

LDA equations

Dianne Cook

$$\begin{aligned} p_1(x_0) &> p_2(x_0) \\ \Rightarrow \frac{\pi_1 \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{1}{2\sigma^2}(x_0 - \mu_1)^2\right)}{\sum_{l=1}^K \pi_l \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{1}{2\sigma^2}(x_0 - \mu_l)^2\right)} &> \frac{\pi_2 \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{1}{2\sigma^2}(x_0 - \mu_2)^2\right)}{\sum_{l=1}^K \pi_l \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{1}{2\sigma^2}(x_0 - \mu_l)^2\right)} \quad \text{common denom} \\ \Rightarrow \pi_1 \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{1}{2\sigma^2}(x_0 - \mu_1)^2\right) &> \pi_2 \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{1}{2\sigma^2}(x_0 - \mu_2)^2\right) \quad \text{rm constant, then log} \\ \Rightarrow \log(\pi_1) - \frac{1}{2\sigma^2}(x_0 - \mu_1)^2 &> \log(\pi_2) - \frac{1}{2\sigma^2}(x_0 - \mu_2)^2 \quad \text{shift sides, expand} \\ \Rightarrow -(x_0^2 - 2x_0\mu_1 + \mu_1^2) + (x_0^2 - 2x_0\mu_2 + \mu_2^2) &> 2\sigma^2(\log(\pi_1) - \log(\pi_2)) \quad \text{cancel and simplify} \\ \Rightarrow 2(\mu_1 - \mu_2)x_0 - (\mu_1^2 + \mu_2^2) &> 2\sigma^2(\log(\pi_1) - \log(\pi_2)) \quad \text{expand} \\ \Rightarrow 2(\mu_1 - \mu_2)x_0 - (\mu_1 - \mu_2)(\mu_1 + \mu_2) &> 2\sigma^2(\log(\pi_1) - \log(\pi_2)) \quad \text{simplify} \\ \Rightarrow 2x_0 - (\mu_1 + \mu_2) &> 2\sigma^2(\log(\pi_1) - \log(\pi_2)) \quad \text{shift sides} \\ \Rightarrow x_0 &> \frac{1}{2}((\mu_1 + \mu_2) + 2\sigma^2(\log(\pi_1) - \log(\pi_2))) \end{aligned}$$

$$\text{if } \pi_1 = \pi_2 \text{ then } x_0 > \frac{\mu_1 + \mu_2}{2}$$