

AMaLGaM IDEAs in Noisy Black-Box Optimization Benchmarking

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ABSTRACT

This paper describes the application of a Gaussian Estimation-of-Distribution (EDA) for real-valued optimization to the noisy part of a benchmark introduced in 2009 called BBOB (Black-Box Optimization Benchmarking). Specifically, the EDA considered here is the recently introduced parameter-free version of the Adapted Maximum-Likelihood Gaussian Model Iterated Density-Estimation Evolutionary Algorithm (AMaLGaM-IDEA). Also the version with incremental model building (iAMaLGaM-IDEA) is considered.

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization Global Optimization, Unconstrained Optimization; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Evolutionary computation

1. METHOD

Estimation-of-distribution algorithms attempt to automatically exploit features of a problem's structure by probabilistically modeling the search space based on previously evaluated solutions and generating new solutions by sampling the probabilistic model.

The EDA considered here is the Adapted Maximum-Likelihood Gaussian Model Iterated Density-Estimation Evolutionary Algorithm (AMaLGaM-IDEA, or AMaLGaM for short). In AMaLGaM, the probability distribution used is the normal, also known as the Gaussian, distribution. This

EDA uses maximum-likelihood estimates for the mean and the covariance matrix, estimated from the selected solutions. It has a mechanism that scales up the covariance matrix when required to prevent premature convergence on slopes. It furthermore has a mechanism that anticipates the mean shift in the next generation to speed up descent (in case of minimization) along slopes. In another paper [1], AMaLGaM, and its incremental-learning variant iAMaLGaM, were tested on the noiseless variant of the BBOB benchmark. Due to space restrictions, we refer the interested reader for more details on AMaLGaM such as the parameters and other settings as well as the CPU timing experiment to the other workshop paper.

2. RESULTS AND CONCLUSION

Results from experiments according to [3] on the benchmark functions given in [2, 4] are presented in Figures 1 and 2 and in Tables 1 and 3 for AMaLGaM and in Figures 3 and 4 and in Tables 2 and 4 for iAMaLGaM.

Problems with severe noise and multimodality appear to be the hardest for (i)AMaLGaM. Even within $10^6 D$ evaluations the optimum cannot be found within a desirable precision for larger D . The difference between AMaLGaM and iAMaLGaM is not large. Most likely due to the larger base population-size, AMaLGaM performs slightly better. The difference is larger for the multi-modal problems, which is consistent with earlier findings.

3. REFERENCES

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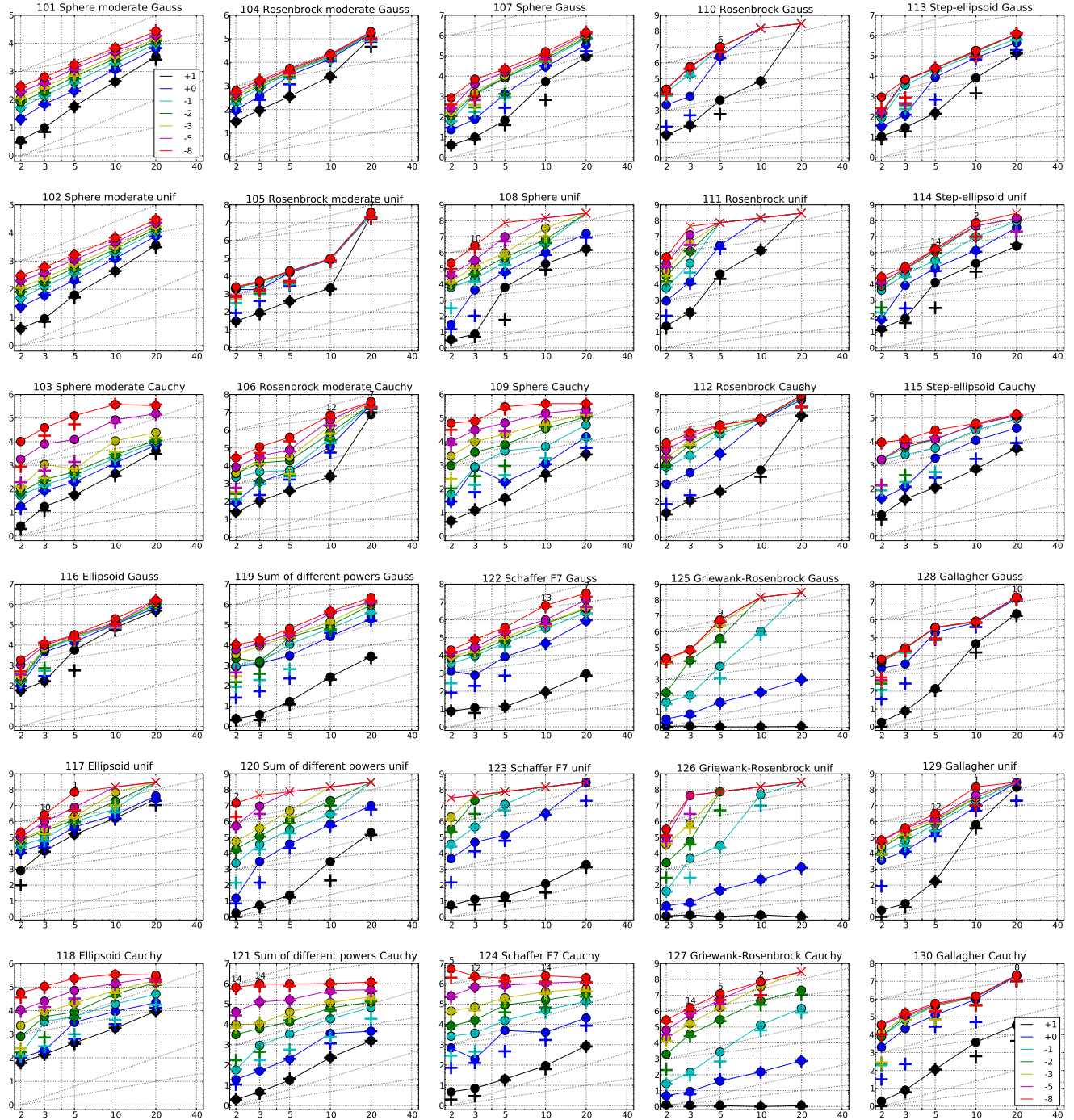


Figure 1: AMaLGaM: Expected Running Time (ERT, ●) to reach $f_{\text{opt}} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_{101} and f_{130}) versus dimension in log-log presentation. The $\text{ERT}(\Delta f)$ equals to $\#FEs(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\text{opt}} + \Delta f$ was surpassed during the trial. The $\#FEs(\Delta f)$ are the total number of function evaluations while $f_{\text{opt}} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and f_{opt} denotes the optimal function value. Crosses (×) indicate the total number of function evaluations $\#FEs(-\infty)$. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.

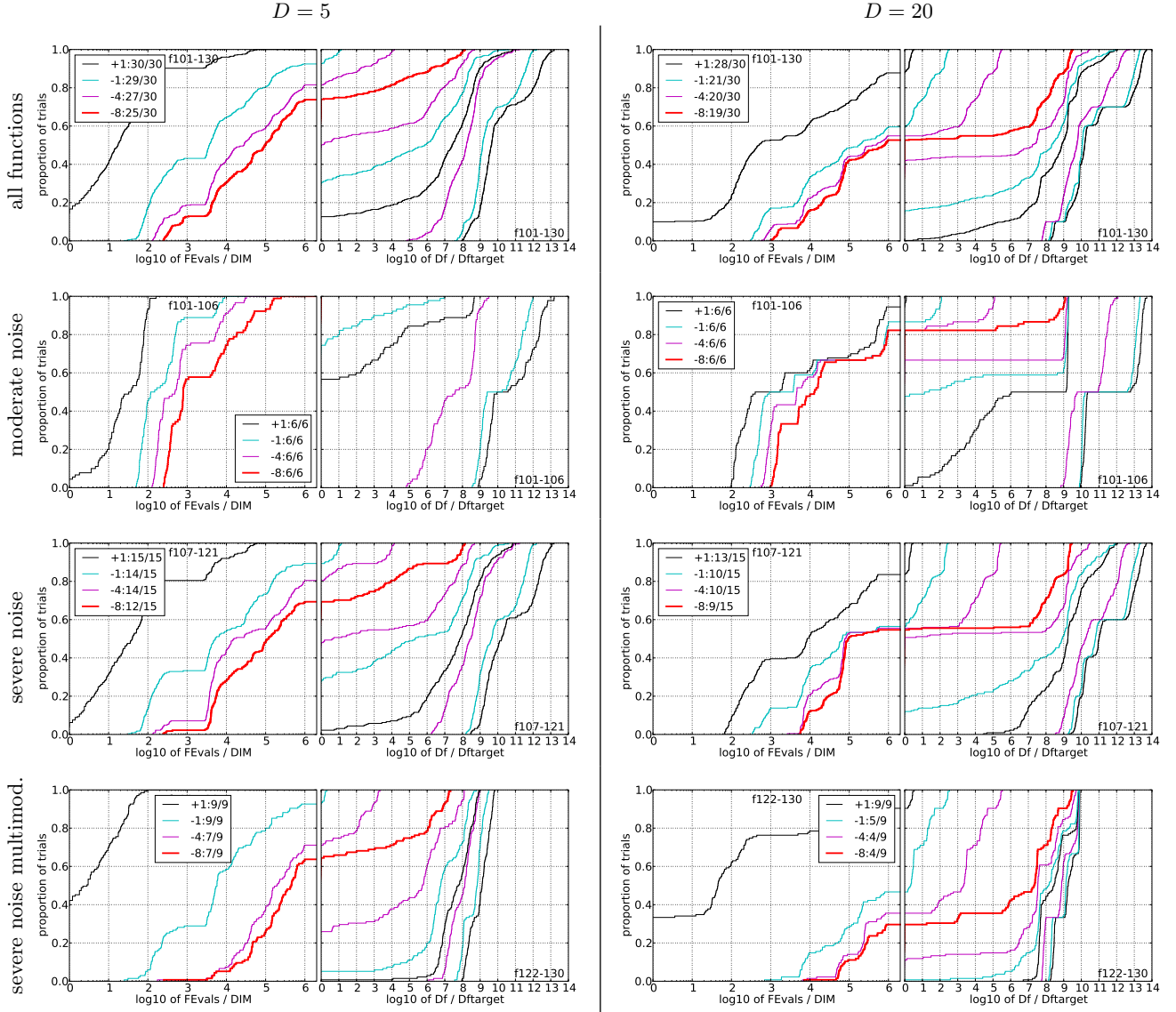


Figure 2: AMaLGaM: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left) or Δf . Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D , to fall below $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: moderate noise functions; third row: severe noise functions; fourth row: severe noise and highly-multimodal functions. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

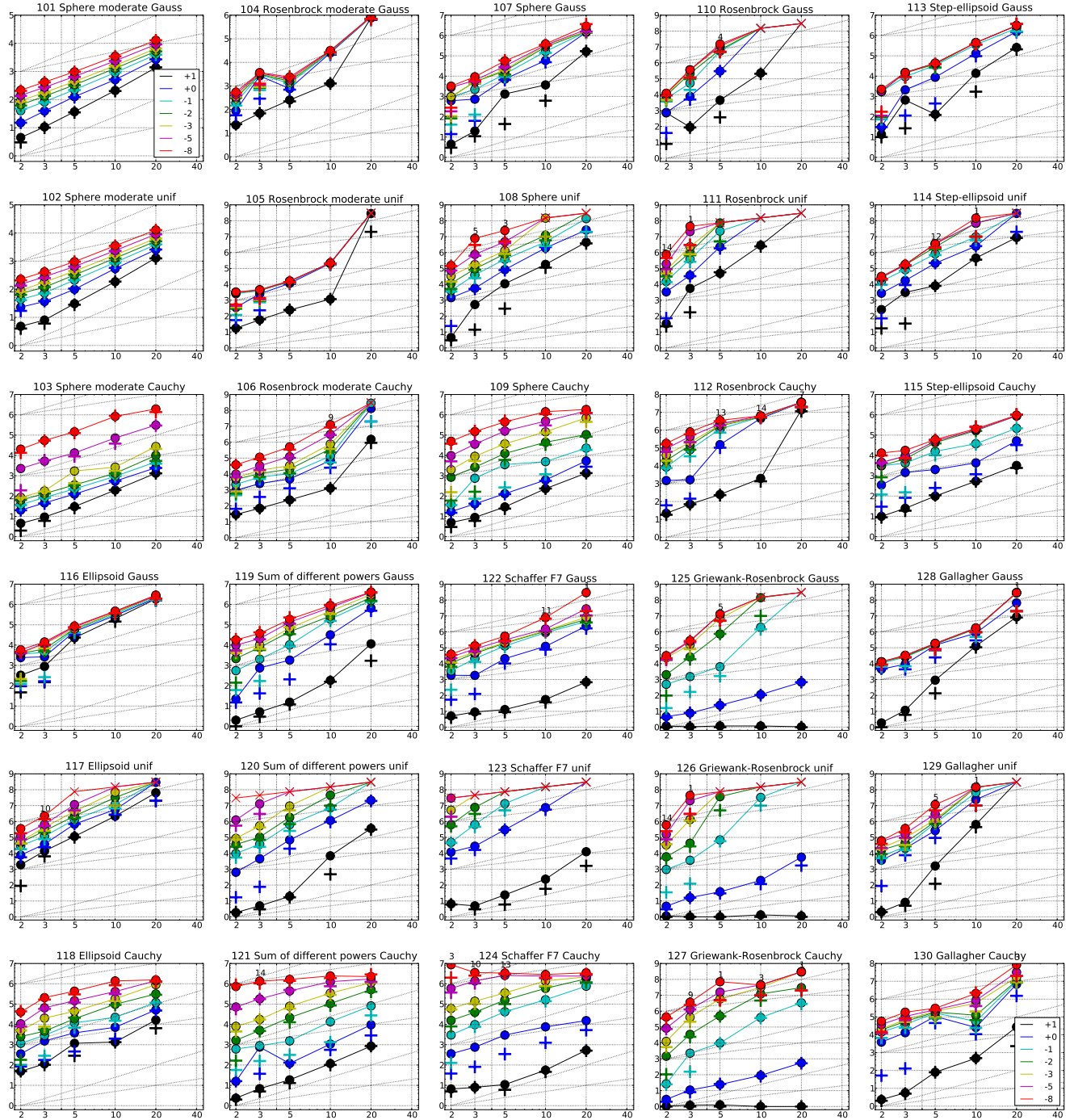


Figure 3: iAMaLGaM: Expected Running Time (ERT, ●) to reach $f_{\text{opt}} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_{101} and f_{130}) versus dimension in log-log presentation. The $\text{ERT}(\Delta f)$ equals to $\#FEs(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\text{opt}} + \Delta f$ was surpassed during the trial. The $\#FEs(\Delta f)$ are the total number of function evaluations while $f_{\text{opt}} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and f_{opt} denotes the optimal function value. Crosses (×) indicate the total number of function evaluations $\#FEs(-\infty)$. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.

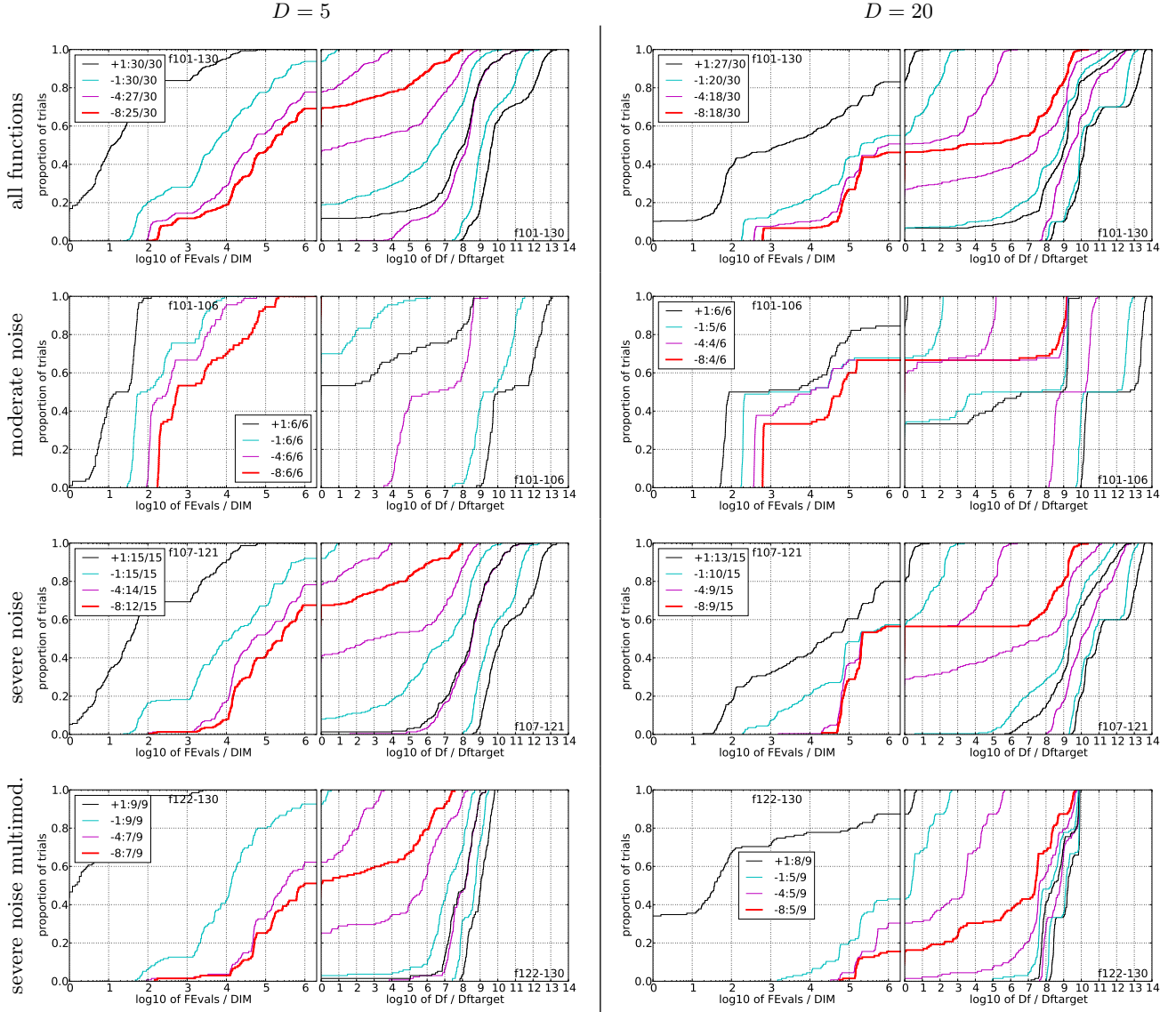


Figure 4: iAMaLGaM: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left) or Δf . Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D , to fall below $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10D, 100D \dots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: moderate noise functions; third row: severe noise functions; fourth row: severe noise and highly-multimodal functions. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

f_{101} in 5-D, N=15, mFE=2892						f_{101} in 20-D, N=15, mFE=31809						f_{102} in 5-D, N=15, mFE=2206						f_{102} in 20-D, N=15, mFE=36921					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	5.8e1	4.5e1	7.0e1	5.8e1	15	3.4e3	2.9e3	3.8e3	3.4e3	10	15	6.3e1	4.7e1	8.0e1	6.3e1	15	3.7e3	3.2e3	4.2e3	3.7e3		
1	15	2.1e2	1.9e2	2.3e2	2.1e2	15	6.5e3	6.0e3	7.0e3	6.5e3	1	15	2.1e2	1.8e2	2.4e2	2.1e2	15	7.8e3	7.1e3	8.5e3	7.8e3		
1e-1	15	4.3e2	3.8e2	4.9e2	4.3e2	15	9.6e3	8.8e3	1.0e4	9.6e3	1e-1	15	3.8e2	3.4e2	4.2e2	3.8e2	15	1.1e4	1.0e4	1.2e4	1.1e4		
1e-3	15	8.4e2	7.5e2	9.3e2	8.4e2	15	1.5e4	1.4e4	1.6e4	1.5e4	1e-3	15	7.5e2	7.0e2	8.1e2	7.5e2	15	1.7e4	1.6e4	1.8e4	1.7e4		
1e-5	15	1.2e3	1.1e3	1.4e3	1.2e3	15	1.9e4	1.8e4	2.1e4	1.9e4	1e-5	15	1.1e3	1.0e3	1.2e3	1.1e3	15	2.2e4	2.1e4	2.4e4	2.2e4		
1e-8	15	1.8e3	1.6e3	1.9e3	1.8e3	15	2.7e4	2.6e4	2.8e4	2.7e4	1e-8	15	1.7e3	1.6e3	1.8e3	1.7e3	15	3.0e4	2.9e4	3.2e4	3.0e4		
f_{103} in 5-D, N=15, mFE=651264						f_{103} in 20-D, N=15, mFE=923247						f_{104} in 5-D, N=15, mFE=31356						f_{104} in 20-D, N=15, mFE=334844					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	5.4e1	4.4e1	6.4e1	5.4e1	15	4.0e3	3.5e3	4.7e3	4.0e3	10	15	3.6e2	3.3e2	3.9e2	3.6e2	15	1.1e5	8.3e4	1.4e5	1.1e5		
1	15	1.9e2	1.8e2	2.1e2	1.9e2	15	7.2e3	6.5e3	7.8e3	7.2e3	1	15	2.8e3	1.0e3	4.5e3	2.8e3	15	1.6e5	1.3e5	1.9e5	1.6e5		
1e-1	15	3.7e2	3.5e2	4.0e2	3.7e2	15	9.5e3	8.7e3	1.0e4	9.5e3	1e-1	15	3.6e3	1.8e3	5.5e3	3.6e3	15	1.7e5	1.4e5	2.0e5	1.7e5		
1e-3	15	6.9e2	6.5e2	7.4e2	6.9e2	15	2.5e4	1.4e4	3.6e4	2.5e4	1e-3	15	4.5e3	2.5e3	6.4e3	4.5e3	15	1.8e5	1.5e5	2.2e5	1.8e5		
1e-5	15	1.2e4	8.2e3	1.7e4	1.2e4	15	1.5e5	1.2e5	1.9e5	1.5e5	1e-5	15	4.9e3	3.0e3	7.0e3	4.9e3	15	1.9e5	1.6e5	2.2e5	1.9e5		
1e-8	15	1.3e5	7.5e4	1.8e5	1.3e5	15	3.4e5	2.7e5	4.1e5	3.4e5	1e-8	15	5.5e3	3.6e3	7.6e3	5.5e3	15	2.0e5	1.7e5	2.4e5	2.0e5		
f_{105} in 5-D, N=15, mFE=48000						f_{105} in 20-D, N=15, mFE=20021580						f_{106} in 5-D, N=15, mFE=1235097						f_{106} in 20-D, N=15, mFE=20013092					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	3.9e2	3.6e2	4.3e2	3.9e2	10	2.3e7	2.1e7	2.5e7	1.6e7	10	15	4.1e2	3.7e2	4.5e2	4.1e2	15	7.4e6	6.1e6	8.5e6	7.4e6		
1	15	1.6e4	1.1e4	2.1e4	1.6e4	9	2.8e7	2.5e7	3.1e7	1.8e7	1	15	4.7e3	1.5e3	7.9e3	4.7e3	9	2.7e7	2.6e7	2.9e7	1.6e7		
1e-1	15	1.7e4	1.2e4	2.3e4	1.7e4	9	2.8e7	2.6e7	3.1e7	1.8e7	1e-1	15	5.6e3	2.4e3	8.7e3	5.6e3	9	2.9e7	2.7e7	3.0e7	1.7e7		
1e-3	15	1.8e4	1.2e4	2.4e4	1.8e4	7	3.7e7	3.3e7	4.0e7	1.8e7	1e-3	15	3.1e4	1.7e4	4.7e4	3.1e4	7	3.9e7	3.7e7	4.0e7	1.8e7		
1e-5	15	1.9e4	1.3e4	2.5e4	1.9e4	7	3.7e7	3.4e7	4.0e7	1.8e7	1e-5	15	8.0e4	6.4e4	9.6e4	8.0e4	7	3.9e7	3.7e7	4.0e7	1.8e7		
1e-8	15	2.0e4	1.4e4	2.6e4	2.0e4	7	3.7e7	3.4e7	4.1e7	1.9e7	1e-8	15	4.0e5	2.9e5	5.1e5	4.0e5	7	3.9e7	3.7e7	4.1e7	1.8e7		
f_{107} in 5-D, N=15, mFE=33716						f_{107} in 20-D, N=15, mFE=1931222						f_{108} in 5-D, N=15, mFE=5009337						f_{108} in 20-D, N=15, mFE=20017285					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	6.7e1	3.8e1	9.7e1	6.7e1	15	8.5e4	6.0e4	1.1e5	8.5e4	10	15	6.5e3	3.0e3	1.0e4	6.5e3	15	1.7e6	1.3e6	2.1e6	1.7e6		
1	15	1.3e3	3.0e2	2.4e3	1.3e3	15	3.4e5	2.0e5	4.9e5	3.4e5	1	15	6.0e4	4.3e4	7.7e4	6.0e4	10	1.6e7	1.2e7	1.9e7	1.0e7		
1e-1	15	8.1e3	5.4e3	1.1e4	8.1e3	15	7.0e5	5.3e5	8.6e5	7.0e5	1e-1	15	2.5e5	1.9e5	3.0e5	2.5e5	0	48e-2	18e-2	19e-1	7.9e6		
1e-3	15	1.0e4	7.3e3	1.3e4	1.0e4	15	9.9e5	8.3e5	1.2e6	9.9e5	1e-3	15	9.6e5	6.6e5	1.3e6	9.6e5			
1e-5	15	1.6e4	1.3e4	1.8e4	1.6e4	15	1.2e6	1.1e6	1.4e6	1.2e6	1e-5	6	9.9e6	8.5e6	1.1e7	3.4e6			
1e-8	15	2.2e4	2.0e4	2.4e4	2.2e4	15	1.4e6	1.3e6	1.5e6	1.4e6	1e-8	0	20e-6	57e-9	16e-5	2.2e6			
f_{109} in 5-D, N=15, mFE=714314						f_{109} in 20-D, N=15, mFE=1118257						f_{110} in 5-D, N=15, mFE=5009199						f_{110} in 20-D, N=15, mFE=20020914					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	4.0e1	3.1e1	4.9e1	4.0e1	15	3.0e3	2.8e3	3.3e3	3.0e3	10	15	4.5e3	1.8e3	7.2e3	4.5e3	0	18e+0	17e+0	18e+0	1.6e7		
1	15	1.9e2	1.8e2	2.1e2	1.9e2	15	1.6e4	6.4e3	2.7e4	1.6e4	1	13	2.5e6	1.8e6	3.2e6	1.9e6			
1e-1	15	4.1e3	9.0e2	7.3e3	4.1e3	15	5.2e4	3.6e4	7.0e4	5.2e4	1e-1	6	1.0e7	8.8e6	1.1e7	4.2e6			
1e-3	15	2.1e4	1.8e4	2.4e4	2.1e4	15	1.4e5	1.2e5	1.6e5	1.4e5	1e-3	6	1.0e7	8.7e6	1.1e7	4.3e6			
1e-5	15	6.2e4	4.2e4	8.1e4	6.2e4	15	2.3e5	1.9e5	2.8e5	2.3e5	1e-5	6	1.0e7	8.8e6	1.1e7	4.3e6			
1e-8	15	3.0e5	2.4e5	3.7e5	3.0e5	15	4.1e5	3.3e5	4.9e5	4.1e5	1e-8	6	1.0e7	9.0e6	1.1e7	4.3e6			
f_{111} in 5-D, N=15, mFE=5009244						f_{111} in 20-D, N=15, mFE=20018706						f_{112} in 5-D, N=15, mFE=3579671						f_{112} in 20-D, N=15, mFE=20002169					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	4.5e4	2.7e4	6.3e4	4.5e4	0	23e+0	20e+0	24e+0	1.0e7	10	15	3.7e2	3.4e2	4.0e2	3.7e2	15	6.7e6	6.2e6	7.2e6	6.7e6		
1	12	2.8e6	2.0e6	3.6e6	2.2e6	1	15	5.0e4	3.4e4	6.6e4	5.0e4	5	5.0e7	4.5e7	5.5e7	1.8e7		
1e-1	0	53e-2	17e-2	10e-1	2.0e6	1e-1	15	6.3e5	4.6e5	8.0e5	6.3e5	4	6.4e7	5.8e7	7.0e7	1.7e7		
1e-3	1e-3	15	1.2e6	9.1e5	1.5e6	1.2e6	3	9.0e7	8.2e7	9.7e7	1.7e7		
1e-5	1e-5	15	2.0e6	1.6e6	2.3e6	2.0e6	3	9.0e7	8.3e7	9.7e7	1.7e7		
1e-8	1e-8	15	2.0e6	1.7e6	2.3e6	2.0e6	3	9.0e7	8.3e7	9.7e7	1.7e7		
f_{113} in 5-D, N=15, mFE=66491						f_{113} in 20-D, N=15, mFE=1702267						f_{114} in 5-D, N=15, mFE=5003184						f_{114} in 20-D, N=15, mFE=20018939					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.5e2	1.3e2	1.8e2	1.5e2	15	1.4e5	1.0e5	1.7e5	1.4e5	10	15	1.3e4	6.2e3	2.0e4	1.3e4	15	2.6e6	2.2e6	2.9e6	2.6e6		
1	15	8.7e3	4.6e3	1.3e4	8.7e3	15	4.4e5	3.1e5	5.6e5	4.4e5	1	15	1.0e5	7.1e4	1.4e5	1.0e5	6	3.8e7	3.3e7	4.3e7	1.3e7		
1e-1	15	1.9e4	1.3e4	2.5e4	1.9e4	15	7.0e5	5.2e5	8.7e5	7.0e5	1e-1	15	3.2e5	2.5e5	4.0e5	3.2e5	3	9.0e7	8.3e7	9.7e7	1.7e7		
1e-3	15	2.4e4	1.9e4	3.0e4	2.4e4	15	1.2e6	1.0e6	1.3e6	1.2e6	1e-3	14	1.6e6	1.1e6	2.0e6	1.5e6	2	1.4e8	1.3e8	1.5e8	1.5e7		
1e-5	15	2.4e4	1.9e4	3.0e4	2.4e4	15	1.2e6	1.0e6	1.3e6	1.2e6	1e-5	14	1.6e6	1.2e6	2.0e6	1.5e6	2	1.4e8	1.3e8	1.5e8	1.5e7		
1e-8	15	2.5e4	2.0e4	3.0e4	2.5e4	15	1.2e6	1.0e6	1.3e6	1.2e6	1e-8	14	1.7e6	1.3e6	2.1e6	1.7e6	0	14e-1	89e-9	32e-1	8.9e6		
f_{115} in 5-D, N=15, mFE=69345						f_{115} in 20-D, N=15, mFE=325587						f_{116} in 5-D, N=15, mFE=77546						f_{116} in 20-D, N=15, mFE=2235403					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.1e2	9.6e1	1.4e2	1.1e2	15	5.2e3	4.6e3	5.7e3	5.2e3	10	15	5.7e3	3.5e3	8.0e3	5.7e3	15	5.0e5	3.9e5	6.1e5	5.0e5		
1	15	2.0e3	3.0e2	3.7e3	2.0e3	15	3.8e4	2.1e4	5.4e4	3.8e4	1	15	1.4e4	9.0e3	2.0e4	1.4e4	15	6.9e5	5.6e5	8.3e5	6.9e5		
1e-1	15	5.3e3	2.6e3	8.1e3	5.3e3	15	9.2e4	7.6e4	1.1e5	9.2e4	1e-1	15	2.2e4	1.6e4	2.9e4	2.2e4	15	8.9e5	7.4e5	1.0e6	8.9e5		
1e-3	15	1.3e4	9.5e3	1.6e4	1.3e4	15	1.3e5	1.2e5	1.3e5	1.3e5	1e-3	15	2.7e4	2.1e4	3.3e4	2.7e4	15	1.2e6	1.1e6	1.4e6	1.2e6		
1e-5	15	1.3e4	9.8e3	1.6e4	1.3e4	15	1.3e5	1.2e5	1.3e5	1.3e5	1e-5	1											

f_{101} in 5-D, N=15, mFE=1072						f_{101} in 20-D, N=15, mFE=13288						f_{102} in 5-D, N=15, mFE=1093						f_{102} in 20-D, N=15, mFE=13675					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	3.7e1	3.1e1	4.3e1	3.7e1	15	1.4e3	1.3e3	1.4e3	1.4e3	10	15	3.1e1	2.8e1	3.4e1	3.1e1	15	1.3e3	1.2e3	1.4e3	1.3e3		
1	15	1.3e2	1.2e2	1.3e2	1.3e2	15	2.7e3	2.6e3	2.7e3	2.7e3	1	15	9.9e1	8.8e1	1.1e2	9.9e1	15	2.6e3	2.5e3	2.6e3	2.6e3		
1e-1	15	2.2e2	2.2e2	2.3e2	2.2e2	15	4.0e3	3.9e3	4.0e3	4.0e3	1e-1	15	2.1e2	2.0e2	2.2e2	2.1e2	15	3.9e3	3.8e3	3.9e3	3.9e3		
1e-3	15	4.4e2	4.3e2	4.5e2	4.4e2	15	6.5e3	6.5e3	6.6e3	6.5e3	1e-3	15	4.4e2	4.3e2	4.5e2	4.4e2	15	6.4e3	6.4e3	6.5e3	6.4e3		
1e-5	15	6.6e2	6.4e2	6.8e2	6.6e2	15	9.1e3	9.0e3	9.2e3	9.1e3	1e-5	15	6.4e2	6.3e2	6.5e2	6.4e2	15	9.0e3	8.9e3	9.1e3	9.0e3		
1e-8	15	9.7e2	9.6e2	9.9e2	9.7e2	15	1.3e4	1.3e4	1.3e4	1.3e4	1e-8	15	9.5e2	9.4e2	9.7e2	9.5e2	15	1.3e4	1.3e4	1.3e4	1.3e4		
f_{103} in 5-D, N=15, mFE=313583						f_{103} in 20-D, N=15, mFE=3182267						f_{104} in 5-D, N=15, mFE=2983						f_{104} in 20-D, N=15, mFE=1889854					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	3.0e1	2.5e1	3.6e1	3.0e1	15	1.3e3	1.3e3	1.4e3	1.3e3	10	15	2.5e2	2.2e2	2.9e2	2.5e2	15	7.7e5	6.3e5	9.2e5	7.7e5		
1	15	1.2e2	1.2e2	1.3e2	1.2e2	15	2.5e3	2.4e3	2.6e3	2.5e3	1	15	7.7e2	6.5e2	8.9e2	7.7e2	15	8.2e5	6.7e5	9.8e5	8.2e5		
1e-1	15	2.2e2	2.1e2	2.4e2	2.2e2	15	4.8e3	3.8e3	5.8e3	4.8e3	1e-1	15	1.3e3	1.2e3	1.4e3	1.3e3	15	8.4e5	6.9e5	9.9e5	8.4e5		
1e-3	15	1.7e3	4.5e2	3.0e3	1.7e3	15	2.8e4	1.9e4	3.7e4	2.8e4	1e-3	15	1.8e3	1.6e3	1.9e3	1.8e3	15	8.6e5	6.9e5	1.0e6	8.6e5		
1e-5	15	1.3e4	7.8e3	1.9e4	1.3e4	15	3.1e5	2.4e5	3.8e5	3.1e5	1e-5	15	2.0e3	1.9e3	2.2e3	2.0e3	15	8.7e5	7.0e5	1.0e6	8.7e5		
1e-8	15	1.5e5	1.1e5	1.8e5	1.5e5	15	1.9e6	1.6e6	2.3e6	1.9e6	1e-8	15	2.4e3	2.3e3	2.5e3	2.4e3	15	8.8e5	7.2e5	1.0e6	8.8e5		
f_{105} in 5-D, N=15, mFE=47725						f_{105} in 20-D, N=15, mFE=20015011						f_{106} in 5-D, N=15, mFE=1072606						f_{106} in 20-D, N=15, mFE=20010297					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	2.4e2	2.3e2	2.6e2	2.4e2	1	2.8e8	2.7e8	3.0e8	2.0e7	10	15	2.3e2	2.2e2	2.4e2	2.3e2	15	1.5e6	8.3e5	2.3e6	1.5e6		
1	15	1.2e4	7.4e3	1.5e4	1.2e4	0	13e+0	10e+0	15e+0	1.0e7	1	15	4.8e3	2.7e3	7.0e3	4.8e3	2	1.3e8	1.1e8	1.5e8	2.0e7		
1e-1	15	1.5e4	1.1e4	1.9e4	1.5e4	1e-1	15	1.0e4	7.6e3	1.3e4	1.0e4	1	2.8e8	2.7e8	3.0e8	2.0e7		
1e-3	15	1.5e4	1.2e4	1.9e4	1.5e4	1e-3	15	3.3e4	2.3e4	4.4e4	3.3e4	0	47e-1	30e-2	81e-1	5.6e6		
1e-5	15	1.6e4	1.2e4	2.0e4	1.6e4	1e-5	15	1.2e5	8.8e4	1.6e5	1.2e5		
1e-8	15	1.6e4	1.2e4	2.1e4	1.6e4	1e-8	15	5.1e5	4.0e5	6.2e5	5.1e5		
f_{107} in 5-D, N=15, mFE=98112						f_{107} in 20-D, N=15, mFE=4298872						f_{108} in 5-D, N=15, mFE=5009730						f_{108} in 20-D, N=15, mFE=20005495					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.3e3	6.2e1	2.6e3	1.3e3	15	1.6e5	1.2e5	2.1e5	1.6e5	10	15	1.1e4	5.7e3	1.6e4	1.1e4	15	4.4e6	3.7e6	5.1e6	4.4e6		
1	15	7.1e3	4.4e3	1.0e4	7.1e3	15	1.4e6	1.2e6	1.6e6	1.4e6	1	15	8.4e4	5.9e4	1.1e5	8.4e4	8	2.6e7	2.2e7	3.0e7	1.2e7		
1e-1	15	1.3e4	9.9e3	1.6e4	1.3e4	15	1.5e6	1.3e6	1.7e6	1.5e6	1e-1	15	3.8e5	2.9e5	4.6e5	3.8e5	2	1.4e8	1.3e8	1.5e8	2.0e7		
1e-3	15	2.1e4	1.6e4	2.6e4	2.1e4	15	1.6e6	1.4e6	1.8e6	1.6e6	1e-3	15	1.3e6	1.1e6	1.6e6	1.3e6	0	85e-2	96e-3	23e-1	7.9e6		
1e-5	15	3.1e4	2.4e4	3.8e4	3.1e4	15	2.0e6	1.6e6	2.3e6	2.0e6	1e-5	10	4.7e6	3.8e6	5.6e6	2.8e6		
1e-8	15	5.6e4	4.7e4	6.6e4	5.6e4	15	2.9e6	2.5e6	3.4e6	2.9e6	1e-8	3	2.4e7	2.4e7	2.5e7	5.0e6		
f_{109} in 5-D, N=15, mFE=799091						f_{109} in 20-D, N=15, mFE=3621383						f_{110} in 5-D, N=15, mFE=5008280						f_{110} in 20-D, N=15, mFE=20009863					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	2.9e1	2.5e1	3.3e1	2.9e1	15	1.3e3	1.3e3	1.4e3	1.3e3	10	15	4.5e3	2.8e3	6.3e3	4.5e3	0	18e+0	18e+0	18e+0	8.9e6		
1	15	1.3e2	1.2e2	1.4e2	1.3e2	15	5.4e3	2.8e3	8.1e3	5.4e3	1	15	3.0e5	2.1e5	4.0e5	3.0e5		
1e-1	15	3.5e3	1.1e3	6.4e3	3.5e3	15	2.3e4	1.5e4	3.1e4	2.3e4	1e-1	9	5.5e6	4.5e6	6.5e6	2.5e6		
1e-3	15	3.7e4	2.7e4	4.9e4	3.7e4	15	7.1e5	5.7e5	8.7e5	7.1e5	1e-3	6	1.0e7	8.8e6	1.1e7	3.2e6		
1e-5	15	1.6e5	1.3e5	1.8e5	1.6e5	15	1.5e6	1.3e6	1.8e6	1.5e6	1e-5	5	1.2e7	1.1e7	1.4e7	3.6e6		
1e-8	15	4.5e5	3.8e5	5.2e5	4.5e5	15	1.8e6	1.6e6	2.1e6	1.8e6	1e-8	4	1.5e7	1.3e7	1.7e7	3.3e6		
f_{111} in 5-D, N=15, mFE=5009373						f_{111} in 20-D, N=15, mFE=20005321						f_{112} in 5-D, N=15, mFE=5008789						f_{112} in 20-D, N=15, mFE=20007211					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	4.9e4	3.9e4	5.9e4	4.9e4	0	27e+0	20e+0	35e+0	7.9e6	10	15	2.5e2	2.3e2	2.7e2	2.5e2	13	1.4e7	1.2e7	1.5e7	1.1e7		
1	14	2.3e6	1.7e6	2.8e6	2.3e6	1	15	1.5e5	1.1e5	2.0e5	1.5e5	7	3.6e7	3.3e7	3.8e7	1.6e7		
1e-1	3	2.2e7	2.0e7	2.4e7	5.0e6	1e-1	15	1.2e6	8.5e5	1.5e6	1.2e6	7	3.6e7	3.3e7	3.9e7	1.6e7		
1e-3	0	26e-2	43e-3	64e-2	2.2e6	1e-3	15	1.9e6	1.6e6	2.3e6	1.9e6	7	3.6e7	3.4e7	3.9e7	1.6e7		
1e-5	1e-5	15	2.2e6	1.9e6	2.6e6	2.2e6	7	3.6e7	3.3e7	3.9e7	1.6e7		
1e-8	1e-8	13	3.6e6	3.1e6	4.0e6	3.0e6	7	3.6e7	3.4e7	3.9e7	1.6e7		
f_{113} in 5-D, N=15, mFE=83906						f_{113} in 20-D, N=15, mFE=4072574						f_{114} in 5-D, N=15, mFE=5008506						f_{114} in 20-D, N=15, mFE=20010822					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.3e2	1.1e2	1.6e2	1.3e2	15	2.6e5	1.9e5	3.3e5	2.6e5	10	15	8.1e3	5.9e3	1.1e4	8.1e3	13	8.8e6	6.7e6	1.1e7	7.9e6		
1	15	8.8e3	3.6e3	1.5e4	8.8e3	15	1.6e6	1.4e6	1.8e6	1.6e6	1	15	2.1e5	1.6e5	2.7e5	2.1e5	1	2.9e8	2.8e8	3.0e8	2.0e7		
1e-1	15	3.4e4	2.4e4	4.4e4	3.4e4	15	2.3e6	1.9e6	2.7e6	2.3e6	1e-1	15	8.3e5	6.3e5	1.1e6	8.3e5	0	27e-1	11e-1	11e+0	7.9e6		
1e-3	15	4.3e4	3.4e4	5.2e4	4.3e4	15	3.1e6	2.7e6	3.4e6	3.1e6	1e-3	12	3.5e6	2.9e6	4.2e6	2.6e6		
1e-5	15	4.3e4	3.3e4	5.2e4	4.3e4	15	3.1e6	2.7e6	3.4e6	3.1e6	1e-5	12	3.5e6	2.9e6	4.2e6	2.6e6		
1e-8	15	4.3e4	3.4e4	5.3e4	4.3e4	15	3.1e6	2.8e6	3.4e6	3.1e6	1e-8	12	3.6e6	3.0e6	4.2e6	2.7e6		
f_{115} in 5-D, N=15, mFE=108673						f_{115} in 20-D, N=15, mFE=1155027						f_{116} in 5-D, N=15, mFE=153370						f_{116} in 20-D, N=15, mFE=4408997					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	9.9e1	8.9e1	1.1e2	9.9e1	15	3.2e3	2.3e3	4.1e3	3.2e3	10	15	2.3e4	1.7e4	2.8e4	2.3e4	15	1.9e6	1.6e6	2.1e6	1.9e6		
1	15	2.0e3	8.6e2	3.3e3	2.0e3	15	5.2e4	3.3e4	7.2e4	5.2e4	1	15	4.7e4	3.3e4	6.1e4	4.7e4	15	2.0e6	1.7e6	2.4e6	2.0e6		
1e-1	15	1.6e4	1.1e4	2.1e4	1.6e4	15	2.2e5	1.6e5	2.7e5	2.2e5	1e-1	15	5.6e4	4.4e4	7.0e4	5.6e4	15	2.2e6	1.9e6	2.6e6	2.2e6		
1e-3	15	5.1e4	4.2e4	6.0e4	5.1e4	15	8.9e5	7.8e5	9.9e5	8.9e5	1e-3	15	6.3e4	5.1e4	7.6e4	6.3e4	15	2.6e6	2.2e6	3.0e6	2.6e6		
1e-5	15	5.1e4	4.1e4	6.0e4	5.1e4	15	8.9e5	7.7e5	9.9e5	8.9e5	1e-5	15	7.8e4	6.6e4	9.0e4	7.8e4	15	2.6e6	2.3e6	3.0e6	2.6e6		
1e-8	15	6.0e4	5.1e4	7.0e4	6.0e4																		

f_{121} in 5-D, N=15, mFE=2708125						f_{121} in 20-D, N=15, mFE=3041481						f_{122} in 5-D, N=15, mFE=733298						f_{122} in 20-D, N=15, mFE=20021683					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.8e1	1.4e1	2.3e1	1.8e1	15	1.6e3	1.5e3	1.6e3	1.6e3	10	15	1.3e1	9.7e0	1.7e1	1.3e1	15	9.5e2	7.9e2	1.1e3	9.5e2		
1	15	2.0e2	1.8e2	2.2e2	2.0e2	15	4.6e3	4.4e3	4.8e3	4.6e3	1	15	8.4e3	4.5e3	1.3e4	8.4e3	15	8.5e5	7.0e5	1.0e6	8.5e5		
1e-1	15	3.3e3	6.4e2	6.1e3	3.3e3	15	6.7e4	4.6e4	8.8e4	6.7e4	1e-1	15	5.1e4	4.1e4	6.3e4	5.1e4	15	2.6e6	2.2e6	3.1e6	2.6e6		
1e-3	15	4.1e4	3.0e4	5.2e4	4.1e4	15	2.3e5	1.9e5	2.7e5	2.3e5	1e-3	15	1.1e5	8.7e4	1.3e5	1.1e5	15	4.6e6	4.5e6	4.7e6	4.6e6		
1e-5	15	1.6e5	1.3e5	2.0e5	1.6e5	15	5.1e5	4.0e5	6.3e5	5.1e5	1e-5	15	2.0e5	1.9e5	2.2e5	2.0e5	11	1.3e7	1.0e7	1.6e7	8.7e6		
1e-8	15	9.7e5	7.6e5	1.2e6	9.7e5	15	1.2e6	1.1e6	1.4e6	1.2e6	1e-8	15	3.7e5	3.0e5	4.5e5	3.7e5	7	3.2e7	2.7e7	3.6e7	1.3e7		
f_{123} in 5-D, N=15, mFE=5009358						f_{123} in 20-D, N=15, mFE=20020903						f_{124} in 5-D, N=15, mFE=3180298						f_{124} in 20-D, N=15, mFE=5042373					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	2.0e1	1.4e1	2.7e1	2.0e1	15	2.0e3	1.4e3	2.6e3	2.0e3	10	15	2.0e1	1.6e1	2.4e1	2.0e1	15	8.8e2	7.9e2	1.0e3	8.8e2		
1	15	1.4e5	9.5e4	1.9e5	1.4e5	1	2.9e8	2.8e8	3.0e8	2.0e7	1	15	5.0e3	2.4e3	7.6e3	5.0e3	15	2.1e4	1.0e4	3.2e4	2.1e4		
1e-1	5	1.2e7	1.0e7	1.3e7	3.4e6	0	23e-1	12e-1	32e-1	8.9e6	1e-1	15	1.5e4	1.1e4	2.0e4	1.5e4	15	1.4e5	1.3e5	1.4e5	1.4e5		
1e-3	0	15e-2	74e-3	17e-2	2.2e6	1e-3	15	2.0e5	1.6e5	2.5e5	2.0e5	15	5.8e5	4.7e5	6.9e5	5.8e5		
1e-5	1e-5	15	8.5e5	7.3e5	9.9e5	8.5e5	15	1.3e6	1.0e6	1.6e6	1.3e6		
1e-8	1e-8	15	1.9e6	1.6e6	2.2e6	1.9e6	15	2.0e6	1.5e6	2.5e6	2.0e6		
f_{125} in 5-D, N=15, mFE=5008084						f_{125} in 20-D, N=15, mFE=20016770						f_{126} in 5-D, N=15, mFE=5008584						f_{126} in 20-D, N=15, mFE=20016867					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.1e0	1.0e0	1.1e0	1.1e0	15	1.1e0	1.0e0	1.3e0	1.1e0	10	15	1.0e0	1.0e0	1.0e0	1.0e0	15	1.0e0	1.0e0	1.0e0	1.0e0		
1	15	3.7e1	2.9e1	4.6e1	3.7e1	15	1.0e3	9.3e2	1.1e3	1.0e3	1	15	4.5e1	3.8e1	5.3e1	4.5e1	15	1.3e3	1.2e3	1.4e3	1.3e3		
1e-1	15	6.8e3	4.0e3	1.0e4	6.8e3	0	24e-2	15e-2	28e-2	1.4e7	1e-1	15	3.0e4	1.7e4	4.4e4	3.0e4	0	31e-2	29e-2	33e-2	8.9e6		
1e-3	12	3.1e6	2.5e6	3.7e6	2.3e6	1e-3	0	15e-3	11e-3	28e-3	1.4e6		
1e-5	9	5.5e6	4.5e6	6.4e6	3.1e6	1e-5		
1e-8	9	5.5e6	4.6e6	6.5e6	3.1e6	1e-8		
f_{127} in 5-D, N=15, mFE=5009449						f_{127} in 20-D, N=15, mFE=20005826						f_{128} in 5-D, N=15, mFE=2115239						f_{128} in 20-D, N=15, mFE=20018320					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.1e0	1.0e0	1.3e0	1.1e0	15	1.1e0	1.0e0	1.3e0	1.1e0	10	15	1.4e2	1.0e2	1.8e2	1.4e2	15	2.2e6	1.7e6	2.7e6	2.2e6		
1	15	4.0e1	3.4e1	4.6e1	4.0e1	15	7.6e2	7.1e2	8.1e2	7.6e2	1	15	2.0e5	1.2e5	2.7e5	2.0e5	11	1.6e7	1.2e7	1.9e7	1.2e7		
1e-1	15	2.7e3	7.0e2	4.7e3	2.7e3	15	1.5e6	9.6e5	2.0e6	1.5e6	1e-1	15	3.6e5	2.0e5	5.6e5	3.6e5	10	1.8e7	1.5e7	2.1e7	1.2e7		
1e-3	14	1.8e6	1.4e6	2.2e6	1.7e6	0	83e-4	36e-4	40e-3	2.0e7	1e-3	15	3.7e5	2.0e5	5.5e5	3.7e5	10	1.8e7	1.5e7	2.2e7	1.2e7		
1e-5	9	5.3e6	4.5e6	6.3e6	3.4e6	1e-5	15	3.7e5	2.1e5	5.5e5	3.7e5	10	1.9e7	1.5e7	2.2e7	1.2e7		
1e-8	5	1.2e7	1.1e7	1.4e7	4.6e6	1e-8	15	3.8e5	2.1e5	5.5e5	3.8e5	10	1.9e7	1.6e7	2.2e7	1.3e7		
f_{129} in 5-D, N=15, mFE=5003969						f_{129} in 20-D, N=15, mFE=20019421						f_{130} in 5-D, N=15, mFE=3516605						f_{130} in 20-D, N=15, mFE=20017912					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.8e2	1.4e2	2.1e2	1.8e2	2	1.4e8	1.4e8	1.5e8	2.0e7	10	15	1.1e2	9.6e1	1.3e2	1.1e2	15	3.5e4	4.8e3	6.5e4	3.5e4		
1	15	2.1e5	1.3e5	2.8e5	2.1e5	1	2.9e8	2.9e8	3.0e8	2.0e7	1	15	1.3e5	6.8e4	1.9e5	1.3e5	8	2.0e7	1.5e7	2.6e7	1.3e7		
1e-1	15	6.9e5	3.8e5	1.0e6	6.9e5	0	23e+0	26e-1	28e+0	1.1e7	1e-1	15	4.2e5	1.5e5	7.2e5	4.2e5	8	2.1e7	1.5e7	2.6e7	1.3e7		
1e-3	15	1.4e6	9.1e5	1.9e6	1.4e6	1e-3	15	4.3e5	1.7e5	7.6e5	4.3e5	8	2.1e7	1.5e7	2.7e7	1.3e7		
1e-5	15	1.7e6	1.2e6	2.1e6	1.7e6	1e-5	15	4.9e5	2.3e5	7.5e5	4.9e5	8	2.1e7	1.5e7	2.6e7	1.3e7		
1e-8	12	3.0e6	2.3e6	3.8e6	2.3e6	1e-8	15	5.7e5	2.9e5	9.1e5	5.7e5	8	2.2e7	1.7e7	2.7e7	1.3e7		

Table 3: AMaLGaM: Shown are, for functions f_{121} - f_{130} and for a given target difference to the optimal function value Δf : the number of successful trials (#); the expected running time to surpass $f_{\text{opt}} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value (RT_{succ}). If $f_{\text{opt}} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf -value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

f_{121} in 5-D, N=15, mFE=3085457						f_{121} in 20-D, N=15, mFE=3595636						f_{122} in 5-D, N=15, mFE=1999552						f_{122} in 20-D, N=15, mFE=20014172					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.9e1	1.3e1	2.5e1	1.9e1	15	8.5e2	7.9e2	9.1e2	8.5e2	10	15	1.3e1	9.5e0	1.7e1	1.3e1	15	6.9e2	5.7e2	8.2e2	6.9e2		
1	15	1.2e2	1.1e2	1.3e2	1.2e2	15	9.5e3	6.5e3	1.2e4	9.5e3	1	15	2.1e4	1.3e4	2.9e4	2.1e4	15	2.3e6	2.0e6	2.6e6	2.3e6		
1e-1	15	1.5e3	3.4e2	2.8e3	1.5e3	15	8.2e4	4.4e4	1.2e5	8.2e4	1e-1	15	1.3e5	1.0e5	1.5e5	1.3e5	15	4.4e6	3.8e6	5.1e6	4.4e6		
1e-3	15	7.8e4	5.7e4	1.0e5	7.8e4	15	1.1e6	9.1e5	1.4e6	1.1e6	1e-3	15	2.1e5	1.9e5	2.3e5	2.1e5	15	8.3e6	7.4e6	9.2e6	8.3e6		
1e-5	15	4.4e5	3.7e5	5.1e5	4.4e5	15	1.7e6	1.4e6	2.0e6	1.7e6	1e-5	15	3.1e5	2.7e5	3.7e5	3.1e5	8	2.8e7	2.5e7	3.1e7	1.5e7		
1e-8	15	1.6e6	1.4e6	1.9e6	1.6e6	15	2.3e6	2.0e6	2.6e6	2.3e6	1e-8	15	5.3e5	3.9e5	6.9e5	5.3e5	1	2.9e8	2.8e8	3.0e8	2.0e7		
f_{123} in 5-D, N=15, mFE=5009700						f_{123} in 20-D, N=15, mFE=20005849						f_{124} in 5-D, N=15, mFE=5008758						f_{124} in 20-D, N=15, mFE=12363963					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	2.4e1	1.2e1	3.7e1	2.4e1	15	1.3e4	5.5e3	2.0e4	1.3e4	10	15	1.1e1	7.5e0	1.4e1	1.1e1	15	5.4e2	4.5e2	6.2e2	5.4e2		
1	15	3.0e5	2.4e5	3.6e5	3.0e5	0	23e-1	13e-1	43e-1	8.9e6	1	15	3.0e3	1.7e3	4.4e3	3.0e3	15	1.5e4	1.1e4	2.0e4	1.5e4		
1e-1	5	1.3e7	1.2e7	1.4e7	4.8e6	1e-1	15	4.2e4	3.5e4	4.9e4	4.2e4	15	7.6e5	6.1e5	8.9e5	7.6e5		
1e-3	0	18e-2	34e-3	24e-2	3.5e6	1e-3	15	3.5e5	2.8e5	4.3e5	3.5e5	15	2.2e6	1.9e6	2.5e6	2.2e6		
1e-5	1e-5	14	2.6e6	2.1e6	3.0e6	2.4e6	15	2.4e6	2.1e6	2.7e6	2.4e6		
1e-8	1e-8	13	3.4e6	2.9e6	3.8e6	3.0e6	15	3.5e6	2.8e6	4.3e6	3.5e6		
f_{125} in 5-D, N=15, mFE=5008580						f_{125} in 20-D, N=15, mFE=20005997						f_{126} in 5-D, N=15, mFE=5008585						f_{126} in 20-D, N=15, mFE=20005928					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.2e0	1.1e0	1.3e0	1.2e0	15	1.1e0	1.0e0	1.1e0	1.1e0	10	15	1.0e0	1.0e0	1.0e0	1.0e0	15	1.1e0	1.0e0	1.3e0	1.1e0		
1	15	2.4e1	2.0e1	2.8e1	2.4e1	15	6.8e2	6.1e2	7.5e2	6.8e2	1	15	3.8e1	2.6e1	5.0e1	3.8e1	15	5.7e3	2.3e3	8.8e3	5.7e3		
1e-1	15	6.4e3	4.0e3	8.7e3	6.4e3	0	24e-2	17e-2	29e-2	7.9e6	1e-1	15	6.9e4	5.1e4	9.0e4	6.9e4	0	34e-2	31e-2	37e-2	1.0e7		
1e-3	8	6.9e6	5.9e6	7.9e6	3.8e6	1e-3	0	16e-3	74e-4	25e-3	2.2e6		
1e-5	5	1.3e7	1.1e7	1.4e7	4.9e6	1e-5		
1e-8	5	1.3e7	1.1e7	1.4e7	4.9e6	1e-8		
f_{127} in 5-D, N=15, mFE=5008766						f_{127} in 20-D, N=15, mFE=20008232						f_{128} in 5-D, N=15, mFE=912091						f_{128} in 20-D, N=15, mFE=20004996					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.3e0	1.0e0	1.5e0	1.3e0	15	1.0e0	1.0e0	1.0e0	1.0e0	10	15	9.1e2	1.1e2	1.7e3	9.1e2	13	8.6e6	6.7e6	1.1e7	7.7e6		
1	15	2.4e1	2.1e1	2.8e1	2.4e1	15	5.6e2	5.0e2	6.2e2	5.6e2	1	15	1.5e5	4.9e4	2.4e5	1.5e5	4	6.6e7	5.9e7	7.2e7	1.6e7		
1e-1	15	9.8e3	7.0e3	1.3e4	9.8e3	15	3.3e6	2.6e6	4.2e6	3.3e6	1e-1	15	1.7e5	6.7e4	2.7e5	1.7e5	1	2.9e8	2.8e8	3.0e8	8.6e6		
1e-3	8	5.9e6	4.5e6	7.9e6	3.8e6	1	3.0e8	3.0e8	3.0e8	2.0e7	1e-3	15	1.8e5	9.5e4	3.8e5	1.8e5	1	2.9e8	2.8e8	3.0e8	8.6e6		
1e-5	5	6.6e7	5.7e7	8.8e7	5.0e6	1	3.0e8	3.0e8	3.0e8	2.0e7	1e-5	15	1.8e5	9.5e4	3.8e5	1.8e5	1	2.9e8	2.8e8	3.0e8	8.6e6		
1e-8	1	7.1e7	6.6e7	7.5e7	5.0e6	1	3.0e8	3.0e8	3.0e8	2.0e7	1e-8	15	1.9e5	1.0e5	2.9e5	1.9e5	1	2.9e8	2.8e8	3.0e8	8.7e6		
f_{129} in 5-D, N=15, mFE=5009247						f_{129} in 20-D, N=15, mFE=20005548						f_{130} in 5-D, N=15, mFE=837591						f_{130} in 20-D, N=15, mFE=200044874					
Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}	Δf	#	ERT	10%	90%	RT _{succ}	#	ERT	10%	90%	RT _{succ}		
10	15	1.6e3	1.3e2	3.1e3	1.6e3	0	31e+0	22e+0	41e+0	8.9e6	10	15	8.0e1	6.6e1	9.4e1	8.0e1	15	2.7e4	1.2e4	4.6e4	2.7e4		
1	15	2.7e5	1.1e5	4.3e5	2.7e5	1	15	1.1e5	5.4e4	1.7e5	1.1e5	13	6.9e6	4.0e6	9.8e6	6.9e6		
1e-1	15	7.7e5	5.5e5	9.9e5	7.7e5	1e-1	15	1.7e5	1.0e5	2.3e5	1.7e5	12	9.5e6	6.2e6	1.3e7	7.8e6		
1e-3	15	1.3e6	9.9e5	1.7e6	1.3e6	1e-3	15	2.1e5	1.4e5	2.8e5	2.1e5	10	1.7e7	1.3e7	2.1e7	1.1e7		
1e-5	12	3.0e6	2.3e6	3.7e6	2.3e6	1e-5	15	2.5e5	1.9e5	3.3e5	2.5e5	7	3.2e7	2.7e7	3.7e7	1.5e7		
1e-8	5	1.2e7	9.9e6	1.3e7	4.0e6	1e-8	13	3.1e5	2.3e5	3.8e5	3.1e5	3	8.3e7	7.2e7	9.4e7	1.4e7		