# The bbob-constrained COCO Test Suite

This document briefly describes the bbob-constrained test suite implemented in the COCO software. It contains everything one needs to know if planning to benchmark an algorithm on the constrained test suite. If you want to know more about the COCO software, please go to the GitHub page and if you want to cite this work, please refer to the following paper:

Paul Dufossé, Nikolaus Hansen, Dimo Brockhoff, Phillipe R. Sampaio, Asma Atamna, and Anne Auger. Building scalable test problems for benchmarking constrained optimizers. *to be submitted to SIAM Journal of Optimization*, 2022

## Contents



| Number of constraints | | | 1 | 3 | 9 | $9+\lfloor 3n/4\rfloor$ | $9+\lfloor 3n/2\rfloor$ | $9+\lfloor 9n/2\rfloor$ |
|---|---|---|---|---|---|---|---|---|
| Number of active constraints | | | 1 | 2 | 6 | $6+\lfloor n/2\rfloor$ | $6+n$ | $6+3n$ |
| Objective | $T$ | $c_{\text{scal}}$ | Function IDs | | | | | |
| $f_{\text{sphere}}$ | id | 10 | 1 | 2 | 3 | 4 | 5 | 6 |
| $f_{\text{ellipsoid}}$ | $T_{\text{osz}}$ | $10^{-4}$ | 7 | 8 | 9 | 10 | 11 | 12 |
| $f_{\text{linear}}$ | id | 10 | 13 | 14 | 15 | 16 | 17 | 18 |
| $f_{\text{elli\_rot}}$ | $T_{\text{osz}}$ | $10^{-4}$ | 19 | 20 | 21 | 22 | 23 | 24 |
| $f_{\text{discus}}$ | $T_{\text{osz}}$ | $10^{-4}$ | 25 | 26 | 27 | 28 | 29 | 30 |
| $f_{\text{bent\_cigar}}$ | $T_{\text{asy}}^{0.5}$ | $10^{-4}$ | 31 | 32 | 33 | 34 | 35 | 36 |
| $f_{\text{diff\_powers}}$ | id | $10^{-2}$ | 37 | 38 | 39 | 40 | 41 | 42 |
| $f_{\text{rastrigin}}$ | $T_{\text{asy}}^{0.2}\circ T_{\text{osz}}$ | 10 | 43 | 44 | 45 | 46 | 47 | 48 |
| $f_{\text{rast\_rot}}$ | $T_{\text{asy}}^{0.2}\circ T_{\text{osz}}$ | 10 | 49 | 50 | 51 | 52 | 53 | 54 |

Table 1: Identifiers of the bbob-constrained problems. The first two rows indicate the number of total and active constraints while the first column indicates the objective functions and the second column the transformations (described in Non-linear transformations) attached to each objective, where $\mathrm{id}(x) = x$. The 2-D contour plots of the first instance of each test problem can be found in the Appendix or by clicking the objective function name. Other instances in 2-D can be visualized at this link.

## Problem definition

A constrained test problem is written in the standard form

$$\underset{x\in\mathcal{X}}{\text{minimize}} \quad f(x) \qquad \text{subject to} \quad g_k(x) \le 0 \quad \text{for } k = 1,\ldots,m \tag{1}$$

where $\mathcal{X} = [-5,5]^n$, $n$ is the dimension of the search space and $m$ is the number of generally non-linear constraints.

To create the constrained problems, each one of the 9 objective functions is composed with a non-linear transformation of the search space defined previously and is combined with $m' \in \{1, 2, 6, 6+\lfloor n/2\rfloor, 6+n, 6+3n\}$ active constraints and

$\lfloor m'/2 \rfloor$ inactive constraints, having overall $m = \lfloor 3m'/2 \rfloor$ constraints. This results in 54 constrained test problems, $f_i$, $i = 1, \ldots, 54$, summarized in Table 1. In practice, $x^{\mathrm{opt}}$ is sampled randomly to define different instances of the same constrained problem. Additionally, the objective functions are scaled down by a factor $c_{\mathrm{scal}} > 0$ which is specific to each function. The final optimization problem writes:

$$\underset{x \in \mathcal{X}}{\text{minimize}} \quad c_{\mathrm{scal}} f(v) \qquad \text{subject to} \quad g_i(v) \leq 0 \quad \text{for } i = 1, \ldots, m \tag{2}$$

where $v = T(x - x^{\mathrm{opt}})$ and $T$ is the non-linear transformation reported in Table 1 along with $c_{\mathrm{scal}}$ and the number of constraints associated to each problem. The considered dimensions are the same as for the bbob test suite, that is $n \in \{2, 3, 5, 10, 20, 40\}$.

# Principles of the construction

The theory allows to control the location of the constrained optimum $y^\star \in \mathbb{R}^n$, provided that its gradient is non-zero. In the construction, $f$ is an already shifted BBOB function, hence taking $y^\star = 0$ generally provides $\nabla f(y^\star) \neq 0$ (in some cases, e.g. the Rastrigin function, some additional conditions on $y^\star$ are needed [2]). From this point, following the gradient direction, an initial solution is determined

$$x^{\mathrm{init}} = y^\star + \rho \nabla f(y^\star) + x^{\mathrm{opt}} \tag{3}$$

where $x^{\mathrm{opt}} \in \mathcal{X}$ defines another (random) translation such that the global minimum of the constrained problem is not in $y^\star = 0$ and $\rho > 0$ is a scaling factor ensuring that $x^{\mathrm{init}} \in \mathcal{X}$.

# Objective functions

The bbob-constrained test suite brings together the aforementioned methodology with several existing COCO "raw" functions already used in the unconstrained setting. They are described in Table 2.

| Function name | Formulation | Transformations |
| --- | --- | --- |
| Sphere | $f_{\text{sphere}}(x) = z^\top z + f_{\text{uopt}}$ | $z = x - x^{\text{uopt}}$ |
| Separable ellipsoid | $f_{\text{ellipsoid}}(x) = \sum_{i=1}^{n} 10^{6\frac{i-1}{n-1}} z_i^2 + f_{\text{uopt}}$ | $z = x - x^{\text{uopt}}$ |
| Linear slope | $f_{\text{linear}}(x) = \sum_{i=1}^{n} 5\lvert s_i\rvert - s_i z_i + f_{\text{uopt}}$ | $s_i = \text{sign}(x_i^{\text{uopt}})10^{\frac{i-1}{n-1}}$ <br> $z_i = \begin{cases} x_i & \text{if } x_i^{\text{uopt}} x_i < 5^2 \\ x_i^{\text{uopt}} & \text{otherwise} \end{cases}$ <br> for $i = 1, \ldots, n$ |
| Rotated ellipsoid | $f_{\text{elli\_rot}}(x) = \sum_{i=1}^{n} 10^{6\frac{i-1}{n-1}} z_i^2 + f_{\text{uopt}}$ | $z = R(x - x^{\text{uopt}})$ |
| Discus | $f_{\text{discus}}(x) = 10^6 z_1^2 + \sum_{i=2}^{n} z_i^2 + f_{\text{uopt}}$ | $z = R(x - x^{\text{uopt}})$ |
| Bent cigar | $f_{\text{bent\_cigar}}(x) = z_1^2 + 10^6 \sum_{i=2}^{n} z_i^2 + f_{\text{uopt}}$ | $z = R(x - x^{\text{uopt}})$ |
| Different powers | $f_{\text{diff\_powers}}(x) = \sqrt{10^6 \sum_{i=1}^{n} \lvert z_i\rvert^{2+4\frac{i-1}{n-1}}} + f_{\text{uopt}}$ | $z = R(x - x^{\text{uopt}})$ |
| Rastrigin | $f_{\text{rastrigin}}(x) = 10\left(n - \sum_{i=1}^{n} \cos(2\pi z_i)\right) + z^\top z + f_{\text{uopt}}$ | $z = x - x^{\text{uopt}}$ |
| Rotated Rastrigin | $f_{\text{rast\_rot}}(x) = 10\left(n - \sum_{i=1}^{n} \cos(2\pi z_i)\right) + z^\top z + f_{\text{uopt}}$ | $z = R(x - x^{\text{uopt}})$ |

Table 2: COCO raw functions definition: formulation and search space transformations, $x^{\text{uopt}} \in \mathbb{R}^n$ is a randomly sampled vector locating the unconstrained minimum of the objective function such that $f(x^{\text{uopt}}) = f_{\text{uopt}} \in \mathbb{R}$. Considering rotations, $R \in \mathbb{R}^{n\times n}$ is a randomly sampled orthogonal matrix.

## Performance Assessment

In order to assess the performance of optimization algorithms, we record the number of objective and constraint evaluations to reach a certain target value.

The target definition encompasses objective minimization and constraint satisfaction together using the merit function

$$\tilde{f}(x) := \max(f_{\text{opt}}, f(x)) + \sum_{k=0}^{m} \max(0, g_k(x)) \ , \tag{4}$$

where $f_{\text{opt}} = f(x^{\text{opt}})$. It is easy to see that $\min \tilde{f} = f_{\text{opt}}$.

The estimated Expected Runtime (ERT) [3] is computed from feasible points only and for 7 target values $f_{\text{target}} = f_{\text{opt}} + 10^i$, with $i \in \{1, 0, -1, -2, -3, -5, -6\}$.

Data points for the ECDFS [3] are collected for 41 target values $f_{\text{target}} = f_{\text{opt}} + 10^i$, with $i$ evenly distributed in $[-6, 2]$.

## Running an experiment

A code snippet of how to run a Python solver on a COCO constrained test problem is reproduced in Listing 1. Running a full experiment requires, among other things, to attach an observer to the current problem, control the budget (number of objective and constraints evaluations) and loop over all problems of the test suite.

The interface yields the feasible starting point $x^{\text{init}}$ under problem.initial_solution. If the experimental design includes restarts, another feasible starting can be sampled from $x^{\text{init}}$ following the procedure described in Algorithm 1.

```python
import cocoex
from numpy import concatenate as concat
from scipy.optimize import fmin_cobyla

def instantiate_constraint(problem):
   def constraint(x):
      # Switches constraint sign and add the problem bounds
      return concat((-problem.constraint(x),
                     x - problem.lower_bounds,
                     problem.upper_bounds - x))
   return constraint

suite = cocoex.Suite('bbob-constrained', '', '')
problem = suite.get_problem_by_function_dimension_instance(
            17, 20, 19)  # instance 19 of function 17 in 20-D
x_start = problem.initial_solution
fmin_cobyla(
   problem, x_start, instantiate_constraint(problem),
   # solver parameters
   rhobeg=2, rhoend=1e-8, maxfun=1e4)
```

Listing 1: A simple run of SciPy's COBYLA on a single test problem.

**Algorithm 1** Sampling a new feasible starting point

**Require:** $x^{\text{init}}$ and $\sigma > 0$
1: **while** True **do**
2: Sample $z \sim \mathcal{N}(0, \text{id})$ and set $x = x^{\text{init}} + \sigma z$
3: **if** $x \notin \mathcal{X}$ or any $g_k(x) \geq 0$ for $k = 1, \ldots, m$ **then**
4: $\sigma \leftarrow \sigma/2$
5: **else**
6: return $x$ as the new starting point

An exhaustive example of how to run a full experiment can be found in the COCO repository[1].

[1] See coco/code-experiments/build/python/example_experiment2.py after installation of COCO

# Supplementary material

## Construction of the feasible region

The construction of any problem of the test suite is detailed in the full paper [2] and relies on Theorem 4.2.16 in [1, p. 195] stating that, if $f$ is *pseudo-convex* at $y^\star$, and $g_k$ is differentiable and *quasi-convex* at $y^\star$ for all $k = 1, \dots, m$, then the KKT conditions are sufficient to ensure that $y^\star$ is a global minimum of the constrained optimization problem. It is easy to show that, if $f$ is *strictly* pseudo-convex at $y^\star$, then the global minimum is also unique [2]. We refer to the textbook of Bazaraa et al. [1] for the definitions and properties of pseudo- and quasi-convexity.

The algorithm to generate an instance of a constrained problem first builds a set of $m$ *linear* constraints, of the form $g_k(x) = \alpha_k a_k^\top (x - y^\star) + \beta_k$ by setting $a_k \in \mathbb{R}^n$ where $\alpha_k > 0, \beta_k \geq 0 \in \mathbb{R}$ are input parameters. The first constraint is chosen such that $a_1 = -\alpha_1 \nabla f(y^\star)/\|\nabla f(y^\star)\|$ and $\beta_1 = 0$ hence $g_1(y^\star) = 0$. If $m = 1$, this is the only choice to satisfy the stationarity condition in the KKT equations. The remaining constraints are randomly sampled from an isotropic multivariate normal distribution. If $g_k(y^\star + \rho \nabla f(y^\star)) > 0$, the sign of $a_k$ is flipped to make sure that $y^\star + \rho \nabla f(y^\star)$ is feasible. A constraint is inactive at the constructed optimum if $\beta_k > 0$.

The Lagrange multipliers associated to $y^\star$ when the constraints are generated according to the aforementioned procedure are $[1, 0, \dots, 0] \in \mathbb{R}^m$. A second step of the algorithm modifies the set of constraints gradients in a randomized manner such that the first constraint is not any more aligned with the gradient direction at the optimum. The constraints' indices are also shuffled.

## Non-linear transformations

Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a bijective transformation of the search space with inverse $T^{-1}$. Then $T^{-1}(y^\star)$ is the global optimum of the problem

$$\underset{x \in \mathcal{X}}{\text{minimize}} \quad f(T(x)) \qquad \text{subject to} \quad g_i(T(x)) \leq 0 \quad i = 1, \dots, m \tag{5}$$

if $y^\star$ is the global optimum of the original problem as described in Equation (1). This allows to apply non-linear transformations of the search space to the set of constructed linear constraints. For practical reasons, we consider in this work bijective transformations $T$ that have $y^\star$ as fixed point, i.e. $T(y^\star) = y^\star$. As a consequence, the optimum of the transformed problem in Equation (5) remains $y^\star$. This way, we avoid computing the inverse transformation at $y^\star$. The non-linear transformations implemented in the test suite are coordinate-wise, defined as $T^\beta_{\text{asy}}$, where $\beta > 0$, and $T_{\text{osz}}$, defined as follows:

$$T^\beta_{\text{asy}}(x) = \left( \begin{cases} x_i^{1+\beta \frac{i-1}{n-1}\sqrt{x_i}} & \text{if } x_i > 0 \\ x_i & \text{otherwise} \end{cases} \right)_{i=1,\dots,n} , \tag{6}$$

$$T_{\text{osz}}(x) = \left( \text{sign}(x_i) \exp(\hat{x}_i + 0.049(\sin(c_1 \hat{x}_i) + \sin(c_2 \hat{x}_i))) \right)_{i=1,\dots,n} , \tag{7}$$

with $\hat{x}_i = \log(|x_i|)\mathbb{1}_{\{x_i \neq 0\}}$ and $\text{sign}(x_i)$ is the sign function with convention $\text{sign}(0) = 0$. The value for the constants $c_1$ and $c_2$ are $c_1 = \begin{cases} 10 & \text{if } x_i > 0 \\ 5.5 & \text{otherwise} \end{cases}$, and $c_2 = \begin{cases} 7.9 & \text{if } x_i > 0 \\ 3.1 & \text{otherwise} \end{cases}$. The idea is that $T^\beta_{\text{asy}}$ introduces asymmetry, as it only changes positive coordinates, and $T_{\text{osz}} : \mathbb{R}^n \to \mathbb{R}^n$ oscillates around the origin. Both transformations are bijective, continuous, sign-preserving hence have fixed point 0.

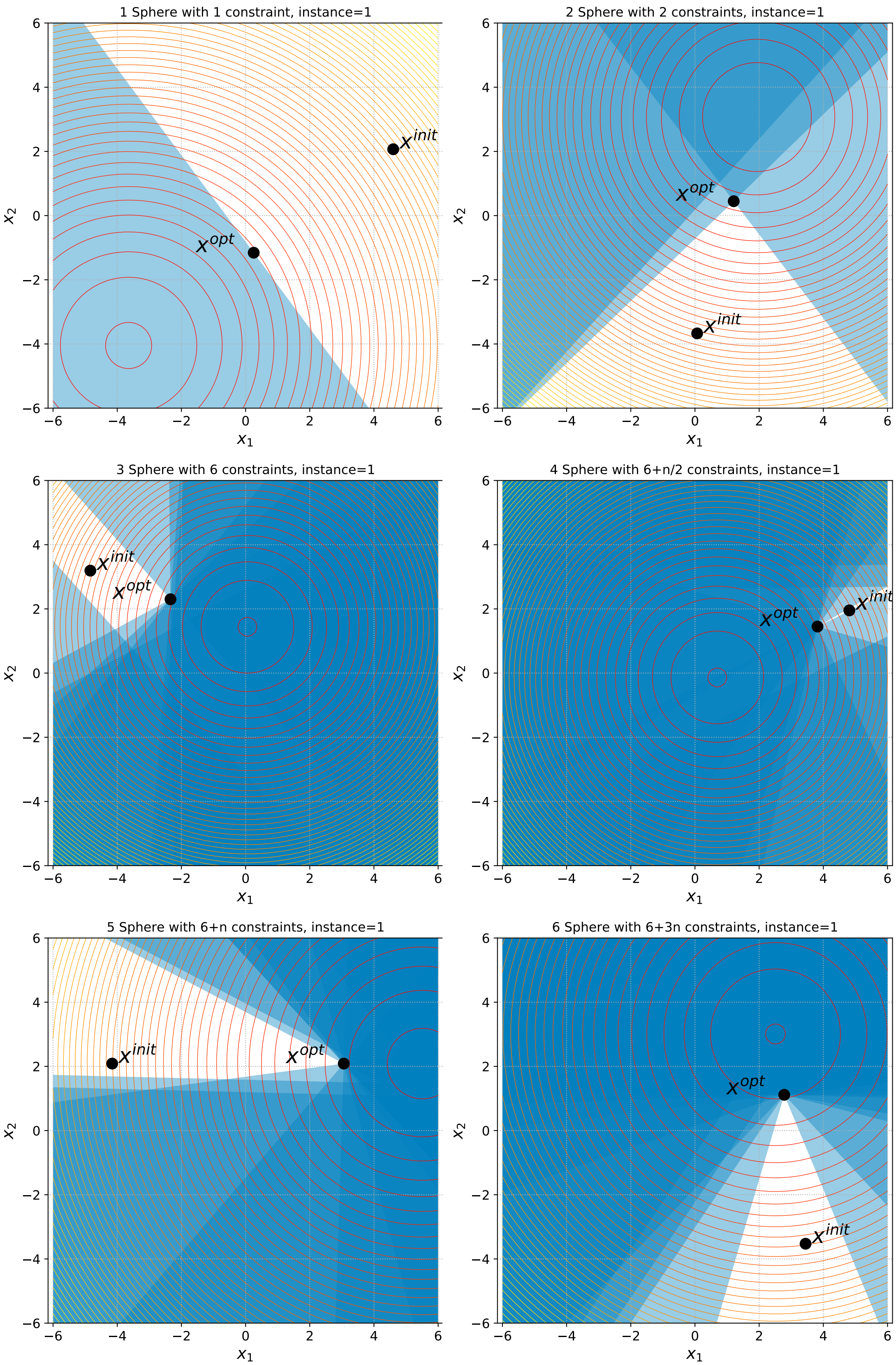
# References


[1] Mokhtar S. Bazaraa, C. M. Shetty, and Hanif D. Sherali. *Nonlinear Programming*. Wiley, 2006.

[2] Paul Dufossé, Nikolaus Hansen, Dimo Brockhoff, Phillipe R. Sampaio, Asma Atamna, and Anne Auger. Building scalable test problems for benchmarking constrained optimizers. *to be submitted to SIAM Journal of Optimization*, 2022.

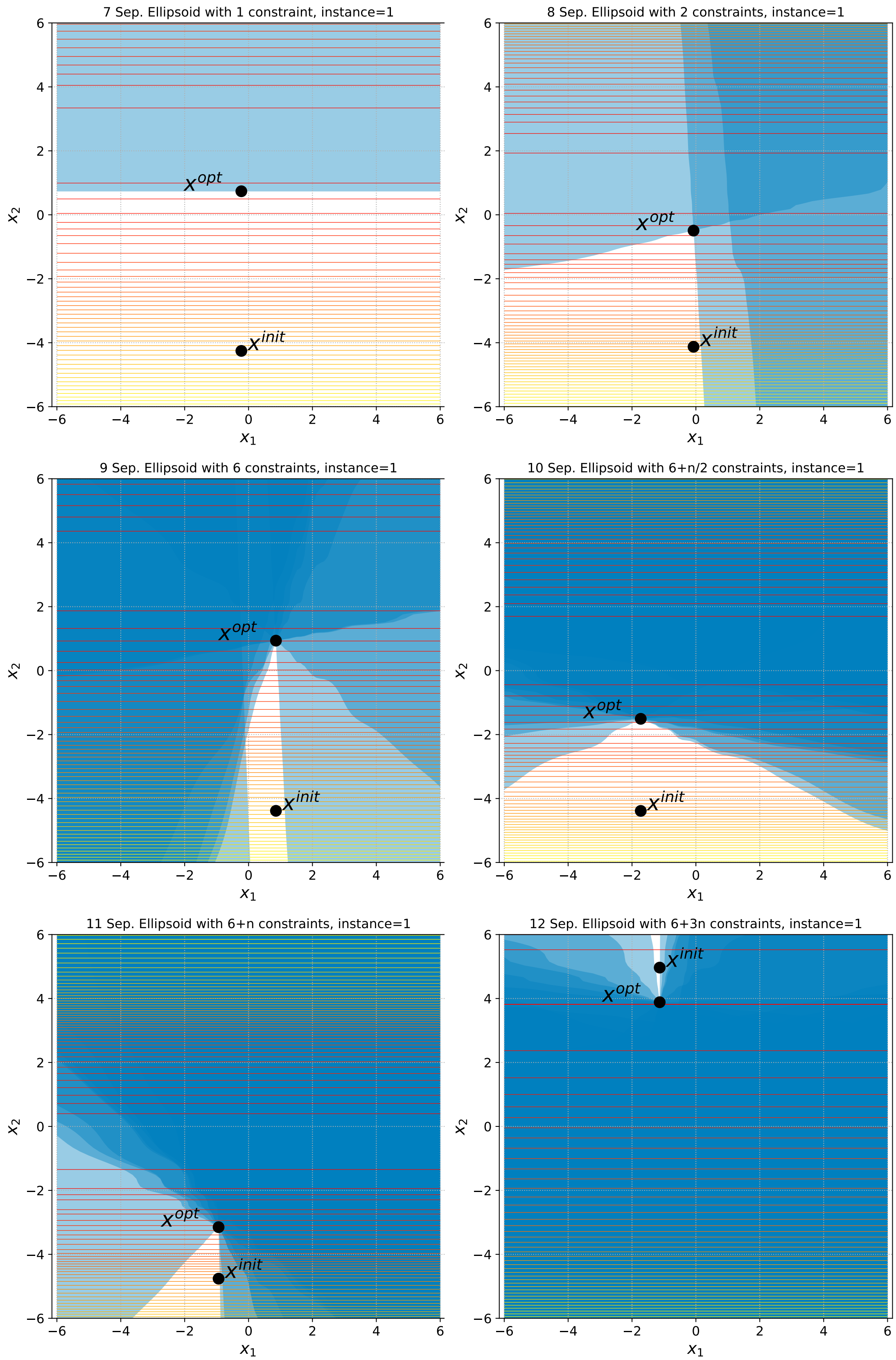
[3] Nikolaus Hansen, Anne Auger, Dimo Brockhoff, Dejan Tušar, and Tea Tušar. COCO: performance assessment. *CoRR*, 2016.

# Sphere



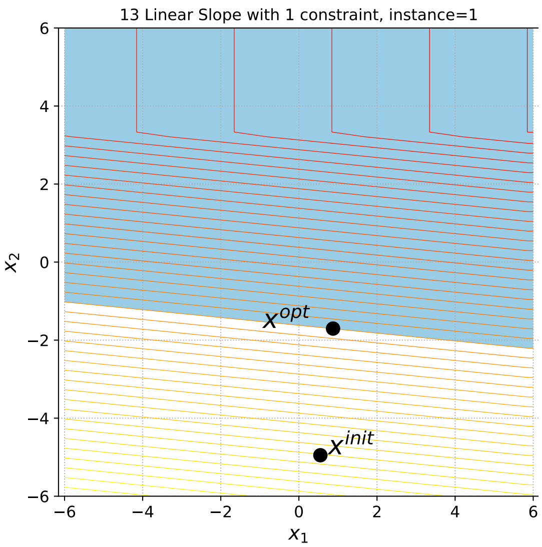
1 Sphere with 1 constraint, instance=1
2 Sphere with 2 constraints, instance=1
3 Sphere with 6 constraints, instance=1
4 Sphere with 6+n/2 constraints, instance=1
5 Sphere with 6+n constraints, instance=1
6 Sphere with 6+3n constraints, instance=1
$x_1$
$x_2$
$x^{init}$
$x^{opt}$

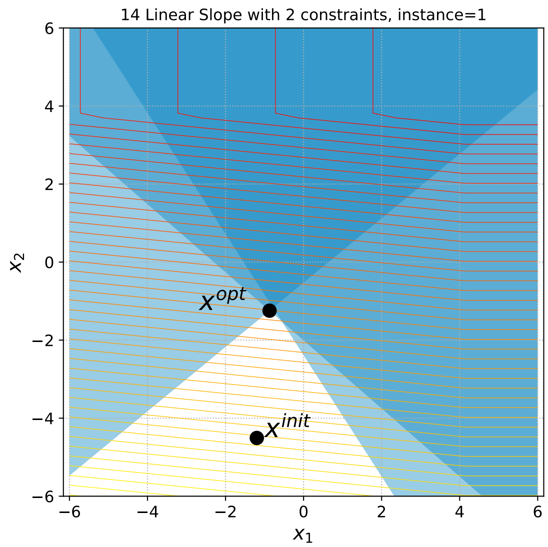
# Ellipsoid

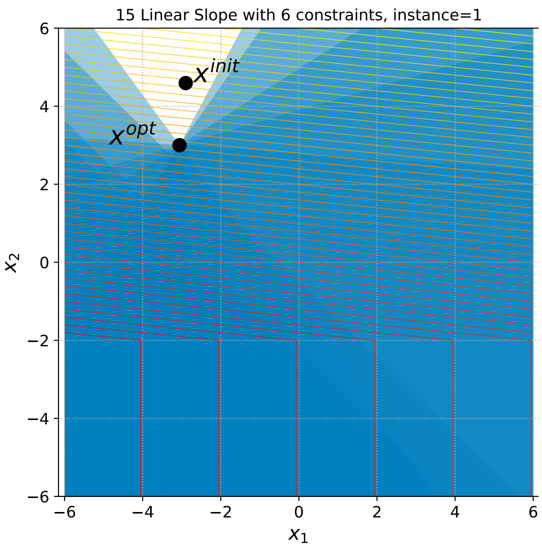


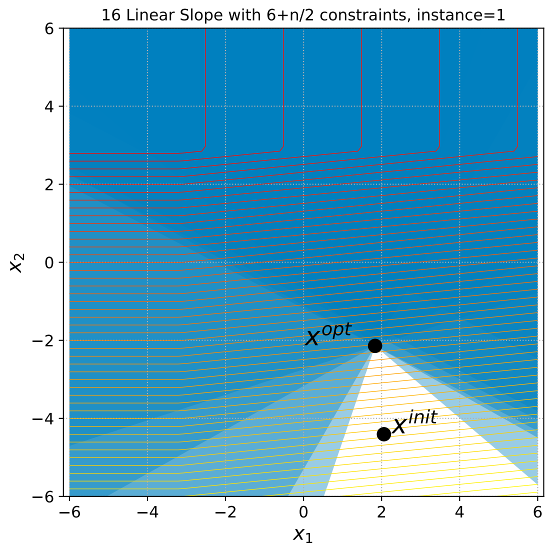
7 Sep. Ellipsoid with 1 constraint, instance=1
8 Sep. Ellipsoid with 2 constraints, instance=1
9 Sep. Ellipsoid with 6 constraints, instance=1
10 Sep. Ellipsoid with 6+n/2 constraints, instance=1
11 Sep. Ellipsoid with 6+n constraints, instance=1
12 Sep. Ellipsoid with 6+3n constraints, instance=1
$x^{opt}$
$x^{init}$
$x_1$
$x_2$

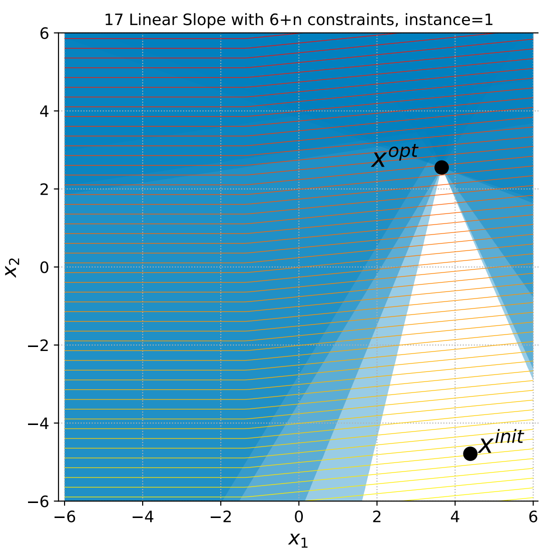
## Linear Slope

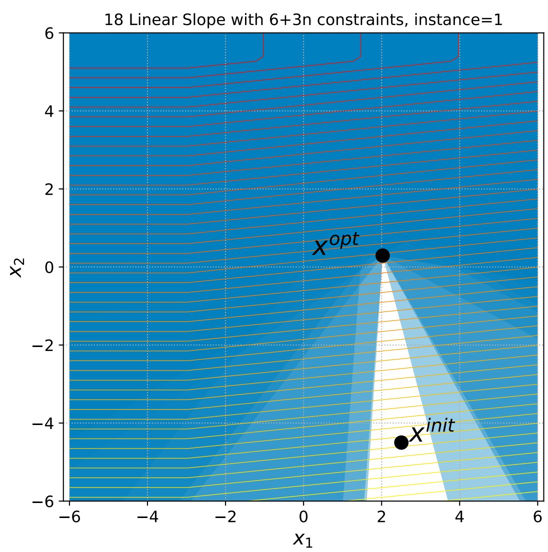


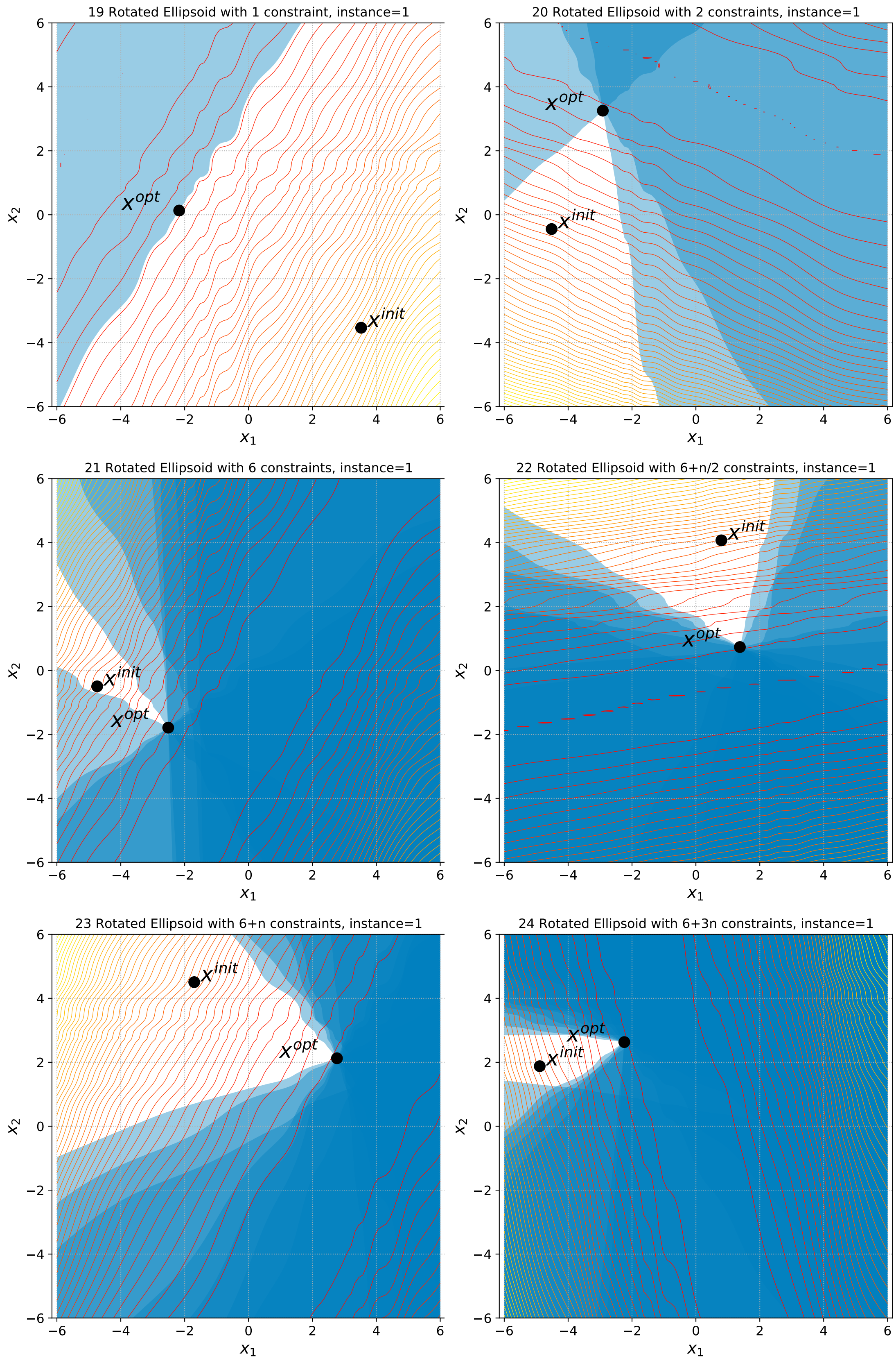



# Rotated Ellipsoid



# Discus

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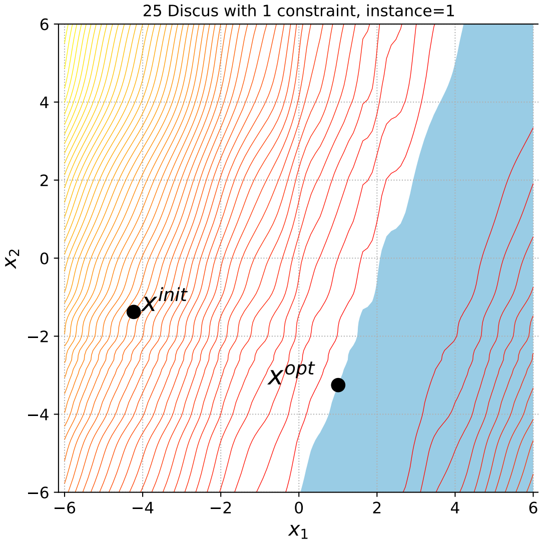
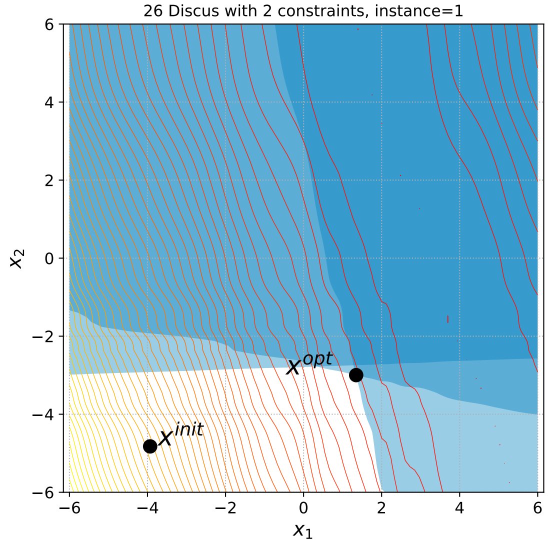
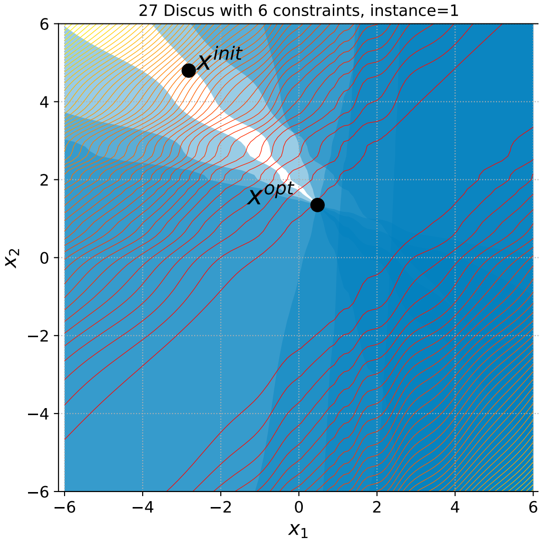



26 Discus with 2 constraints, instance=1




27 Discus with 6 constraints, instance=1

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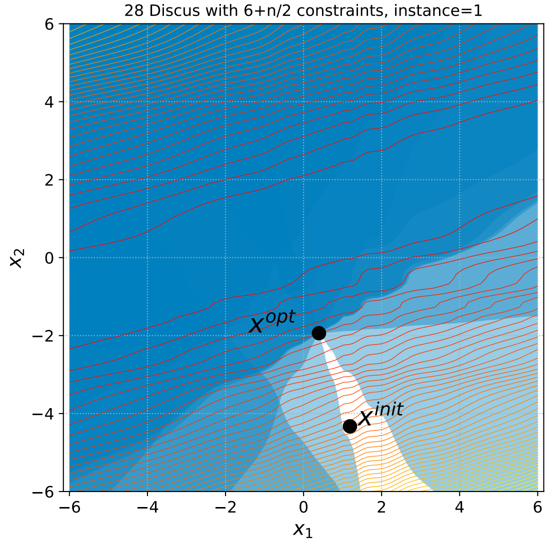


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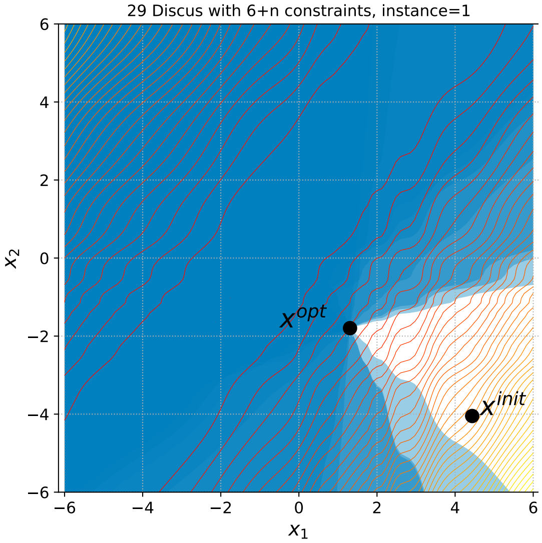
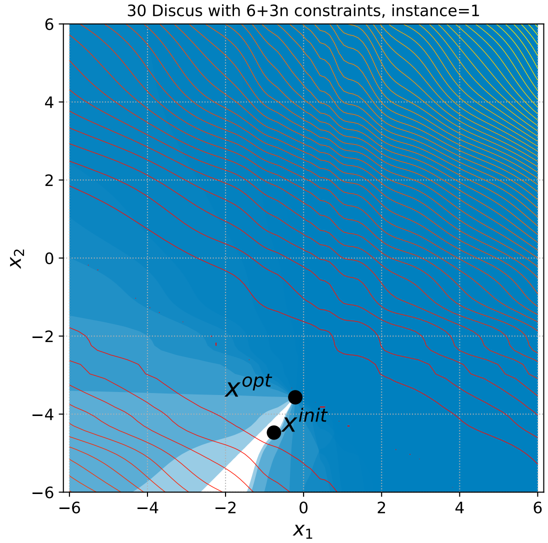
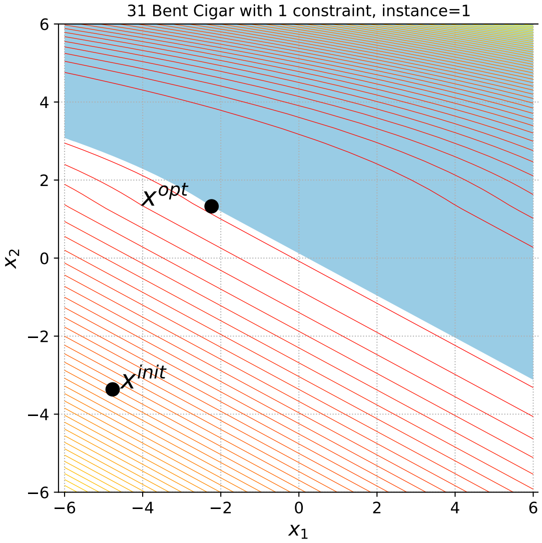



30 Discus with 6+3n constraints, instance=1



# Bent Cigar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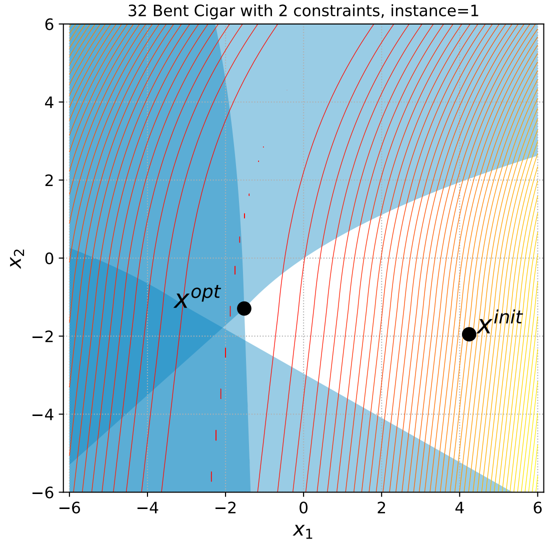
$x^{init}$
$x_1$
$x_2$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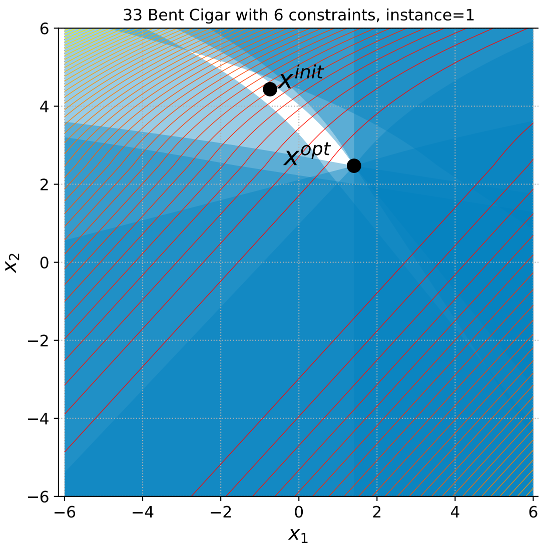
$x_2$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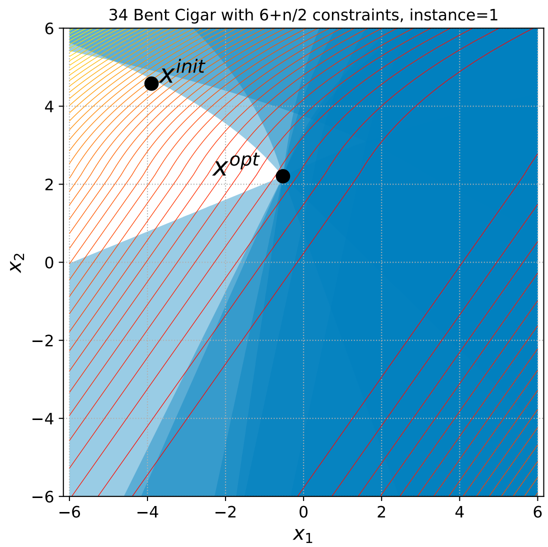
$x^{opt}$
$x_1$
$x_2$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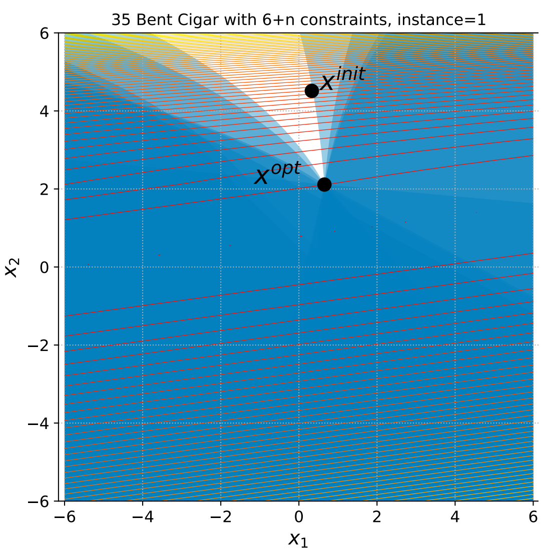
$x^{opt}$
$x_1$
$x_2$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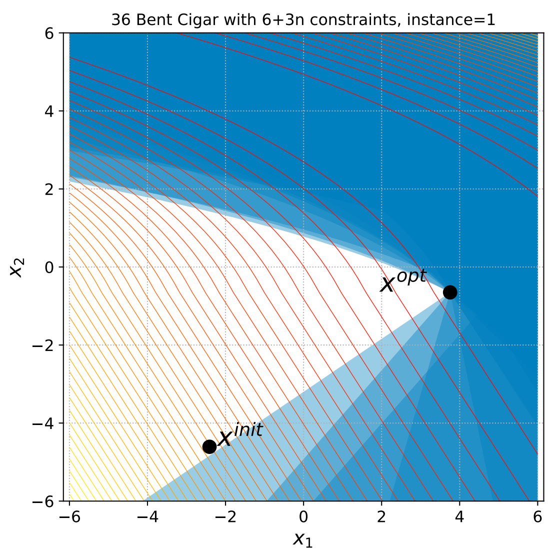
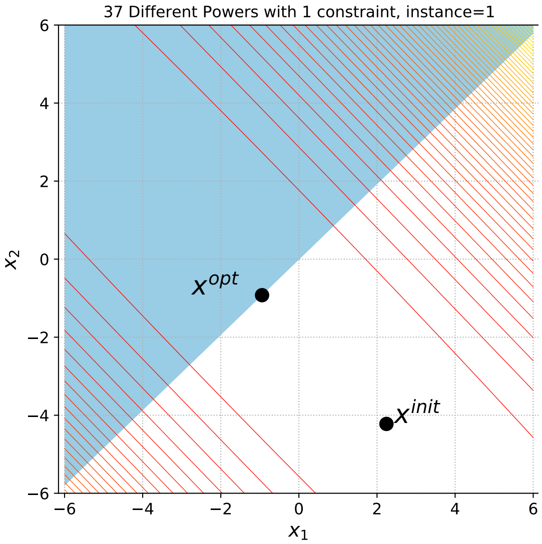
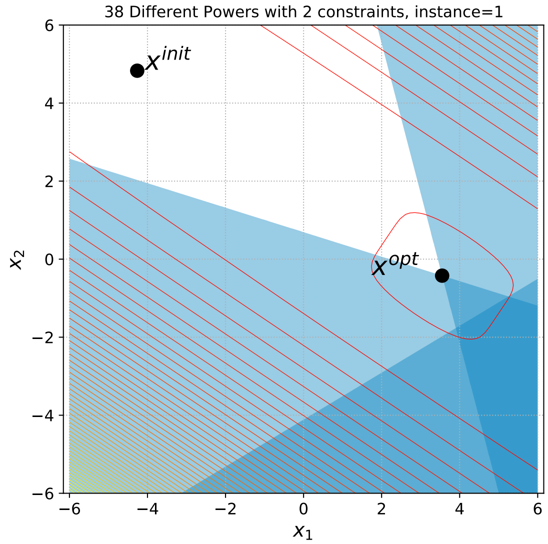
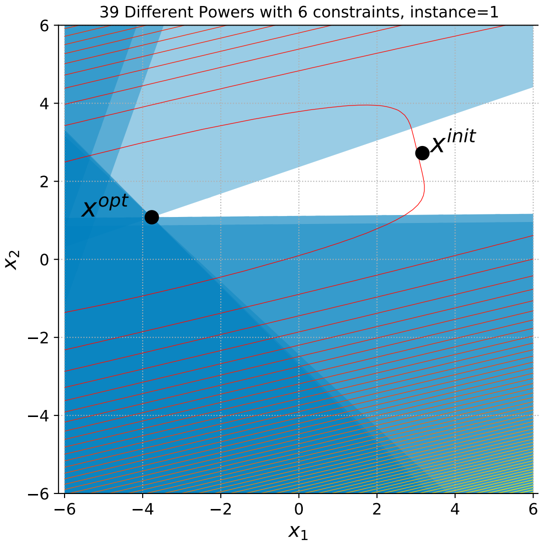
$x^{opt}$
$x_1$
$x_2$

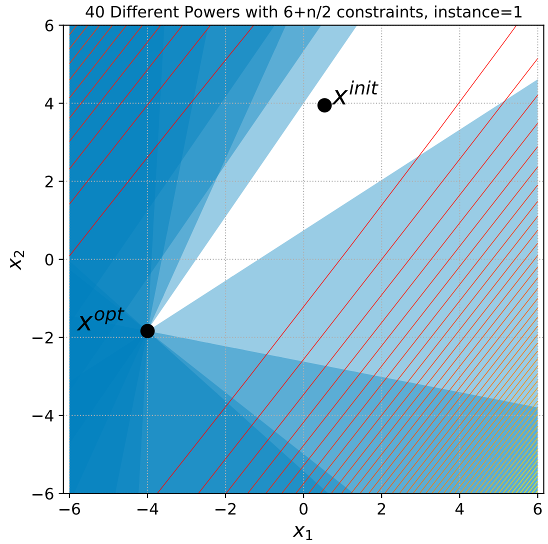

36 Bent Cigar with 6+3n constraints, instance=1
$x^{opt}$
$x^{init}$
$x_1$
$x_2$

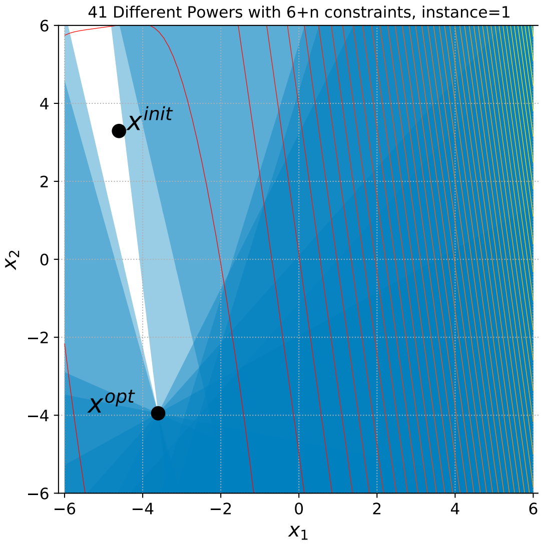
# Different Powers

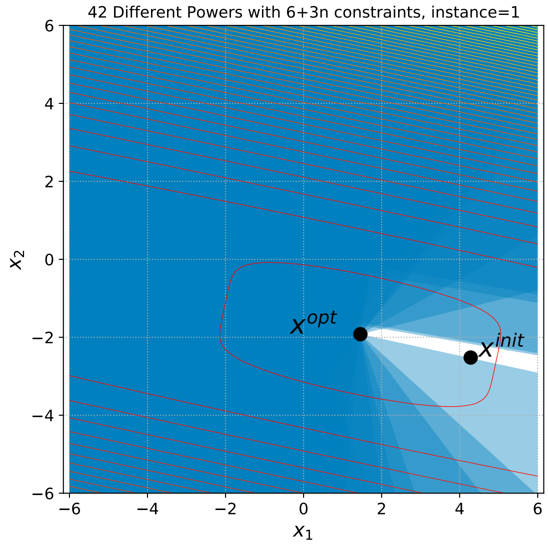


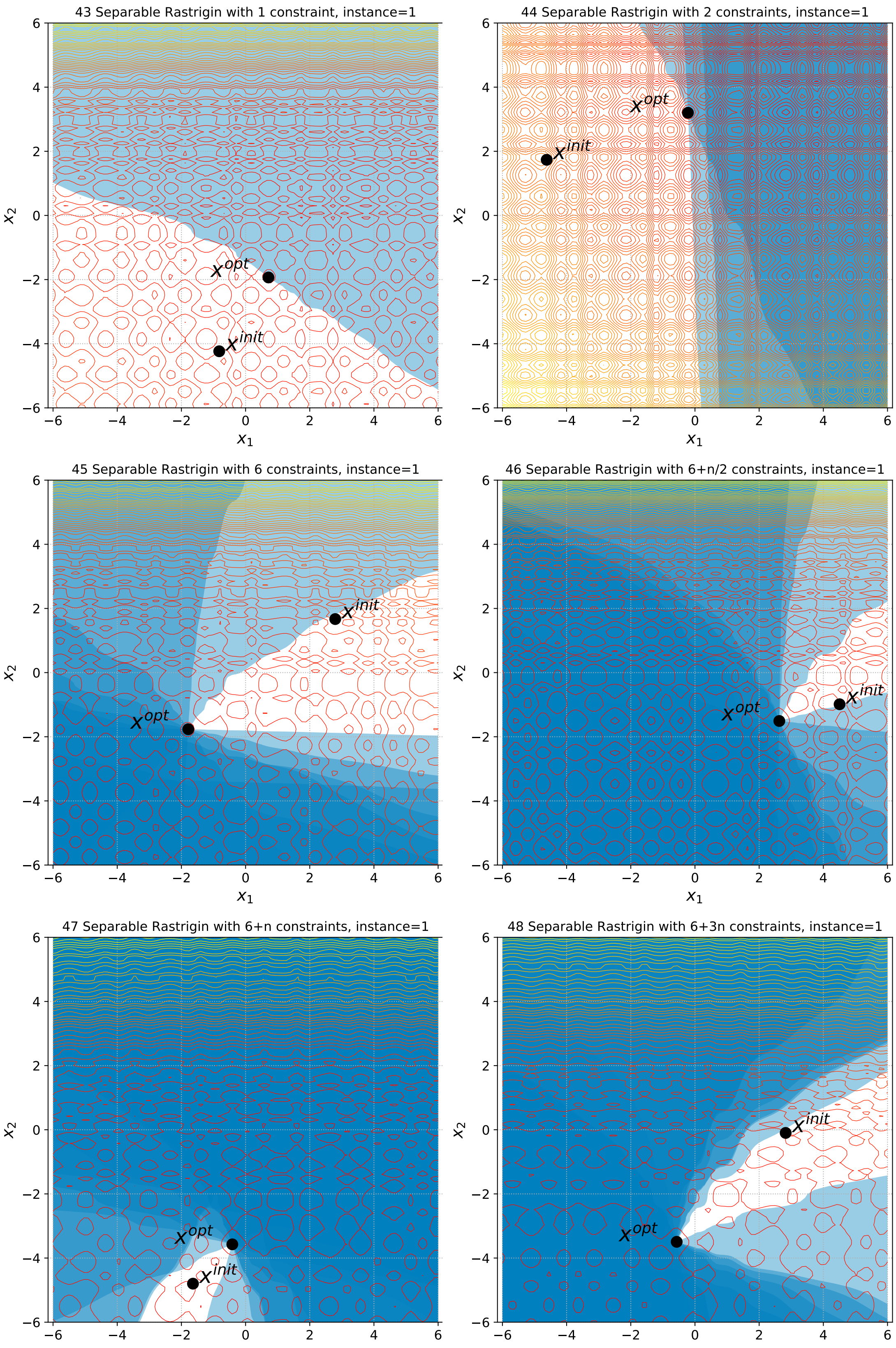



# Rastrigin


43 Separable Rastrigin with 1 constraint, instance=1
44 Separable Rastrigin with 2 constraints, instance=1
45 Separable Rastrigin with 6 constraints, instance=1
46 Separable Rastrigin with 6+n/2 constraints, instance=1
47 Separable Rastrigin with 6+n constraints, instance=1
48 Separable Rastrigin with 6+3n constraints, instance=1
$x_1$
$x_2$
$x^{opt}$
$x^{init}$
−6
−4
−2
0
2
4
6

# Rotated Rastrigin