepsilon_t \sim NID(0, \sigma^2)$ $|\theta| < 1$

$E(y_t) = \mu = c$

$var(y_t) = \gamma_0 = E(y_t - \mu)^2 = E(\varepsilon_t)^2 + \theta^2 E(\varepsilon_{t-1})^2 + \cancel{cov(\varepsilon_t, \varepsilon_{t-1})} \xrightarrow{} 0 = \sigma^2 + \theta^2 \sigma^2 = (1 + \theta^2)\sigma^2$

$cov(y_t, y_{t-1}) = \gamma_1 = E(y_t - \mu)(y_{t-1} - \mu) = E(\varepsilon_t + \underline{\theta \varepsilon_{t-1}})(\underline{\varepsilon_{t-1}} + \theta \varepsilon_{t-2}) = \theta E(\varepsilon_{t-1})^2 = \theta \sigma^2$

$\gamma_2 = E(y_t - \mu)(y_{t-2} - \mu) = E(\varepsilon_t + \theta \varepsilon_{t-1})(\varepsilon_{t-2} - \mu) = 0$

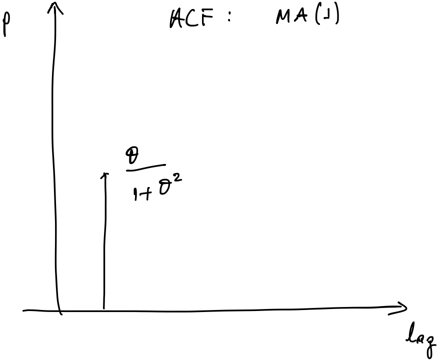
In fact for an MA(q) all $\gamma_j = 0$ $j > q$

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{\theta \cancel{\sigma^2}}{(1+\theta^2)\cancel{\sigma^2}}$$

$$\rho_2 = 0$$

$$\vdots$$

$$\rho_k = 0$$



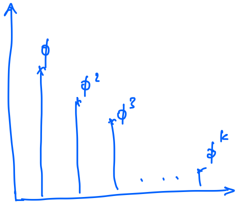
AR(1) $\quad y_t = c + \phi y_{t-1} + \varepsilon_t \qquad |\phi| < 1$ (stationary)

$$E(y_t) = \mu = \frac{c}{1-\phi}$$

$$var(y_t) = \gamma_0 = \frac{\sigma^2}{1-\phi^2}$$

$$\gamma_k = \phi^k \gamma_0 \qquad \rho_k = \frac{\gamma_k}{\gamma_0} = \phi^k$$

ACF



MA(1) $\quad y_t = c + \theta \varepsilon_{t-1} + \varepsilon_t \qquad |\theta| < 1$ (invertible)

$$E(y_t) = \mu = c$$

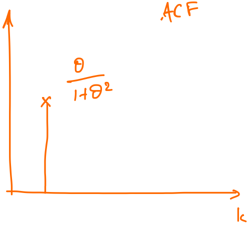
$$var(y_t) = \gamma_0 = (1+\theta^2)\sigma^2$$

$$\gamma_1 = \theta\sigma^2 \qquad \gamma_2 = \dots = \gamma_k = 0$$

$$\boxed{\text{in general } \gamma_j = 0 \quad j > q}$$

$$\rho_1 = \frac{\theta}{1+\theta^2}\ ; \quad \rho_2 = \dots = \rho_k = 0$$

ACF



AR(1) $\quad y_t = c + \phi y_{t-1} + \varepsilon_t \qquad |\phi| < 1$

(stationary)

$$E(y_t) = \mu = \frac{c}{1-\phi}$$

$$var(y_t) = \gamma_0 = \frac{\sigma^2}{1-\phi^2}$$

$$\gamma_k = \phi^k \gamma_0 \qquad \rho_k = \frac{\gamma_k}{\gamma_0} = \phi^k$$

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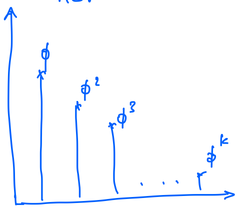


MA(1) $\quad y_t = c + \theta \varepsilon_{t-1} + \varepsilon_t \qquad |\theta| < 1$

(invertible)

$$E(y_t) = \mu = c$$

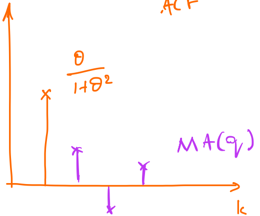
$$var(y_t) = \gamma_0 = (1+\theta^2)\sigma^2$$

$$\gamma_1 = \theta\sigma^2 \qquad \gamma_2 = \ldots = \gamma_k = 0$$

$$\boxed{\text{in general } \gamma_j = 0 \quad j > q}$$

$$\rho_1 = \frac{\theta}{1+\theta^2} \;;\quad \rho_2 = \ldots = \rho_k = 0$$

ACF



ARIMA models

ARIMA(p, d, q) model: $\phi(B)(1 - B)^d y_t = c + \theta(B)\varepsilon_t$

AR:	p = order of the autoregressive part
I:	d = degree of first differencing involved
MA:	q = order of the moving average part.

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MA: q = order of the moving average part.

- Conditions on AR coefficients ensure stationarity.
- Conditions on MA coefficients ensure invertibility.
- White noise model: ARIMA(0,0,0) $y_t = \varepsilon_t$
- $I(1)$ Random walk: ARIMA(0,1,0) with no constant $y_t' = \varepsilon_t \Rightarrow y_t = y_{t-1} + \varepsilon_t$
- Random walk with drift: ARIMA(0,1,0) with const. $y_t' = c + \varepsilon_t \Rightarrow y_t = c + y_{t-1} + \varepsilon_t$
- AR(p): ARIMA(p,0,0)
- MA(q): ARIMA(0,0,q)

R model

Intercept form

*

$$(1 - \phi_1 B - \cdots - \phi_p B^p) y'_t = c + (1 + \theta_1 B + \cdots + \theta_q B^q)\varepsilon_t$$

Mean form

$$(1 - \phi_1 B - \cdots - \phi_p B^p)(y'_t - \mu) = (1 + \theta_1 B + \cdots + \theta_q B^q)\varepsilon_t$$

* Other books use this. Don't get confused.

- $y'_t = (1 - B)^d y_t$
- μ is the mean of y'_t.
- $c = \mu(1 - \phi_1 - \cdots - \phi_p)$.
- `ARIMA()` in the fable package uses intercept form.

Recall : the mean of the (differenced) data is not the contront.

Understanding ARIMA models

* VERY IMPORTANT TO UNDERSTAND

(p, q) → short-run

(c, d) → long-run

- If $c = 0$ and $d = 0$, the long-term forecasts will go to zero.
- If $c = 0$ and $d = 1$, the long-term forecasts will go to a non-zero constant.
- If $c = 0$ and $d = 2$, the long-term forecasts will follow a straight line.
- If $c \neq 0$ and $d = 0$, the long-term forecasts will go to the mean of the data.
- If $c \neq 0$ and $d = 1$, the long-term forecasts will follow a straight line.
- If $c \neq 0$ and $d = 2$, the long-term forecasts will follow a quadratic trend.

SUMMARY

constant

$c = 0$	$d = 0$	$\hat{y}_{T+\infty} \to 0$	ARMA stat around	$E(y_t) = 0$
$c \neq 0$	$d = 0$	$\hat{y}_{T+\infty} \to E(y_t)$	" " "	$E(y_t) \neq 0$
$c = 0$	$d = 1$	$\hat{y}_{T+\infty} \to$ const	RW + ARMA	• diff of RW is stat and the ARMA part will converge to const.

linear trend

$c \neq 0$	$d = 1$		RW + drift + ARMA
		$\hat{y}_{T+\infty} \to t$	
$c = 0$	$d = 2$		Two unit roots

quadratic trend $c \neq 0$ $d = 2$

DO IT AT YOUR OWN RISK.

Understanding ARIMA models

Forecast variance and d

- The higher the value of d, the more rapidly the prediction intervals increase in size.
- For $d = 0$, the long-term forecast standard deviation will go to the standard deviation of the historical data.

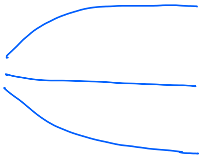
Cyclic behaviour

- For cyclic forecasts, $p \geq 2$ and some restrictions on coefficients are required.
- If $p = 2$, we need $\phi_1^2 + 4\phi_2 < 0$. Then average cycle of length

$$(2\pi)/\left[\text{arc}\cos(-\phi_1(1-\phi_2)/(4\phi_2))\right].$$

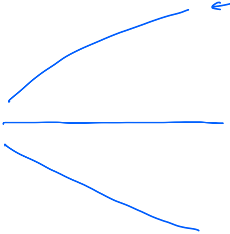
$d = 0$

constant (in sample st. dev).



$d = 1$

keep increasing



Outline

Partial autocorrelations

Partial autocorrelations measure relationship between y_t and y_{t-k}, when the effects of other time lags — $1, 2, 3, \ldots, k-1$ — are removed. why. PTO →

Partial autocorrelations

Partial autocorrelations measure relationship between y_t and y_{t-k}, when the effects of other time lags — $1, 2, 3, \dots, k-1$ — are removed.

α_k = kth partial autocorrelation coefficient

= equal to the estimate of ϕ_k in regression:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_k y_{t-k}. \neq \rho_1, \dots, \rho_k \text{ in ACF}$$

Partial autocorrelations

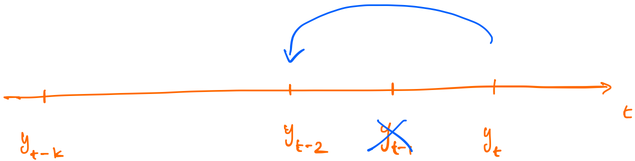
Partial autocorrelations measure relationship between y_t and y_{t-k}, when the effects of other time lags — $1, 2, 3, \dots, k-1$ — are removed.

$$\alpha_k = k\text{th partial autocorrelation coefficient}$$
$$= \text{equal to the estimate of } \phi_k \text{ in regression:}$$
$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_k y_{t-k}.$$

- Varying number of terms on RHS gives α_k for different values of k.
- $\boxed{\alpha_1 = \rho_1}$ ALWAYS
- same critical values of $\pm 1.96/\sqrt{T}$ as for ACF.
- * Last significant α_k indicates the order of an AR model. *

AR(1) $\quad y_t = \phi y_{t-1} + \varepsilon_t \quad \Rightarrow \quad y_{t-1} = \phi y_{t-2} + \varepsilon_{t-1}$

$$\Rightarrow y_t = \phi(\phi y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t = \phi^2 y_{t-2} + \phi \varepsilon_{t-1} + \varepsilon_t$$



- correlation between y_t & y_{t-k} conditional on all other regressors
- this is what regression does

ACF

$y_t = c + \rho_1 y_{t-1} + \varepsilon_t$

$y_t = c + \rho_2 y_{t-2} + \varepsilon_t$

PACF

$y_t \quad \boxed{\begin{matrix} y_{t-1} \\ y_{t-1} \end{matrix}} \quad y_{t-2}$

$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_2$

$\alpha_1 = \phi_1 = \rho_1$ (Nothing between y_t & y_{t-1}) ALWAYS

$\alpha_2 = \phi_2 < \rho_2$ (takes out the chaining / link of y_{t-1})

$\alpha_k = \phi_k = 0$ for $k > p$, hence max $\phi_k \neq 0$ gives you the AR order

Note: the purpose of the PACF is largely to select the AR order. If ACF shows WN, PACF will also show WN ($\alpha_1 = \phi_1 = 0$).

ACF and PACF interpretation

AR(1)

$$\rho_k = \phi_1^k \qquad \text{for } k = 1, 2, \dots;$$
$$\alpha_1 = \phi_1 \qquad \alpha_k = 0 \qquad \text{for } k = 2, 3, \dots.$$

So we have an AR(1) model when

- autocorrelations exponentially decay
- there is a single significant partial autocorrelation.

ACF and PACF interpretation

AR(p)

- ACF dies out in an exponential or damped sine-wave manner
- PACF has all zero spikes beyond the pth spike

So we have an AR(p) model when

- the ACF is exponentially decaying or sinusoidal
- there is a significant spike at lag p in PACF, but none beyond p

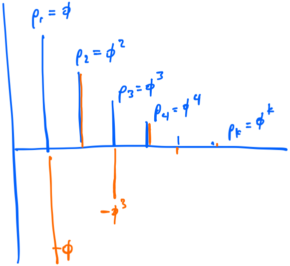
ACF

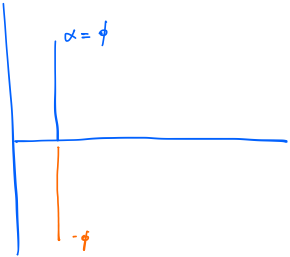
$\boxed{\text{PACF}}$ *

$0 < \phi < 1$

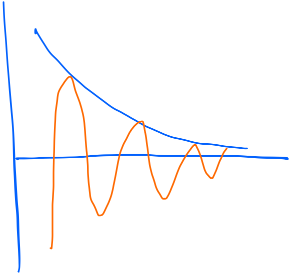
$-1 < \phi < 0$

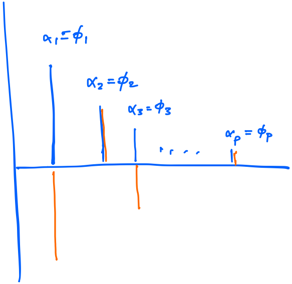
AR(1)





AR(p)





ACF and PACF interpretation

MA(1)

$$\rho_1 = \theta_1 \qquad \rho_k = 0 \qquad \text{for } k = 2, 3, \ldots;$$
$$\alpha_k = -(-\theta_1)^k$$

So we have an MA(1) model when

- the PACF is exponentially decaying and
- there is a single significant spike in ACF

ACF and PACF interpretation

MA(q)

- PACF dies out in an exponential or damped sine-wave manner
- ACF has all zero spikes beyond the qth spike

So we have an MA(q) model when

- the PACF is exponentially decaying or sinusoidal
- there is a significant spike at lag q in ACF, but none beyond q

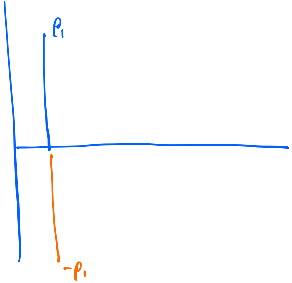
$\boxed{\text{ACF}}$ *

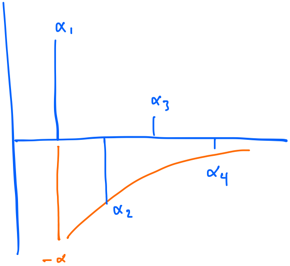
PACF

$0 < \theta < 1$

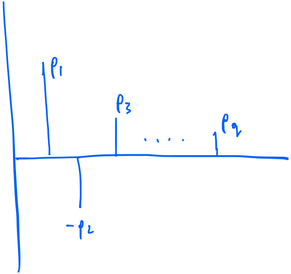
$-1 < \theta < 0$

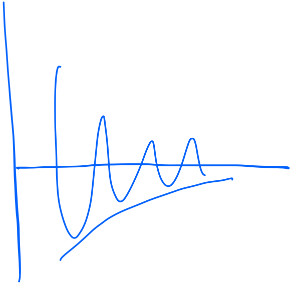
MA(1)



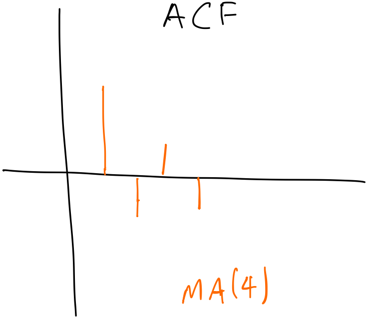


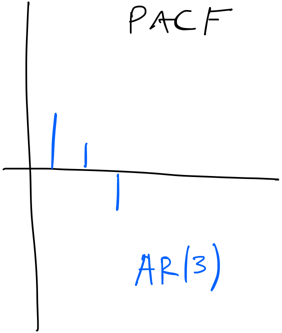
MA(q)





EXTRA JUST TO RECAP *





* from theory / reading ACF / PACF we can tell either AR or MA orders

EGYPTIAN EXPORTS

ARIMA(2,0,1) with constant.

$$(1 - \phi_1 B - \phi_2 B^2)\, y_t = c + (1 + \theta_1 B)\, \varepsilon_t$$

EGYPTIAN EXPORTS

ARIMA(2,0,1) with constant.

$$(1 - \phi_1 B - \phi_2 B^2)\, y_t = c + (1 + \theta_1 B)\, \varepsilon_t$$

$$y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2} = c + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

$$\Rightarrow \quad y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \theta_1 \varepsilon_{t-1} + \varepsilon_t$$

$$\Rightarrow \quad y_t = 2.562 + 1.676\, y_{t-1} - 0.803\, y_{t-2} - 0.69\, \varepsilon_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim NID(0, 8.046)$

or $\hat{\sigma} = \sqrt{8.046} = 2.837$