AMaLGaM IDEAs in Noisy Black-Box Optimization Benchmarking

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ABSTRACT

This paper describes the application of a Gaussian Estimation-of-Distribution (EDA) for real-valued optimization to the noisy part of a benchmark introduced in 2009 called BBOB (Black-Box Optimization Benchmarking). Specifically, the EDA considered here is the recently introduced parameter-free version of the Adapted Maximum-Likelihood Gaussian Model Iterated Density-Estimation Evolutionary Algorithm (AMaLGaM-IDEA). Also the version with incremental model building (iAMaLGaM-IDEA) is considered.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: OptimizationGlobal Optimization, Unconstrained Optimization; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Evolutionary computation

1. METHOD

Estimation-of-distribution algorithms attempt to automatically exploit features of a problem's structure by probabilistically modeling the search space based on previously evaluated solutions and generating new solutions by sampling the probabilistic model.

The EDA considered here is the Adapted Maximum-Likelihood Gaussian Model Iterated Density-Estimation Evolutionary Algorithm (AMaLGaM-IDEA, or AMaLGaM for short). In AMaLGaM, the probability distribution used is the normal, also known as the Gaussian, distribution. This EDA uses maximum–likelihood estimates for the mean and the covariance matrix, estimated from the selected solutions. It has a mechanism that scales up the covariance matrix when required to prevent premature convergence on slopes. It furthermore has a mechanism that anticipates the mean shift in the next generation to speed up descent (in case of minimization) along slopes. In another paper [1], AMaLGaM, and its incremental-learning variant iAMaLGaM, were tested on the noiseless variant of the BBOB benchmark. Due to space restrictions, we refer the interested reader for more details on AMaLGaM such as the parameters and other settings as well as the CPU timing experiment to the other workshop paper.

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GECCO'09, July 8–12, 2009, Montréal Québec, Canada.
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2. RESULTS AND CONCLUSION

Results from experiments according to [3] on the benchmark functions given in [2, 4] are presented in Figures 1 and 2 and in Tables 1 and 3 for AMaLGaM and in Figures 3 and 4 and in Tables 2 and 4 for iAMaLGaM.

Problems with severe noise and multimodality appear to be the hardest for (i)AMaLGaM. Even within $10^6 D$ evaluations the optimum cannot be found within a desirable precision for larger D. The difference between AMaLGaM and iAMaLGaM is not large. Most likely due to the larger base population-size, AMaLGaM performs slightly better. The difference is larger for the multi-modal problems, which is consistent with earlier findings.

3. REFERENCES

[1] P. A. N. Bosman, J. Grahl, and D. Thierens. AMaLGaM IDEAs in noiseless black-box optimization benchmarking. In A. Auger et al., editors, *Proceedings of the Black Box Optimization Benchmarking BBOB Workshop at the Genetic and Evolutionary Computation Conference — GECCO–2009*, New York, New York, 2009. ACM Press. (*To Appear*).

[2] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Presentation of the noiseless functions. Technical Report 2009/20, Research Center PPE, 2009.

[3] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2009: Experimental setup. Technical Report RR-6828, INRIA, 2009.

[4] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noiseless functions definitions. Technical Report RR-6829, INRIA, 2009.

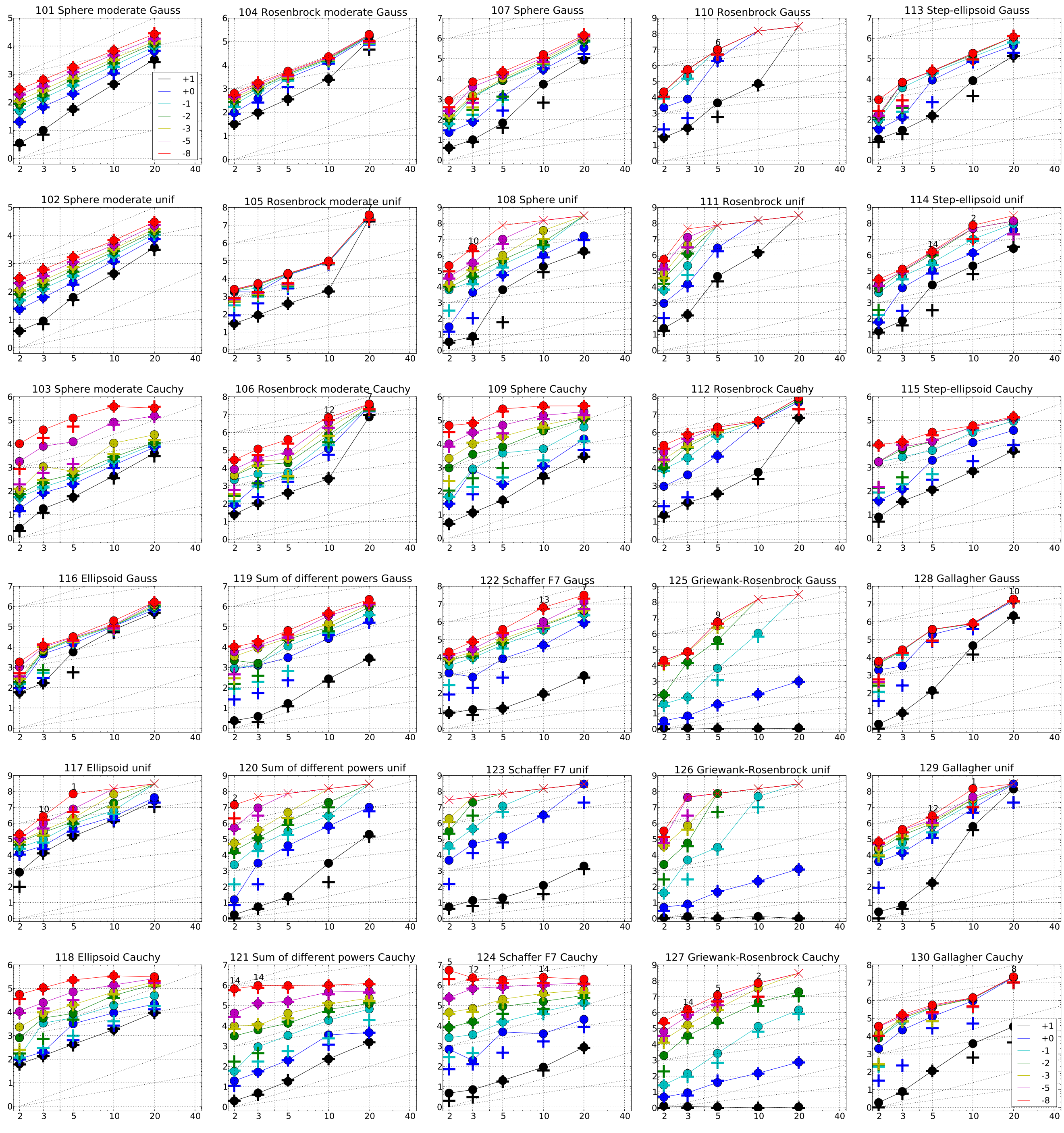

Figure 1: AMaLGaM: Expected Running Time (ERT, ●) to reach $f_{\rm opt} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_{101} and f_{130}) versus dimension in log-log presentation. The ERT(Δf) equals to #FEs(Δf) divided by the number of successful trials, where a trial is successful if $f_{\rm opt} + \Delta f$ was surpassed during the trial. The #FEs(Δf) are the total number of function evaluations while $f_{\rm opt} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and $f_{\rm opt}$ denotes the optimal function value. Crosses ($\times$) indicate the total number of function evaluations #FEs($-\infty$). Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.

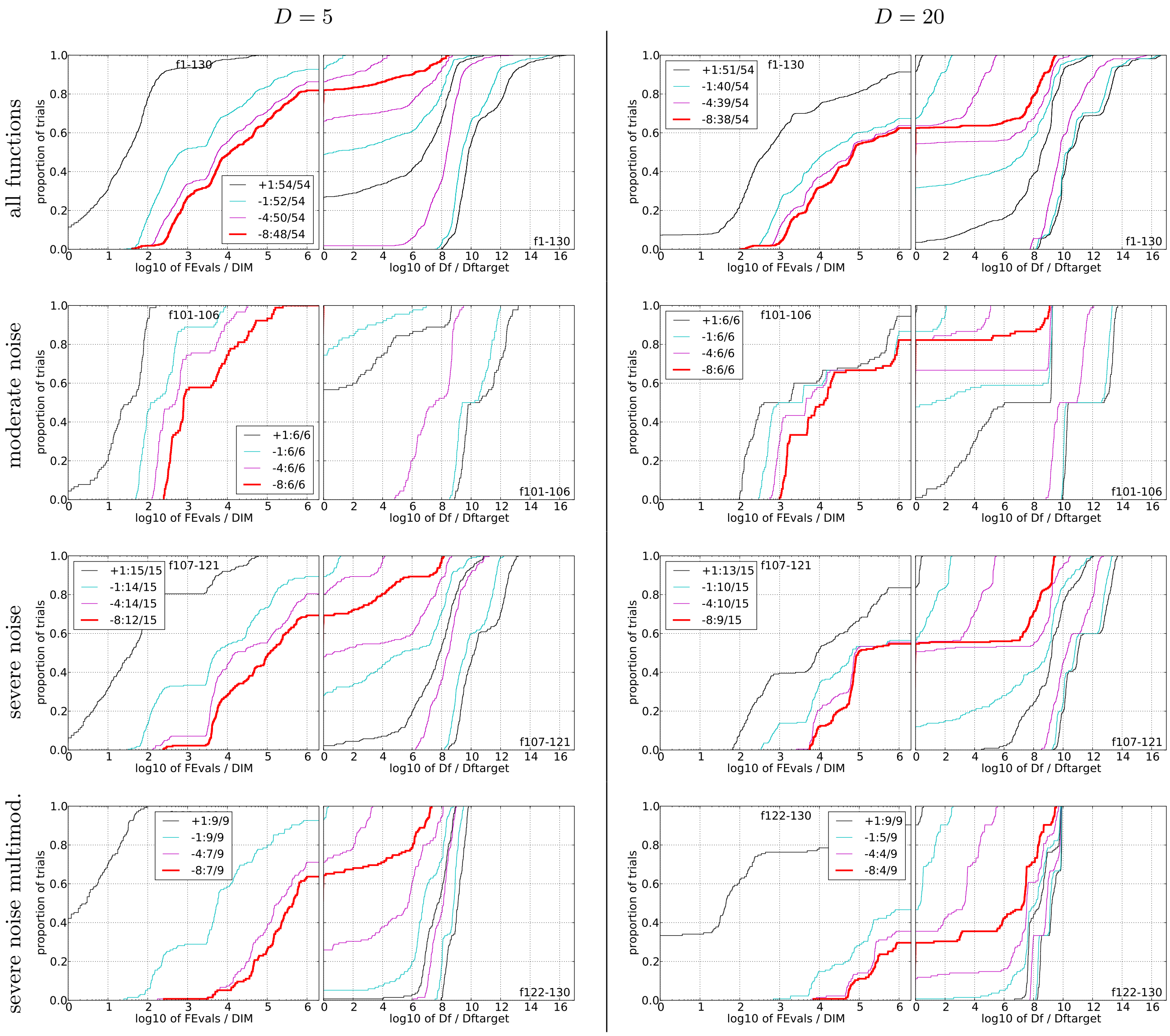

Figure 2: AMaLGaM: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left) or Δf. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10\,D, 100\,D \ldots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: moderate noise functions; third row: severe noise functions; fourth row: severe noise and highly-multimodal functions. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

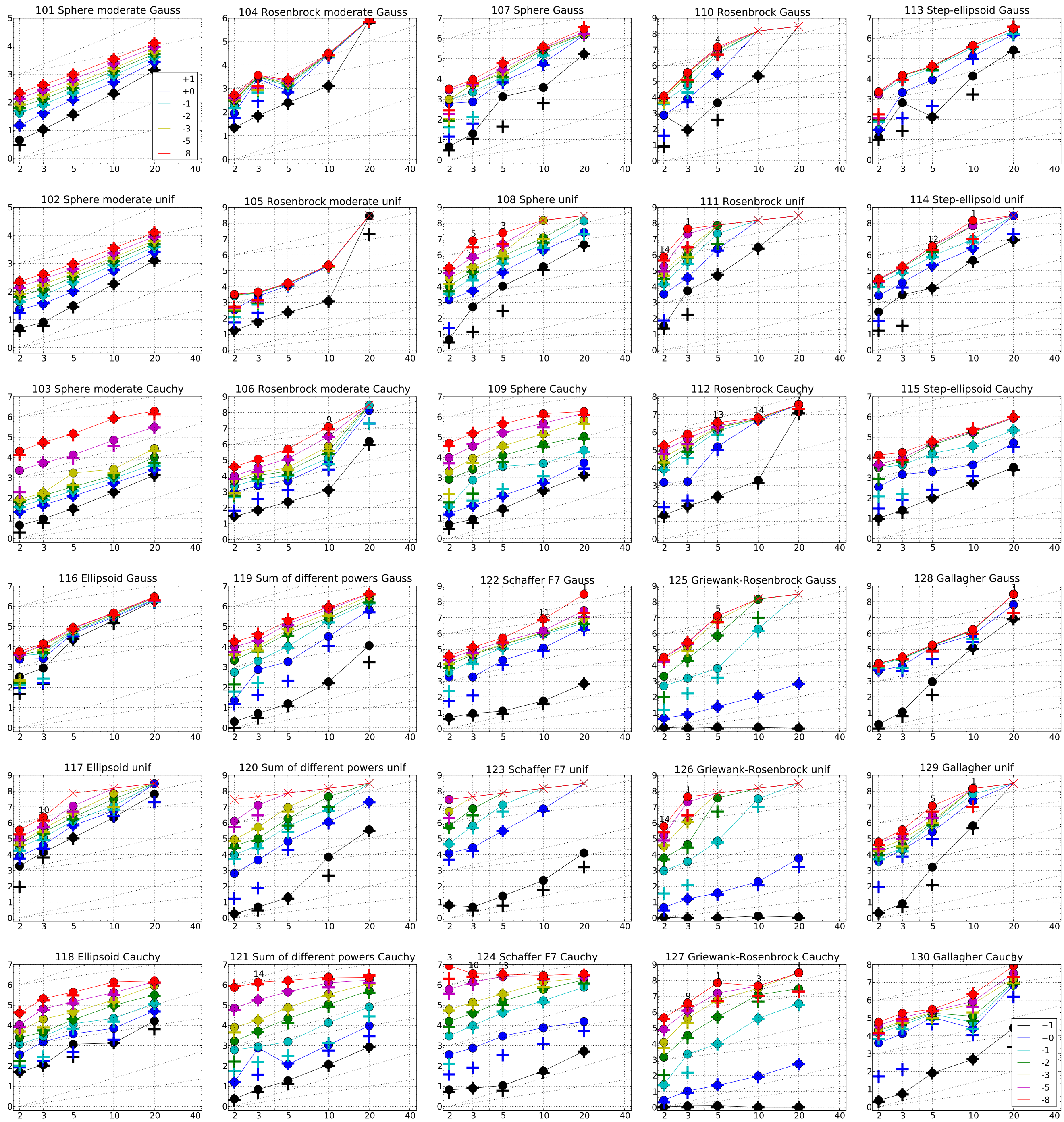

Figure 3: iAMaLGaM: Expected Running Time (ERT, ●) to reach $f_{\rm opt} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_{101} and f_{130}) versus dimension in log-log presentation. The $\mathrm{ERT}(\Delta f)$ equals to $\#\mathrm{FEs}(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\rm opt} + \Delta f$ was surpassed during the trial. The $\#\mathrm{FEs}(\Delta f)$ are the total number of function evaluations while $f_{\rm opt} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and $f_{\rm opt}$ denotes the optimal function value. Crosses ($\times$) indicate the total number of function evaluations $\#\mathrm{FEs}(-\infty)$. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.

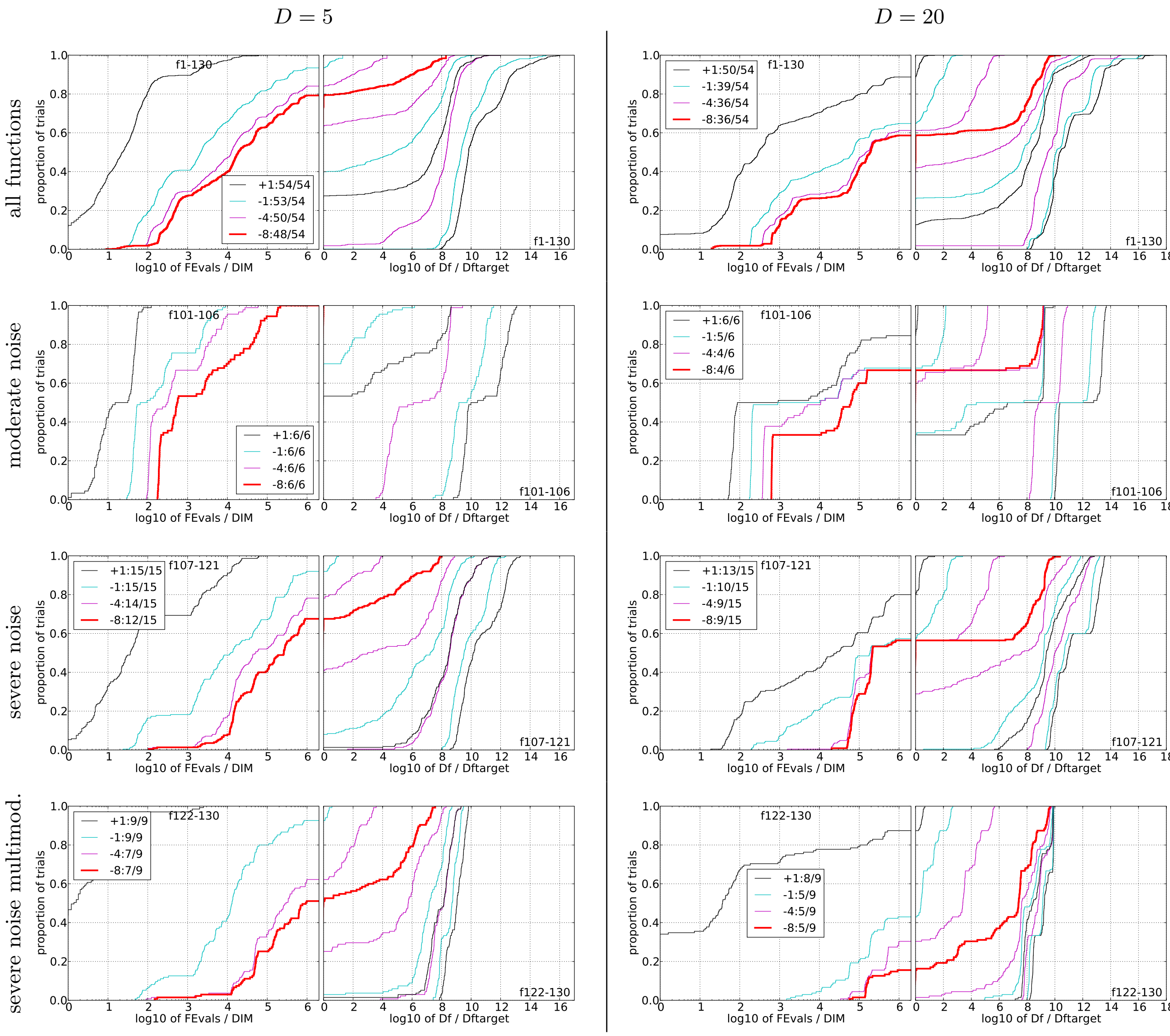

Figure 4: iAMaLGaM: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left) or Δf. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10\,D, 100\,D\ldots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: moderate noise functions; third row: severe noise functions; fourth row: severe noise and highly-multimodal functions. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

Δf	f101 in 5-D, N=15, mFE=2892					f101 in 20-D, N=15, mFE=31809				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	5.8e1	4.5e1	7.1e1	5.8e1	15	3.4e3	2.9e3	3.8e3	3.4e3
1	15	2.1e2	1.8e2	2.3e2	2.1e2	15	6.5e3	6.0e3	7.0e3	6.5e3
1e−1	15	4.3e2	3.7e2	4.9e2	4.3e2	15	9.6e3	8.8e3	1.0e4	9.6e3
1e−3	15	8.4e2	7.5e2	9.3e2	8.4e2	15	1.5e4	1.4e4	1.6e4	1.5e4
1e−5	15	1.2e3	1.1e3	1.4e3	1.2e3	15	1.9e4	1.8e4	2.1e4	1.9e4
1e−8	15	1.8e3	1.6e3	1.9e3	1.8e3	15	2.7e4	2.6e4	2.8e4	2.7e4

Δf	f102 in 5-D, N=15, mFE=2206					f102 in 20-D, N=15, mFE=36921				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	6.3e1	4.6e1	7.8e1	6.3e1	15	3.7e3	3.2e3	4.2e3	3.7e3
1	15	2.1e2	1.8e2	2.4e2	2.1e2	15	7.8e3	7.1e3	8.4e3	7.8e3
1e−1	15	3.8e2	3.5e2	4.2e2	3.8e2	15	1.1e4	1.0e4	1.2e4	1.1e4
1e−3	15	7.5e2	7.0e2	8.1e2	7.5e2	15	1.7e4	1.6e4	1.8e4	1.7e4
1e−5	15	1.1e3	1.0e3	1.2e3	1.1e3	15	2.2e4	2.1e4	2.4e4	2.2e4
1e−8	15	1.7e3	1.6e3	1.8e3	1.7e3	15	3.0e4	2.9e4	3.2e4	3.0e4

Δf	f103 in 5-D, N=15, mFE=651264					f103 in 20-D, N=15, mFE=923247				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	5.4e1	4.4e1	6.4e1	5.4e1	15	4.0e3	3.5e3	4.7e3	4.0e3
1	15	1.9e2	1.8e2	2.1e2	1.9e2	15	7.2e3	6.5e3	7.8e3	7.2e3
1e−1	15	3.7e2	3.5e2	3.9e2	3.7e2	15	9.5e3	8.6e3	1.0e4	9.5e3
1e−3	15	6.9e2	6.5e2	7.4e2	6.9e2	15	2.5e4	1.4e4	3.6e4	2.5e4
1e−5	15	1.2e4	8.1e3	1.7e4	1.2e4	15	1.5e5	1.2e5	1.9e5	1.5e5
1e−8	15	1.3e5	7.5e4	1.8e5	1.3e5	15	3.4e5	2.7e5	4.0e5	3.4e5

Δf	f104 in 5-D, N=15, mFE=31356					f104 in 20-D, N=15, mFE=334844				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	3.6e2	3.3e2	3.9e2	3.6e2	15	1.1e5	8.4e4	1.4e5	1.1e5
1	15	2.8e3	1.0e3	4.5e3	2.8e3	15	1.6e5	1.3e5	1.9e5	1.6e5
1e−1	15	3.6e3	1.8e3	5.5e3	3.6e3	15	1.7e5	1.3e5	2.0e5	1.7e5
1e−3	15	4.5e3	2.5e3	6.5e3	4.5e3	15	1.8e5	1.5e5	2.2e5	1.8e5
1e−5	15	4.9e3	3.0e3	7.0e3	4.9e3	15	1.9e5	1.6e5	2.3e5	1.9e5
1e−8	15	5.5e3	3.6e3	7.6e3	5.5e3	15	2.0e5	1.7e5	2.3e5	2.0e5

Δf	f105 in 5-D, N=15, mFE=48000					f105 in 20-D, N=15, mFE=20021580				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	3.9e2	3.6e2	4.3e2	3.9e2	10	2.3e7	2.1e7	2.5e7	1.6e7
1	15	1.6e4	1.0e4	2.1e4	1.6e4	9	2.8e7	2.6e7	3.0e7	1.8e7
1e−1	15	1.7e4	1.2e4	2.3e4	1.7e4	9	2.8e7	2.6e7	3.1e7	1.8e7
1e−3	15	1.8e4	1.3e4	2.4e4	1.8e4	7	3.7e7	3.3e7	4.0e7	1.8e7
1e−5	15	1.9e4	1.3e4	2.5e4	1.9e4	7	3.7e7	3.3e7	4.0e7	1.8e7
1e−8	15	2.0e4	1.4e4	2.6e4	2.0e4	7	3.7e7	3.4e7	4.0e7	1.9e7

Δf	f106 in 5-D, N=15, mFE=1235097					f106 in 20-D, N=15, mFE=20013092				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	4.1e2	3.7e2	4.6e2	4.1e2	15	7.4e6	6.1e6	8.6e6	7.4e6
1	15	4.7e3	1.5e3	7.9e3	4.7e3	9	2.7e7	2.6e7	2.9e7	1.6e7
1e−1	15	5.6e3	2.5e3	9.0e3	5.6e3	9	2.9e7	2.7e7	3.0e7	1.7e7
1e−3	15	3.1e4	1.7e4	4.6e4	3.1e4	7	3.9e7	3.7e7	4.0e7	1.8e7
1e−5	15	8.0e4	6.3e4	9.6e4	8.0e4	7	3.9e7	3.7e7	4.0e7	1.8e7
1e−8	15	4.0e5	2.9e5	5.2e5	4.0e5	7	3.9e7	3.7e7	4.0e7	1.8e7

Δf	f107 in 5-D, N=15, mFE=33716					f107 in 20-D, N=15, mFE=1931222				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	6.7e1	3.9e1	1.0e2	6.7e1	15	8.5e4	6.4e4	1.1e5	8.5e4
1	15	1.3e3	3.0e2	2.3e3	1.3e3	15	3.4e5	2.1e5	5.0e5	3.4e5
1e−1	15	8.1e3	5.5e3	1.1e4	8.1e3	15	7.0e5	5.3e5	8.8e5	7.0e5
1e−3	15	1.0e4	7.6e3	1.3e4	1.0e4	15	9.9e5	8.3e5	1.2e6	9.9e5
1e−5	15	1.6e4	1.3e4	1.9e4	1.6e4	15	1.2e6	1.1e6	1.4e6	1.2e6
1e−8	15	2.2e4	2.0e4	2.4e4	2.2e4	15	1.4e6	1.3e6	1.5e6	1.4e6

Δf	f108 in 5-D, N=15, mFE=5009337					f108 in 20-D, N=15, mFE=20017285				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	6.5e3	3.0e3	1.0e4	6.5e3	15	1.7e6	1.3e6	2.1e6	1.7e6
1	15	6.0e4	4.1e4	7.7e4	6.0e4	10	1.6e7	1.2e7	2.0e7	1.0e7
1e−1	15	2.5e5	1.9e5	3.1e5	2.5e5	0	*48e−2*	*18e−2*	*19e−1*	7.9e6
1e−3	15	9.6e5	6.5e5	1.3e6	9.6e5	.	.	.	.	.
1e−5	6	9.9e6	8.6e6	1.1e7	3.4e6	.	.	.	.	.
1e−8	0	*20e−6*	*57e−9*	*16e−5*	2.2e6	.	.	.	.	.

Δf	f109 in 5-D, N=15, mFE=714314					f109 in 20-D, N=15, mFE=1118257				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	4.0e1	3.1e1	5.0e1	4.0e1	15	3.0e3	2.8e3	3.3e3	3.0e3
1	15	1.9e2	1.8e2	2.1e2	1.9e2	15	1.6e4	6.7e3	2.7e4	1.6e4
1e−1	15	4.1e3	1.1e3	6.9e3	4.1e3	15	5.2e4	3.6e4	7.1e4	5.2e4
1e−3	15	2.1e4	1.8e4	2.4e4	2.1e4	15	1.4e5	1.2e5	1.6e5	1.4e5
1e−5	15	6.2e4	4.3e4	8.2e4	6.2e4	15	2.3e5	1.9e5	2.8e5	2.3e5
1e−8	15	3.0e5	2.3e5	3.7e5	3.0e5	15	4.1e5	3.3e5	5.0e5	4.1e5

Δf	f110 in 5-D, N=15, mFE=5009199					f110 in 20-D, N=15, mFE=20020914				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	4.5e3	1.8e3	7.3e3	4.5e3	0	*18e+0*	*17e+0*	*18e+0*	1.6e7
1	13	2.5e6	1.7e6	3.2e6	1.9e6	.	.	.	.	.
1e−1	6	1.0e7	8.8e6	1.1e7	4.2e6	.	.	.	.	.
1e−3	6	1.0e7	8.7e6	1.1e7	4.3e6	.	.	.	.	.
1e−5	6	1.0e7	8.9e6	1.1e7	4.3e6	.	.	.	.	.
1e−8	6	1.0e7	9.0e6	1.1e7	4.3e6	.	.	.	.	.

Δf	f111 in 5-D, N=15, mFE=5009244					f111 in 20-D, N=15, mFE=20018706				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	4.5e4	2.8e4	6.3e4	4.5e4	0	*23e+0*	*20e+0*	*24e+0*	1.0e7
1	12	2.8e6	2.0e6	3.6e6	2.2e6	.	.	.	.	.
1e−1	0	*53e−2*	*17e−2*	*10e−1*	2.0e6	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f112 in 5-D, N=15, mFE=3579671					f112 in 20-D, N=15, mFE=20002169				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	3.7e2	3.4e2	4.0e2	3.7e2	15	6.7e6	6.3e6	7.2e6	6.7e6
1	15	5.0e4	3.4e4	6.6e4	5.0e4	5	5.0e7	4.4e7	5.5e7	1.8e7
1e−1	15	6.3e5	4.7e5	8.0e5	6.3e5	4	6.4e7	5.9e7	7.0e7	1.7e7
1e−3	15	1.2e6	9.0e5	1.5e6	1.2e6	3	9.0e7	8.3e7	9.7e7	1.7e7
1e−5	15	2.0e6	1.6e6	2.3e6	2.0e6	3	9.0e7	8.3e7	9.7e7	1.7e7
1e−8	15	2.0e6	1.7e6	2.3e6	2.0e6	3	9.0e7	8.4e7	9.7e7	1.7e7

Δf	f113 in 5-D, N=15, mFE=66491					f113 in 20-D, N=15, mFE=1702267				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.5e2	1.3e2	1.8e2	1.5e2	15	1.4e5	1.0e5	1.7e5	1.4e5
1	15	8.7e3	4.5e3	1.3e4	8.7e3	15	4.4e5	3.2e5	5.6e5	4.4e5
1e−1	15	1.9e4	1.3e4	2.5e4	1.9e4	15	7.0e5	5.2e5	8.9e5	7.0e5
1e−3	15	2.4e4	1.9e4	2.9e4	2.4e4	15	1.2e6	1.0e6	1.3e6	1.2e6
1e−5	15	2.4e4	1.9e4	3.0e4	2.4e4	15	1.2e6	1.0e6	1.3e6	1.2e6
1e−8	15	2.5e4	1.9e4	3.0e4	2.5e4	15	1.2e6	1.0e6	1.3e6	1.2e6

Δf	f114 in 5-D, N=15, mFE=5003184					f114 in 20-D, N=15, mFE=20018939				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.3e4	6.4e3	2.0e4	1.3e4	15	2.6e6	2.2e6	2.9e6	2.6e6
1	15	1.0e5	7.3e4	1.4e5	1.0e5	6	3.8e7	3.3e7	4.3e7	1.3e7
1e−1	15	3.2e5	2.5e5	4.0e5	3.2e5	3	9.0e7	8.3e7	9.7e7	1.7e7
1e−3	14	1.6e6	1.2e6	2.0e6	1.5e6	2	1.4e8	1.3e8	1.5e8	1.5e7
1e−5	14	1.6e6	1.2e6	2.0e6	1.5e6	2	1.4e8	1.3e8	1.5e8	1.5e7
1e−8	14	1.7e6	1.3e6	2.1e6	1.7e6	0	*14e−1*	*89e−9*	*32e−1*	8.9e6

Δf	f115 in 5-D, N=15, mFE=69345					f115 in 20-D, N=15, mFE=325587				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.1e2	9.4e1	1.3e2	1.1e2	15	5.2e3	4.7e3	5.7e3	5.2e3
1	15	2.0e3	3.0e2	3.7e3	2.0e3	15	3.8e4	2.2e4	5.6e4	3.8e4
1e−1	15	5.3e3	2.7e3	8.1e3	5.3e3	15	9.2e4	7.5e4	1.1e5	9.2e4
1e−3	15	1.3e4	9.8e3	1.6e4	1.3e4	15	1.3e5	1.2e5	1.3e5	1.3e5
1e−5	15	1.3e4	9.7e3	1.6e4	1.3e4	15	1.3e5	1.2e5	1.3e5	1.3e5
1e−8	15	3.1e4	2.5e4	3.8e4	3.1e4	15	1.4e5	1.3e5	1.6e5	1.4e5

Δf	f116 in 5-D, N=15, mFE=77546					f116 in 20-D, N=15, mFE=2235403				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	5.7e3	3.4e3	8.0e3	5.7e3	15	5.0e5	3.9e5	6.1e5	5.0e5
1	15	1.4e4	9.5e3	2.0e4	1.4e4	15	6.9e5	5.6e5	8.4e5	6.9e5
1e−1	15	2.2e4	1.6e4	2.9e4	2.2e4	15	8.9e5	7.4e5	1.0e6	8.9e5
1e−3	15	2.7e4	2.1e4	3.3e4	2.7e4	15	1.2e6	1.0e6	1.4e6	1.2e6
1e−5	15	3.0e4	2.5e4	3.6e4	3.0e4	15	1.5e6	1.5e6	1.6e6	1.5e6
1e−8	15	3.2e4	2.7e4	3.8e4	3.2e4	15	1.6e6	1.5e6	1.7e6	1.6e6

Δf	f117 in 5-D, N=15, mFE=5006519					f117 in 20-D, N=15, mFE=20021072				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.5e5	1.3e5	1.8e5	1.5e5	8	2.6e7	2.2e7	3.0e7	1.4e7
1	15	4.5e5	3.2e5	6.1e5	4.5e5	6	4.1e7	3.7e7	4.5e7	1.6e7
1e−1	15	9.3e5	7.7e5	1.1e6	9.3e5	0	*90e−1*	*54e−2*	*20e+0*	1.0e7
1e−3	15	2.1e6	1.7e6	2.4e6	2.1e6	.	.	.	.	.
1e−5	7	7.8e6	6.6e6	9.0e6	4.8e6	.	.	.	.	.
1e−8	1	7.1e7	6.7e7	7.5e7	5.0e6	.	.	.	.	.

Δf	f118 in 5-D, N=15, mFE=500283					f118 in 20-D, N=15, mFE=737362				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	4.3e2	3.9e2	4.6e2	4.3e2	15	9.4e3	8.8e3	1.0e4	9.4e3
1	15	3.2e3	8.4e2	5.5e3	3.2e3	15	2.2e4	1.2e4	3.2e4	2.2e4
1e−1	15	5.7e3	3.2e3	8.3e3	5.7e3	15	5.0e4	3.2e4	6.8e4	5.0e4
1e−3	15	2.1e4	1.7e4	2.4e4	2.1e4	15	1.9e5	1.5e5	2.5e5	1.9e5
1e−5	15	7.2e4	5.1e4	9.3e4	7.2e4	15	2.6e5	2.2e5	3.1e5	2.6e5
1e−8	15	2.3e5	2.0e5	2.7e5	2.3e5	15	3.2e5	2.6e5	3.7e5	3.2e5

Δf	f119 in 5-D, N=15, mFE=178282					f119 in 20-D, N=15, mFE=5624208				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.6e1	1.1e1	2.2e1	1.6e1	15	2.8e3	2.4e3	3.2e3	2.8e3
1	15	3.0e3	3.3e2	5.7e3	3.0e3	15	2.0e5	1.7e5	2.3e5	2.0e5
1e−1	15	1.1e4	6.5e3	1.6e4	1.1e4	15	4.3e5	3.2e5	5.3e5	4.3e5
1e−3	15	2.8e4	2.1e4	3.5e4	2.8e4	15	1.3e6	1.2e6	1.3e6	1.3e6
1e−5	15	3.5e4	2.9e4	4.2e4	3.5e4	15	1.4e6	1.3e6	1.5e6	1.4e6
1e−8	15	6.5e4	5.0e4	8.0e4	6.5e4	15	2.2e6	1.8e6	2.7e6	2.2e6

Δf	f120 in 5-D, N=15, mFE=5008513					f120 in 20-D, N=15, mFE=20021059				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.3e1	1.7e1	3.0e1	2.3e1	15	2.0e5	1.4e5	2.6e5	2.0e5
1	15	3.8e4	2.2e4	5.7e4	3.8e4	12	1.0e7	7.6e6	1.3e7	7.5e6
1e−1	15	3.0e5	2.1e5	3.8e5	3.0e5	0	*37e−2*	*12e−2*	*25e−1*	8.9e6
1e−3	10	4.8e6	4.0e6	5.5e6	2.6e6	.	.	.	.	.
1e−5	0	*68e−5*	*89e−6*	*24e−4*	2.5e6	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 1: AMaLGaM: Shown are, for functions f_{101}-f_{120} and for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\rm opt} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value ($\mathbf{RT_{succ}}$). If $f_{\rm opt} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

Δf	**f_{101} in 5-D**, N=15, mFE=1072					**f_{101} in 20-D**, N=15, mFE=13288				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	3.7e1	3.1e1	4.3e1	3.7e1	15	1.4e3	1.3e3	1.4e3	1.4e3
1	15	1.3e2	1.2e2	1.3e2	1.3e2	15	2.7e3	2.6e3	2.7e3	2.7e3
1e−1	15	2.2e2	2.2e2	2.3e2	2.2e2	15	4.0e3	3.9e3	4.0e3	4.0e3
1e−3	15	4.4e2	4.3e2	4.5e2	4.4e2	15	6.5e3	6.5e3	6.6e3	6.5e3
1e−5	15	6.6e2	6.5e2	6.8e2	6.6e2	15	9.1e3	9.0e3	9.2e3	9.1e3
1e−8	15	9.7e2	9.6e2	9.9e2	9.7e2	15	1.3e4	1.3e4	1.3e4	1.3e4

Δf	**f_{103} in 5-D**, N=15, mFE=313583					**f_{103} in 20-D**, N=15, mFE=3182267				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	3.0e1	2.4e1	3.5e1	3.0e1	15	1.3e3	1.3e3	1.4e3	1.3e3
1	15	1.2e2	1.1e2	1.3e2	1.2e2	15	2.5e3	2.4e3	2.5e3	2.5e3
1e−1	15	2.2e2	2.1e2	2.4e2	2.2e2	15	4.8e3	3.8e3	5.8e3	4.8e3
1e−3	15	1.7e3	4.5e2	3.0e3	1.7e3	15	2.8e4	1.9e4	3.7e4	2.8e4
1e−5	15	1.3e4	7.9e3	1.9e4	1.3e4	15	3.1e5	2.3e5	3.9e5	3.1e5
1e−8	15	1.5e5	1.2e5	1.8e5	1.5e5	15	1.9e6	1.6e6	2.3e6	1.9e6

Δf	**f_{105} in 5-D**, N=15, mFE=47725					**f_{105} in 20-D**, N=15, mFE=20015011				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.4e2	2.3e2	2.6e2	2.4e2	1	2.8e8	2.7e8	3.0e8	2.0e7
1	15	1.2e4	7.9e3	1.6e4	1.2e4	0	*13e+0*	*10e+0*	*15e+0*	1.0e7
1e−1	15	1.5e4	1.1e4	1.9e4	1.5e4	.	.	.	.	.
1e−3	15	1.5e4	1.2e4	1.9e4	1.5e4	.	.	.	.	.
1e−5	15	1.6e4	1.2e4	2.0e4	1.6e4	.	.	.	.	.
1e−8	15	1.6e4	1.3e4	2.1e4	1.6e4	.	.	.	.	.

Δf	**f_{107} in 5-D**, N=15, mFE=98112					**f_{107} in 20-D**, N=15, mFE=4298872				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.3e3	5.9e1	2.6e3	1.3e3	15	1.6e5	1.2e5	2.0e5	1.6e5
1	15	7.1e3	4.5e3	1.0e4	7.1e3	15	1.4e6	1.2e6	1.6e6	1.4e6
1e−1	15	1.3e4	9.8e3	1.6e4	1.3e4	15	1.5e6	1.3e6	1.7e6	1.5e6
1e−3	15	2.1e4	1.7e4	2.6e4	2.1e4	15	1.6e6	1.4e6	1.8e6	1.6e6
1e−5	15	3.1e4	2.4e4	3.8e4	3.1e4	15	2.0e6	1.7e6	2.3e6	2.0e6
1e−8	15	5.6e4	4.6e4	6.5e4	5.6e4	15	2.9e6	2.5e6	3.3e6	2.9e6

Δf	**f_{109} in 5-D**, N=15, mFE=799091					**f_{109} in 20-D**, N=15, mFE=3621383				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.9e1	2.5e1	3.4e1	2.9e1	15	1.3e3	1.3e3	1.4e3	1.3e3
1	15	1.3e2	1.2e2	1.4e2	1.3e2	15	5.4e3	2.8e3	8.1e3	5.4e3
1e−1	15	3.5e3	1.0e3	6.2e3	3.5e3	15	2.3e4	1.5e4	3.1e4	2.3e4
1e−3	15	3.7e4	2.6e4	4.8e4	3.7e4	15	7.1e5	5.5e5	8.7e5	7.1e5
1e−5	15	1.6e5	1.4e5	1.8e5	1.6e5	15	1.5e6	1.2e6	1.8e6	1.5e6
1e−8	15	4.5e5	3.8e5	5.1e5	4.5e5	15	1.8e6	1.5e6	2.1e6	1.8e6

Δf	**f_{111} in 5-D**, N=15, mFE=5009373					**f_{111} in 20-D**, N=15, mFE=20005321				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	4.9e4	4.0e4	5.9e4	4.9e4	0	*27e+0*	*20e+0*	*35e+0*	7.9e6
1	14	2.3e6	1.7e6	2.9e6	2.3e6	.	.	.	.	.
1e−1	3	2.2e7	2.0e7	2.4e7	5.0e6	.	.	.	.	.
1e−3	0	*26e−2*	*43e−3*	*64e−2*	2.2e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{113} in 5-D**, N=15, mFE=83906					**f_{113} in 20-D**, N=15, mFE=4072574				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.3e2	1.1e2	1.5e2	1.3e2	15	2.6e5	1.9e5	3.3e5	2.6e5
1	15	8.8e3	3.6e3	1.5e4	8.8e3	15	1.6e6	1.4e6	1.8e6	1.6e6
1e−1	15	3.4e4	2.5e4	4.4e4	3.4e4	15	2.3e6	2.0e6	2.6e6	2.3e6
1e−3	15	4.3e4	3.3e4	5.2e4	4.3e4	15	3.1e6	2.8e6	3.4e6	3.1e6
1e−5	15	4.3e4	3.3e4	5.2e4	4.3e4	15	3.1e6	2.7e6	3.4e6	3.1e6
1e−8	15	4.3e4	3.3e4	5.3e4	4.3e4	15	3.1e6	2.8e6	3.4e6	3.1e6

Δf	**f_{115} in 5-D**, N=15, mFE=108673					**f_{115} in 20-D**, N=15, mFE=1155027				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	9.9e1	8.8e1	1.1e2	9.9e1	15	3.2e3	2.3e3	4.1e3	3.2e3
1	15	2.0e3	8.7e2	3.3e3	2.0e3	15	5.2e4	3.2e4	7.3e4	5.2e4
1e−1	15	1.6e4	1.1e4	2.1e4	1.6e4	15	2.2e5	1.7e5	2.7e5	2.2e5
1e−3	15	5.1e4	4.2e4	6.0e4	5.1e4	15	8.9e5	7.8e5	9.9e5	8.9e5
1e−5	15	5.1e4	4.2e4	5.9e4	5.1e4	15	8.9e5	7.7e5	9.9e5	8.9e5
1e−8	15	6.0e4	5.2e4	7.0e4	6.0e4	15	9.6e5	8.8e5	1.0e6	9.6e5

Δf	**f_{117} in 5-D**, N=15, mFE=5008014					**f_{117} in 20-D**, N=15, mFE=20015008				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.2e5	8.6e4	1.5e5	1.2e5	4	6.4e7	5.9e7	7.0e7	1.7e7
1	15	6.8e5	5.6e5	8.1e5	6.8e5	1	2.9e8	2.8e8	3.0e8	2.0e7
1e−1	15	1.3e6	1.1e6	1.5e6	1.3e6	0	*19e+0*	*13e−1*	*77e+0*	8.9e6
1e−3	11	4.4e6	3.6e6	5.1e6	2.9e6	.	.	.	.	.
1e−5	6	1.2e7	1.1e7	1.2e7	4.4e6	.	.	.	.	.
1e−8	0	*14e−6*	*57e−8*	*26e−4*	4.0e6	.	.	.	.	.

Δf	**f_{119} in 5-D**, N=15, mFE=290580					**f_{119} in 20-D**, N=15, mFE=4424520				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.6e1	1.2e1	1.9e1	1.6e1	15	1.2e4	4.2e3	1.9e4	1.2e4
1	15	1.8e3	7.1e2	3.0e3	1.8e3	15	6.6e5	4.8e5	8.5e5	6.6e5
1e−1	15	1.0e4	7.1e3	1.3e4	1.0e4	15	1.6e6	1.4e6	1.7e6	1.6e6
1e−3	15	6.9e4	5.9e4	7.9e4	6.9e4	15	3.1e6	2.7e6	3.4e6	3.1e6
1e−5	15	1.3e5	1.1e5	1.6e5	1.3e5	15	3.9e6	3.9e6	4.0e6	3.9e6
1e−8	15	1.9e5	1.6e5	2.1e5	1.9e5	15	4.1e6	4.0e6	4.1e6	4.1e6

Δf	**f_{102} in 5-D**, N=15, mFE=1093					**f_{102} in 20-D**, N=15, mFE=13675				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	3.1e1	2.8e1	3.4e1	3.1e1	15	1.3e3	1.2e3	1.4e3	1.3e3
1	15	9.9e1	8.9e1	1.1e2	9.9e1	15	2.6e3	2.5e3	2.6e3	2.6e3
1e−1	15	2.1e2	2.0e2	2.2e2	2.1e2	15	3.9e3	3.8e3	3.9e3	3.9e3
1e−3	15	4.4e2	4.3e2	4.5e2	4.4e2	15	6.4e3	6.4e3	6.5e3	6.4e3
1e−5	15	6.4e2	6.3e2	6.5e2	6.4e2	15	9.0e3	8.9e3	9.1e3	9.0e3
1e−8	15	9.5e2	9.4e2	9.7e2	9.5e2	15	1.3e4	1.3e4	1.3e4	1.3e4

Δf	**f_{104} in 5-D**, N=15, mFE=2983					**f_{104} in 20-D**, N=15, mFE=1889854				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.5e2	2.2e2	2.9e2	2.5e2	15	7.7e5	6.2e5	9.0e5	7.7e5
1	15	7.7e2	6.5e2	8.9e2	7.7e2	15	8.2e5	6.8e5	9.8e5	8.2e5
1e−1	15	1.3e3	1.2e3	1.4e3	1.3e3	15	8.4e5	6.9e5	1.0e6	8.4e5
1e−3	15	1.8e3	1.6e3	1.9e3	1.8e3	15	8.6e5	7.1e5	1.0e6	8.6e5
1e−5	15	2.0e3	1.9e3	2.2e3	2.0e3	15	8.7e5	7.0e5	1.0e6	8.7e5
1e−8	15	2.4e3	2.3e3	2.5e3	2.4e3	15	8.8e5	7.3e5	1.0e6	8.8e5

Δf	**f_{106} in 5-D**, N=15, mFE=1072606					**f_{106} in 20-D**, N=15, mFE=20010297				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.3e2	2.2e2	2.4e2	2.3e2	15	1.5e6	8.2e5	2.3e6	1.5e6
1	15	4.8e3	2.6e3	7.1e3	4.8e3	2	1.3e8	1.1e8	1.5e8	2.0e7
1e−1	15	1.0e4	7.6e3	1.3e4	1.0e4	1	2.8e8	2.7e8	3.0e8	2.0e7
1e−3	15	3.3e4	2.3e4	4.5e4	3.3e4	0	*47e−1*	*30e−2*	*81e−1*	5.6e6
1e−5	15	1.2e5	8.5e4	1.6e5	1.2e5	.	.	.	.	.
1e−8	15	5.1e5	4.0e5	6.1e5	5.1e5	.	.	.	.	.

Δf	**f_{108} in 5-D**, N=15, mFE=5009730					**f_{108} in 20-D**, N=15, mFE=20005495				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.1e4	6.0e3	1.6e4	1.1e4	15	4.4e6	3.8e6	5.1e6	4.4e6
1	15	8.4e4	6.0e4	1.1e5	8.4e4	8	2.6e7	2.2e7	3.0e7	1.2e7
1e−1	15	3.8e5	2.9e5	4.6e5	3.8e5	2	1.4e8	1.2e8	1.5e8	2.0e7
1e−3	15	1.3e6	1.0e6	1.6e6	1.3e6	0	*85e−2*	*96e−3*	*23e−1*	7.9e6
1e−5	10	4.7e6	3.9e6	5.6e6	2.8e6	.	.	.	.	.
1e−8	3	2.4e7	2.4e7	2.5e7	5.0e6	.	.	.	.	.

Δf	**f_{110} in 5-D**, N=15, mFE=5008280					**f_{110} in 20-D**, N=15, mFE=20009863				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	4.5e3	2.8e3	6.6e3	4.5e3	0	*18e+0*	*18e+0*	*18e+0*	8.9e6
1	15	3.0e5	2.1e5	3.9e5	3.0e5	.	.	.	.	.
1e−1	9	5.5e6	4.5e6	6.5e6	2.5e6	.	.	.	.	.
1e−3	6	1.0e7	8.8e6	1.1e7	3.2e6	.	.	.	.	.
1e−5	5	1.2e7	1.1e7	1.4e7	3.6e6	.	.	.	.	.
1e−8	4	1.5e7	1.3e7	1.7e7	3.3e6	.	.	.	.	.

Δf	**f_{112} in 5-D**, N=15, mFE=5008789					**f_{112} in 20-D**, N=15, mFE=20007211				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.5e2	2.3e2	2.7e2	2.5e2	13	1.4e7	1.3e7	1.5e7	1.1e7
1	15	1.5e5	1.1e5	2.0e5	1.5e5	7	3.6e7	3.3e7	3.8e7	1.6e7
1e−1	15	1.2e6	8.6e5	1.5e6	1.2e6	7	3.6e7	3.3e7	3.8e7	1.6e7
1e−3	15	1.9e6	1.6e6	2.3e6	1.9e6	7	3.6e7	3.3e7	3.9e7	1.6e7
1e−5	15	2.2e6	1.9e6	2.6e6	2.2e6	7	3.6e7	3.4e7	3.9e7	1.6e7
1e−8	13	3.6e6	3.1e6	4.0e6	3.0e6	7	3.6e7	3.4e7	3.9e7	1.6e7

Δf	**f_{114} in 5-D**, N=15, mFE=5008506					**f_{114} in 20-D**, N=15, mFE=20010822				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	8.1e3	5.9e3	1.0e4	8.1e3	13	8.8e6	6.8e6	1.1e7	7.9e6
1	15	2.1e5	1.6e5	2.7e5	2.1e5	1	2.9e8	2.8e8	3.0e8	2.0e7
1e−1	15	8.3e5	6.2e5	1.1e6	8.3e5	0	*27e−1*	*11e−1*	*11e+0*	7.9e6
1e−3	12	3.5e6	2.9e6	4.1e6	2.6e6	.	.	.	.	.
1e−5	12	3.5e6	2.9e6	4.2e6	2.6e6	.	.	.	.	.
1e−8	12	3.6e6	2.9e6	4.3e6	2.7e6	.	.	.	.	.

Δf	**f_{116} in 5-D**, N=15, mFE=153370					**f_{116} in 20-D**, N=15, mFE=4408997				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.3e4	1.7e4	2.8e4	2.3e4	15	1.9e6	1.6e6	2.2e6	1.9e6
1	15	4.7e4	3.3e4	6.1e4	4.7e4	15	2.0e6	1.7e6	2.4e6	2.0e6
1e−1	15	5.6e4	4.4e4	6.9e4	5.6e4	15	2.2e6	1.9e6	2.6e6	2.2e6
1e−3	15	6.3e4	5.2e4	7.6e4	6.3e4	15	2.6e6	2.2e6	3.0e6	2.6e6
1e−5	15	7.8e4	6.6e4	9.0e4	7.8e4	15	2.6e6	2.2e6	3.1e6	2.6e6
1e−8	15	8.6e4	7.7e4	9.7e4	8.6e4	15	2.9e6	2.4e6	3.3e6	2.9e6

Δf	**f_{118} in 5-D**, N=15, mFE=756125					**f_{118} in 20-D**, N=15, mFE=3288169				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.2e3	2.9e2	2.1e3	1.2e3	15	1.6e4	1.1e4	2.2e4	1.6e4
1	15	3.8e3	2.3e3	5.2e3	3.8e3	15	5.2e4	3.8e4	6.7e4	5.2e4
1e−1	15	1.2e4	7.8e3	1.6e4	1.2e4	15	1.2e5	8.8e4	1.5e5	1.2e5
1e−3	15	4.4e4	3.6e4	5.1e4	4.4e4	15	9.1e5	7.6e5	1.1e6	9.1e5
1e−5	15	1.5e5	1.1e5	1.9e5	1.5e5	15	1.4e6	1.3e6	1.6e6	1.4e6
1e−8	15	4.3e5	3.5e5	5.2e5	4.3e5	15	1.6e6	1.3e6	1.8e6	1.6e6

Δf	**f_{120} in 5-D**, N=15, mFE=5009454					**f_{120} in 20-D**, N=15, mFE=20004625				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.0e1	1.5e1	2.4e1	2.0e1	15	3.7e5	2.8e5	4.6e5	3.7e5
1	15	7.0e4	2.5e4	1.1e5	7.0e4	10	2.1e7	1.8e7	2.4e7	1.4e7
1e−1	15	6.3e5	3.4e5	9.6e5	6.3e5	0	*39e−2*	*12e−2*	*24e−1*	8.9e6
1e−3	7	9.4e6	8.9e6	1.0e7	4.6e6	.	.	.	.	.
1e−5	0	*13e−4*	*22e−5*	*67e−4*	3.5e6	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 2: iAMaLGaM: Shown are, for functions f_{101}-f_{120} and for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\mathrm{opt}} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value ($\mathbf{RT_{succ}}$). If $f_{\mathrm{opt}} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

Δf	f_{121} **in 5-D**, N=15, mFE=2708125					f_{121} **in 20-D**, N=15, mFE=3041481				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.8e1	1.3e1	2.3e1	1.8e1	15	1.6e3	1.5e3	1.6e3	1.6e3
1	15	2.0e2	1.8e2	2.2e2	2.0e2	15	4.6e3	4.4e3	4.8e3	4.6e3
1e−1	15	3.3e3	8.1e2	6.0e3	3.3e3	15	6.7e4	4.6e4	9.1e4	6.7e4
1e−3	15	4.1e4	3.0e4	5.3e4	4.1e4	15	2.3e5	1.9e5	2.7e5	2.3e5
1e−5	15	1.6e5	1.3e5	2.0e5	1.6e5	15	5.1e5	4.1e5	6.2e5	5.1e5
1e−8	15	9.7e5	7.6e5	1.2e6	9.7e5	15	1.2e6	1.1e6	1.4e6	1.2e6

Δf	f_{123} **in 5-D**, N=15, mFE=5009358					f_{123} **in 20-D**, N=15, mFE=20020903				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.0e1	1.4e1	2.6e1	2.0e1	15	2.0e3	1.4e3	2.6e3	2.0e3
1	15	1.4e5	9.1e4	1.9e5	1.4e5	1	2.9e8	2.8e8	3.0e8	2.0e7
1e−1	5	1.2e7	1.0e7	1.3e7	3.4e6	0	*23e−1*	*12e−1*	*32e−1*	8.9e6
1e−3	0	*15e−2*	*74e−3*	*17e−2*	2.2e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{125} **in 5-D**, N=15, mFE=5008084					f_{125} **in 20-D**, N=15, mFE=20016770				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.1e0	1.0e0	1.1e0	1.1e0	15	1.1e0	1.0e0	1.3e0	1.1e0
1	15	3.7e1	2.9e1	4.5e1	3.7e1	15	1.0e3	9.3e2	1.1e3	1.0e3
1e−1	15	6.8e3	3.7e3	9.9e3	6.8e3	0	*24e−2*	*15e−2*	*28e−2*	1.4e7
1e−3	12	3.1e6	2.5e6	3.7e6	2.3e6	.	.	.	.	.
1e−5	9	5.5e6	4.5e6	6.5e6	3.1e6	.	.	.	.	.
1e−8	9	5.5e6	4.6e6	6.5e6	3.1e6	.	.	.	.	.

Δf	f_{127} **in 5-D**, N=15, mFE=5009449					f_{127} **in 20-D**, N=15, mFE=20005826				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.1e0	1.0e0	1.3e0	1.1e0	15	1.1e0	1.0e0	1.3e0	1.1e0
1	15	4.0e1	3.3e1	4.5e1	4.0e1	15	7.6e2	7.1e2	8.1e2	7.6e2
1e−1	15	2.7e3	6.9e2	4.7e3	2.7e3	15	1.5e6	9.8e5	2.0e6	1.5e6
1e−3	14	1.8e6	1.3e6	2.2e6	1.7e6	0	*83e−4*	*36e−4*	*40e−3*	2.0e7
1e−5	9	5.3e6	4.5e6	6.2e6	3.4e6	.	.	.	.	.
1e−8	5	1.2e7	1.1e7	1.4e7	4.6e6	.	.	.	.	.

Δf	f_{129} **in 5-D**, N=15, mFE=5003969					f_{129} **in 20-D**, N=15, mFE=20019421				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.8e2	1.4e2	2.1e2	1.8e2	2	1.4e8	1.4e8	1.5e8	2.0e7
1	15	2.1e5	1.3e5	2.9e5	2.1e5	1	2.9e8	2.9e8	3.0e8	2.0e7
1e−1	15	6.9e5	3.9e5	1.0e6	6.9e5	0	*23e+0*	*26e−1*	*28e+0*	1.1e7
1e−3	15	1.4e6	9.3e5	1.8e6	1.4e6	.	.	.	.	.
1e−5	15	1.7e6	1.3e6	2.1e6	1.7e6	.	.	.	.	.
1e−8	12	3.0e6	2.3e6	3.7e6	2.3e6	.	.	.	.	.

Δf	f_{122} **in 5-D**, N=15, mFE=733298					f_{122} **in 20-D**, N=15, mFE=20021683				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.3e1	9.9e0	1.7e1	1.3e1	15	9.5e2	8.0e2	1.1e3	9.5e2
1	15	8.4e3	4.5e3	1.3e4	8.4e3	15	8.5e5	7.0e5	1.0e6	8.5e5
1e−1	15	5.1e4	4.1e4	6.2e4	5.1e4	15	2.6e6	2.2e6	3.1e6	2.6e6
1e−3	15	1.1e5	8.7e4	1.3e5	1.1e5	15	4.6e6	4.5e6	4.7e6	4.6e6
1e−5	15	2.0e5	1.9e5	2.2e5	2.0e5	11	1.3e7	1.0e7	1.6e7	8.7e6
1e−8	15	3.7e5	3.1e5	4.5e5	3.7e5	7	3.2e7	2.7e7	3.6e7	1.3e7

Δf	f_{124} **in 5-D**, N=15, mFE=3180298					f_{124} **in 20-D**, N=15, mFE=5042373				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.0e1	1.6e1	2.4e1	2.0e1	15	8.8e2	7.8e2	9.9e2	8.8e2
1	15	5.0e3	2.3e3	7.7e3	5.0e3	15	2.1e4	1.1e4	3.2e4	2.1e4
1e−1	15	1.5e4	1.1e4	2.0e4	1.5e4	15	1.4e5	1.3e5	1.4e5	1.4e5
1e−3	15	2.0e5	1.6e5	2.5e5	2.0e5	15	5.8e5	4.6e5	6.8e5	5.8e5
1e−5	15	8.5e5	7.2e5	9.8e5	8.5e5	15	1.3e6	1.0e6	1.6e6	1.3e6
1e−8	15	1.9e6	1.6e6	2.2e6	1.9e6	15	2.0e6	1.5e6	2.4e6	2.0e6

Δf	f_{126} **in 5-D**, N=15, mFE=5008584					f_{126} **in 20-D**, N=15, mFE=20016867				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.0e0	1.0e0	1.0e0	1.0e0	15	1.0e0	1.0e0	1.0e0	1.0e0
1	15	4.5e1	3.8e1	5.3e1	4.5e1	15	1.3e3	1.2e3	1.4e3	1.3e3
1e−1	15	3.0e4	1.8e4	4.3e4	3.0e4	0	*31e−2*	*29e−2*	*33e−2*	8.9e6
1e−3	0	*15e−3*	*11e−3*	*28e−3*	1.4e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{128} **in 5-D**, N=15, mFE=2115239					f_{128} **in 20-D**, N=15, mFE=20018320				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.4e2	1.0e2	1.8e2	1.4e2	15	2.2e6	1.7e6	2.7e6	2.2e6
1	15	2.0e5	1.2e5	2.7e5	2.0e5	11	1.6e7	1.3e7	1.9e7	1.2e7
1e−1	15	3.6e5	2.1e5	5.3e5	3.6e5	10	1.8e7	1.5e7	2.1e7	1.2e7
1e−3	15	3.7e5	2.0e5	5.6e5	3.7e5	10	1.8e7	1.5e7	2.2e7	1.2e7
1e−5	15	3.7e5	2.0e5	5.6e5	3.7e5	10	1.9e7	1.5e7	2.2e7	1.2e7
1e−8	15	3.8e5	2.0e5	5.6e5	3.8e5	10	1.9e7	1.5e7	2.2e7	1.3e7

Δf	f_{130} **in 5-D**, N=15, mFE=3516605					f_{130} **in 20-D**, N=15, mFE=20017912				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.1e2	9.5e1	1.3e2	1.1e2	15	3.5e4	5.3e3	6.5e4	3.5e4
1	15	1.3e5	6.3e4	1.9e5	1.3e5	8	2.0e7	1.4e7	2.6e7	1.3e7
1e−1	15	4.2e5	1.6e5	7.2e5	4.2e5	8	2.1e7	1.6e7	2.7e7	1.3e7
1e−3	15	4.3e5	1.7e5	7.6e5	4.3e5	8	2.1e7	1.6e7	2.7e7	1.3e7
1e−5	15	4.9e5	2.2e5	8.0e5	4.9e5	8	2.1e7	1.6e7	2.6e7	1.3e7
1e−8	15	5.7e5	2.8e5	8.6e5	5.7e5	8	2.2e7	1.6e7	2.7e7	1.3e7

Table 3: AMaLGaM: Shown are, for functions f_{121}-f_{130} and for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\rm opt} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value (RT_{succ}). If $f_{\rm opt} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

Δf	f_{121} **in 5-D**, N=15, mFE=3085457					f_{121} **in 20-D**, N=15, mFE=3595636				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.9e1	1.3e1	2.4e1	1.9e1	15	8.5e2	8.0e2	9.1e2	8.5e2
1	15	1.2e2	1.1e2	1.3e2	1.2e2	15	9.5e3	6.6e3	1.3e4	9.5e3
1e−1	15	1.5e3	3.3e2	2.8e3	1.5e3	15	8.2e4	4.4e4	1.3e5	8.2e4
1e−3	15	7.8e4	5.7e4	1.0e5	7.8e4	15	1.1e6	9.0e5	1.4e6	1.1e6
1e−5	15	4.4e5	3.8e5	5.1e5	4.4e5	15	1.7e6	1.5e6	2.0e6	1.7e6
1e−8	15	1.6e6	1.4e6	1.9e6	1.6e6	15	2.3e6	1.9e6	2.6e6	2.3e6

Δf	f_{123} **in 5-D**, N=15, mFE=5009700					f_{123} **in 20-D**, N=15, mFE=20005849				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.4e1	1.2e1	3.5e1	2.4e1	15	1.3e4	5.5e3	2.0e4	1.3e4
1	15	3.0e5	2.4e5	3.7e5	3.0e5	0	*23e−1*	*13e−1*	*43e−1*	8.9e6
1e−1	5	1.3e7	1.2e7	1.4e7	4.8e6	.	.	.	.	.
1e−3	0	*18e−2*	*34e−3*	*24e−2*	3.5e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{125} **in 5-D**, N=15, mFE=5008580					f_{125} **in 20-D**, N=15, mFE=20005997				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.2e0	1.1e0	1.3e0	1.2e0	15	1.1e0	1.0e0	1.1e0	1.1e0
1	15	2.4e1	2.0e1	2.8e1	2.4e1	15	6.8e2	6.1e2	7.5e2	6.8e2
1e−1	15	6.4e3	4.1e3	8.7e3	6.4e3	0	*24e−2*	*17e−2*	*29e−2*	7.9e6
1e−3	8	6.9e6	5.9e6	8.0e6	3.8e6	.	.	.	.	.
1e−5	5	1.3e7	1.1e7	1.4e7	4.9e6	.	.	.	.	.
1e−8	5	1.3e7	1.1e7	1.4e7	4.9e6	.	.	.	.	.

Δf	f_{127} **in 5-D**, N=15, mFE=5008766					f_{127} **in 20-D**, N=15, mFE=20008232				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.3e0	1.0e0	1.5e0	1.3e0	15	1.0e0	1.0e0	1.0e0	1.0e0
1	15	2.4e1	2.0e1	2.8e1	2.4e1	15	5.6e2	5.0e2	6.2e2	5.6e2
1e−1	15	9.8e3	7.1e3	1.2e4	9.8e3	15	3.3e6	2.6e6	4.2e6	3.3e6
1e−3	8	5.8e6	4.6e6	7.2e6	3.8e6	1	3.0e8	3.0e8	3.0e8	2.0e7
1e−5	4	1.6e7	1.4e7	1.8e7	3.1e6	1	3.0e8	3.0e8	3.0e8	2.0e7
1e−8	1	7.1e7	6.6e7	7.5e7	5.0e6	1	3.0e8	3.0e8	3.0e8	2.0e7

Δf	f_{129} **in 5-D**, N=15, mFE=5009247					f_{129} **in 20-D**, N=15, mFE=20005448				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.6e3	1.2e2	3.1e3	1.6e3	0	*31e+0*	*22e+0*	*41e+0*	8.9e6
1	15	2.7e5	1.2e5	4.3e5	2.7e5	.	.	.	.	.
1e−1	15	7.7e5	5.6e5	1.0e6	7.7e5	.	.	.	.	.
1e−3	15	1.3e6	1.0e6	1.7e6	1.3e6	.	.	.	.	.
1e−5	12	3.0e6	2.3e6	3.6e6	2.3e6	.	.	.	.	.
1e−8	5	1.2e7	9.8e6	1.3e7	4.0e6	.	.	.	.	.

Δf	f_{122} **in 5-D**, N=15, mFE=1999552					f_{122} **in 20-D**, N=15, mFE=20014172				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.3e1	9.4e0	1.7e1	1.3e1	15	6.9e2	5.7e2	8.2e2	6.9e2
1	15	2.1e4	1.2e4	2.9e4	2.1e4	15	2.3e6	2.0e6	2.6e6	2.3e6
1e−1	15	1.3e5	1.0e5	1.5e5	1.3e5	15	4.4e6	3.8e6	5.0e6	4.4e6
1e−3	15	2.1e5	1.9e5	2.4e5	2.1e5	15	8.3e6	7.4e6	9.2e6	8.3e6
1e−5	15	3.1e5	2.5e5	3.7e5	3.1e5	8	2.8e7	2.5e7	3.1e7	1.5e7
1e−8	15	5.3e5	3.9e5	6.9e5	5.3e5	1	2.9e8	2.8e8	3.0e8	2.0e7

Δf	f_{124} **in 5-D**, N=15, mFE=5008758					f_{124} **in 20-D**, N=15, mFE=12363963				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.1e1	7.5e0	1.5e1	1.1e1	15	5.4e2	4.6e2	6.2e2	5.4e2
1	15	3.0e3	1.7e3	4.3e3	3.0e3	15	1.5e4	1.1e4	2.0e4	1.5e4
1e−1	15	4.2e4	3.5e4	4.9e4	4.2e4	15	7.6e5	6.2e5	9.0e5	7.6e5
1e−3	15	3.5e5	2.8e5	4.2e5	3.5e5	15	2.2e6	1.9e6	2.5e6	2.2e6
1e−5	14	2.6e6	2.2e6	3.0e6	2.4e6	15	2.4e6	2.1e6	2.7e6	2.4e6
1e−8	13	3.4e6	2.9e6	3.8e6	3.0e6	15	3.5e6	2.7e6	4.3e6	3.5e6

Δf	f_{126} **in 5-D**, N=15, mFE=5008585					f_{126} **in 20-D**, N=15, mFE=20005928				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.0e0	1.0e0	1.0e0	1.0e0	15	1.1e0	1.0e0	1.3e0	1.1e0
1	15	3.8e1	2.7e1	4.9e1	3.8e1	15	5.7e3	2.4e3	9.2e3	5.7e3
1e−1	15	6.9e4	5.1e4	9.0e4	6.9e4	0	*34e−2*	*31e−2*	*37e−2*	1.0e7
1e−3	0	*16e−3*	*74e−4*	*25e−3*	2.2e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{128} **in 5-D**, N=15, mFE=912091					f_{128} **in 20-D**, N=15, mFE=20004996				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	9.1e2	1.1e2	1.7e3	9.1e2	13	8.6e6	6.5e6	1.1e7	7.7e6
1	15	1.5e5	6.0e4	2.5e5	1.5e5	4	6.6e7	5.9e7	7.2e7	1.6e7
1e−1	15	1.7e5	7.5e4	2.6e5	1.7e5	1	2.9e8	2.8e8	3.0e8	8.6e6
1e−3	15	1.8e5	9.3e4	2.8e5	1.8e5	1	2.9e8	2.8e8	3.0e8	8.6e6
1e−5	15	1.8e5	9.9e4	2.7e5	1.8e5	1	2.9e8	2.8e8	3.0e8	8.7e6
1e−8	15	1.9e5	1.0e5	2.9e5	1.9e5	1	2.9e8	2.8e8	3.0e8	8.7e6

Δf	f_{130} **in 5-D**, N=15, mFE=837591					f_{130} **in 20-D**, N=15, mFE=20044874				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	8.0e1	6.8e1	9.2e1	8.0e1	15	2.7e4	1.2e4	4.5e4	2.7e4
1	15	1.1e5	5.6e4	1.7e5	1.1e5	13	6.9e6	4.0e6	9.8e6	6.9e6
1e−1	15	1.7e5	1.1e5	2.3e5	1.7e5	12	9.5e6	6.2e6	1.3e7	7.8e6
1e−3	15	2.1e5	1.3e5	2.9e5	2.1e5	10	1.7e7	1.3e7	2.1e7	1.1e7
1e−5	15	2.5e5	1.8e5	3.2e5	2.5e5	7	3.2e7	2.7e7	3.7e7	1.5e7
1e−8	15	3.1e5	2.3e5	3.8e5	3.1e5	3	8.3e7	7.2e7	9.4e7	1.4e7

Table 4: iAMaLGaM: Shown are, for functions f_{121}-f_{130} and for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\rm opt} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value (RT_{succ}). If $f_{\rm opt} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.