Example paper: Black-Box Optimization Benchmarking Template for the Comparison of More than Two Algorithms on the NoisyTestbed

Draft version *

BBOBies

ABSTRACT

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—global optimization, unconstrained optimization; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. RESULTS

Results from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures 2 and 3, and Figure 1. The expected running time (ERT), used in the figures and table, depends on a given target function value, $f_{\rm t} = f_{\rm opt} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach $f_{\rm t}$, summed over all trials and divided by the number of trials that actually reached f_t [?, ?]. Statistical significance is tested with the rank-sum test for a given target Δf_t using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration if available. Tables 1 and 2 give the Expected Running Time (ERT) for targets $10^{1,-1,-3,-5,-7}$ divided by the best ERT obtained during BBOB-2009 (given in the ERT_{best} row), respectively in 5-D and 20-D. Bold entries correspond to the best (or 3-best if there are more than 3 algorithms) values.

Entries with the \downarrow symbol are statistically significantly better (according to the rank-sum test) compared to the best algorithm in BBOB-2009, with p=0.05 or $p=10^{-k}$ where k>1 is the number following the \downarrow symbol, with Bonferroni correction of 30.

The median number of conducted function evaluations is additionally given in *italics*, if $ERT(10^{-7}) = \infty$. #succ is the

number of trials that reached the final target $f_{\rm opt} + 10^{-8}$.

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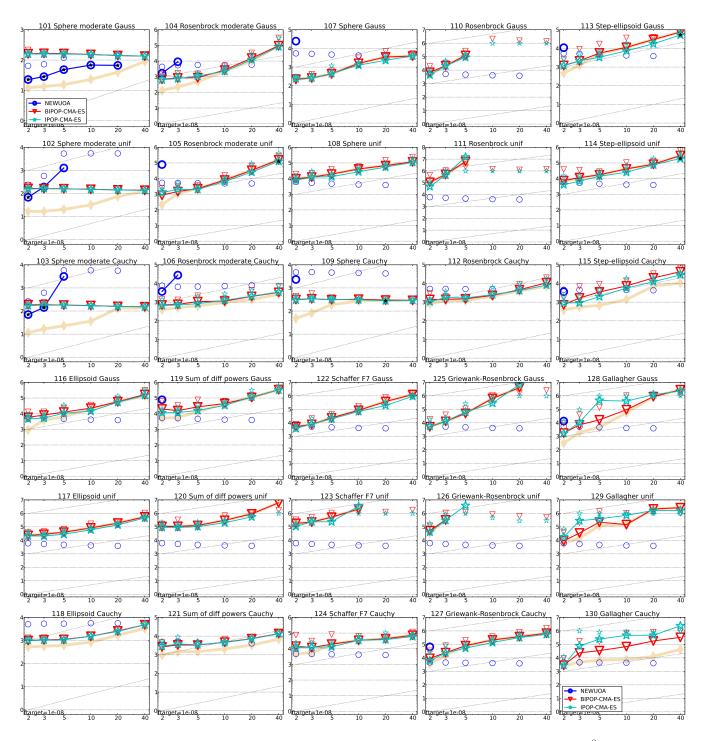


Figure 1: Expected running time (ERT) divided by dimension for target function value 10^{-8} as \log_{10} values versus dimension. Different symbols correspond to different algorithms given in legend of f_{101} and f_{130} . Light symbols give the maximum number of function evaluations from all trials divided by the dimension. Horizontal lines give linear scaling, the slanted dotted lines give quadratic scaling. Legend: \circ : NEWUOA, ∇ : BIPOP-CMA-ES, \star : IPOP-CMA-ES

$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7		1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f101 11	37	44 2.1(0.9)*2	62	69	75	15/15 f116	5730	14472	22311	26868	30329	31661	15/15
NEWUOA 2.5(1) BIPOP-C 3.2(2)	1.6(0.7)* 3.1(0.8)	4.6(0.9)	2.6 (2)*2 6.1(0.5)	3.0(2)*3 8.0(0.4)	3.1(2)*4 10(0.7)	15/18EWUOA 15/18IPOP-C		∞ 2.0 (2)	∞ 1.9(2)	∞ 2.1(2)	$_{2.0(2)}^{\infty}$	$\infty 2e4$ 2.0(2)	0/15 15/15
IPOP-CM 3.3(3)	3.4(1)	4.7(1)	6.0(1)	7.8(1)	9.3(0.5)	15/1 P OP-CM		2.3(2)	1.9(2)	1.8(1)	1.7(1)	1.7(1)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\# \mathrm{succ} \Delta f_{\mathrm{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f102 11	35	50	72	86	99	15/15 f117	26686	76052	1.1e5	1.4e5	1.7e5	1.9e5	15/15
NEWUOA 6.3(11) BIPOP-C 2.7 (2)	6.0(7) 3.0(1)	7.0(9) 4.0(0.6)	20(19) 5.1(0.5)	33(32) 6.3(0.5)	41(57) 7.2(0.7)	15/18 EWUOA 15/18 IPOP-C	. ∞ 1(0.7)	∞ $1(0.8)$	$\infty \\ 1(0.7)$	$\infty \\ 1(0.6)$	∞ $1(0.6)$	$\infty 2e4$ 1(0.5)	0/15 15/15
IPOP-CM 3.4(2)	3.1(2)	4.1(0.9)	5.1 (0.8)	6.5(0.9)	7.3(0.6)	15/18POP-CM		0.95(0.8)	0.77(0.6)	0.73(0.5)	0.67(0.5)	0.69 (0.5)	15/15
$\Delta f_{ m opt}$ [1e1]	1e0	1e-1 1	.e-3	1e-5	1e-7		1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f103 11				35	115	15/15 f118	429	1217	1555	1998	2430	2913	15/15
NEWUOA 2.4(1.0) BIPOP-C 3.5(4)	1.9(0.7)*2			178(172)	136(176)	12/18 EWUOA 15/18 IPOP-C	13.2(1)	10(11) 2.0(0.7)	116(134) 1.9(0.7)	∞ 2.1(0.4)	$_{2.0(0.4)}^{\infty}$	$\infty 3e4$ 1.8(0.3)	0/15 $15/15$
IPOP-CM 3.6(2)		7.4(1) 1 6.6(1) 1	3(1) 2(2)	17(2) 17(3)	6.9(0.9) 7.1(0.6)	15/15 OP-CM	3.2(1)	2.0(0.9)	1.9(0.8)	2.0(0.4)	1.9(0.3)	1.7(0.2)	15/15
$\Delta f_{ m opt}$ [1e1]	1e0	1e-1	1e-3	1e-5	1e-7	$I_{\#_{SUCC}}\Delta f_{ODt}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f104 173	773	1287	1768	2040	2284	15/15 f119	12	657	1136	10372	35296	49747	15/15
NEWUOA 1.2(2) BIPOP-C 1.4(0.3)	3.4(4) 1.9(0.6)	6.0(8) 2.0 (0.3)	24(27) 2.0(0.2)	∞ 1.9(0.2)	$\infty 3e4$ 1.8(0.2)	0/18EWUOA 15/18IPOP-C	1.9(3)	35(41) $1(2)$	$\frac{\infty}{1(2)}$	0.000	∞ 1.5(0.8)	$\infty 2e4$ 2.3(1)	$0/15 \\ 15/15$
IPOP-CM 1.4(0.4)	3.4(3)	2.9(2)	2.7(1)	2.5(1)	2.4(1)	15/15 OP-CM	1.1(1)	0.35(0.2)		0.83(0.7)	1.0(0.4)	1.4(0.7)	15/15
$\Delta f_{ m opt}$ [1e1]	1e0	1e-1	1e-3	1e-5	1e-7	$ _{\#_{SUCC}}\Delta f_{ODt} $	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f105 167	1436	5174	10388	10824	11202	15/15 f120	16	2900	18698	72438	3.3e5	5.5e5	15/15
NEWUOA 1.7(2) BIPOP-C 1.7(0.4)	2.7(3) 3.7(2)	3.3(4) $1.7(0.9)$	$\infty \\ 1(0.4)$	∞ $1(0.4)$	$\infty 3e4$ 1(0.4)	0/19EWUOA 15/18IPOP-C	17(16)	55(67) 1.1(1)	$_{1(0.6)}^{\infty}$	$_{1(0.8)}^{\infty}$	0.5 $1(0.5)$	$\infty 2e4$ 1(0.4)	0/15 15/15
IPOP-CM 1.6(0.7)	3.8(3)	1.6(0.9)	0.90(0.3)	0.90(0.3)	0.90(0.3)	15/15 OP-CM	6.0(8)	1.6(2)	0.68(0.4				15/15
$\Delta f_{ m opt}$ [1e1]	1e0	1e-1	1e-3	1e-5	1e-7	$\#_{\text{succ}}\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f106 92	529	1050	2666	2887	3087	15/15 f121	8.6	111	273	1583	3870	6195	15/15
NEWUOA 0.93(0.7)*	³ 2.2 (3)	5.0(6)	59(63)	∞	∞ 3e4	0/18EWUOA	4.8(13) 2.7(3)	15(24) 1.1(0.4)	76(82) 1(0.2)	∞ 1.1(0.5)	∞ 2.0 (0.2)	$\infty 2e4$ 2.2(0.2)	0/15 $15/15$
BIPOP-C 3.3(0.9) IPOP-CM 3.1(1)	4.3(5) 2.5(1)	3.2(3) 2.2(0.6)	1.6(1) 1.2(0.2)	1.7(1) 1.3(0.2)	1.7(1) 1.3(0.2)	15/15 OP-CM	1.9(2)	1.1(0.5)	1.0(0.3)	1.1(0.4)	2.1(0.4)	2.3(0.4)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$ _{\# \text{succ}} \Delta f_{\text{opt}} $	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f107 40	228	453	940	1376	1850	15/15 f122	10	1727	9190	30087	53743	1.1e5	15/15
NEWUOA 60(50)	194(204)	∞ 1(0.5)	∞	∞ 1(0,0)	∞ 2e4	0/19EWUOA 15/18IPOP-C	114(23) 2.2(2)	91(104) 1(1)	$\infty \\ 1(0.8)$	∞ $1(0.5)$	$\infty \\ 1(0.6)$	$\infty 2e4$ 1(0.6)	$0/15 \\ 15/15$
BIPOP-C 1.7(2) IPOP-CM 2.1(3)	1(0.7) 0.98 (0.	1(0.5) 4) 1.1(0.7)	1(0.3) 1.3(1)	1(0.2) 1.2(1.0)	1(0.2) 1.1(0.7)	15/15 OP-CM	4.8(5)	0.94(0.7)		0.56(0.3)			15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\#\operatorname{succ}\Delta f_{\operatorname{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f108 87	5144	14469	30935	58628	80667	15/15 f123	11	16066	81505	3.4e5	6.7e5	2.2e6	15/15
NEWUOA 77(89) BIPOP-C 6.1 (10)	64(74) 1.0(0.9)	$\infty \\ 1(0.8)$	$\infty \\ 1(0.6)$	$ \begin{array}{c} \infty \\ 1(0.4) \end{array} $	$\infty 2e4$ 1(0.3)	0/1NEWUOA 15/1BIPOP-C		$_{1(0.8)}^{\infty}$	1(0.6)	$_{1(0.6)}^{\infty}$	$\infty \\ 1(0.6)$	$\infty 2e4$ 1(0.9)	$0/15 \\ 15/15$
IPOP-CM 9.1(13)	0.80(0.9)				1(0.3) 1 0.69 (0.3)	15/18POP-CM	23(38)	0.62(0.5)	0.52(0.3)	0.74(0.5)	0.65(0.4)	0.45(0.3)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\#_{\operatorname{succ}}\Delta f_{\operatorname{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f109 11	57	216	572	873	946	15/15 f124	10	202	1040	20478	45337	95200	15/15
NEWUOA 4.8(10)	13(10) 2.2(0.9)	83(108) 1.1(0.3)	∞ 1.1(0.2)	∞ 1.1(0.3)	$\infty 2e4$ 1.5(0.3)	0/19EWUOA 15/1BIPOP-C	3.0(1) 1.5(2)	158(177) 1.1(0.4)	∞ 1(0.3)	∞ 1.1(0.7)	∞ 1.2(1.0)	$\infty 2e4$ 1(0.5)	0/15 $15/15$
BIPOP-C 3.5(2) IPOP-CM 2.9 (3)	2.2(0.9) 2.2(0.9)	1.1(0.3) $1.2(0.5)$	1.1(0.2) 1.0(0.3)	1.1(0.3) 1.1(0.2)	1.5(0.3)	15/15 OP-CM	2.8(3)	1.3(0.6)	4.0(8)	1.2(0.8)	0.93(0.2)	0.65(0.4)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\#_{\text{succ}}\Delta f_{\text{opt}}$		1e0	1e-1	1e-3	1e-5	1e-7	#succ
f110 949	33625	1.2e5	5.9e5	6.0e5	6.1e5	15/15 f125	1	1	1	2.4e5	2.4e5	2.5e5	15/15
NEWUOA 118(111) BIPOP-C 1(1)	10(12) 4.8(7)	∞ $3.7(4)$	0.000	0.7	$\infty 2e4$ 1(0.6)	0/19EWUOA	11.1(0)	15(6) 17(18)	6088(8822) 3443(2609)		${f 1}(0.7)$	$\infty 2e4$ 1(0.7)	0/15 15/15
IPOP-CM 0.73 (0.8		3.4(2)	0.72(0.4			15/15 OP-CM	1(0)	27(28)	2599 (2294)			1.3(0.6)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$I_{\#_{\text{SUCC}}} \Delta f_{\text{ODt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f111 6856	6.1e5	8.8e6	2.3e7	3.1e7	3.1e7	3/15 f126 0/15NEWUOA	1 2(0)	1 1053(1172)	1 3.5e5(3	∞ .5)	∞	∞	$0 \\ 0/15$
NEWUOA ∞ BIPOP-C 1(1.0)	∞ 2.5 (3)	∞ 1(1)	∞ 1(0.9)	∞ 1(1.0)	$\infty 2e4$ 1(1.0)	0/15 BIPOP-C	1(0)	160(130)	1.3e4(1				0/15
IPOP-CM 0.78 (0.8)	15(19)	3.9(5)	3.2(3)	2.4(3)	2.4(3)	1/15POP-CM	1(0)	63(59)			e7) 1.9e7 (2e	e7) 1.9e7 (2e7	2/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7		1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f112 107	1684	3421	4502	5132	5596	15/15 f127 0/18EWUOA	2.5(6)	1 14(4)	1 7248(1e4)	$3.4e5$ ∞	3.9e5 ∞	4.0e5 ∞ 2e4	15/15 0/15
NEWUOA 1.9(3) BIPOP-C 4.0(2)	7.7(9) 1(0.6)	105(112) 1.2(0.2)	∞ 1.3(0.2)	∞ 1.3(0.2)	$\infty 2e4$ 1.3(0.2)	In E / ABIPOP-C	H(0)	19(24)	2136(1530)	1(1.0)	1(0.8)	1(0.8)	15/15
IPOP-CM 2.1(1)	1.4(0.5)	1.4(0.2)	1.5(0.2)	1.5(0.2)	1.5(0.2)	15/15POP-CM	1 (0)	15(18)	1542 (1498)	•			15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\# \operatorname{su}_{\operatorname{CC}} \Delta f_{\operatorname{opt}}$	1e1			1e-3	1e-5	1e-7	#succ
f113 133 NEWUOA13(16)	1883	8081	24128	24128 ~	24402 \(\times 2e \lambda \)	15/15 f128 0/19EWUOA	111	4248 17(19)		12447 ∞	17217 ∞	21162 $\infty 2e4$	15/15 0/15
BIPOP-C 1.5(1.0)	44(43) 1.3(2)	$\frac{\infty}{1.7(2)}$	$\frac{\infty}{1.1(1)}$	∞ 1.1(1)	∞ 2e4 1.1(1)	15/1BIPOP-C	2.2(2)	6.9(9)	10(17)	6.6(11)	4.8(8)	3.9(6)	15/15
IPOP-CM 3.7(8)	1.4(1)	1.4(1)	0.67(0.4)	0.67 (0.4)	0.67(0.4)	15/1FOP-CM	1.0(0.7)				132(150)	108(196)	10/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\# \operatorname{succ}^{\Delta f_{\text{opt}}}$	1e1 64	1e0 10710	1e-1 59443	1e-3 2.8e5	1e-5	1e-7 5.8e5	#succ
f114 767 NEWUOA43(40)	14720 ∞	56311 ∞	83272 ∞	83272 ∞	84949 ∞ 2e4	15/15 f129 0/18 EWUOA	124(124)	10710 16(16)	59443 ∞	$2.8e5$ ∞	$5.1e5$ ∞	5.8e5 ∞ 2e4	15/15 0/15
BIPOP-C 2.2(2)	$\frac{\infty}{1(0.6)}$	$\frac{\infty}{1(0.7)}$	1(0.7)	$_{1(0.7)}^{\infty}$	∞ ze4 1(0.7)	In a 74BIPOP-C	I 12(15)	7.1(8)	9.2(2)	3.9(12)	2.2(7)	1.9(6)	13/15
IPOP-CM $3.2(4)$	0.45(0.5)			0.79 (0.7)	0.80(0.7)	15/15POP-CM	8.5(10)	13(18)	18(35)	6.7(9)	3.8(5)	3.3(4)	11/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\# \operatorname{succ}^{\Delta} f_{\text{opt}}$	1e1 55	1e0 812	1e-1 3034	1e-3 32823	1e-5 33889	1e-7 34528	#succ 10/15
f115 64 NEWUOA 2.9(3)	485 14(20)	1829 42(48)	2550 ∞	2550	2970 ∞ 2e4	15/15 f130 0/18 EWUOA	2.3(4)	11(10)	10(10)	∞	∞	∞ 2e4	0/15
BIPOP-C 1.5(0.8)	2.6(2)	6.5(7)	5.9(6)	5.9(6)	5.7(5)	15/1RIPOP-C	1.9(1)	57(82)	55(101)	5.1 (9)	5.0 (9)	5.0 (9)	15/15
IPOP-CM 1.7(2)	2.4(4)	2.7(4)	3.1 (3)	3.1 (3)	2.7 (3)	15/14POP-CM	µ.2(1)	59(10)	321(634)	37(62)	36(60)	35(59)	12/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 5. The inter-80%tile range divided by two is given in braces. The median number of conducted function evaluations is additionally given in *italics*, if $\text{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\text{opt}} + 10^{-8}$. Best results are printed in bold.

$\Delta f_{ m opt}$ [1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\#\operatorname{succ}\Delta f_{\operatorname{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f101 59	425	571	700	739	783	15/15 f116	5.0e5	6.9e5	8.9e5	1.0e6	1.1e6	1.1e6	15/15
NEWUOA 3.1 (0.8)*3 BIPOP-C 6.1(1)	0.85(0.2)* 1.5(0.2)	³ 0.90(0.2)* 1.6(0.1)	⁴ 1.1(0.2)*4 2.1(0.1)	1.5(0.3)*2 2.7(0.1)	1.6(0.4)*2 3.3(0.2)	15/1SEWUOA 15/1BIPOP-C		∞ 1.2(0.6)	∞ 1.1(0.5)	$\frac{\infty}{1(0.4)}$	∞ $1(0.4)$	$\infty 8e4$ 1(0.4)	0/15 15/15
IPOP-CM 6.0(2)	1.5(0.2)	1.5(0.2)	2.0(0.2)	2.6(0.1)	3.2(0.2)	15/18POP-CM		1.1(0.5)	1.00(1)	0.92(1.0)	0.93(0.9)	0.93(0.9)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\#_{\mathrm{succ}}\Delta f_{\mathrm{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f102 231	399	579	921	1157	1407	15/15 f117	1.8e6	2.5e6	2.6e6	2.9e6	3.2e6	3.6e6	15/15
NEWUOA 2.9(4) BIPOP-C 1.6 (0.3)	6.1(9) 1.6(0.2)	6.3(7)	45(49) 1.6(0.1)	∞ 1.8(0.1)	$\infty 1e5$ 1.8(0.1)	0/1SEWUOA 15/1SIPOP-C	∞ 1(0.5)	∞ $1(0.2)$	$\infty \\ 1(0.2)$	∞ $1(0.2)$	$\infty \\ 1(0.2)$	$\infty 8e4$ 1(0.2)	0/15 15/15
IPOP-CM 1.6(0.2)	1.6(0.2) 1.6(0.3)	1.6(0.2) 1.6(0.2)	1.6(0.1) 1.6(0.1)	1.8(0.1) 1.7(0.1)	1.8(0.1)	15/15POP-CM						20.72(0.3)	
$\Delta f_{ m opt}$ 1e1	1e0		1e-3	1e-5	1e-7	$\# \mathrm{succ} \Delta f_{\mathrm{opt}}$	11e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f103 65	417	629	1313	1893	2464	14/15 f118	6908	11786	17514	26342	30062	32659	15/15
NEWUOA 2.3(0.9)*4	1.00(0.2)*		1231(1324)	∞	$\infty~1e5$	0/1SEWUOA	∞	∞	∞	∞	∞	∞ 1e5	0/15
BIPOP-C 5.5(1) IPOP-CM 5.5(1)		1.5(0.1) 1.4(0.2)	1.2(0.1) 1.2(0.1)	1.2(0.1) 1.2(0.1)	1.2(0.1) 1.2(0.1)	15/1BIPOP-C 15/1POP-CM		1.8(0.4) 1.8(0.5)	1.6(0.2) 1.7(0.2)	1.5(0.1) 1.5(0.1)	1.6(0.1) 1.5(0.2)	1.6(0.1) 1.5(0.1)	$\frac{15}{15}$
! ` '	1.5(0.2) 1e0	1.4(0.2) 1e-1	1.2(0.1) 1e-3	1.2(0.1) 1e-5	1.2(0.1) 1e-7	$\#_{\text{succ}}\Delta f_{\text{opt}}$	12.0(0.4)	1.8(0.5) 1e0	1.7(0.2) 1e-1	1.5(0.1) 1e-3	1.3(0.2) 1e-5	1.3(0.1) 1e-7	#succ
$\frac{\Delta f_{ m opt}}{{ m f104}}$ 1e1	85656	1.7e5	1.8e5	1.9e5	2.0e5	15/15 f119	2771	29365	35930	4.1e5	1.4e6	1.9e6	15/15
NEWUOA 68(78)	∞	∞	∞	∞	∞ 1e5	0/1MEWUOA		∞	∞	∞	∞	∞ 8e4	0/15
BIPOP-C 10(7)	3.2(2)	1.7(1)	1.6(1)	1.6(1.0)	1.6(0.9)	15/1BIPOP-C	1.6(1) 1.9(0.6)	1(1) 0.58(0.4)	1(1) 0.69(0.3)	1(0.5) 0.58 (0.3)	1.3(0.3)	1.1(0.2)	15/15
IPOP-CM 7.5(6)	2.5(2)	1.3(0.9)	1.3(0.9)	1.3(0.9)	1.2(0.8)	15/18POP-CM		` ′	` ′	` ′		↓2 0.97 (0.5)	15/15
$\frac{\Delta f_{\text{opt}}}{\text{f105}}$ 1e1	1e0 6.1e5	1e-1 6.3e5	1e-3 6.5e5	1e-5 6.6e5	1e-7 6.7e5	$\# succ \Delta f_{opt}$ 15/15 f120	1e1 36040	1e0 1.8e5	1e-1 2.8e5	1e-3 1.6e6	1e-5 6.7e6	1e-7	#succ 13/15
NEWUOA ∞	∞	∞	∞	∞	∞ 9e4	0/18 EWUOA		∞	∞	∞	∞	∞ 8e4	0/15
BIPOP-C 2.7(2)	1(0.6)	1(0.6)	1(0.6)	1(0.6)	1(0.6)	15/1BIPOP-C	1(0.6)	1(0.9)	1(0.6)	1(0.6)	1(0.4)	1(0.4)	13/15
IPOP-CM 1.9(0.9)	0.76 (0.3)	0.76 (0.3)	0.77 (0.3)	0.77 (0.3)	0.76(0.2)	15/15POP-CM	0.69 (0.4)	0.60 (0.4)	0.74 (0.5)	0.67(0.4) _↓	0.69(0.4)	0.69 (0.3)	15/15
$\frac{\Delta f_{\text{opt}}}{\text{f106}}$ 1e1	1e0	1e-1 23746	1e-3 25470	1e-5 26492	1e-7 27360	$\#\operatorname{succ}\Delta f_{\operatorname{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f106 11480 NEWUOA 7.0(5)	21668 31(32)	23740	25470	20492 ∞	27360 $\infty 2e5$	15/15 f121 0/1 Ñ EWUOA	249	769 ∞	1426 ∞	9304 ∞	34434 ∞	57404 ∞ 8e4	15/15 0/15
BIPOP-C 1.0(0.3)	1.3(0.3)	1.4(1)	1.5(1)	1.5(1)	1.5(1)	15/1 § IPOP-C	1.2(0.5)	1.0(0.2)	1.2(0.3)	1.1(0.2)	1.3(0.1)	1.9(0.1)	15/15
IPOP-CM 1.0(0.4)	1.4(1)	1.5(1.0)	1.5(0.9)	1.5(0.9)	1.5(0.9)	15/1 f POP-CM	1.3(0.4)	1.1(0.2)	1.1(0.2)	1.1(0.1)	1.4(0.1)	1.9(0.2)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\#\operatorname{succ}\Delta f_{\operatorname{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f107 8571 NEWUOA ∞	13582 ∞	16226 ∞	27357 ∞	52486 ∞	65052 $\infty 8e4$	15/15 f122 0/1 % EWUOA	692	52008	1.4e5 ∞	7.9e5 ∞	2.0e6 ∞	5.8e6 ∞ 8e4	15/15 0/15
BIPOP-C 1(0.4)	1(0.7)	1(0.6)	1(0.4)	1(0.8)	1(0.8)	15/15IPOP-C	1.8(2)	1(0.5)	1(0.7)	1(0.7)	1(0.5)	1(0.8)	15/15
IPOP-CM 1.1(0.6)	0.95 (0.4)	1.1(0.7)	0.96 (0.5)	0.68 (0.3)	0.65 (0.3)	15/15POP-CM	2.0(2)	0.92 (0.5)	0.74 (0.2)	0.63 (0.5)	0.95 (0.5)	0.64 (0.4)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7		1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f108 58063 NEWUOA ∞	97228 ∞	2.0e5 ∞	4.5e5 ∞	6.3e5 ∞	9.0e5 ∞ 8e4	15/15 f123	1063	5.3e5	1.5e6	5.3e6	2.7e7	1.6e8	0 /15
BIPOP-C 1(0.5)	1(0.4)	1(0.5)	1(0.5)	1(0.5)	1(0.4)	0/15EWUOA 15/15IPOP-C	5.7(4)	$_{1(0.8)}^{\infty}$	$\infty \\ 1(0.7)$	$\infty \\ 1(0.6)$	$_{1(0.8)}^{\infty}$	∞ 8e4 1(1)	$0/15 \ 0/15$
IPOP-CM 0.72(0.2)	0.87 (0.6)	0.66 (0.3)	0.77 (0.4)	0.94 (0.4)	1.0(0.6)	15/15POP-CM	7.2(5)	0.72(0.4)			0.62 (0.4)	∞ $2e7$	0/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\#\operatorname{succ}\Delta f_{\mathrm{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f109 333 NEWUOA17(23)	632 ∞	1138 ∞	2287 ∞	3583 ∞	4952 ∞ 8e4	15/15 f124 0/1 N EWUOA	192	1959 ∞	40840 ∞	1.3e5	3.9e5	8.0e5 ∞ 8e4	15/15 0/15
BIPOP-C 1.2(0.3)	1.2(0.2)	1.1(0.2)	1.1(0.1)	1.1(0.1)	1.0(0.1)	15/1BIPOP-C		1.0(0.5)	1(1.0)	$_{1(0.9)}^{\infty}$	∞ 1(0.8)	0.4	15/15
IPOP-CM 1.1(0.2)	1.2(0.1)	1.1 (0.1)	1.1(0.1)	1.0(0.1)	1.00(0.1)	15/1POP-CM	1.1(0.5)	0.99 (0.7)	0.75 (0.7)	0.98 (0.5)	0.84 (0.4)	0.78 (0.3)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7		1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f110 ∞ NEWUOA .	∞	∞	∞	∞	∞	0 f125 0/15NEWUOA	1	1	1	2.5e7	8.0e7	8.1e7	4/15
BIPOP-C .						0/15BIPOP-C	1(0)	414(426) 383(356)	∞ 9.8e6(7e	∞ 6)1(0.9)	∞ 1(1)	$\infty 8e4$ 1(1.0)	0/15 $4/15$
IPOP-CM .						0/15IPOP-CM		957(1360)		e6) 0.79 (0.6)	1.8(2)	1.8(2)	2/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\# \operatorname{succ} \Delta f_{\mathrm{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f111 ∞ NEWUOA	∞	∞	∞	∞	∞	0 f126	1	1	1	∞	∞	∞	0
BIPOP-C .						0/15NEWUOA 0/15BIPOP-C	4.2(0) 1(0)	1.3e5(1e5) 5781(4226)	∞ ∞				0/15 0/15
IPOP-CM .						0/15IPOP-CM	1(0)	5759 (3156)					0/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\#\operatorname{succ}\Delta f_{\operatorname{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f112 25552 NEWUOA ∞	64124 ∞	69621 ∞	73557 ∞	76137 ∞	78238 ∞ 1e5	15/15 f127	1	1	1	4.4e6	7.3e6	7.4e6	15/15
BIPOP-C 1(0.3)	1.1(0.8)	1.1(0.8)	1.2(0.7)	1.2(0.7)	1.2(0.7)	0/15EWUOA 15/15IPOP-C	3.7(0) 1(0)	253(389) 176(91)	∞ 9.0e5(1e	∞ e6)1(0.6)	0.000	$\infty 8e4$ 1(0.7)	0/15 15/15
IPOP-CM 0.95 (0.4)	0.94 (0.2)	1.0(0.1)	1.1(0.1)	1.1(0.1)	1.1 (0.1)	15/15POP-CM	1(0)	267(132)		6) 0.81 (0.6)	0.84(0.5)	0.85 (0.5)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7		1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f113 50123 NEWUOA ∞	$3.6e5$ ∞	$5.6e5$ ∞	5.9e5 ∞	5.9e5 ∞	5.9e5 $\infty 8e4$	15/15 f128	1.4e5	1.3e7	1.7e7	1.7e7	1.7e7	1.7e7	9/15
BIPOP-C 1(1.0)	1(0.7)	1(0.4)	1(0.4)	1(0.4)	1(0.4)	0/15EWUOA 15/15IPOP-C		${\bf 1}(2)$	∞ 1(1)	${f 1}(1)$	∞ 1(1)	∞ 8e4 1(1)	0/15 9/15
IPOP-CM 1.0(0.8)	0.53 (0.4)	0.58(0.3) _↓	0.59(0.2) _↓		0.59(0.2)	15/15POP-CM		1.0(1)	1.4(2)	1.4(2)	1.4(2)	1.4(2)	6/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\# \operatorname{succ} \Delta f_{\operatorname{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f114 2.1e5	1.1e6	1.4e6	1.6e6	1.6e6	1.6e6	15/15 f129	7.8e6	4.1e7	4.2e7	4.2e7	4.2e7	4.2e7	5/15
NEWUOA ∞ BIPOP-C 1(0.4)	$\infty \\ 1(0.5)$	$\infty \\ 1(0.5)$	$\infty \\ 1(0.5)$	$\infty \\ 1(0.5)$	$\infty 8e4$ 1(0.5)	0/1%EWUOA 15/1BIPOP-C		∞ $1(1)$	∞ $1(1)$	∞ $1(1)$	∞ 1(1)	∞ 8e4 1(1)	$0/15 \\ 5/15$
IPOP-CM 0.59(0.3)		0.84(0.3)	0.91(0.3)	0.91(0.3)	0.92 (0.4)	15/15POP-CM		0.30(0.4)	0.77 (1.0)	0.77 (0.9)	0.77 (1.0)	0.77 (0.9)	$\frac{3}{15}$
Δf_{opt} 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\#_{\text{succ}}\Delta f_{\text{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f115 2405	30268	91749	1.3e5	1.3e5	1.3e5	15/15 f130	4904	93149	2.5e5	2.5e5	2.6e5	2.6e5	7/15
NEWUOA 236(283) BIPOP-C 1(1)	$\infty \\ 6.5(7)$	∞ $3.9(2)$	∞ $3.0(1)$	$_{3.0(1)}^{\infty}$	$\infty 8e4 \\ 3.0(2)$	0/1NEWUOA 15/1BIPOP-C	9.1(9) 1.9(4)	∞ 33(75)	∞ 14(28)	∞ 14(27)	$\infty \\ 14(27)$	$\infty 8e4$ 14(27)	0/15 15/15
IPOP-CM 1.1(2)	4.8(3)	2.2(1.0)	1.8(0.8)	1.8(0.8)	1.9(0.8)	15/18POP-CM			37(57)	37(56)	37(57)	37(55)	9/15
						•							

Table 2: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 20. The inter-80%tile range divided by two is given in braces. The median number of conducted function evaluations is additionally given in italics, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\mathrm{opt}} + 10^{-8}$. Best results are printed in bold.

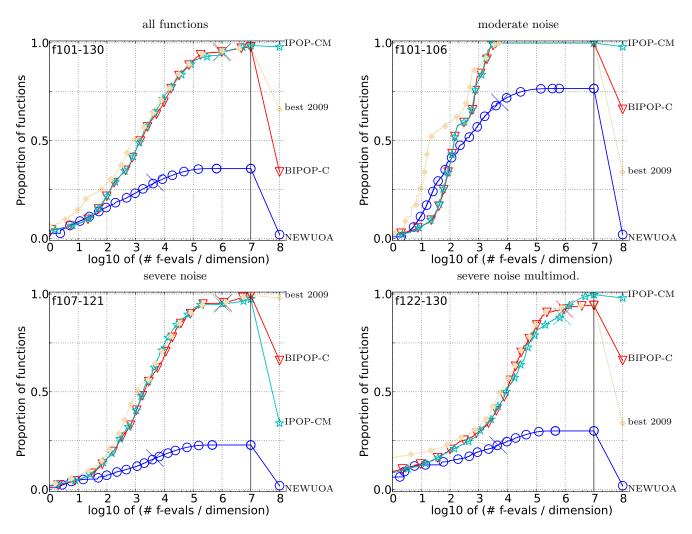


Figure 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

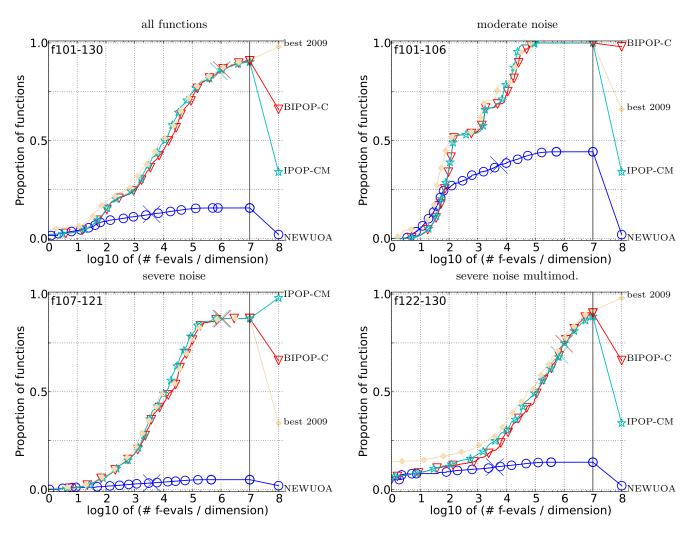


Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.