# Black-Box Optimization Benchmarking for Noiseless Function Testbed using Particle Swarm Optimization

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## ABSTRACT

This paper benchmarks the particle swarm optimizer (PSO) algorithm using the noise-free BBOB 2009 testbed.

## Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: OptimizationGlobal Optimization, Unconstrained Optimization; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

## General Terms

Algorithms

## Keywords

Benchmarking, Black-box optimization, Evolutionary computation, Particle Swarm Optimization

## 1. INTRODUCTION

Particle Swarm Optimization (PSO) [1, 5] is an optimization method widely used to solve continuous nonlinear functions. It is a stochastic optimization technique that emerged from simulations of the birds flocking and fish schooling behaviors.

The algorithm used is a simple PSO algorithm utilizing the *gbest* (global best) model.

## 2. ALGORITHM PRESENTATION

The only design choice made was to select the absorbing boundaries to handle any particles leaving the search space, where the position is set to the boundary and the velocity is reset to zeros. Fig. 1 shows the MATLAB code for PSO algorithm.

The simulations for 2; 5; 10 and 20 D were done with the MATLAB-code and took 13 hours and 28 minutes. No parameter tuning was done and the crafting effort CrE [4] is computed to zero.

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## 3. RESULTS

The parameters are set as $c1 = c2 = 2$ and $w$ linearly decreases from 0.9 to 0.1.

Results from experiments according to [3] on the benchmark functions given in [2, 4] are presented in Figures 2 and 3 and in Table 1.

## 4. CPU TIMING EXPERIMENT

For the timing experiment, PSO was run with a maximum of $10^4$ function evaluations and restarted until 30 seconds has passed (according to Figure 2 in [4]). The experiments have been conducted with an Intel Core 2 Quad 2.4 GHz under Windows XP using the MATLAB-code provided. The time per function evaluation was 1.1; 1.1; 1.3; 1.4; 1.6 times $10^{-5}$ seconds in dimensions 2; 3; 5; 10; 20 respectively.

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| $\Delta f$ | # | ERT | 10% | 90% | $\mathrm{RT_{succ}}$ |
|---|---|---|---|---|---|
| **$f_1$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 3.7e1 | 2.6e1 | 4.8e1 | 3.7e1 |
| 1 | 15 | 4.8e2 | 4.1e2 | 5.2e2 | 4.8e2 |
| 1e−1 | 15 | 2.7e3 | 2.1e3 | 2.9e3 | 2.7e3 |
| 1e−3 | 15 | 2.1e4 | 1.7e4 | 2.3e4 | 2.1e4 |
| 1e−5 | 15 | 3.9e4 | 3.4e4 | 4.3e4 | 3.9e4 |
| 1e−8 | 14 | 9.2e4 | 6.0e4 | 1.3e5 | 8.8e4 |
| **$f_1$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 15 | 3.6e3 | 3.3e3 | 4.1e3 | 3.6e3 |
| 1 | 14 | 1.8e5 | 3.5e4 | 3.2e5 | 1.7e5 |
| 1e−1 | 14 | 2.2e5 | 8.3e4 | 3.6e5 | 2.2e5 |
| 1e−3 | 8 | 1.9e6 | 1.3e6 | 2.4e6 | 8.7e5 |
| 1e−5 | 0 | *71e−5* | *46e−6* | *68e−4* | 4.0e5 |
| 1e−8 | . | . | . | . | . |
| **$f_3$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 7.3e3 | 5.8e3 | 8.7e3 | 7.3e3 |
| 1 | 9 | 3.9e5 | 2.3e5 | 4.9e5 | 2.5e5 |
| 1e−1 | 4 | 1.4e6 | 1.2e6 | 1.5e6 | 3.9e5 |
| 1e−3 | 4 | 1.4e6 | 1.1e6 | 1.7e6 | 3.9e5 |
| 1e−5 | 4 | 1.4e6 | 1.3e6 | 1.8e6 | 4.0e5 |
| 1e−8 | 4 | 1.5e6 | 1.2e6 | 1.7e6 | 4.0e5 |
| **$f_3$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 0 | *41e+0* | *33e+0* | *56e+0* | 2.8e5 |
| 1 | . | . | . | . | . |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_5$ in 5-D**, N=15, mFE=280 | | | | | |
| 10 | 15 | 9.5e1 | 8.8e1 | 1.0e2 | 9.5e1 |
| 1 | 15 | 1.5e2 | 1.4e2 | 1.6e2 | 1.5e2 |
| 1e−1 | 15 | 1.6e2 | 1.4e2 | 1.8e2 | 1.6e2 |
| 1e−3 | 15 | 1.6e2 | 1.5e2 | 1.8e2 | 1.6e2 |
| 1e−5 | 15 | 1.6e2 | 1.3e2 | 1.8e2 | 1.6e2 |
| 1e−8 | 15 | 1.6e2 | 1.5e2 | 1.8e2 | 1.6e2 |
| **$f_5$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 1 | 2.8e7 | 2.6e7 | 3.0e7 | 2.0e6 |
| 1 | 0 | *30e+0* | *10e+0* | *92e+0* | 5.0e2 |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_7$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 1.9e2 | 1.6e2 | 2.3e2 | 1.9e2 |
| 1 | 15 | 3.4e3 | 2.1e3 | 5.9e3 | 3.4e3 |
| 1e−1 | 14 | 4.7e4 | 1.1e4 | 1.2e5 | 4.6e4 |
| 1e−3 | 14 | 7.6e4 | 4.5e4 | 1.7e5 | 7.4e4 |
| 1e−5 | 14 | 7.6e4 | 4.0e4 | 1.2e5 | 7.4e4 |
| 1e−8 | 14 | 8.0e4 | 4.6e4 | 1.1e5 | 7.7e4 |
| **$f_7$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 8 | 1.9e6 | 1.4e6 | 2.1e6 | 1.1e6 |
| 1 | 15 | 3.4e3 | 2.1e3 | 5.9e3 | 3.4e3 |
| 1e−1 | 14 | 4.7e4 | 1.1e4 | 1.2e5 | 4.6e4 |
| 1e−3 | 14 | 7.6e4 | 4.5e4 | 1.7e5 | 7.4e4 |
| 1e−5 | 14 | 7.6e4 | 4.0e4 | 1.2e5 | 7.4e4 |
| 1e−8 | 14 | 8.0e4 | 4.6e4 | 1.1e5 | 7.7e4 |
| **$f_9$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 3.5e3 | 3.1e3 | 4.2e3 | 3.5e3 |
| 1 | 14 | 6.1e4 | 2.1e4 | 8.6e4 | 5.9e4 |
| 1e−1 | 13 | 1.3e5 | 6.5e4 | 1.9e5 | 1.2e5 |
| 1e−3 | 12 | 1.8e5 | 1.1e5 | 2.6e5 | 1.7e5 |
| 1e−5 | 11 | 2.5e5 | 1.7e5 | 3.3e5 | 1.8e5 |
| 1e−8 | 11 | 2.6e5 | 1.4e5 | 3.4e5 | 1.9e5 |
| **$f_9$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 4 | 5.6e6 | 4.7e6 | 6.1e6 | 2.0e6 |
| 1 | 0 | *13e+0* | *42e−1* | *67e+0* | 2.8e5 |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{11}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 1.3e4 | 1.3e4 | 1.5e4 | 1.3e4 |
| 1 | 15 | 4.1e4 | 3.9e4 | 4.6e4 | 4.1e4 |
| 1e−1 | 15 | 6.7e4 | 6.1e4 | 7.2e4 | 6.7e4 |
| 1e−3 | 15 | 8.1e4 | 7.9e4 | 8.6e4 | 8.1e4 |
| 1e−5 | 14 | 1.2e5 | 9.5e4 | 1.3e5 | 1.2e5 |
| 1e−8 | 12 | 2.2e5 | 1.5e5 | 2.6e5 | 2.0e5 |
| **$f_{11}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 14 | 3.2e5 | 1.8e5 | 4.5e5 | 3.0e5 |
| 1 | 2 | 1.3e7 | 1.2e7 | 1.5e7 | 2.0e6 |
| 1e−1 | 0 | *22e−1* | *98e−2* | *69e−1* | 3.2e5 |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{13}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 10 | 2.7e5 | 1.2e5 | 3.7e5 | 1.7e5 |
| 1 | 6 | 7.9e5 | 6.4e5 | 8.7e5 | 3.4e5 |
| 1e−1 | 4 | 1.4e6 | 1.1e6 | 1.8e6 | 3.9e5 |
| 1e−3 | 3 | 2.1e6 | 1.9e6 | 2.4e6 | 5.0e5 |
| 1e−5 | 3 | 2.1e6 | 1.6e6 | 2.4e6 | 5.0e5 |
| 1e−8 | 1 | 7.1e6 | 6.7e6 | 7.5e6 | 5.0e5 |
| **$f_{13}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 3 | 8.2e6 | 7.6e6 | 9.4e6 | 8.3e5 |
| 1 | 0 | *15e+0* | *80e−1* | *48e+0* | 2.8e5 |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{15}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 8.4e3 | 6.6e3 | 9.8e3 | 8.4e3 |
| 1 | 4 | 1.4e6 | 1.3e6 | 1.7e6 | 3.9e5 |
| 1e−1 | 2 | 3.3e6 | 2.7e6 | 3.8e6 | 5.0e5 |
| 1e−3 | 1 | 7.1e6 | 6.2e6 | 7.5e6 | 5.0e5 |
| 1e−5 | 1 | 7.1e6 | 6.3e6 | 7.5e6 | 5.0e5 |
| 1e−8 | 1 | 7.1e6 | 6.7e6 | 7.5e6 | 5.0e5 |
| **$f_{15}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 0 | *66e+0* | *46e+0* | *89e+0* | 3.2e5 |
| 1 | . | . | . | . | . |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{17}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 1.3e1 | 8.9e0 | 1.8e1 | 1.3e1 |
| 1 | 15 | 1.5e3 | 1.1e3 | 1.8e3 | 1.5e3 |
| 1e−1 | 15 | 2.4e4 | 2.1e4 | 2.9e4 | 2.4e4 |
| 1e−3 | 4 | 1.4e6 | 1.2e6 | 1.8e6 | 3.9e5 |
| 1e−5 | 1 | 7.1e6 | 6.6e6 | 7.5e6 | 5.0e5 |
| 1e−8 | 0 | *67e−4* | *11e−6* | *44e−3* | 1.1e5 |
| **$f_{17}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 15 | 2.8e2 | 2.2e2 | 3.6e2 | 2.8e2 |
| 1 | 9 | 1.4e6 | 1.0e6 | 2.1e6 | 9.4e5 |
| 1e−1 | 0 | *89e−2* | *50e−2* | *19e−1* | 3.2e5 |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{19}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 3.7e1 | 3.2e1 | 4.2e1 | 3.7e1 |
| 1 | 15 | 3.1e3 | 1.7e3 | 3.5e3 | 3.1e3 |
| 1e−1 | 14 | 6.7e4 | 2.6e4 | 9.8e4 | 6.5e4 |
| 1e−3 | 0 | *16e−3* | *16e−3* | *25e−3* | 7.9e4 |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{19}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 15 | 2.0e2 | 1.5e2 | 2.5e2 | 2.0e2 |
| 1 | 11 | 8.1e5 | 4.6e5 | 1.2e6 | 4.4e5 |
| 1e−1 | 1 | 2.8e7 | 2.5e7 | 3.0e7 | 2.0e6 |
| 1e−3 | 0 | *18e−2* | *12e−2* | *16e−1* | 4.0e5 |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{21}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 9.9e1 | 7.5e1 | 1.2e2 | 9.9e1 |
| 1 | 10 | 2.5e5 | 1.0e5 | 3.0e5 | 2.0e5 |
| 1e−1 | 9 | 3.4e5 | 2.3e5 | 5.6e5 | 2.2e5 |
| 1e−3 | 9 | 3.4e5 | 2.3e5 | 4.5e5 | 2.3e5 |
| 1e−5 | 9 | 3.5e5 | 2.4e5 | 5.2e5 | 2.3e5 |
| 1e−8 | 9 | 3.7e5 | 2.6e5 | 4.7e5 | 2.4e5 |
| **$f_{21}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 7 | 2.3e6 | 1.4e6 | 2.9e6 | 1.1e6 |
| 1 | 0 | *15e+0* | *18e−1* | *24e+0* | 2.5e5 |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{23}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 1.1e1 | 5.9e0 | 1.5e1 | 1.1e1 |
| 1 | 15 | 1.9e4 | 1.3e4 | 2.5e4 | 1.9e4 |
| 1e−1 | 2 | 3.3e6 | 3.1e6 | 3.5e6 | 5.0e5 |
| 1e−3 | 0 | *24e−2* | *71e−3* | *40e−2* | 4.5e5 |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{23}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 15 | 7.5e0 | 6.1e0 | 9.4e0 | 7.5e0 |
| 1 | 11 | 1.3e6 | 9.4e5 | 1.5e6 | 1.1e6 |
| 1e−1 | 0 | *70e−2* | *43e−2* | *11e−1* | 1.8e6 |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |

| $\Delta f$ | # | ERT | 10% | 90% | $\mathrm{RT_{succ}}$ |
|---|---|---|---|---|---|
| **$f_2$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 2.6e4 | 2.1e4 | 3.0e4 | 2.6e4 |
| 1 | 15 | 3.6e4 | 3.2e4 | 4.0e4 | 3.6e4 |
| 1e−1 | 15 | 4.3e4 | 4.0e4 | 4.9e4 | 4.3e4 |
| 1e−3 | 15 | 6.0e4 | 5.4e4 | 6.2e4 | 6.0e4 |
| 1e−5 | 15 | 6.9e4 | 6.4e4 | 7.3e4 | 6.9e4 |
| 1e−8 | 14 | 1.1e5 | 7.7e4 | 1.7e5 | 1.1e5 |
| **$f_2$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 1 | 2.8e7 | 2.5e7 | 3.0e7 | 2.0e6 |
| 1 | 0 | *21e+1* | *45e+0* | *37e+3* | 2.8e5 |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_4$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 1.3e4 | 9.1e3 | 1.6e4 | 1.3e4 |
| 1 | 6 | 8.0e5 | 6.6e5 | 1.1e6 | 2.0e5 |
| 1e−1 | 1 | 7.1e6 | 6.6e6 | 7.5e6 | 5.0e5 |
| 1e−3 | 1 | 7.1e6 | 6.6e6 | 7.5e6 | 5.0e5 |
| 1e−5 | 1 | 7.1e6 | 6.7e6 | 7.5e6 | 5.0e5 |
| 1e−8 | 1 | 7.1e6 | 6.7e6 | 7.5e6 | 5.0e5 |
| **$f_4$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 0 | *61e+0* | *34e+0* | *85e+0* | 2.8e5 |
| 1 | . | . | . | . | . |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_6$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 14 | 3.7e4 | 1.1e3 | 3.7e4 | 3.7e4 |
| 1 | 13 | 8.4e4 | 4.5e4 | 1.6e5 | 4.5e4 |
| 1e−1 | 13 | 9.5e4 | 2.5e4 | 1.7e5 | 5.6e4 |
| 1e−3 | 13 | 1.2e5 | 5.1e4 | 2.3e5 | 8.1e4 |
| 1e−5 | 13 | 1.4e5 | 7.8e4 | 1.8e5 | 1.0e5 |
| 1e−8 | 6 | 8.2e5 | 6.1e5 | 9.6e5 | 3.6e5 |
| **$f_6$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 6 | 3.1e6 | 2.5e6 | 3.5e6 | 1.1e6 |
| 1 | 2 | 1.3e7 | 1.1e7 | 1.4e7 | 1.1e6 |
| 1e−1 | 0 | *23e+0* | *42e−2* | *16e+1* | 2.8e5 |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_8$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 3.2e3 | 2.4e3 | 3.6e3 | 3.2e3 |
| 1 | 13 | 1.1e5 | 3.4e4 | 1.5e5 | 1.0e5 |
| 1e−1 | 13 | 1.2e5 | 8.5e4 | 2.2e5 | 1.2e5 |
| 1e−3 | 12 | 1.9e5 | 1.2e5 | 2.3e5 | 1.7e5 |
| 1e−5 | 11 | 2.5e5 | 1.3e5 | 3.3e5 | 1.9e5 |
| 1e−8 | 11 | 2.6e5 | 1.5e5 | 3.4e5 | 1.9e5 |
| **$f_8$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 0 | *19e+0* | *18e+0* | *79e+0* | 2.8e5 |
| 1 | . | . | . | . | . |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{10}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 8 | 4.9e5 | 3.1e5 | 5.5e5 | 3.3e5 |
| 1 | 7 | 6.3e5 | 5.6e5 | 7.6e5 | 3.7e5 |
| 1e−1 | 7 | 6.5e5 | 5.8e5 | 7.7e5 | 3.8e5 |
| 1e−3 | 5 | 1.1e6 | 9.0e5 | 1.2e6 | 4.1e5 |
| 1e−5 | 5 | 1.1e6 | 9.2e5 | 1.3e6 | 4.2e5 |
| 1e−8 | 3 | 2.1e6 | 1.8e6 | 2.2e6 | 3.6e5 |
| **$f_{10}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 0 | *38e+2* | *19e+2* | *10e+3* | 2.8e5 |
| 1 | . | . | . | . | . |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{12}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 11 | 2.2e5 | 1.4e5 | 3.5e5 | 1.6e5 |
| 1 | 8 | 5.0e5 | 3.5e5 | 6.1e5 | 2.3e5 |
| 1e−1 | 3 | 2.1e6 | 1.8e6 | 2.5e6 | 5.0e5 |
| 1e−3 | 3 | 2.1e6 | 1.7e6 | 2.4e6 | 5.0e5 |
| 1e−5 | 0 | *73e−2* | *32e−5* | *19e+3* | 1.3e5 |
| 1e−8 | . | . | . | . | . |
| **$f_{12}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 0 | *55e+1* | *70e+0* | *58e+2* | 1.1e6 |
| 1 | . | . | . | . | . |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{14}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 1.8e1 | 1.3e1 | 2.5e1 | 1.8e1 |
| 1 | 15 | 4.0e2 | 2.7e2 | 5.5e2 | 4.0e2 |
| 1e−1 | 15 | 2.9e3 | 2.3e3 | 3.7e3 | 2.9e3 |
| 1e−3 | 15 | 2.6e4 | 2.4e4 | 2.9e4 | 2.6e4 |
| 1e−5 | 14 | 9.9e4 | 6.3e4 | 1.0e5 | 9.3e4 |
| 1e−8 | 7 | 6.5e5 | 4.7e5 | 7.8e5 | 2.0e5 |
| **$f_{14}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 15 | 1.2e3 | 9.7e2 | 1.5e3 | 1.2e3 |
| 1 | 14 | 1.6e5 | 1.7e4 | 4.4e5 | 1.6e5 |
| 1e−1 | 14 | 2.2e5 | 7.9e4 | 3.7e5 | 2.2e5 |
| 1e−3 | 0 | *42e−4* | *22e−4* | *17e−3* | 4.5e5 |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{16}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 2.9e2 | 2.3e2 | 3.4e2 | 2.9e2 |
| 1 | 15 | 1.4e4 | 1.2e4 | 1.6e4 | 1.4e4 |
| 1e−1 | 8 | 4.8e5 | 3.6e5 | 6.5e5 | 3.3e5 |
| 1e−3 | 1 | 7.1e6 | 6.6e6 | 7.5e6 | 5.0e5 |
| 1e−5 | 0 | *66e−3* | *17e−4* | *53e−2* | 1.0e5 |
| 1e−8 | . | . | . | . | . |
| **$f_{16}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 15 | 1.0e5 | 6.9e4 | 1.3e5 | 1.0e5 |
| 1 | 0 | *46e−1* | *20e−1* | *85e−1* | 4.5e5 |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{18}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 2.2e2 | 1.8e2 | 2.7e2 | 2.2e2 |
| 1 | 15 | 1.6e4 | 9.0e3 | 2.0e4 | 1.6e4 |
| 1e−1 | 7 | 6.1e5 | 4.8e5 | 7.5e5 | 3.1e5 |
| 1e−3 | 0 | *12e−2* | *37e−4* | *44e−2* | 1.1e5 |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{18}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 15 | 1.9e4 | 1.1e4 | 2.2e4 | 1.9e4 |
| 1 | 0 | *32e−1* | *18e−1* | *55e−1* | 3.2e5 |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{20}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 9.6e1 | 8.6e1 | 1.1e2 | 9.6e1 |
| 1 | 15 | 1.3e4 | 1.1e4 | 1.5e4 | 1.3e4 |
| 1e−1 | 7 | 6.1e5 | 4.8e5 | 7.5e5 | 2.3e5 |
| 1e−3 | 7 | 6.3e5 | 4.9e5 | 8.2e5 | 2.4e5 |
| 1e−5 | 6 | 8.1e5 | 6.6e5 | 1.0e6 | 2.7e5 |
| 1e−8 | 6 | 8.2e5 | 7.5e5 | 9.7e5 | 2.8e5 |
| **$f_{20}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 15 | 3.3e3 | 2.5e3 | 4.1e3 | 3.3e3 |
| 1 | 1 | 2.8e7 | 2.6e7 | 3.0e7 | 2.0e6 |
| 1e−1 | 0 | *15e−1* | *11e−1* | *17e−1* | 3.2e5 |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{22}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 2.4e2 | 2.0e2 | 2.6e2 | 2.4e2 |
| 1 | 11 | 1.8e5 | 9.2e4 | 2.7e5 | 9.2e4 |
| 1e−1 | 9 | 3.4e5 | 1.8e5 | 4.5e5 | 1.7e5 |
| 1e−3 | 9 | 3.5e5 | 1.9e5 | 4.6e5 | 1.8e5 |
| 1e−5 | 9 | 3.6e5 | 2.1e5 | 4.7e5 | 1.9e5 |
| 1e−8 | 9 | 3.8e5 | 3.3e5 | 5.4e5 | 2.0e5 |
| **$f_{22}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 12 | 5.1e5 | 1.8e5 | 8.4e5 | 3.4e5 |
| 1 | 1 | 2.8e7 | 2.4e7 | 3.0e7 | 2.0e6 |
| 1e−1 | 0 | *78e−1* | *20e−1* | *30e+0* | 2.8e5 |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{24}$ in 5-D**, N=15, mFE=500000 | | | | | |
| 10 | 15 | 3.1e4 | 2.6e4 | 3.5e4 | 3.1e4 |
| 1 | 0 | *62e−1* | *25e−1* | *74e−1* | 8.9e4 |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |
| **$f_{24}$ in 20-D**, N=15, mFE=2000000 | | | | | |
| 10 | 0 | *65e+0* | *41e+0* | *75e+0* | 3.5e5 |
| 1 | . | . | . | . | . |
| 1e−1 | . | . | . | . | . |
| 1e−3 | . | . | . | . | . |
| 1e−5 | . | . | . | . | . |
| 1e−8 | . | . | . | . | . |

**Table 1: Shown are, for a given target difference to the optimal function value $\Delta f$: the number of successful trials (#); the expected running time to surpass $f_{\mathrm{opt}}+\Delta f$ (ERT, see Figure 2); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value ($\mathrm{RT_{succ}}$). If $f_{\mathrm{opt}}+\Delta f$ was never reached, figures in *italics* denote the best achieved $\Delta f$-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 2 for the names of functions.**

```
function PSO(FUN, DIM, ftarget, maxfunevals)

% Set algorithm parameters
popsize = 40;
c1 = 2;
c2 = 2;
xbound = 5;
vbound = 5;

% Allocate memory and initialize
xmin = -xbound * ones(1,DIM);
xmax = xbound * ones(1,DIM);
vmin = -vbound * ones(1,DIM);
vmax = vbound * ones(1,DIM);

x = 2 * xbound * rand(popsize,DIM) - xbound;
v = 2 * vbound * rand(popsize,DIM) - vbound;
pbest = x;

% update pbest and gbest
cost_p = feval(FUN, pbest');
[cost,index] = min(cost_p);
gbest = pbest(index,:);

%maxfunevals = min(1e5 * DIM, maxfunevals);
maxfunevals = min(10000, maxfunevals);
maxiterations = ceil(maxfunevals/popsize);

for iter = 2 : maxiterations
    % Update inertia weight
    w = 0.9 - 0.8*(iter-2)/(maxiterations-2);

    % Update velocity
    v = w*v + c1*rand*(pbest-x) + c2*rand*(repmat(gbest,popsize,1)-x);

    % Clamp veloctiy
    s = v < repmat(vmin,popsize,1);
    v = (1-s).*v + s.*repmat(vmin,popsize,1);
    b = v > repmat(vmax,popsize,1);
    v = (1-b).*v + b.*repmat(vmax,popsize,1);

    % Update position
    x = x + v;

    % Clamp position - Absorbing boundaries
    % Set x to the boundary
    s = x < repmat(xmin,popsize,1);
    x = (1-s).*x + s.*repmat(xmin,popsize,1);
    b = x > repmat(xmax,popsize,1);
    x = (1-b).*x + b.*repmat(xmax,popsize,1);

    % Clamp position - Absorbing boundaries
```



```
    % Set v to zero
    b = s | b;
    v = (1-b).*v + b.*zeros(popsize,DIM);

    % Update pbest and gbest if necessary
    cost_x = feval(FUN, x');
    s = cost_x<cost_p;
    cost_p = (1-s).*cost_p + s.*cost_x;
    s = repmat(s',1,DIM);
    pbest = (1-s).*pbest + s.*x;
    [cost,index] = min(cost_p);
    gbest = pbest(index,:);

    % Exit if target is reached
    if feval(FUN, 'fbest') < ftarget
      break;
    end
end
```

**Figure 1: PSO MATLAB-code.**

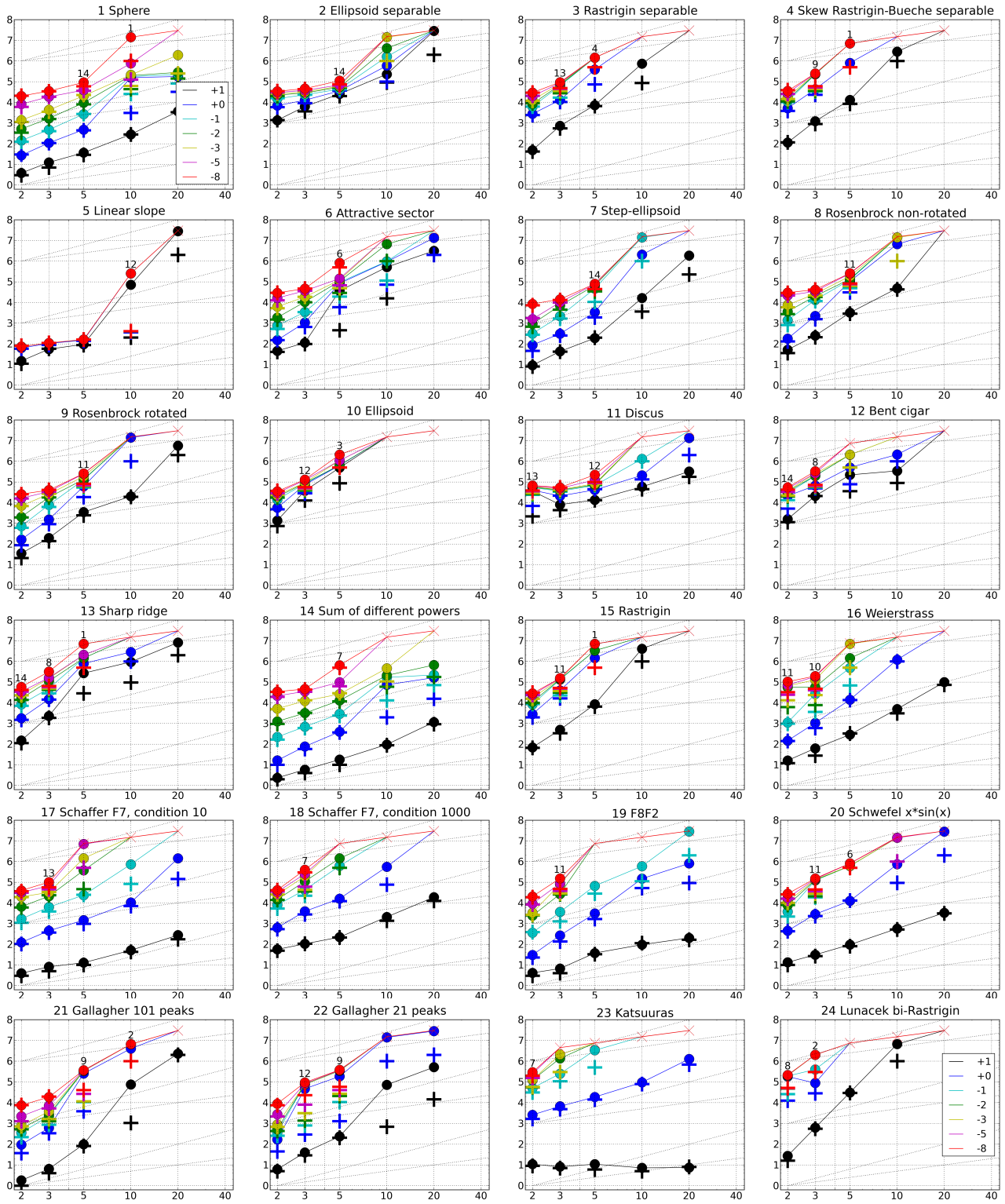


**Figure 2: Expected Running Time** (ERT, ●) **to reach** $f_{\mathrm{opt}} + \Delta f$ **and median number of function evaluations of successful trials** (+), **shown for** $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ **(the exponent is given in the legend of** $f_1$ **and** $f_{24}$**) versus dimension in log-log presentation. The** $\mathrm{ERT}(\Delta f)$ **equals to** $\#\mathrm{FEs}(\Delta f)$ **divided by the number of successful trials, where a trial is successful if** $f_{\mathrm{opt}} + \Delta f$ **was surpassed during the trial. The** $\#\mathrm{FEs}(\Delta f)$ **are the total number of function evaluations while** $f_{\mathrm{opt}} + \Delta f$ **was not surpassed during the trial from all respective trials (successful and unsuccessful), and** $f_{\mathrm{opt}}$ **denotes the optimal function value. Crosses** (×) **indicate the total number of function evaluations** $\#\mathrm{FEs}(-\infty)$**. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.**

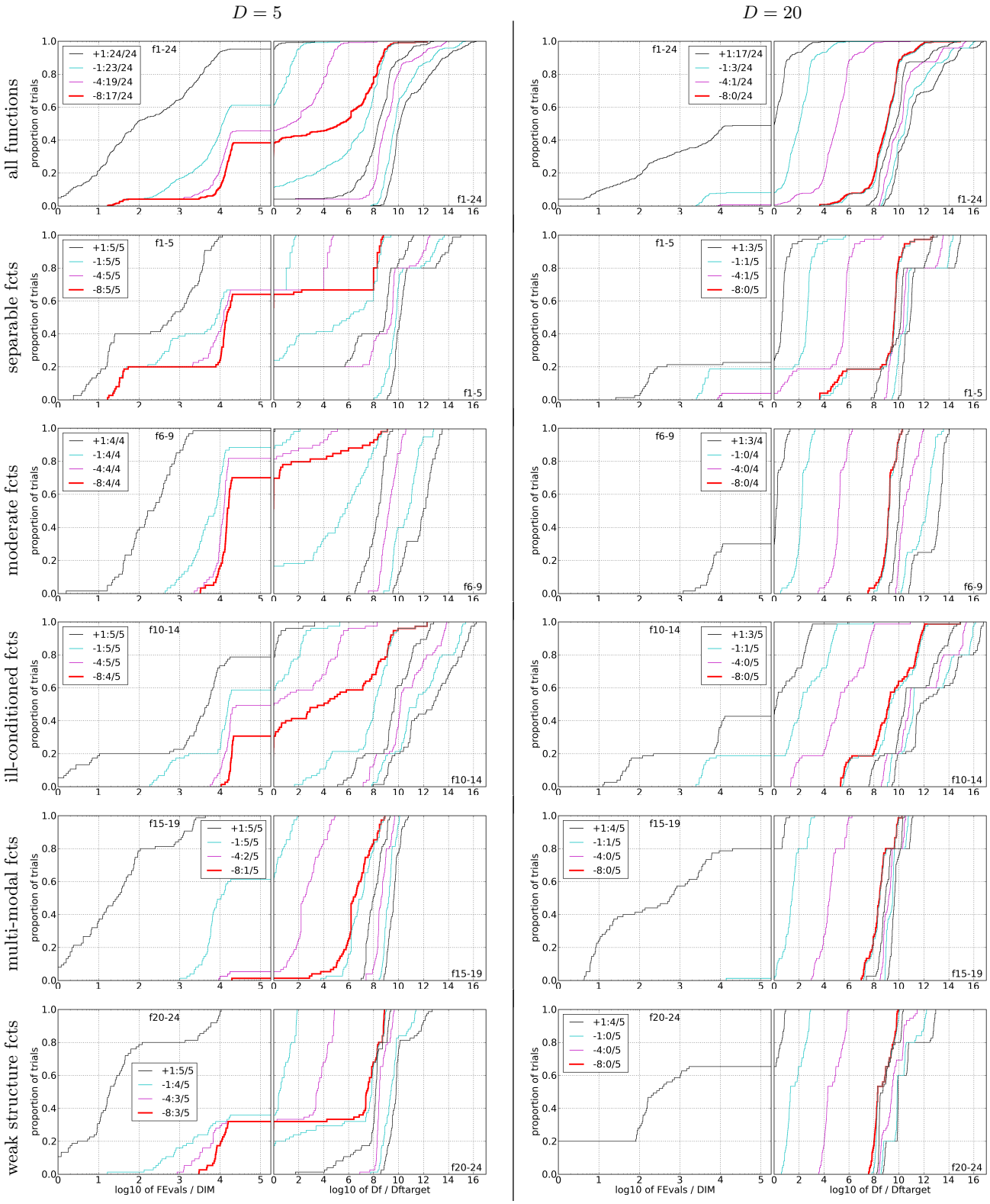


**Figure 3: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left) or $\Delta f$. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension $D$, to fall below $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where $k$ is the first value in the legend. Right subplots: ECDF of the best achieved $\Delta f$ divided by $10^k$ (upper left lines in continuation of the left subplot), and best achieved $\Delta f$ divided by $10^{-8}$ for running times of $D, 10\,D, 100\,D \ldots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: separable functions; third row: misc. moderate functions; fourth row: ill-conditioned functions; fifth row: multi-modal functions with adequate structure; last row: multi-modal functions with weak structure. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, $D$ and DIM denote search space dimension, and $\Delta f$ and Df denote the difference to the optimal function value.**