

Recap

- Models as functions
- Linear models

Overview

- Feedback from the tests
- What is R^2
 - (pull examples from exercise)
- short exercise in class to calculate correlation and r2 and answer questions
- augment?
- understanding residuals
- components of variation?

Other Admin

Project deadline (Next Week)

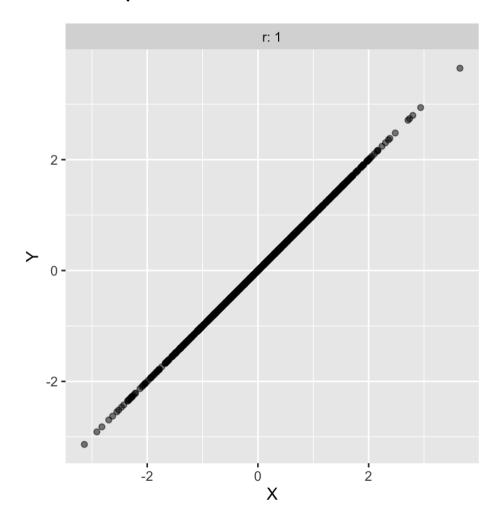
Find team members, and potential topics to study (ed quiz will be posted soon)

What is correlation?

- Linear association between two variables can be described by correlation
- Ranges from -1 to +1

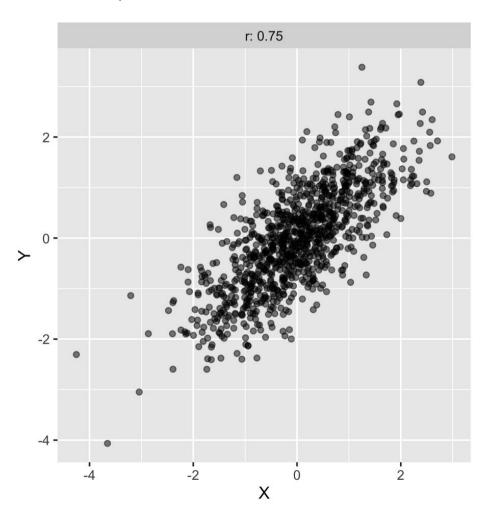
Strong Positive correlation

As one variable increases, so does another

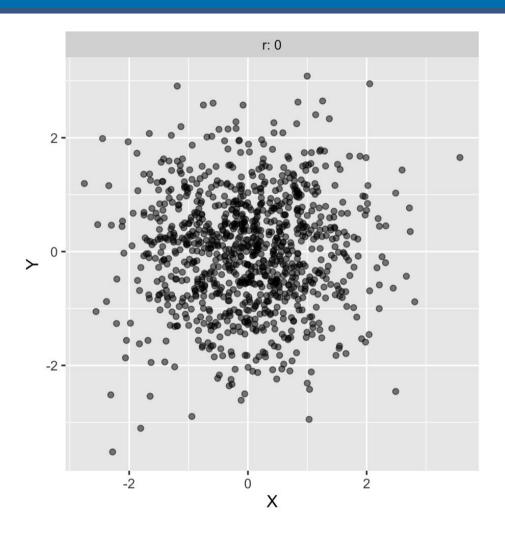


Strong Positive correlation

As one variable increases, so does another variable

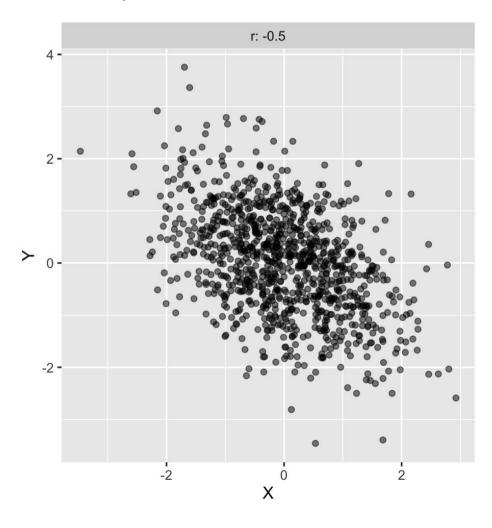


Zero correlation: neither variables are related



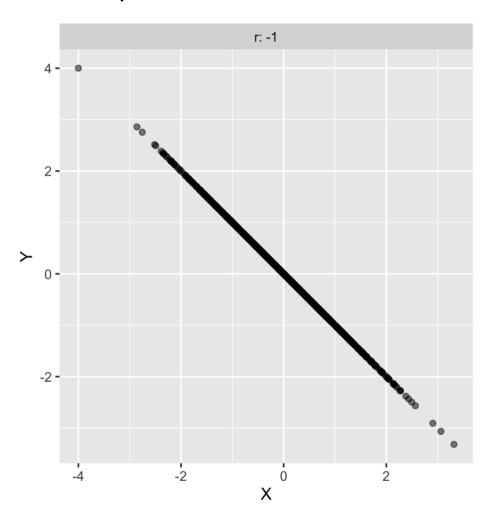
Strong negative correlation

As one variable increases, another decreases

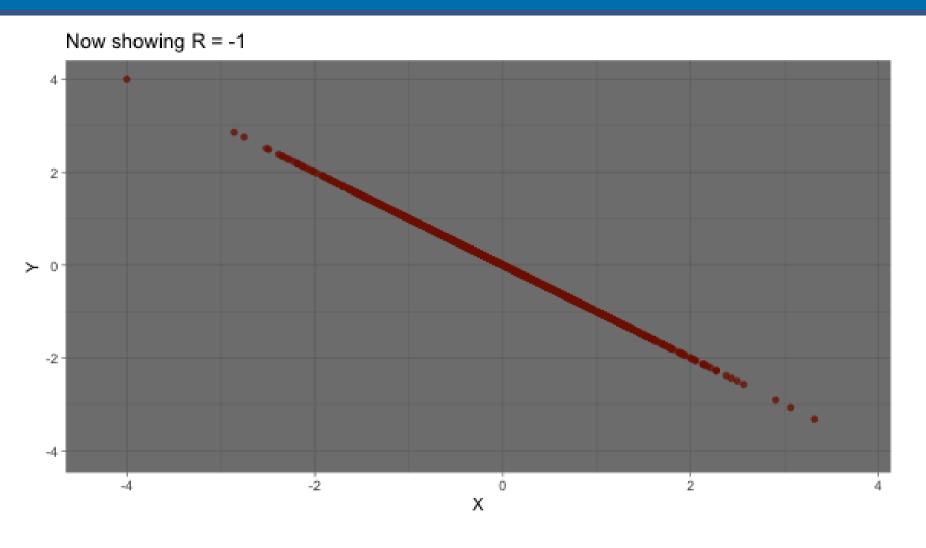


STRONG negative correlation

As one variable increases, another decreases



Correlation: The animation

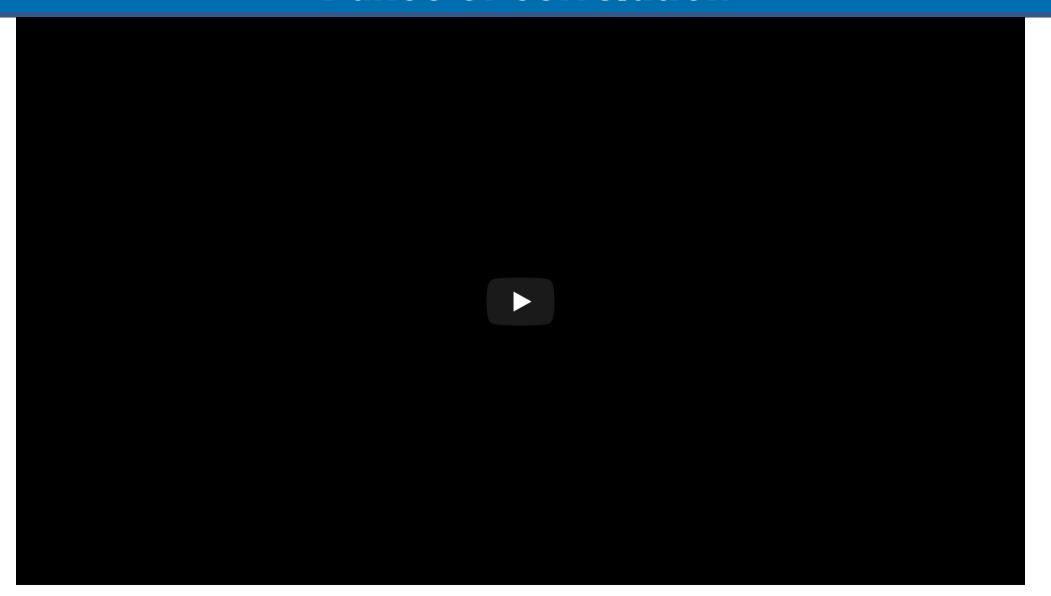


definition of correlation

For two variables X, Y, correlation is:

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2}} = \frac{cov(X, Y)}{s_x s_y}$$

Dance of correlation



Remember! Correlation does not equal causation

What is R^2 ?

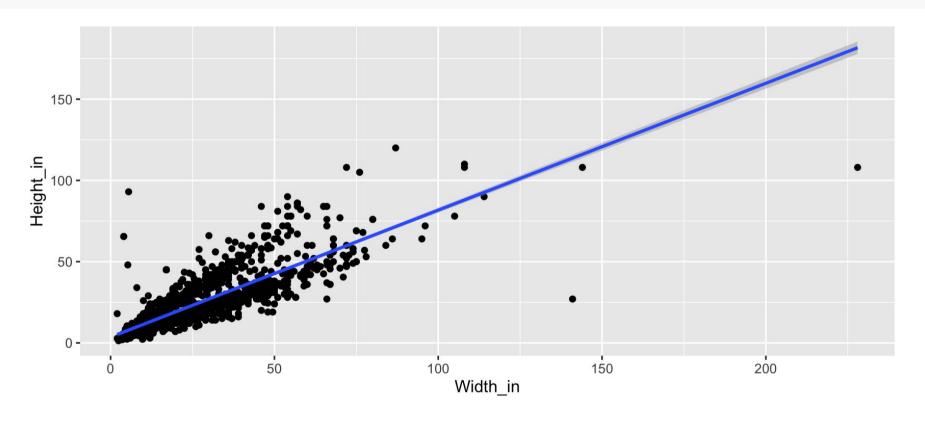
- (model variance)/(total variance), the amount of variance in response explained by the model.
- Always ranges between 0 and 1, with 1 indicating a perfect fit.
- Adding more variables to the model will always increase \mathbb{R}^2 , so what is important is how big an increase is gained. Adjusted \mathbb{R}^2 reduces this for every additional variable added.

unpacking Im and model objects

```
pp <- read_csv("data/paris-paintings.csv", na = c("n/a", "", "NA"))</pre>
pp
## # A tibble: 3,393 x 61
##
     name sale lot
                     position dealer year origin_author origin_cat school_pntg
     <chr> <chr> <chr> <dbl> <chr> <dbl> <chr>
                                                       <chr>
                                                                 <chr>
   1 L176... L1764 2 0.0328 L
                                     1764 F
                                                       0
                                                                 F
   2 L176... L1764 3 0.0492 L
                                     1764 I
   3 L176... L1764 4 0.0656 L
                                     1764 X
                                                                 D/FL
   4 L176... L1764 5 0.0820 L
                                     1764 F
   5 L176... L1764 5 0.0820 L
                                     1764 F
   6 L176... L1764 6 0.0984 L
                                     1764 X
   7 L176... L1764 7 0.115 L
                                     1764 F
   8 L176... L1764 7 0.115 L
                                     1764 F
   9 L176... L1764 8
                       0.131 L
                                     1764 X
## 10 L176... L1764 9
                       0.148 L
                                     1764 D/FL
                                                                 D/FL
## # ... with 3,383 more rows, and 52 more variables: diff_origin <dbl>, logprice <dbl>,
## #
      price <dbl>, count <dbl>, subject <chr>, authorstandard <chr>, artistliving <db
## #
      authorstyle <chr>, author <chr>, winningbidder <chr>, winningbiddertype <chr>,
      endbuyer <chr>, Interm <dbl>, type_intermed <chr>, Height_in <dbl>, Width_in <d 16/79
## #
```

unpacking linear models

```
ggplot(data = pp, aes(x = Width_in, y = Height_in)) +
  geom_point() +
  geom_smooth(method = "lm") # lm for linear model
```



template for linear model

```
lm(<FORMULA>, <DATA>)
```

<FORMULA>

RESPONSE ~ EXPLANATORY VARIABLES

Fitting a linear model

```
m_ht_wt <- lm(Height_in ~ Width_in, data = pp)

m_ht_wt

##

## Call:

## lm(formula = Height_in ~ Width_in, data = pp)

##

## Coefficients:

## (Intercept) Width_in

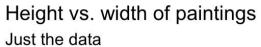
## 3.6214 0.7808</pre>
```

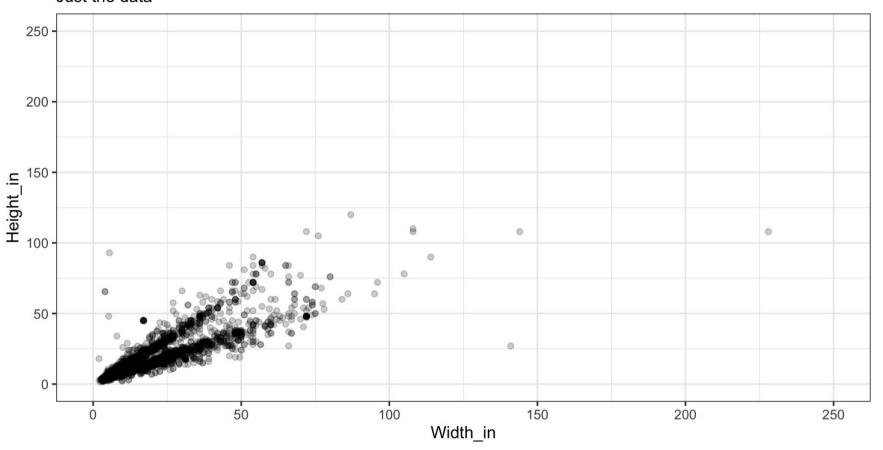
using tidy, augment, glance

tidy: return a tidy table of model information

tidy(<MODEL OBJECT>)

Visualizing residuals

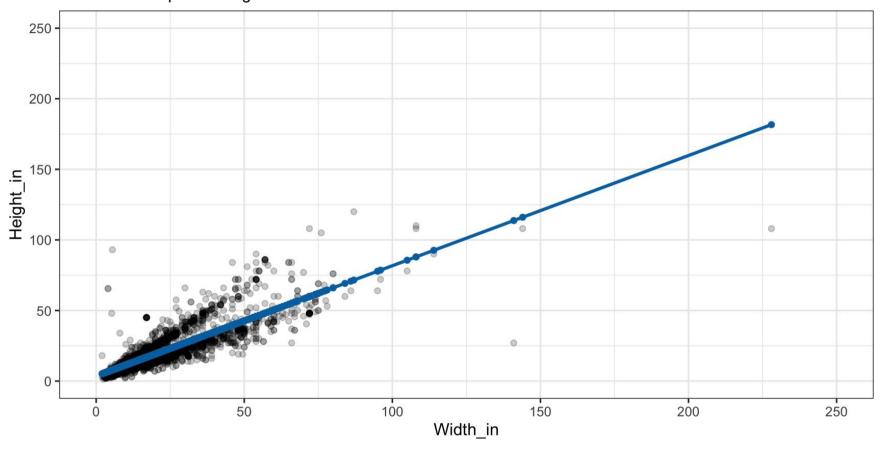




Visualizing residuals (cont.)

Height vs. width of paintings

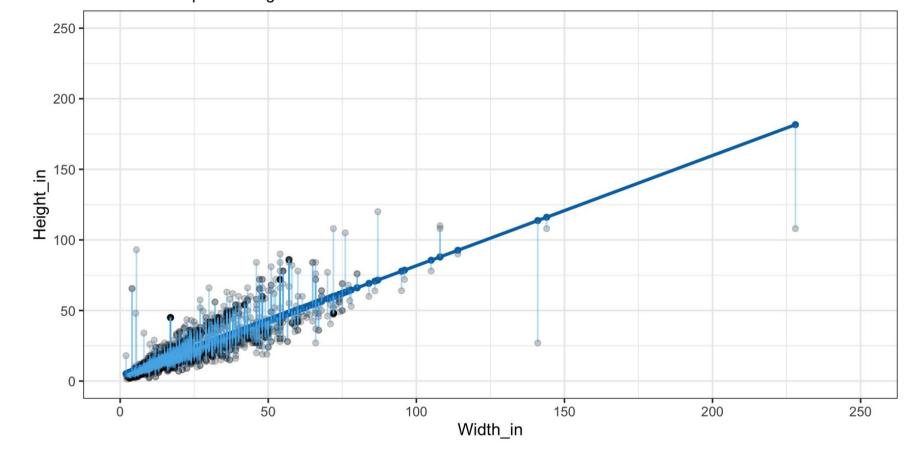
Data + least squares resgression line



Visualizing residuals (cont.)

Height vs. width of paintings

Data + least squares resgression line + residuals



glance: get a one-row summary out

glance(<MODEL OBJECT>)

AIC, BIC, Deviance

- AIC, BIC, and Deviance are evidence to make a decision
- Deviance is the residual variation, how much variation in response that IS NOT explained by the model. The close to 0 the better, but it is not on a standard scale. In comparing two models if one has substantially lower deviance, then it is a better model.
- Similarly BIC (Bayes Information Criterion) indicates how well the model fits, best used to compare two models. Lower is better.

augment: get the data

```
augment<MODEL>
or
augment(<MODEL>, <DATA>)
```

augment

```
augment(m_ht_wt)
## # A tibble: 3,135 x 10
##
      .rownames Height_in Width_in .fitted .se.fit .resid .hat .sigma .cooksd .st
                                                              <dbl> <dbl> <dbl> <dbl>
                   <dbl>
                             <db1>
                                     <dbl>
                                             <dbl> <dbl>
##
    <chr>
                                             0.166 10.3
                                                           0.000399
                                                                      8.30 3.10e-4
##
                       37
                              29.5
                                     26.7
                                     14.6
                                             0.165 3.45
                                                           0.000396
                                                                      8.31 3.42e-5
##
   2 2
                       18
                              14
##
   3 3
                       13
                              16
                                     16.1
                                             0.158 - 3.11
                                                           0.000361
                                                                      8.31 2.54e-5
##
    4 4
                       14
                              18
                                     17.7
                                             0.152 - 3.68
                                                           0.000337
                                                                      8.31 3.30e-5
##
    5 5
                       14
                              18
                                     17.7
                                             0.152 - 3.68
                                                           0.000337
                                                                      8.31 3.30e-5
                                                           0.000498
                                                                      8.31 7.09e-5
##
    6 6
                              10
                                     11.4
                                             0.185 - 4.43
##
   7 7
                        6
                              13
                                     13.8
                                             0.170 - 7.77
                                                           0.000418
                                                                      8.30 1.83e-4
                                             0.170 - 7.77
##
    8 8
                        6
                              13
                                     13.8
                                                           0.000418
                                                                      8.30 1.83e-4
##
    9 9
                       15
                              15
                                     15.3
                                             0.161 - 0.333
                                                           0.000377
                                                                      8.31 3.04e-7
## 10 10
                        9
                                      9.09
                                             0.204 -0.0870 0.000601
                                                                      8.31 3.30e-8
## # ... with 3,125 more rows
```

understanding residuals

- variation explained by the model
- residual variation: what's left over after fitting the model

Your turn: go to rstudio cloud and get started on exercise 7B

Going beyond a single model

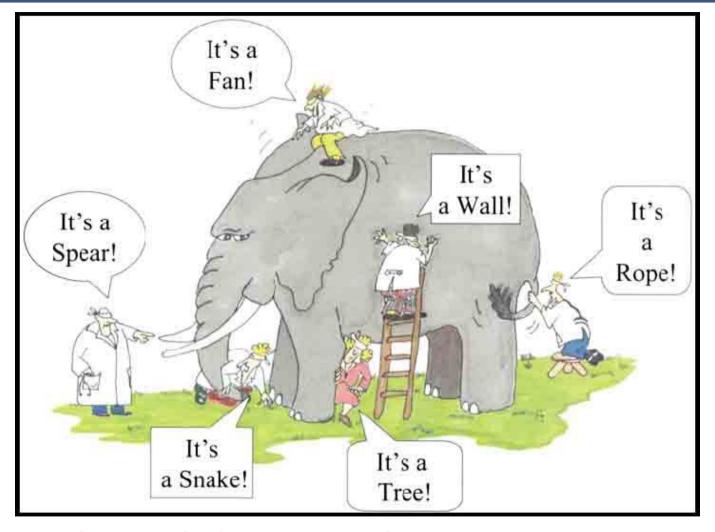


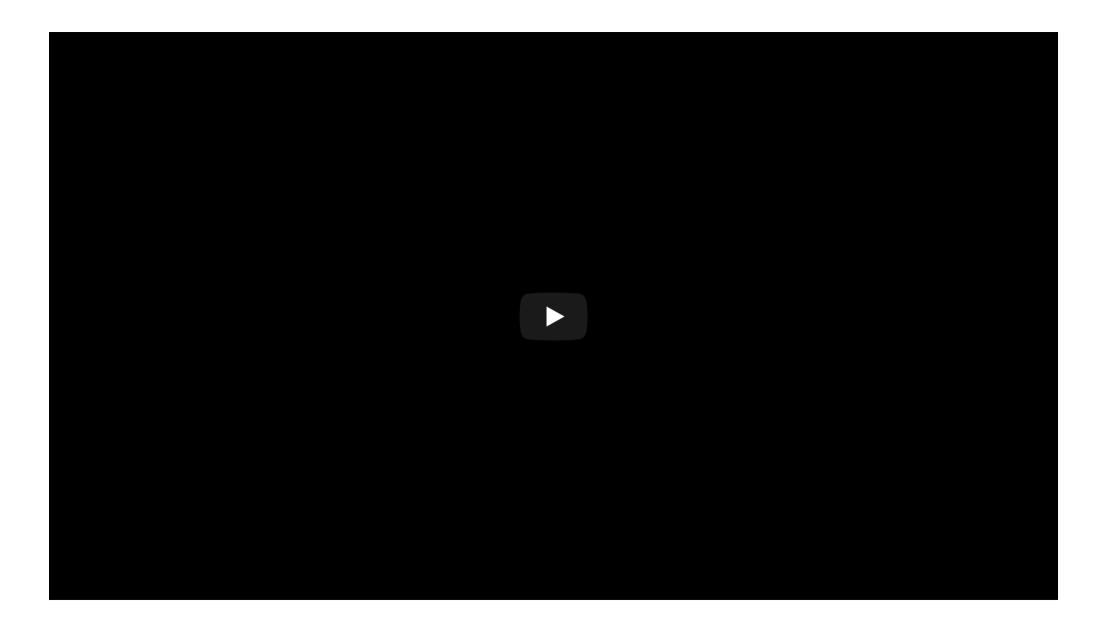
Image source: https://balajiviswanathan.quora.com/Lessons-from-the-Blind-men-and-the-elephant

Going beyond a single model

- Beyond a single model
- Fitting many models

Gapminder

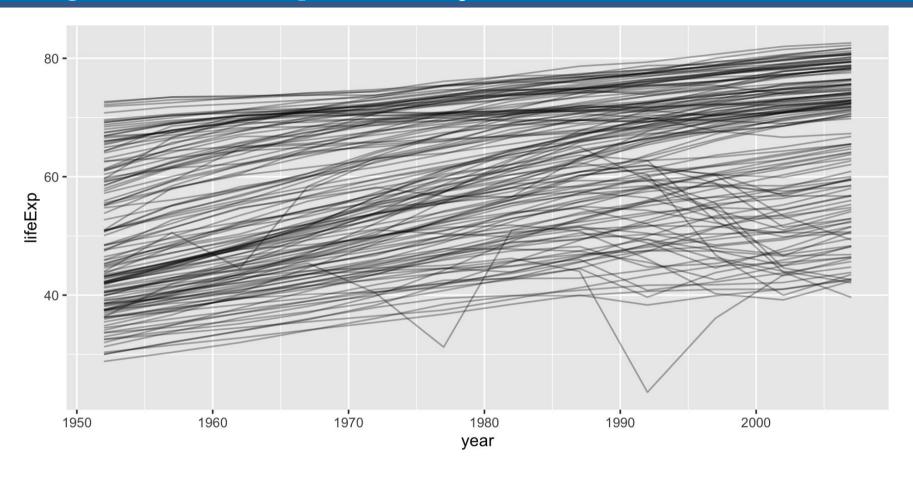
- Hans Rosling was a Swedish doctor, academic and statistician, Professor of International Health at Karolinska Institute. Sadley he passed away in 2017.
- He developed a keen interest in health and wealth across the globe, and the relationship with other factors like agriculture, education, energy.
- You can play with the gapminder data using animations at https://www.gapminder.org/tools/.



R package: gapminder

Contains subset of the data on five year intervals from 1952 to 2007.

"Change in life expectancy in countries over time?"



"Change in life expectancy in countries over time?"

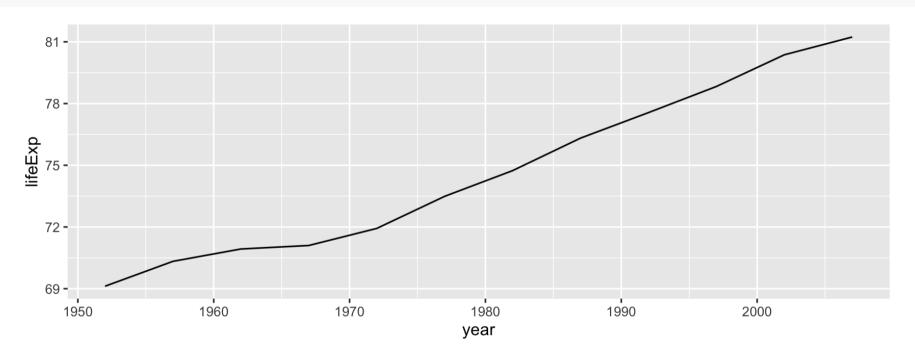
- There generally appears to be an increase in life expectancy
- A number of countries have big dips from the 70s through 90s
- a cluster of countries starts off with low life expectancy but ends up close to the highest by the end of the period.

Gapminder: Australia

Australia was already had one of the top life expectancies in the 1950s.

```
oz <- gapminder %>% filter(country == "Australia")
ΟZ
## # A tibble: 12 x 6
##
     country continent year lifeExp pop gdpPercap
     <fct>
               <fct>
                                                    <db1>
##
                         <int>
                                 <db1>
                                       <int>
   1 Australia Oceania
                          1952
                                  69.1
                                       8691212
                                                   10040.
   2 Australia Oceania
                          1957
                                  70.3
                                       9712569
                                                   10950.
   3 Australia Oceania
                          1962
                                  70.9 10794968
                                                   12217.
   4 Australia Oceania
                                                   14526.
                          1967
                                  71.1 11872264
   5 Australia Oceania
                          1972
                                  71.9 13177000
                                                   16789.
   6 Australia Oceania
                          1977
                                  73.5 14074100
                                                   18334.
   7 Australia Oceania
                          1982
                                  74.7 15184200
                                                   19477.
   8 Australia Oceania
                          1987
                                  76.3 16257249
                                                   21889.
   9 Australia Oceania
                          1992
                                  77.6 17481977
                                                   23425.
   10 Australia Oceania
                          1997
                                  78.8 18565243
                                                   26998.
```

Gapminder: Australia



Gapminder: Australia

Tidy Gapminder Australia

$$\widehat{lifeExp} = -376.1163 - 0.2277 \ year$$

Center year

- Let us treat 1950 is the first year
- so for model fitting we are going to shift year to begin in 1950
- This improved interpretability.

```
gap <- gapminder %>% mutate(year1950 = year - 1950)
oz <- gap %>% filter(country == "Australia")
```

Model for centered year

```
oz_lm <- lm(lifeExp ~ year1950, data = oz)

oz_lm

##

## Call:
## lm(formula = lifeExp ~ year1950, data = oz)
##

## Coefficients:
## (Intercept) year1950
## 67.9451 0.2277</pre>
```

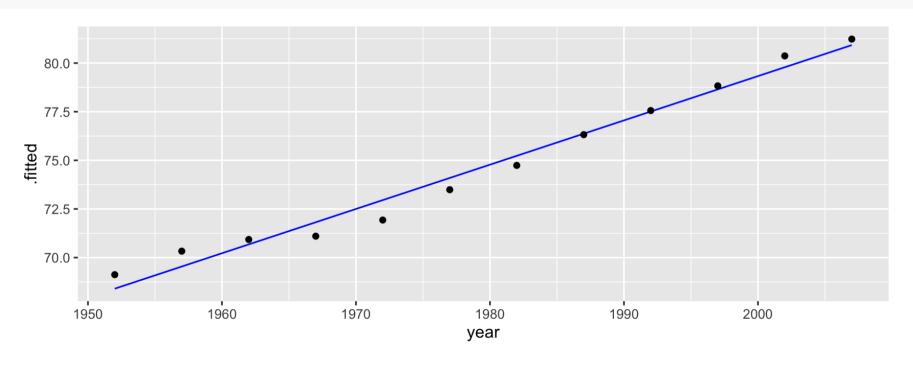
Tidy the model

$$\widehat{lifeExp} = 67.9 + 0.2277 \ year$$

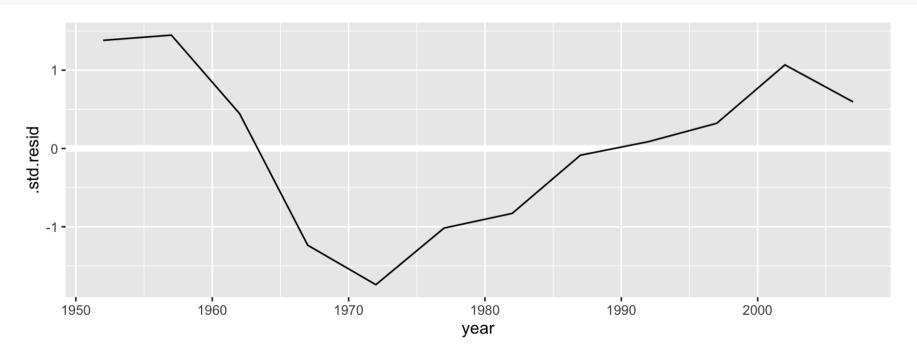
Augment

```
oz_aug <- augment(oz_lm, oz)
oz_aug
## # A tibble: 12 x 14
                                           pop gdpPercap year1950 .fitted .se.fit
##
      country continent
                          year lifeExp
##
      <fct> <fct>
                         <int>
                                  <dbl> <int>
                                                    <db1>
                                                              <db1>
                                                                       <db1>
                                                                               <db1>
                                                                                        <dbl
                                                                               0.337
##
    1 Austra... Oceania
                          1952
                                   69.1 8.69e6
                                                   10040.
                                                                        68.4
                                                                                       0.719
                                   70.3 9.71e6
##
    2 Austra... Oceania
                          1957
                                                   10950.
                                                                        69.5
                                                                               0.294
                                                                                       0.791
##
    3 Austra... Oceania
                          1962
                                   70.9 1.08e7
                                                   12217.
                                                                 12
                                                                        70.7
                                                                               0.255
                                                                                       0.252
##
    4 Austra... Oceania
                          1967
                                   71.1 1.19e7
                                                   14526.
                                                                 17
                                                                        71.8
                                                                               0.221 - 0.716
##
    5 Austra... Oceania
                          1972
                                   71.9 1.32e7
                                                   16789.
                                                                 22
                                                                        73.0
                                                                               0.195 - 1.02
##
    6 Austra... Oceania
                          1977
                                   73.5 1.41e7
                                                   18334.
                                                                 27
                                                                        74.1
                                                                               0.181 - 0.604
    7 Austra... Oceania
                          1982
                                   74.7 1.52e7
                                                   19477.
                                                                 32
                                                                        75.2
                                                                               0.181 - 0.492
##
##
    8 Austra... Oceania
                          1987
                                   76.3 1.63e7
                                                   21889.
                                                                 37
                                                                        76.4
                                                                               0.195 - 0.050
    9 Austra... Oceania
                          1992
                                   77.6 1.75e7
                                                   23425.
                                                                 42
                                                                        77.5
                                                                               0.221
                                                                                       0.050
   10 Austra... Oceania
                          1997
                                   78.8 1.86e7
                                                   26998.
                                                                 47
                                                                        78.6
                                                                               0.255
                                                                                      0.182
   11 Austra... Oceania
                          2002
                                   80.4 1.95e7
                                                   30688.
                                                                 52
                                                                        79.8
                                                                               0.294
                                                                                      0.583
   12 Austra... Oceania
                          2007
                                   81.2 2.04e7
                                                   34435.
                                                                 57
                                                                        80.9
                                                                               0.337
                                                                                       0.310
## # ... with 4 more variables: .hat <dbl>, .sigma <dbl>, .cooksd <dbl>, .std.resid <dbl
```

Plot fitted against values



Plot standardised residuals against year



Making inferences from this

- Life expectancy has increased 2.3 years every decade, on average.
- There was a slow period from 1960 through to 1972, probably related to mortality during the Vietnam war.

Can we fit for New Zealand?

Can we fit for Japan?

Can we fit for Italy?

Is there a better way?

Like, what if we wanted to fit a model for ALL countries? Let's tinker with the data.

nest() country level data (one row = one country)

```
by_country <- gap %>%
  select(country, year1950, lifeExp, continent) %>%
 group_by(country, continent) %>%
  nest()
by_country
## # A tibble: 142 x 3
## # Groups: country, continent [710]
##
    country continent data
   <fct> <fct> <fct> <fct> <
##
   1 Afghanistan Asia <tibble [12 × 2]>
   2 Albania Europe \langle \text{tibble } [12 \times 2] \rangle
   3 Algeria Africa <tibble [12 \times 2]>
##
   4 Angola Africa <tibble [12 × 2]>
##
   5 Argentina Americas <tibble [12 \times 2]>
##
   6 Australia Oceania
                         <tibble [12 × 2]>
   7 Austria Europe
                         <tibble [12 × 2]>
##
   8 Bahrain Asia <tibble [12 × 2]>
##
   9 Bangladesh Asia <tibble [12 × 2]>
```

What is in data?

```
by_country$data[[1]]
## # A tibble: 12 x 2
      year1950 lifeExp
##
         <db1>
##
                 <db1>
                  28.8
##
##
                  30.3
            12
                  32.0
##
            17
                  34.0
            22
                  36.1
##
            27
                  38.4
##
            32
                  39.9
            37
                  40.8
##
                  41.7
                  41.8
##
                  42.1
## 12
            57
                  43.8
```

It's a list!

fit a linear model to each one?

```
lm_afganistan <- lm(lifeExp ~ year1950, data = by_country$data[[1]])
lm_albania <- lm(lifeExp ~ year1950, data = by_country$data[[2]])
lm_algeria <- lm(lifeExp ~ year1950, data = by_country$data[[3]])</pre>
```

But we are copying and pasting this code **more than twice**...is there a better way?

A case for our friend, map ... ???

```
map(<data object>, <function>)
```

A case for map ???

```
mapped_lm \leftarrow map(.x = by_country$data,
                  .f = function(x){
                    lm(lifeExp \sim year1950, data = x)
                    })
mapped_lm
## [[1]]
##
## Call:
## lm(formula = lifeExp \sim year1950, data = x)
##
## Coefficients:
## (Intercept) year1950
   29.3566 0.2753
##
##
##
## [[2]]
##
## Call:
```

Map inside the data?

```
country_model <- by_country %>%
 mutate(model = map(.x = data,
                 .f = function(x){
                   lm(lifeExp \sim year1950, data = x)
   }))
country_model
## # A tibble: 142 x 4
## # Groups: country, continent [710]
##
  country continent data model
## <fct> <fct> <list> <list>
  2 Albania Europe <tibble [12 × 2]> <lm>
##
  3 Algeria Africa <tibble [12 × 2]> <lm>
##
##
   4 Angola Africa <tibble [12 × 2]> <lm>
   5 Argentina
             Americas <tibble [12 × 2]> <lm>
##
  6 Australia Oceania <tibble [12 × 2]> <lm>
##
             Europe <tibble [12 \times 2]> <lm>
##
  7 Austria
##
   8 Bahrain Asia <tibble [12 × 2]> <lm>
```

A case for map (shorthand function)

```
country_model <- by_country %>%
 mutate(model = map(.x = data,
                  .f = \sim lm(lifeExp \sim year1950, data = .)))
country_model
## # A tibble: 142 x 4
## # Groups: country, continent [710]
## country continent data model
## <fct> <fct> <list> <list>
  1 Afghanistan Asia <tibble [12 × 2]> <lm>
   2 Albania Europe <tibble [12 × 2]> <lm>
##
  3 Algeria Africa <tibble [12 × 2]> <lm>
##
   4 Angola Africa <tibble [12 × 2]> <lm>
##
  5 Argentina Americas <tibble [12 × 2]> <lm>
##
   6 Australia Oceania
                        <tibble [12 × 2]> <lm>
                       <tibble [12 × 2]> <lm>
##
   7 Austria Europe
  8 Bahrain Asia <tibble [12 × 2]> <lm>
##
   9 Bangladesh Asia <tibble [12 × 2]> <lm>
                      <tibble [12 × 2]> <lm>
  10 Belgium
             Europe
```

Where's the model?

```
country_model$model[[1]]

##

## Call:
## lm(formula = lifeExp ~ year1950, data = .)
##

## Coefficients:
## (Intercept)  year1950
## 29.3566  0.2753
```

We need to summarise this content

So should we repeat it for each one?

```
tidy(country_model$model[[1]])
## # A tibble: 2 x 5
  term estimate std.error statistic p.value
##
## <chr> <dbl> <dbl> <dbl> <dbl>
## 1 (Intercept) 29.4 0.699 42.0 1.40e-12
## 2 year1950 0.275 0.0205 13.5 9.84e- 8
tidy(country_model$model[[2]])
## # A tibble: 2 x 5
## term estimate std.error statistic p.value
## <chr> <dbl> <dbl> <dbl> <dbl>
## 1 (Intercept) 58.6 1.13 51.7 1.79e-13
## 2 year1950 0.335 0.0332 10.1 1.46e- 6
tidy(country_model$model[[3]])
## # A tibble: 2 x 5
##
   term estimate std.error statistic p.value
##
   <chr> <dbl>
                    <dbl> <dbl> <dbl> <dbl>
```

Use map

```
country_model %>%
   mutate(tidy = map(model, tidy))
## # A tibble: 142 x 5
## # Groups: country, continent [710]
## country continent data model tidy
## <fct> <fct> <list> <list>
    1 Afghanistan Asia \langle \text{tibble } [12 \times 2] \rangle \langle \text{lm} \rangle \langle \text{tibble } [2 \times 5] \rangle
     2 Albania Europe \langle \text{tibble } [12 \times 2] \rangle \langle \text{lm} \rangle \langle \text{tibble } [2 \times 5] \rangle
##
## 3 Algeria Africa \langle \text{tibble } [12 \times 2] \rangle \langle \text{lm} \rangle \langle \text{tibble } [2 \times 5] \rangle
##
     4 Angola Africa \langle \text{tibble } [12 \times 2] \rangle \langle \text{lm} \rangle \langle \text{tibble } [2 \times 5] \rangle
    5 Argentina Americas <tibble [12 \times 2]> <lm> <tibble [2 \times 5]>
##
## 6 Australia Oceania
                                     <tibble [12 × 2]> <lm> <tibble [2 × 5]>
    7 Austria Europe
                                      <tibble [12 × 2]> <lm> <tibble [2 × 5]>
##
    8 Bahrain Asia \langle \text{tibble } [12 \times 2] \rangle \langle \text{lm} \rangle \langle \text{tibble } [2 \times 5] \rangle
##
## 9 Bangladesh Asia \langle \text{tibble } [12 \times 2] \rangle \langle \text{lm} \rangle \langle \text{tibble } [2 \times 5] \rangle
## 10 Belgium Europe \langle \text{tibble } [12 \times 2] \rangle \langle \text{lm} \rangle \langle \text{tibble } [2 \times 5] \rangle
## # ... with 132 more rows
```

unnest

```
country_coefs <- country_model %>%
 mutate(tidy = map(model, tidy)) %>%
 unnest(tidy) %>%
 select(country, continent, term, estimate)
country_coefs
## # A tibble: 284 x 4
## # Groups: country, continent [710]
##
    country continent term estimate
  <fct> <fct> <fct> <dbl>
##
  1 Afghanistan Asia (Intercept) 29.4
   2 Afghanistan Asia year1950 0.275
   3 Albania Europe
                      (Intercept) 58.6
##
   4 Albania Europe
                      year1950 0.335
##
   5 Algeria Africa
                      (Intercept) 42.2
##
   6 Algeria Africa
                      year1950 0.569
##
   7 Angola Africa
                      (Intercept) 31.7
##
   8 Angola Africa
                      year1950 0.209
##
   9 Argentina Americas
                       (Intercept)
                                  62.2
```

Pivot the term

```
tidy_country_coefs <- country_coefs %>%
 pivot_wider(id_cols = c(term, country, continent),
            names_from = term,
            values_from = estimate) %>%
 rename(intercept = `(Intercept)`)
tidy_country_coefs
## # A tibble: 142 x 4
## # Groups: country, continent [710]
##
     country continent intercept year1950
##
  <fct> <fct>
                            <db1>
                                    <db1>
   1 Afghanistan Asia
                            29.4
                                   0.275
##
   2 Albania Europe
                            58.6
                                   0.335
##
   3 Algeria Africa
                            42.2
                                    0.569
   4 Angola Africa
##
                            31.7
                                    0.209
   5 Argentina
              Americas
                            62.2
                                    0.232
##
   6 Australia Oceania
                                   0.228
##
                            67.9
##
  7 Austria
              Europe
                            66.0
                                    0.242
##
   8 Bahrain
               Asia
                            51.8
                                    0.468
```

Filter to only Australia

```
tidy_country_coefs %>%
  filter(country == "Australia")

## # A tibble: 1 x 4

## Groups: country, continent [710]

## country continent intercept year1950

## <fct> <fct> <dbl> <dbl>
## 1 Australia Oceania 67.9 0.228
```

Your turn: Five minute challenge

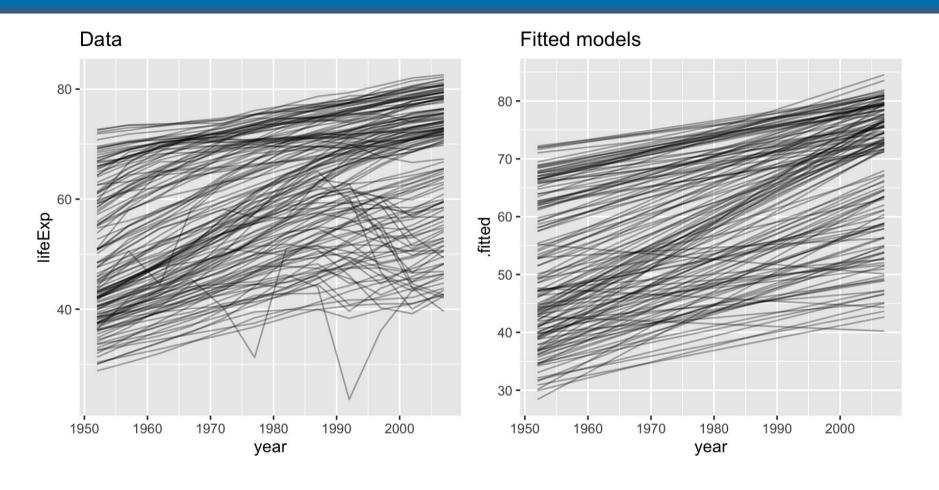
- Fit the models to all countries
- Pick your favourite country (not Australia), print the coefficients, and make a hand sketch of the the model fit.

Plot all the models

```
country_aug <- country_model %>%
  mutate(augmented = map(model, augment)) %>%
  unnest(augmented)
country_aug
## # A tibble: 1,704 x 13
  # Groups: country, continent [710]
##
      country continent data model lifeExp year1950 .fitted .se.fit .resid
                                                                                   .hat .s
##
      <fct>
              <fct>
                         <</li>
                                        <db1>
                                                 <db1>
                                                          <db1>
                                                                  <dbl>
                                                                          <db1>
                                                                                  <dbl> <
##
    1 Afghan... Asia
                         <tib... <1m>
                                         28.8
                                                          29.9
                                                                  0.664 - 1.11
                                                                                 0.295
##
    2 Afghan... Asia
                         <tib... <1m>
                                        30.3
                                                           31.3
                                                                  0.580 - 0.952
                                                                                 0.225
##
    3 Afghan... Asia
                         <tib... <lm>
                                         32.0
                                                    12
                                                           32.7
                                                                  0.503 - 0.664
                                                                                 0.169
##
    4 Afghan... Asia
                         <tib... <1m>
                                         34.0
                                                    17
                                                           34.0
                                                                  0.436 - 0.0172
                                                                                 0.127
##
    5 Afghan... Asia
                         <tib... <1m>
                                         36.1
                                                    22
                                                           35.4
                                                                  0.385
                                                                         0.674
                                                                                 0.0991
##
    6 Afghan... Asia
                         <tib... <1m>
                                         38.4
                                                    27
                                                           36.8
                                                                  0.357
                                                                         1.65
                                                                                 0.0851
    7 Afghan... Asia
                         <tib... <lm>
                                         39.9
                                                    32
                                                           38.2
                                                                  0.357
                                                                         1.69
                                                                                 0.0851
##
##
    8 Afghan... Asia
                         <tib... <1m>
                                         40.8
                                                    37
                                                           39.5
                                                                  0.385
                                                                         1.28
                                                                                 0.0991
##
    9 Afghan... Asia
                         <tib... <1m>
                                         41.7
                                                    42
                                                           40.9
                                                                  0.436
                                                                         0.754
                                                                                 0.127
   10 Afghan... Asia
                         <tib... <1m>
                                         41.8
                                                    47
                                                           42.3
                                                                  0.503 - 0.534
                                                                                 0.169
```

Plot all the models

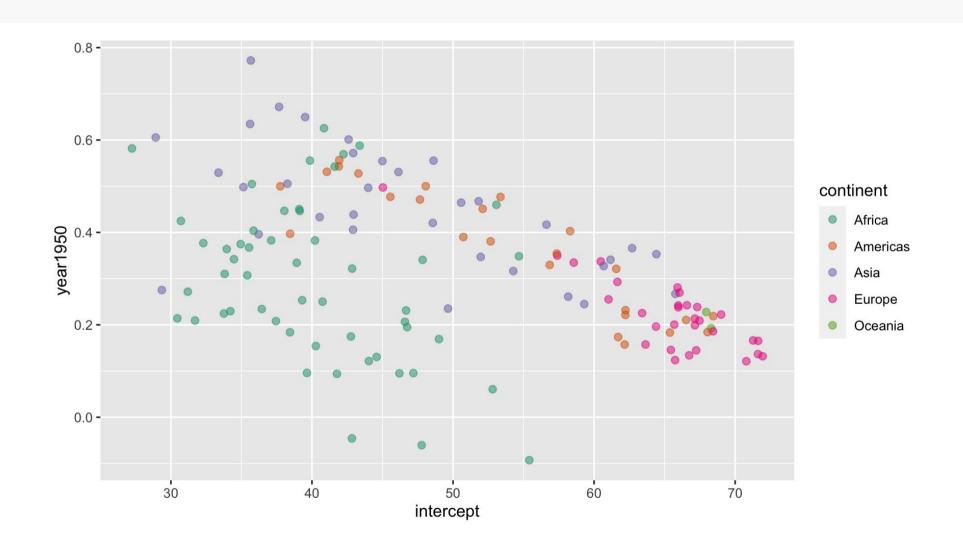
Plot all the models



Plot all the model coefficients

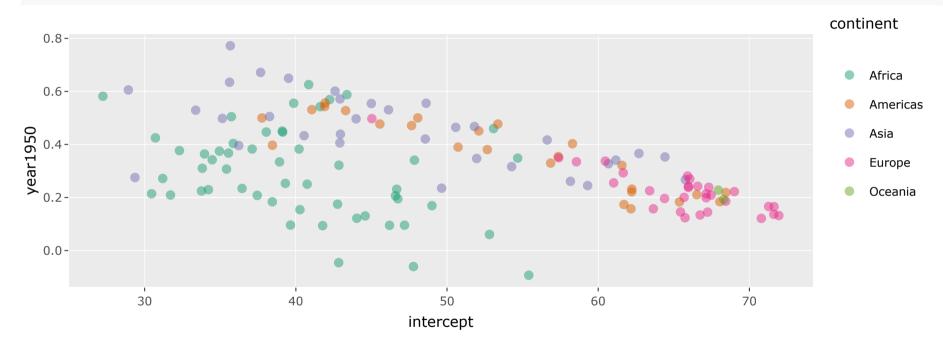
Plot all the model coefficients





Make it interactive!

library(plotly)
ggplotly(p)



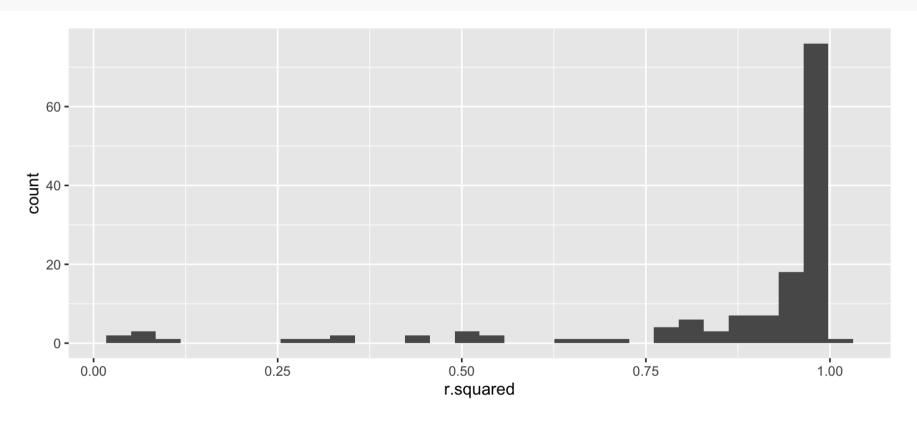
Let's summarise the information learned from the model coefficients.

- Generally the relationship is negative: this means that if a country started with a high intercept tends to have lower rate of increase.
- There is a difference across the continents: Countries in Europe and Oceania tended to start with higher life expectancy and increased; countries in Asia and America tended to start lower but have high rates of improvement; Africa tends to start lower and have a huge range in rate of change.
- Three countries had negative growth in life expectancy: Rwand, Zimbabwe, Zambia

Model diagnostics by country

```
country_glance <- country_model %>%
  mutate(glance = map(model, glance)) %>%
  unnest(glance)
country_glance
## # A tibble: 142 x 15
## # Groups: country, continent [710]
##
     country continent data model r.squared adj.r.squared sigma statistic p.value
##
     <fct> <fct>
                       <</li>
                                      <db1>
                                                    <dbl> <dbl>
                                                                   <db1>
                                                                            <dbl> <
   1 Afghan... Asia <tib... <lm>
                                      0.948
                                                    0.942 1.22
                                                                   181.
                                                                         9.84e-8
##
   2 Albania Europe <tib... <lm>
                                      0.911
                                                    0.902 1.98
                                                                   102. 1.46e- 6
##
   3 Algeria Africa <tib... <lm>
                                      0.985
                                                    0.984 1.32
                                                                   662. 1.81e-10
##
   4 Angola Africa
                       <tib... <1m>
                                      0.888
                                                    0.877 1.41
                                                                    79.1 4.59e- 6
                                                                  2246. 4.22e-13
##
   5 Argent... Americas
                       <tib... <1m>
                                      0.996
                                                    0.995 0.292
##
   6 Austra... Oceania
                       <tib... <1m>
                                      0.980
                                                    0.978 0.621
                                                                   481.
                                                                         8.67e-10
   7 Austria Europe
                       <tib... <1m>
                                      0.992
                                                    0.991 0.407
                                                                  1261. 7.44e-12
##
                       <tib... <1m>
##
   8 Bahrain Asia
                                      0.967
                                                    0.963 1.64
                                                                   291. 1.02e- 8
##
   9 Bangla… Asia
                       <tib... <1m>
                                      0.989
                                                    0.988 0.977
                                                                   930. 3.37e-11
   10 Belgium Europe
                     <tib... <1m>
                                      0.995
                                                    0.994 0.293
                                                                  1822. 1.20e-12
```

Plot the \mathbb{R}^2 values as a histogram.



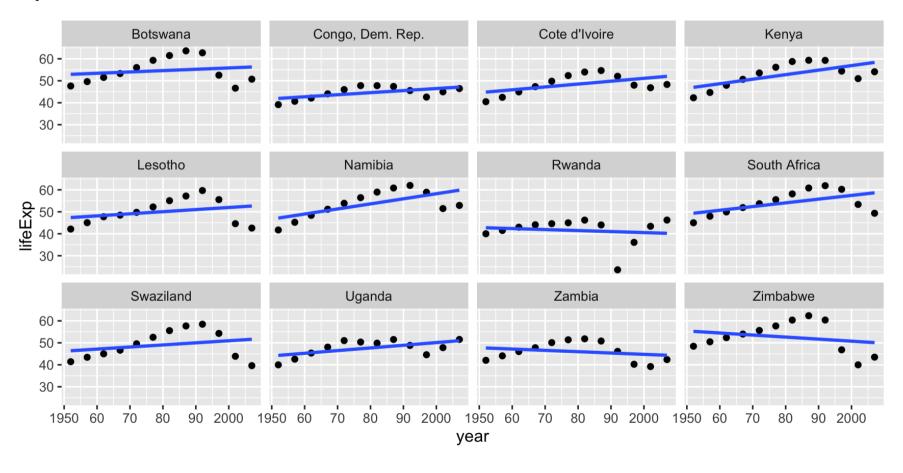
Countries with worst fit

Examine the countries with the worst fit, countries with $R^2 < 0.45$, by making scatterplots of the data, with the linear model overlaid.

```
badfit <- country_glance %>% filter(r.squared <= 0.45)</pre>
gap_bad <- gap %>% filter(country %in% badfit$country)
gg_bad_fit <-
ggplot(data = gap_bad,
       aes(x = year)
           y = lifeExp)) +
         geom_point() +
  facet_wrap(~country) +
  scale_x_continuous(breaks = seq(1950, 2000, 10),
                      labels = c("1950", "60", "70", "80", "90", "2000")) +
  geom_smooth(method = "lm",
              se = FALSE
```

Countries with worst fit

Each of these countries had been moving on a nice trajectory of increasing life expectancy, and then suffered a big dip during the time period.



Your Turn:

- Use google to explain these dips using world history and current affairs information.
- finish the lab exercise (with new data)
- once you are done, you can collect mid semester exam
- remember the project deadline: Find team members, and potential topics to study (List of groups will be posted here)