

Noiseless Functions Black-Box Optimization: Evaluation of a Hybrid Particle Swarm with Differential Operators

José García-Nieto
Dept. de Lenguajes y Ciencias
de la Computación
University of Málaga,
ETSI Informática,
Málaga - 29071, Spain
jniето@lcc.uma.es

Enrique Alba
Dept. de Lenguajes y Ciencias
de la Computación
University of Málaga,
ETSI Informática,
Málaga - 29071, Spain
eat@lcc.uma.es

Javier Apolloni
LIDIC - Departamento de
Informática
University of San Luis,
Ejército de los Andes 950,
5700, Argentina
javierma@unsl.edu.ar

ABSTRACT

In this work we evaluate a Particle Swarm Optimizer hybridized with Differential Evolution and apply it to the Black-Box Optimization Benchmarking for noiseless functions (BBOB 2009). We have performed the complete procedure established in this special session dealing with noiseless functions with dimension: 2, 3, 5, 10, 20, and 40 variables. Our proposal obtained an accurate level of coverage rate, despite the simplicity of the model and the relatively small number of function evaluations used.

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization, Global Optimization, Unconstrained Optimization; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Noiseless Functions, Hybrid Algorithms, Particle Swarm, Differential Evolution

1. INTRODUCTION

Particle Swarm Optimization (PSO) [8] and Differential Evolution (DE) [9] have been successfully used on real parameter function optimization since they are two well adapted algorithms for continuous solution encoding. Real parameter optimization problems consist basically in this: *find* \mathbf{x}^* such that $\forall \mathbf{x} \ f(\mathbf{x}^*) \leq f(\mathbf{x})$ (minimization). Here, $f(\cdot)$ is a function in a real space domain that models an optimization problem, $\mathbf{x} = \{x_1, x_2, \dots, x_{DIM}\}$ is a solution for such problem, and DIM is the number of variables with

$x_i \in [x_i^{low}, x_i^{upp}]$ ($1 \leq i \leq DIM$). Finally, x_i^{low} , $x_i^{upp} \in \mathbb{R}$ correspond to lower (*low*) and upper (*upp*) limits of the variable domain, respectively.

Swagatam Das *et al.* [3] proposed an initial hybridization of PSO and DE for continuous optimization. Based on this first idea, in this work we propose a PSO algorithm which basically uses the differential variation scheme employed in DE for adjusting the velocity of particles. This way, by combining search strategies, parameter adaptation, and differential operators present in PSO and DE we expect to improve the performance of the resulting technique. Our proposal, called DEPSO, is evaluated by means of the Black-Box Optimization Benchmarking for noiseless functions (BBOB 2009) [4, 7] according to the experimental design from [6]. We have performed the complete procedure established in this GECCO'09 workshop dealing with noiseless functions with dimension: 2, 3, 5, 10, 20, and 40 variables. Our proposal will be shown to obtain an accurate level of coverage rate, even for the higher dimensions, and despite the relatively small number of function evaluations used ($1000 \times DIM$).

The remaining of the paper is organized as follows. In Section 2 the DEPSO algorithm is briefly described. Sections 3 and 4 presents the experimentation procedure and the results obtained, respectively. In Section 5, a brief analysis of the CPU Timing experiment is reported. Finally, conclusions and remarks are given in Section 6.

2. THE ALGORITHM: DEPSO

Our proposal, DEPSO, is basically running a PSO algorithm in which we incorporate ideas of DE. For each particle \mathbf{p} (with velocity and position vectors \mathbf{v} and \mathbf{x} , respectively) the velocity is updated according to two main influences: social and differential variation factors. The social factor concerns the local or global best position \mathbf{g} of a given neighborhood of particles (being global when this neighborhood is the entire swarm). The differential variation factor is composed by using two randomly selected particle positions as made in DE. This way, for each particle \mathbf{x}_i of the swarm, a differential vector $\mathbf{w} = \mathbf{x}_{r1} - \mathbf{x}_{r2}$ is generated where particles \mathbf{x}_{r1} and \mathbf{x}_{r2} are randomly (uniformly) selected (with $\mathbf{x}_i \neq \mathbf{x}_{r1} \neq \mathbf{x}_{r2}$). Then, the new velocity \mathbf{v}' of a particle i is calculated by means of the following equation:

$$\mathbf{v}'_i \leftarrow \omega \cdot \mathbf{v}_i + \mu \cdot \mathbf{w} + \varphi \cdot (\mathbf{g}_i - \mathbf{x}_i), \quad (1)$$

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.

GECCO'09, July 8–12, 2009, Montréal Québec, Canada.
Copyright 2009 ACM 978-1-60558-505-5/09/07 ...\$5.00.

where ω is the inertia weight of the particle that controls the trade-off between global and local experience and μ is a scaling factor applied to the differential vector ($\mu = UN(0, 1)$). The third component corresponds to the social factor which is influenced by the global best \mathbf{g} of the swarm and directly proportional to the social coefficient φ (in this case $\varphi = UN(0, 1)$). Therefore, in the velocity calculation, the standard historical influence used in PSO is replaced in our proposal by the differential vector \mathbf{w} .

Similarly to DE, the update of each j component of the velocity vector of a given particle i is carried out by means of the Equation 2 as follows:

$$v'_i(j) = \begin{cases} v'_i(j) & \text{if } r \leq Cr, \\ v_i(j) & \text{otherwise.} \end{cases} \quad (2)$$

Here, $r \in [0, 1]$ is a uniformly distributed value which determines if the j^{th} component is selected from the new velocity or is selected from the current velocity, based on the crossover probability $Cr \in [0, 1]$. This mechanism is used to increase the exploitation ability of the algorithm through the search space [9]. Finally, a particle i changes its position (moves) only if the new one \mathbf{x}'_i is lower or equal than the current position \mathbf{x}_i (minimizing in this work). In other case, the particle keeps its current position (equations 3 and 4).

$$\mathbf{x}''_i = \begin{cases} \mathbf{x}'_i & \text{if } f(\mathbf{x}'_i) \leq f(\mathbf{x}_i) \\ \mathbf{x}_i & \text{otherwise,} \end{cases} \quad (3)$$

being

$$\mathbf{x}'_i \leftarrow \mathbf{x}_i + \mathbf{v}'_i \quad (4)$$

Additionally, with certain probability p_{mut} , a mutation operation is made on each particle in order to avoid an early convergence to a local optimum. The new mutated position \mathbf{x}' is generated by means of the Equation 5.

$$\mathbf{x}'' \leftarrow \mathbf{x}^{low} + UN(0, 1) \cdot (\mathbf{x}^{upp} - \mathbf{x}^{low}) \quad (5)$$

Vectors \mathbf{x}^{low} and \mathbf{x}^{upp} correspond to lower and upper limits of each dimension of the function to optimize, respectively.

Algorithm 1 shows pseudocode of the hybrid DEPSO algorithm developed for this work. First, an initialization process of all particles in the swarm S (as stated in [6]), and their initial evaluation (line 1) are carried out. After this, each evolution step the particle's positions are updated following the differential variation model of the equations previously explained (lines 4 to 18). In addition, the global best position reached at the moment is updated in order to guide the rest of the swarm. Eventually, a mutation operation is made (lines 20 to 24). Finally, the algorithm returns the best solution found during the whole process.

3. EXPERIMENTAL PROCEDURE

The experimentation procedure has been carried out according to [6] on the benchmark of 24 noiseless functions given in [4, 7]. The DEPSO algorithm was implemented in C++ using the MALLBA library [1] of metaheuristics. The noiseless functions were tackled connecting the C-code of the Black-Box Optimization Benchmarking to our implementation of DEPSO. Each candidate solution was sampled

Algorithm 1 Pseudocode of DEPSO

```

1: initialize( $S$ )
2: while not stop condition is met do
3:   for each particle position  $\mathbf{x}_i$  of the swarm  $S$  do
4:     /* Differential Variation */
5:     for each dimension  $j$  of the particle position  $\mathbf{x}_i$  do
6:        $w(j) \leftarrow x_{r1}(j) - x_{r2}(j)$ 
7:       if  $r \leq Cr$  then
8:          $v'_i(j) \leftarrow \omega \cdot v_i(j) + \mu \cdot w(j) + \varphi \cdot (g(j) - x_i(j))$ 
9:       end if
10:    end for
11:    for each dimension  $j$  of the particle position  $\mathbf{x}_i$  do
12:       $x'_i(j) \leftarrow x_i(j) + v'_i(j)$ 
13:    end for
14:    if  $f(\mathbf{x}'_i) \leq f(\mathbf{x}_i)$  then
15:       $\mathbf{x}''_i \leftarrow \mathbf{x}'_i$ 
16:    else
17:       $\mathbf{x}''_i \leftarrow \mathbf{x}_i$ 
18:    end if
19:    /* Mutation */
20:    if  $UN(0, 1) < p_{mut}$  then
21:      for each dimension  $j$  of the particle position  $\mathbf{x}_i$  do
22:         $\mathbf{x}''_i(j) \leftarrow \mathbf{x}^{inf}(j) + UN(0, 1) \cdot (\mathbf{x}^{sup}(j) - \mathbf{x}^{inf}(j))$ 
23:      end for
24:    end if
25:  end for
26: end while
27: Output: Best solution found

```

uniformly in $[-5, 5]^{DIM}$, where DIM represents the search space dimension. The maximal number of function evaluation was set to $1000 \times DIM$.

Our proposal was tested performing 15 independent runs for each noiseless function and each dimension. Table 1 shows the parameter setting used to configure DEPSO. These parameters were tuned in the context of the special session of CEC'05 for real parameter optimization [11, 5] reaching results statistically similar to the best participant algorithms (G-CMA-ES [2] and K-PCX [10]) in that session. This parameterization was kept the same for all the experiments.

Table 1: Parameter setting used in DEPSO

Description	Parameter	Value
Swarm size	Ss	20
Crossover probability	Cr	0.9
Inertia weight	ω	0.1
Differential mutation	μ	$UN(0, 1)$
Social coefficient	φ	$1 \cdot UN(0, 1)$
Mutation probability	p_{mut}	$\frac{1}{DIM}$

4. RESULTS AND DISCUSSION

In this section, the results are presented and some discussions are made in terms of the number of successful trials ($f_{opt} + \Delta f \leq 10^{-8}$) reached for each kind of noiseless function: separable (f1-f5), moderate (f6-f9), ill-conditioned (f10-f14), multimodal (f15-f19), and weak structure (f20-f24).

Figure 1 shows the Expected Running Time (ERT, ●) to reach $f_{opt} + \Delta f$ and median number of function evaluations of successful trials (+). As we can observe, our proposal obtains the highest number of successful trials in separable, moderate and weak structure noiseless functions, specifically in f1, f2, f5, f6, f7, f21, and f22 for dimensions 2, 3, and 5. We

can notice that for f5 our DEPSO reaches the target function for all dimensions. In the scope of multimodal functions, a lower number of successful trials was found for dimension 2 and 3 in f15, f17, and f19. For ill-conditioned functions, only f12 shows successful trials (six) with dimension 2. As a summary, in Table 2 the functions are ranked by means of the number of successful trials that DEPSO has obtained with them (with the best ranked functions in the top).

In the analysis of the results with higher dimensions (20 and 40), the best behavior of our proposal can be observed when facing separable noiseless functions as shown in Table 3. Concretely with functions f1 and f5 where error precisions close to 10^{-8} were reached. In addition, DEPSO obtained error values lower than 10^{-7} in the weak structured function f21 with dimension 20. Concerning the remaining of functions, the best achieved Δf precisions (of the median trials) obtained by our proposal are always lower than $e+0$, although being some of them close to $e-3$ as in f2 and f14. Only for f10, f15, and f14 our DEPSO obtained Δf precisions higher than $e+0$.

The empirical cumulative distribution function (ECDF) of the running time (in terms of function evaluations) is showed in Figure 2 left. We can easily observe that the ECDF tends to get reduced with the number of function evaluations for all functions. For separable, moderate, multimodal and weak structured noiseless functions, the ECDF of the best achieved Δf (Figure 2 right) tends to reduce noticeably the difference to the optimal function value (with the proportion of trials).

Table 2: Noiseless Functions ranked by the number of successful trials obtained by DEPSO. Columns indicate dimensions

	2	3	5	10	20	40
f1	f1	f1	f1	f1	f5	f5
f2	f2	f2	f2	f5	-	-
f3	f5	f5	f5	f2	-	-
f5	f6	f7	f21	-	-	-
f6	f21	f21	-	-	-	-
f7	f7	f6	-	-	-	-
f21	f3	f22	-	-	-	-
f22	f20	-	-	-	-	-
f20	f22	-	-	-	-	-
f12	f4	-	-	-	-	-
f15	f17	-	-	-	-	-
f9	-	-	-	-	-	-
f17	-	-	-	-	-	-
f19	-	-	-	-	-	-

5. CPU TIMING EXPERIMENT

For the timing experiment, the same DEPSO algorithm was run on f8 until at least 30 seconds had passed (according to the `exampletiming` procedure available in BBOB 2009 [6]). These experiments have been conducted with an Intel(R) Core(TM)2 CPU processor with 1.66 GHz and 1GB RAM; O.S Linux Ubuntu version 8.10 using the C-code provided. The results were 1.1, 0.9, 1.3, 2.3, 3.9, and $7.1 \times e-06$ seconds per function evaluation in dimension 2, 3, 5, 10, 20, and 40, respectively.

6. CONCLUSION

In this work we have experimentally studied DEPSO, a simple and easy to implement optimization algorithm constructed by hybridizing the Particle Swarm Optimizer with Differential Evolution operations. The experiments have been made in the context of the special session of real parameter Black-Box Optimization Benchmarking (GECCO BBOB 2009), performing the complete procedure previously established, and dealing with noiseless functions with dimension: 2, 3, 5, 10, 20, and 40 variables. Our proposal obtained an accurate level of coverage rate for dimensions 2, 3, 5, and 10, specifically with separable and weak structured noiseless functions. Successful trials were found for 14 out of 24 functions. Specifically for f5, our proposal obtained successful trials for all dimensions. The fact of using the same parameter setting for all functions (and dimensions), together with the relatively small number of function evaluations used ($1000 \times DIM$), lead us to think that DEPSO can be easily improved for better covering noiseless functions with higher dimensions. Future experiments with different parameter settings depending of each family of noiseless functions, and performing larger number of evaluations can be made in this sense.

7. ACKNOWLEDGMENTS

Authors acknowledge funds from the Spanish Ministry of Sciences and Innovation European FEDER under contract TIN2008-06491-C04-01 (M* project, available in URL <http://mstar.lcc.uma.es>) and CICE, Junta de Andalucía under contract P07-TIC-03044 (DIRICOM project, available in URL <http://diricom.lcc.uma.es>).

8. REFERENCES

- [1] E. Alba, G. Luque, J. García-Nieto, G. Ordóñez, and G. Leguizamón. Mallba: a software library to design efficient optimisation algorithms. *Int. J. of Innovative Computing and Applications 2007 (IJICA)*, 1(1):74-85, 2007.
- [2] A. Auger and N. Hansen. A restart cma evolution strategy with increasing population size. *IEEE Congress on Evolutionary Computation*, 2:1769-1776, 2005.
- [3] S. Das, A. Abraham, and A. Konar. Particle swarm optimization and differential evolution algorithms: Technical analysis, applications and hybridization perspectives. In *Advances of Computational Intelligence in Industrial Systems*, 2008.
- [4] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Presentation of the noiseless functions. Technical Report 2009/20, Research Center PPE, 2009.
- [5] J. Garcia-Nieto, E. J. Apolloni, Alba, and G. Leguizamón. Algoritmo Basado en Cúmulos de Partículas y Evolución Diferencial para la Resolución de Problemas de Optimización Continua. In *VI Congreso Español sobre Metaheurísticas, Algoritmos Evolutivos y Bioinspirados (MAEB'09)*, page 433.440, Málaga, 11 a 13 de Febrero, 2009.
- [6] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2009: Experimental setup. Technical Report RR-6828, INRIA, 2009.

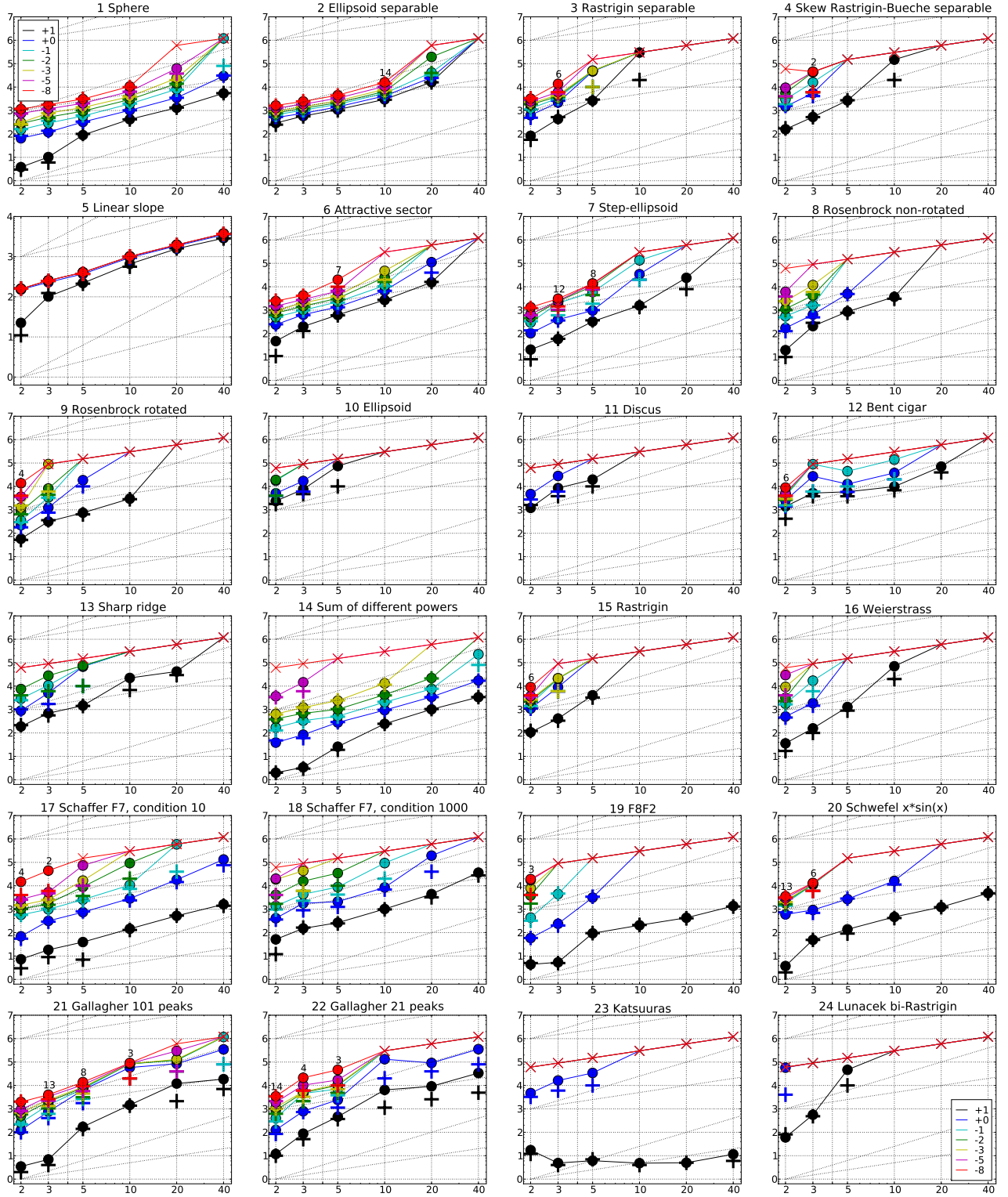


Figure 1: Expected Running Time (ERT, \bullet) to reach $f_{\text{opt}} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_1 and f_{24}) versus dimension in log-log presentation. The ERT(Δf) equals to $\#Fes(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\text{opt}} + \Delta f$ was surpassed during the trial. The $\#Fes(\Delta f)$ are the total number of function evaluations while $f_{\text{opt}} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and f_{opt} denotes the optimal function value. Crosses (\times) indicate the total number of function evaluations $\#Fes(-\infty)$. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.

f1 in 5-D, N=15, mFE=3560						f1 in 20-D, N=15, mFE=40040						f2 in 5-D, N=15, mFE=5400						f2 in 20-D, N=15, mFE=40040					
Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc	Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc		
10	15	8.9e1	6.8e1	1.1e2	8.9e1	15	1.3e3	1.2e3	1.4e3	1.3e3	10	15	1.1e3	1.0e3	1.1e3	1.1e3	15	1.6e4	1.5e4	1.7e4	1.6e4		
1	15	3.2e2	3.0e2	3.5e2	3.2e2	15	3.5e3	3.3e3	3.7e3	3.5e3	1	15	1.4e3	1.4e3	1.5e3	1.4e3	15	2.6e4	2.4e4	2.8e4	2.6e4		
1e-1	15	5.8e2	5.3e2	6.3e2	5.8e2	15	8.1e3	7.8e3	8.4e3	8.1e3	1e-1	15	1.8e3	1.8e3	1.9e3	1.8e3	12	4.2e4	3.9e4	4.4e4	3.5e4		
1e-3	15	1.3e3	1.3e3	1.4e3	1.3e3	14	2.4e4	2.2e4	2.6e4	2.3e4	1e-3	15	2.6e3	2.5e3	2.6e3	2.6e3	0	39e-3	12e-4	21e-2	4.0e4		
1e-5	15	2.0e3	2.0e3	2.1e3	2.0e3	9	6.2e4	6.0e4	6.4e4	3.7e4	1e-5	15	3.4e3	3.3e3	3.5e3	3.4e3		
1e-8	15	3.1e3	3.0e3	3.1e3	3.1e3	0	44e-7	83e-8	70e-5	4.0e4	1e-8	15	4.5e3	4.4e3	4.6e3	4.5e3		
f3 in 5-D, N=15, mFE=10040						f3 in 20-D, N=15, mFE=40040						f4 in 5-D, N=15, mFE=10040						f4 in 20-D, N=15, mFE=40040					
Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc	Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc		
10	15	2.6e3	2.3e3	2.9e3	2.6e3	0	84e+0	63e+0	12e+1	3.2e4	10	15	2.7e3	2.5e3	2.9e3	2.7e3	0	10e+1	91e+0	12e+1	3.2e4		
1	3	4.7e4	4.4e4	4.9e4	1.0e4	1	0	30e-1	10e-1	50e-1	8.9e3		
1e-1	3	4.8e4	4.7e4	5.0e4	1.0e4	1e-1		
1e-3	3	5.0e4	5.0e4	5.0e4	1.0e4	1e-3		
1e-5	0	23e-1	35e-5	50e-1	8.9e3	1e-5		
1e-8	1e-8		
f5 in 5-D, N=15, mFE=520						f5 in 20-D, N=15, mFE=2080						f6 in 5-D, N=15, mFE=10040						f6 in 20-D, N=15, mFE=40040					
Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc	Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc		
10	15	2.2e2	2.0e2	2.4e2	2.2e2	15	1.6e3	1.5e3	1.6e3	1.6e3	10	15	6.3e2	5.8e2	6.8e2	6.3e2	15	1.5e4	1.5e4	1.6e4	1.5e4		
1	15	3.7e2	3.5e2	3.9e2	3.7e2	15	1.9e3	1.8e3	1.9e3	1.9e3	1	15	1.4e3	1.3e3	1.5e3	1.4e3	5	1.1e5	1.1e5	1.2e5	3.9e4		
1e-1	15	4.1e2	3.9e2	4.2e2	4.1e2	15	2.0e3	1.9e3	2.0e3	2.0e3	1e-1	15	2.3e3	2.1e3	2.4e3	2.3e3	0	13e-1	49e-2	19e-1	4.0e4		
1e-3	15	4.1e2	3.9e2	4.2e2	4.1e2	15	2.0e3	1.9e3	2.0e3	2.0e3	1e-3	15	4.1e3	3.9e3	4.3e3	4.1e3		
1e-5	15	4.1e2	3.9e2	4.2e2	4.1e2	15	2.0e3	1.9e3	2.0e3	2.0e3	1e-5	15	6.5e3	6.2e3	6.8e3	6.5e3		
1e-8	15	4.1e2	3.9e2	4.3e2	4.1e2	15	2.0e3	1.9e3	2.0e3	2.0e3	1e-8	7	2.0e4	1.9e4	2.1e4	9.4e3		
f7 in 5-D, N=15, mFE=10040						f7 in 20-D, N=15, mFE=40040						f8 in 5-D, N=15, mFE=10040						f8 in 20-D, N=15, mFE=40040					
Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc	Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc		
10	15	3.3e2	3.0e2	3.6e2	3.3e2	11	2.4e4	1.8e4	3.1e4	1.6e4	10	15	8.7e2	8.0e2	9.3e2	8.7e2	0	17e+0	17e+0	73e+0	4.0e4		
1	15	9.6e2	8.7e2	1.1e3	9.6e2	0	77e-1	31e-1	14e+0	2.8e4	1	14	5.0e3	4.3e3	5.6e3	4.8e3		
1e-1	11	5.9e3	4.2e3	7.5e3	3.7e3	1e-1	0	23e-2	17e-2	60e-2	8.9e3		
1e-3	8	1.3e4	1.1e4	1.5e4	6.9e3	1e-3		
1e-5	8	1.3e4	1.1e4	1.5e4	6.9e3	1e-5		
1e-8	8	1.4e4	1.2e4	1.6e4	7.1e3	1e-8		
f9 in 5-D, N=15, mFE=10040						f9 in 20-D, N=15, mFE=40040						f10 in 5-D, N=15, mFE=10040						f10 in 20-D, N=15, mFE=40040					
Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc	Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc		
10	15	7.5e2	6.6e2	8.4e2	7.5e2	0	18e+0	16e+0	72e+0	4.0e4	10	2	7.4e4	7.3e4	7.5e4	1.0e4	0	17e+3	40e+2	50e+3	3.5e4		
1	7	1.9e4	1.6e4	2.0e4	7.7e3	1	0	88e+0	90e-1	33e+1	8.9e3		
1e-1	0	10e-1	28e-2	23e-1	1.0e4	1e-1		
1e-3	1e-3		
1e-5	1e-5		
1e-8	1e-8		
f11 in 5-D, N=15, mFE=10040						f11 in 20-D, N=15, mFE=40040						f12 in 5-D, N=15, mFE=10040						f12 in 20-D, N=15, mFE=40040					
Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc	Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc		
10	6	2.0e4	1.7e4	2.2e4	8.8e3	0	95e+0	71e+0	15e+1	3.2e4	10	12	5.9e3	4.6e3	7.2e3	5.0e3	8	7.1e4	6.9e4	7.2e4	3.8e4		
1	0	10e+0	46e-1	27e+0	7.1e3	1	8	1.3e4	1.1e4	1.5e4	7.0e3	0	77e-1	26e-1	42e+0	4.0e4		
1e-1	1e-1	3	4.5e4	4.1e4	4.8e4	8.2e3		
1e-3	1e-3	0	95e-2	65e-3	14e+0	8.9e3		
1e-5	1e-5		
1e-8	1e-8		
f13 in 5-D, N=15, mFE=10040						f13 in 20-D, N=15, mFE=40040						f14 in 5-D, N=15, mFE=10040						f14 in 20-D, N=15, mFE=40040					
Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc	Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc		
10	15	1.5e3	1.3e3	1.7e3	1.5e3	11	4.2e4	3.9e4	4.5e4	3.1e4	10	15	2.6e1	2.1e1	3.1e1	2.6e1	15	1.0e3	9.5e2	1.1e3	1.0e3		
1	2	6.8e4	6.0e4	7.5e4	1.0e4	0	53e-1	27e-1	17e+0	4.0e4	1	15	2.8e2	2.6e2	3.0e2	2.8e2	15	3.5e3	3.3e3	3.6e3	3.5e3		
1e-1	2	6.9e4	6.3e4	7.5e4	1.0e4	1e-1	15	5.2e2	4.8e2	5.5e2	5.2e2	15	7.6e3	7.3e3	7.9e3	7.6e3		
1e-3	0	21e-1	10e-3	48e-1	8.9e3	1e-3	15	2.4e3	2.3e3	2.5e3	2.4e3	0	44e-4	29e-4	73e-4	4.0e4		
1e-5	1e-5	0	47e-6	19e-6	20e-5	8.9e3		
1e-8	1e-8		
f15 in 5-D, N=15, mFE=10040						f15 in 20-D, N=15, mFE=40040						f16 in 5-D, N=15, mFE=10040						f16 in 20-D, N=15, mFE=40040					
Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc	Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc		
10	15	4.0e3	3.3e3	4.8e3	4.0e3	0	12e+1	11e+1	14e+1	2.5e4	10	15	1.3e3	9.0e2	1.7e3	1.3e3	0	23e+0	22e+0	25e+0	2.5e4		
1	0	41e-1	20e-1	77e-1	7.9e3	1	0	42e-1	22e-1	58e-1	5.6e3		
1e-1	1e-1		
1e-3	1e-3		
1e-5	1e-5		
1e-8	1e-8		
f17 in 5-D, N=15, mFE=10040						f17 in 20-D, N=15, mFE=40040						f18 in 5-D, N=15, mFE=10040						f18 in 20-D, N=15, mFE=40040					
Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc	Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc		
10	15	3.9e1	1.8e1	6.2e1	3.9e1	15	5.2e2	4.4e2	6.0e2	5.2e2	10	15	2.6e2	2.2e2	3.1e2	2.6e2	15	4.4e3	3.4e3	5.5e3	4.4e3		
1	15	7.4e2	6.9e2	8.1e2	7.4e2	13	1.8e4	1.4e4	2.3e4	1.4e4	1	15	2.1e3	1.6e3	2.7e3	2.1e3	3	1.9e5	1.9e5	2.0e5	4.0e4		
1e-1	15	2.6e3	2.2e3	2.9e3	2.6e3	1	5.9e5	5.8e5	6.0e5	4.0e4	1e-1	10	8.8e3	7.2e3	1.0e4	5.8e3	0	16e-1	77e-2	37e-1	3.5e4		
1e-3	7	1.7e4	1.5e4	1.8e4	7.7e3	0	43e-2	11e-2	11e-1	3.5e4	1e-3	0	57e-3	18e-4	87e-2	8.9e3		
1e-5	2	7.4e4	7.2e4	7.5e4	1.0e4	1e-5		
1e-8	0	31e-4	37e-7	23e-3	8.9e3	1e-8		
f19 in 5-D, N=15, mFE=10040						f19 in 20-D, N=15, mFE=40040						f20 in 5-D, N=15, mFE=10040						f20 in 20-D, N=15, mFE=40040					
Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc	Δf	#	ERT	10%	90%	RTsucc	#	ERT	10%	90%	RTsucc		
10	15	9.3e1	8.2e1	1.1e2	9.3e1	15	4.3e2	3.9e2	4.7e2	4.3e2	10	15	1.4e2	1.1e2	1.6e2	1.4e2	15	1.2e3	1.2e3	1.3e3	1.2e3		
1	15	3.1e3	2.7e3	3.6e3	3.1e3	0	50e-1	44e-1	54e-1	2.5e4	1	15	2.7e3	2.5e3	2.9e3	2.7e3	0	21e-1	14e-1	24e-1	3		

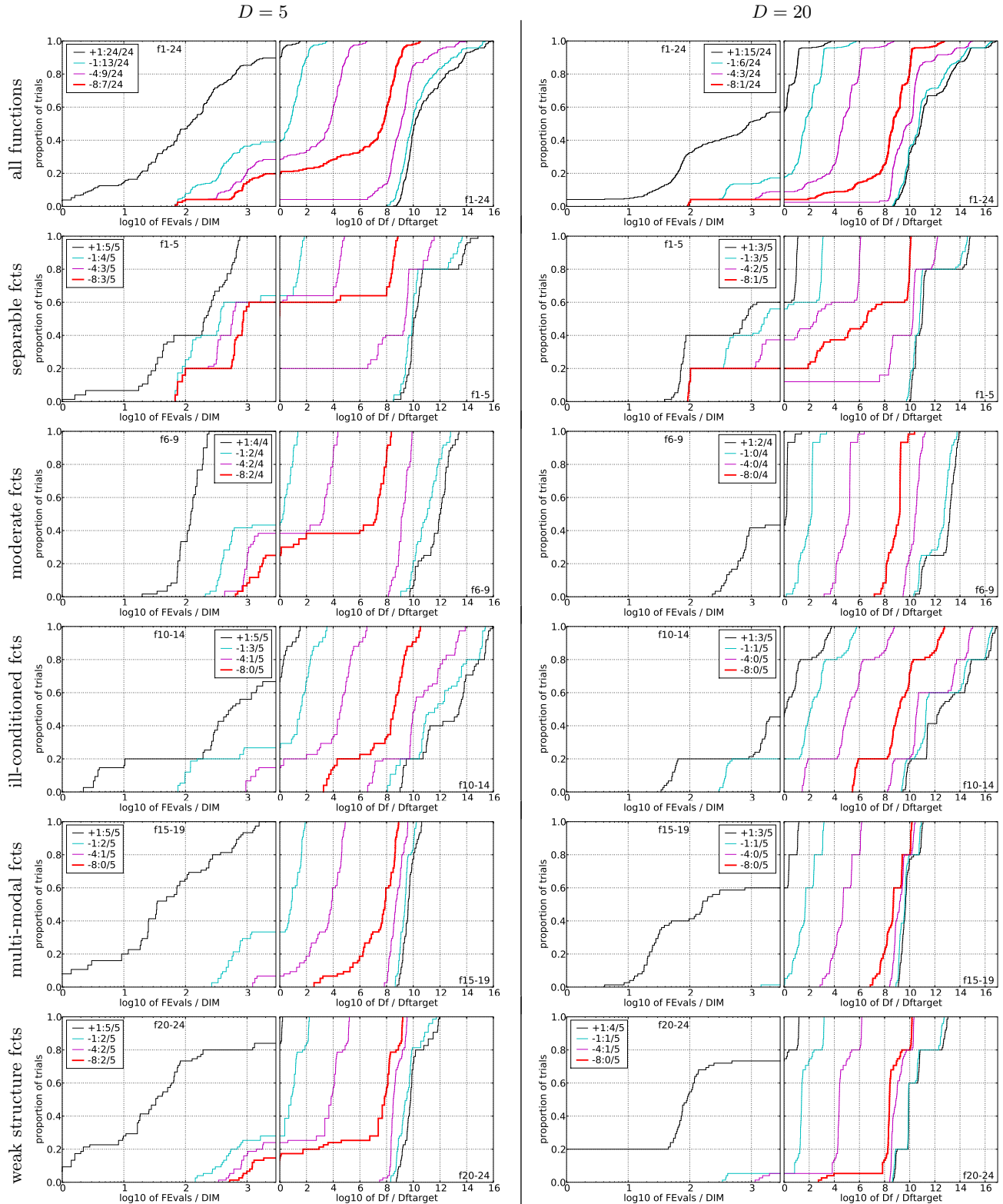


Figure 2: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left) or Δf . Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D , to fall below $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10D, 100D \dots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: separable functions; third row: misc. moderate functions; fourth row: ill-conditioned functions; fifth row: multi-modal functions with adequate structure; last row: multi-modal functions with weak structure. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

- [7] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noiseless functions definitions. Technical Report RR-6829, INRIA, 2009.
- [8] J. Kennedy and R. Eberhart. Particle swarm optimization. *Neural Networks, Piscataway, NJ., Proceedings of IEEE International Conference on*, pages 1942–1948, 1995.
- [9] K. V. Price, R. Storn, and J. Lampinen. *Differential Evolution: A practical Approach to Global Optimization*. Springer-Verlag, London, UK, 2005.
- [10] A. Sinha, S. Tiwari, and K. Deb. A population-based, steady-state procedure for real-parameter optimization. *IEEE Congress on Evolutionary Computation*, 1:514–521, 2005.
- [11] P. N. Suganthan, N. Hansen, J. J. Liang, K. Deb, Y.-P. Chen, A. Auger, and S. Tiwari. Problem definitions and evaluation criteria for the cec 2005 special session on real-parameter optimization. Technical report, Nanyang Technological University, 2005.