Black-Box Optimization Benchmarking for Noiseless Function Testbed using Harmony Search

Draft version *

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ABSTRACT

This paper benchmarks the harmony search (HS) algorithm using the noise-free BBOB 2010 testbed.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. HARMONY SEARCH

HS [6, 10] was introduced as an imitation of the musical process that searches for a harmony balance. In HS, each variable of the problem is regarded as the pitch of a different musical instrument and the complete solution is referred to as a harmony vector. If the pitch (decision variable value) makes good harmony (good objective function value), it is stored in memory.

The memory part is modeled using a memory structure referred to as the Harmony Memory (HM). Initially, HS is initialized with random harmonies. During the algorithm's progress, when a new harmony is found, it is inserted in HM replacing the worst harmony if it is better than it.

In HS, new harmonies are developed by producing a series of new pitches. Each new pitch is produced following one of three rules. Playing one pitch from memory, picking one pitch from memory and adjusting it to play a new pitch, playing a totally random pitch. These rules are applied using two parameters known as the Harmony Memory Considering Rate (HCMR) and the Pitch Adjusting Rate (PAR). Algorithm 1 shows the basic steps of the HS algorithm.

If the second rule is applied, a random pitch p_i is picked from memory and adjusted according to the following equation:

$$p_j = p_j + \phi \times b, \tag{1}$$

for $j \in \{1 \ldots d\}$ where d is the number of dimensions, ϕ is a random number uniformly distributed in the range [-1,1], and b is an arbitrary distance bandwidth.

If a totally random new pitch is to be generated, it is generated randomly in the allowable domain using the following equation:

$$p_j = lb_j + r \times (ub_j - lb_j), \tag{2}$$

where lb_j and ub_j are the lower and upper domains for decision variable j and r is a random number uniformly distributed in the range [0,1].

Algorithm 1 The HS algorithm

Require: Max_Function_Evaluations, memory size, $HCMR$, PAR, b
Initialize the harmony memory
Evaluate the solutions in the memory
Max_Iterations = Max_Function_Evaluations
Iter_number=1
while $iter\_number \leq Max\_Iterations$ **do**
  for each dimension d **do**
    if $U(0,1) \leq HCMR$ **then**
      Choose a randomly selected pitch from the memory
      if $U(0,1) \leq PAR$ **then**
        Adjust the selected pitch
      end if
    else
      Improvise a new pitch
    end if
  end for
  Evaluate the new harmony
  if the new harmony is better than the worst one in memory **then**
    Replace the worst harmony in memory
  end if
  Iter_number = Iter_number + 1
end while
return best harmony

Recent modifications for the HS algorithm include the proposed Improved HS (IHS) [7], in which the parameters PAR and b are not fixed during the algorithm's progress. PAR

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is increased linearly with the iterations while b is decreased in a logarithmic approach. IHS was shown to produce good results, when compared to other algorithms, for some constrained optimization applications including the minimization of the weight of a spring, pressure vessel design and welded beam design.

Another modification is the Global-best HS (GHS) proposed by [8]. The modification was proposed to imitate how the particles in PSO follow the global best in the swarm. The pitch adjustment step is updated so that the new harmony can mimic the best harmony in memory. The GHS was shown to outperform HS and IHS on almost all of the classical benchmark functions tested. GHS also remained as the best performer under the influence of noise or increasing dimensionality.

The Differential HS (DHS) was reported in [1]. In DHS, the pitch adjustment operation is replaced by a mutation strategy borrowed from DE. It was mathematically shown that DHS under certain circumstances can have more population variance over the generations when compared to the classical HS. When applied to a set of classical benchmark functions, DHS outperformed the classical HS, IHS and GHS in terms of the solution reached, the speed of convergence and the robustness.

1.1 Parameter Tuning

HS code was obtained by private communication with Z. W. Geem. The parameters were set as $HMS = 5, HCMR = 0.9$ and $PAR = 0.3$ following [8]. The value of b was reduced from 0.01 to 0.001 as it provided better results in earlier experiments conducted on the CEC05 benchmarks [9].

2. RESULTS

Results from experiments according to [3] on the benchmark functions given in [2, 5] are presented in Figures 1, 2 and 3 and in Tables 1 and 2.

3. CPU TIMING EXPERIMENT

For the timing experiment, HS was run on f8 and restarted until at least 30 seconds had passed (according to Figure 2 in [4]). The experiments have been conducted with an Intel Core 2 Quad 2.4 GHz under Windows Vista using the MATLAB-code provided. The results were 3.1×10^{-4} seconds per function evaluation in dimensions 2 up to 20. A dependency of CPU time on the search space dimensionality is not visible.

4. REFERENCES

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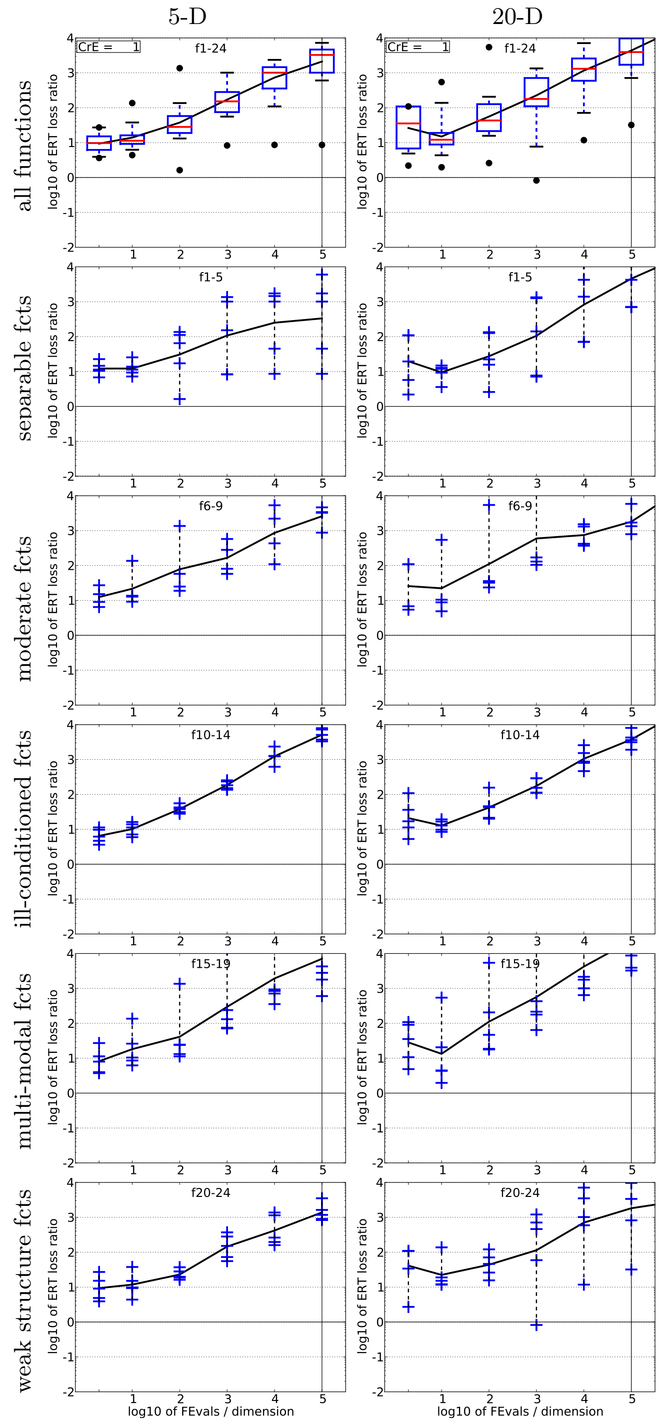

Figure 3: ERT loss ratio versus given budget FEvals. The target value f_t for ERT (see Figure 1) is the smallest (best) recorded function value such that $\mathrm{ERT}(f_t) \le \mathrm{FEvals}$ for the presented algorithm. Shown is FEvals divided by the respective best $\mathrm{ERT}(f_t)$ from BBOB-2009 for functions f_1–f_{24} in 5-D and 20-D. Each ERT is multiplied by $\exp(\mathrm{CrE})$ correcting for the parameter crafting effort. Line: geometric mean. Box-Whisker error bar: 25-75%-ile with median (box), 10-90%-ile (caps), and minimum and maximum ERT loss ratio (points). The vertical line gives the maximal number of function evaluations in this function subset.

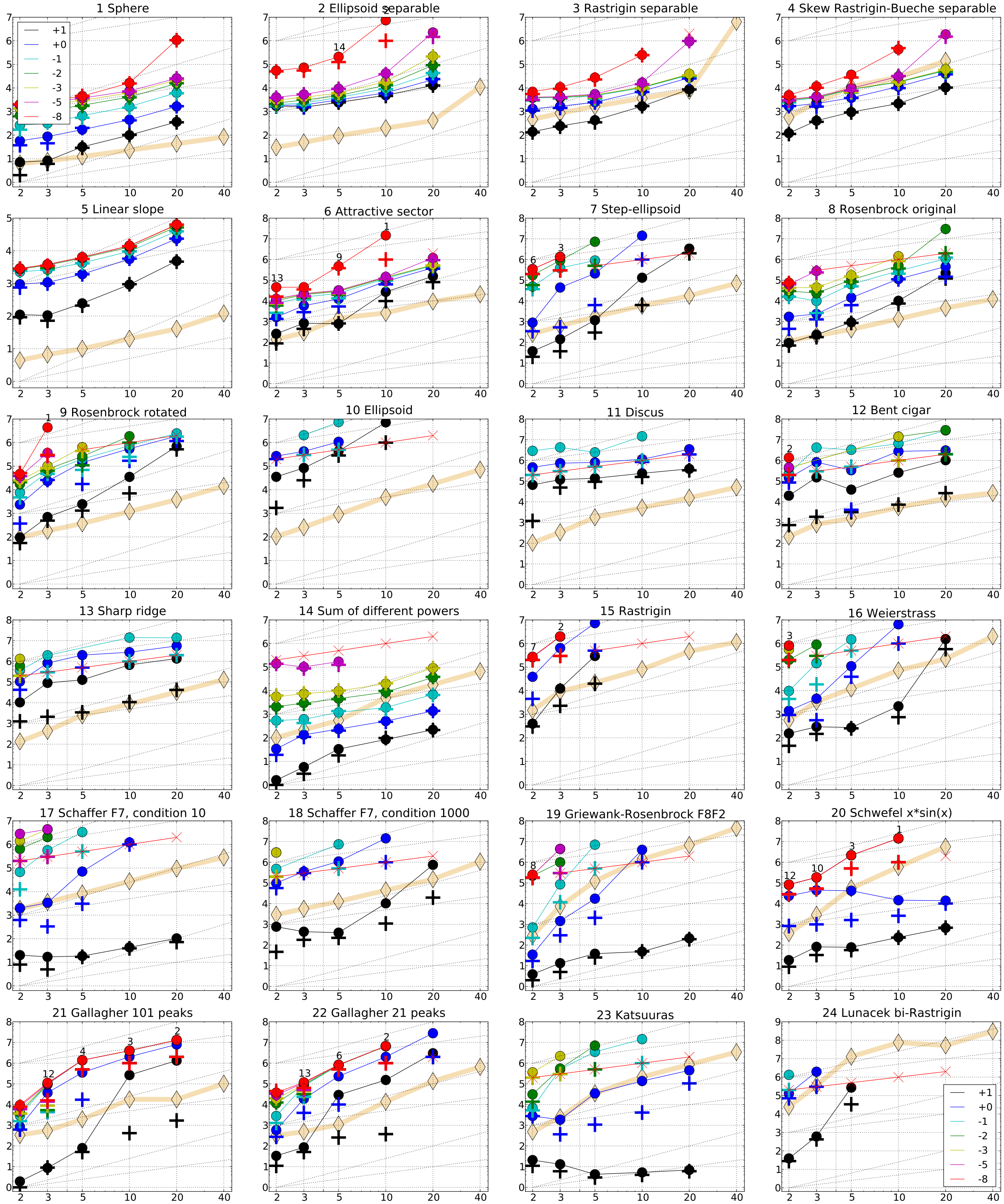

Figure 1: Expected Running Time (ERT, ●) to reach $f_{\rm opt} + \Delta f$ and median number of f-evaluations from successful trials (+), for $\Delta f = 10^{\{+1,0,-1,-2,-3,-5,-8\}}$ (the exponent is given in the legend of f_1 and f_{24}) versus dimension in log-log presentation. For each function and dimension, $\mathrm{ERT}(\Delta f)$ equals to $\#\mathrm{FEs}(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\rm opt} + \Delta f$ was surpassed. The $\#\mathrm{FEs}(\Delta f)$ are the total number (sum) of f-evaluations while $f_{\rm opt} + \Delta f$ was not surpassed in the trial, from all (successful and unsuccessful) trials, and $f_{\rm opt}$ is the optimal function value. Crosses (×) indicate the total number of f-evaluations, $\#\mathrm{FEs}(-\infty)$, divided by the number of trials. Numbers above ERT-symbols indicate the number of successful trials. Y-axis annotations are decimal logarithms. The thick light line with diamonds shows the single best results from BBOB-2009 for $\Delta f = 10^{-8}$. Additional grid lines show linear and quadratic scaling.

Δf	f_1 in 5-D, N=15, mFE=5813					f_1 in 20-D, N=15, mFE=1.39e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.2e1	1.0e1	6.4e1	3.2e1	15	3.6e2	1.9e2	4.8e2	3.6e2
1	15	1.7e2	5.8e1	3.0e2	1.7e2	15	1.6e3	1.3e3	2.0e3	1.6e3
1e−1	15	6.7e2	2.9e2	1.0e3	6.7e2	15	6.2e3	5.0e3	7.5e3	6.2e3
1e−3	15	3.0e3	1.5e3	4.0e3	3.0e3	15	2.2e4	2.0e4	2.5e4	2.2e4
1e−5	15	3.7e3	2.0e3	4.9e3	3.7e3	15	2.6e4	2.3e4	2.9e4	2.6e4
1e−8	15	4.5e3	3.1e3	5.5e3	4.5e3	15	1.1e6	7.8e5	1.3e6	1.1e6

Δf	f_2 in 5-D, N=15, mFE=500000					f_2 in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.4e3	1.5e3	3.9e3	2.4e3	15	1.3e4	1.1e4	1.6e4	1.3e4
1	15	3.1e3	2.2e3	4.6e3	3.1e3	15	2.3e4	1.6e4	2.7e4	2.3e4
1e−1	15	3.9e3	2.9e3	4.9e3	3.9e3	15	4.2e4	3.2e4	6.8e4	4.2e4
1e−3	15	6.1e3	4.3e3	8.3e3	6.1e3	15	2.2e5	1.4e5	3.3e5	2.2e5
1e−5	15	9.3e3	6.7e3	1.2e4	9.3e3	10	2.3e6	8.4e5	3.8e6	1.3e6
1e−8	14	2.1e5	5.2e4	4.8e5	1.7e5	0	*58e−7*	*35e−7*	*17e−6*	2.0e6

Δf	f_3 in 5-D, N=15, mFE=82140					f_3 in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	4.3e2	2.2e2	7.0e2	4.3e2	15	8.8e3	6.5e3	1.2e4	8.8e3
1	15	2.5e3	1.7e3	3.3e3	2.5e3	15	2.7e4	2.1e4	3.8e4	2.7e4
1e−1	15	4.6e3	2.8e3	5.7e3	4.6e3	15	3.6e4	2.3e4	6.3e4	3.6e4
1e−3	15	5.1e3	3.2e3	5.9e3	5.1e3	15	3.9e4	2.7e4	7.3e4	3.9e4
1e−5	15	5.5e3	3.6e3	6.7e3	5.5e3	15	9.5e5	6.2e5	1.3e6	9.5e5
1e−8	15	2.8e4	1.4e4	6.2e4	2.8e4	0	*37e−7*	*20e−7*	*56e−7*	2.0e6

Δf	f_4 in 5-D, N=15, mFE=78670					f_4 in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	9.5e2	4.6e2	1.5e3	9.5e2	15	1.1e4	7.2e3	1.4e4	1.1e4
1	15	3.8e3	2.4e3	6.0e3	3.8e3	15	4.0e4	2.5e4	5.7e4	4.0e4
1e−1	15	8.6e3	2.9e3	1.7e4	8.6e3	15	5.4e4	2.8e4	8.0e4	5.4e4
1e−3	15	9.0e3	3.2e3	1.7e4	9.0e3	15	6.1e4	3.3e4	8.4e4	6.1e4
1e−5	15	9.7e3	3.9e3	1.9e4	9.7e3	12	1.9e6	1.0e6	3.6e6	1.4e6
1e−8	15	3.6e4	1.6e4	7.8e4	3.6e4	0	*63e−7*	*31e−7*	*11e−6*	2.0e6

Δf	f_5 in 5-D, N=15, mFE=11740					f_5 in 20-D, N=15, mFE=83550				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.4e2	1.3e2	4.0e2	2.4e2	15	4.9e3	3.4e3	6.4e3	4.9e3
1	15	1.9e3	9.4e2	2.5e3	1.9e3	15	2.3e4	2.0e4	2.8e4	2.3e4
1e−1	15	4.2e3	2.9e3	6.0e3	4.2e3	15	4.0e4	3.2e4	4.8e4	4.0e4
1e−3	15	6.3e3	4.3e3	8.7e3	6.3e3	15	6.1e4	5.1e4	8.1e4	6.1e4
1e−5	15	6.4e3	4.4e3	8.8e3	6.4e3	15	6.3e4	5.2e4	7.6e4	6.3e4
1e−8	15	6.4e3	4.4e3	8.8e3	6.4e3	15	6.3e4	5.2e4	7.6e4	6.3e4

Δf	f_6 in 5-D, N=15, mFE=500000					f_6 in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	7.9e2	4.5e2	1.4e3	7.9e2	15	1.6e5	1.2e4	3.4e5	1.6e5
1	15	1.2e4	2.3e3	4.0e4	1.2e4	15	4.0e5	1.5e5	6.5e5	4.0e5
1e−1	15	2.3e4	7.7e3	6.0e4	2.3e4	15	5.1e5	2.2e5	7.7e5	5.1e5
1e−3	15	3.1e4	1.0e4	5.9e4	3.1e4	15	5.6e5	2.5e5	8.3e5	5.6e5
1e−5	15	3.3e4	1.3e4	7.3e4	3.3e4	13	1.2e6	5.2e5	2.9e6	8.9e5
1e−8	9	4.7e5	6.2e4	1.4e6	1.4e5	0	*40e−7*	*31e−7*	*11e−6*	2.0e6

Δf	f_7 in 5-D, N=15, mFE=500000					f_7 in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.2e3	2.1e1	3.2e3	1.2e3	6	3.4e6	9.8e4	1.0e7	3.7e5
1	11	2.1e5	5.7e2	5.3e5	3.0e4	0	*12e+0*	*57e−1*	*17e+0*	6.2e5
1e−1	6	8.9e5	9.1e4	2.2e6	1.4e5	.	.	.	.	.
1e−3	0	*17e−2*	*13e−3*	*45e−1*	1.0e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_8 in 5-D, N=15, mFE=500000					f_8 in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	9.3e2	4.9e2	1.7e3	9.3e2	15	2.2e5	3.6e4	5.3e5	2.2e5
1	15	1.4e4	2.3e3	3.3e4	1.4e4	15	4.6e5	7.4e4	1.1e6	4.6e5
1e−1	15	6.3e4	3.4e3	1.6e5	6.3e4	14	1.1e6	7.2e4	1.8e6	9.6e5
1e−3	15	1.8e5	9.3e4	2.8e5	1.8e5	0	*31e−3*	*13e−3*	*96e−3*	2.0e6
1e−5	0	*47e−6*	*34e−6*	*12e−5*	5.0e5	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_9 in 5-D, N=15, mFE=500000					f_9 in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.4e3	4.8e2	4.1e3	2.4e3	14	6.9e5	4.4e5	1.7e6	5.5e5
1	12	1.4e5	5.2e3	5.1e5	1.7e4	11	1.9e6	1.0e6	3.2e6	1.1e6
1e−1	12	2.0e5	3.1e3	6.0e5	7.6e4	11	2.5e6	1.6e6	3.8e6	1.7e6
1e−3	9	6.4e5	1.7e5	1.6e6	3.1e5	0	*80e−3*	*70e−3*	*41e−1*	2.0e6
1e−5	0	*87e−5*	*42e−6*	*39e−1*	5.0e5	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{10} in 5-D, N=15, mFE=500000					f_{10} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	11	4.4e5	1.9e5	8.8e5	2.5e5	0	*31e+1*	*70e+0*	*57e+1*	2.0e6
1	6	1.1e6	3.3e5	2.4e6	3.3e5	.	.	.	.	.
1e−1	1	7.5e6	9.7e5	1.7e7	4.7e5	.	.	.	.	.
1e−3	0	*20e−1*	*16e−2*	*29e+0*	5.0e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{11} in 5-D, N=15, mFE=500000					f_{11} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	14	1.4e5	1.1e4	2.1e5	1.0e5	15	3.9e5	8.6e4	6.4e5	3.9e5
1	7	8.0e5	1.1e5	1.8e6	2.3e5	8	3.5e6	1.6e6	5.8e6	1.7e6
1e−1	3	2.5e6	4.7e5	7.0e6	4.6e5	0	*95e−2*	*64e−2*	*29e−1*	2.0e6
1e−3	0	*10e−1*	*76e−3*	*48e−1*	5.0e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{12} in 5-D, N=15, mFE=500000					f_{12} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	14	3.9e4	2.1e3	4.9e3	3.2e3	10	1.0e6	2.4e4	4.0e6	2.6e4
1	9	3.4e5	2.2e3	1.0e6	3.2e3	6	3.0e6	3.1e4	6.0e6	3.6e4
1e−1	2	3.3e6	2.5e5	7.5e6	3.3e3	1	2.8e7	2.1e6	7.4e7	1.4e5
1e−3	0	*65e−2*	*97e−3*	*98e−1*	5.0e5	0	*14e−1*	*24e−2*	*31e+0*	2.0e6
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{13} in 5-D, N=15, mFE=500000					f_{13} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	12	1.3e5	2.1e3	5.0e5	3.5e3	9	1.4e6	3.0e4	4.0e6	3.6e4
1	3	2.0e6	7.1e3	4.0e6	5.6e3	4	5.5e6	5.4e4	1.8e7	4.9e4
1e−1	0	*18e−1*	*34e−2*	*33e+0*	4.2e5	2	1.4e7	7.8e5	3.1e7	7.8e5
1e−3	.	.	.	.	.	0	*52e−1*	*92e−3*	*28e+0*	2.0e6
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{14} in 5-D, N=15, mFE=500000					f_{14} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.3e1	5.0e0	9.1e1	3.3e1	15	2.2e2	1.1e2	3.1e2	2.2e2
1	15	2.3e2	1.4e2	3.9e2	2.3e2	15	1.4e3	8.9e2	1.8e3	1.4e3
1e−1	15	1.2e3	2.9e2	1.9e3	1.2e3	15	6.9e3	4.5e3	9.8e3	6.9e3
1e−3	15	9.7e3	6.5e3	1.4e4	9.7e3	15	9.0e4	8.2e4	1.0e5	9.0e4
1e−5	15	1.7e5	3.7e4	3.0e5	1.7e5	0	*11e−5*	*10e−5*	*12e−5*	2.0e6
1e−8	0	*30e−7*	*14e−7*	*62e−7*	5.0e5	.	.	.	.	.

Δf	f_{15} in 5-D, N=15, mFE=500000					f_{15} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	10	3.0e5	8.1e2	1.0e6	4.6e4	0	*94e+0*	*60e+0*	*13e+1*	2.0e6
1	1	7.3e6	8.4e5	1.5e7	3.4e5	.	.	.	.	.
1e−1	0	*40e−1*	*20e−1*	*18e+0*	2.2e5	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{16} in 5-D, N=15, mFE=500000					f_{16} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.7e2	9.9e1	4.7e2	2.7e2	9	1.5e6	8.7e3	4.1e6	2.1e5
1	14	1.1e5	3.2e3	3.6e5	7.4e4	0	*83e−1*	*41e−1*	*12e+0*	1.9e6
1e−1	4	1.5e6	7.0e3	3.9e6	1.1e5	.	.	.	.	.
1e−3	0	*41e−2*	*60e−3*	*92e−2*	4.9e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{17} in 5-D, N=15, mFE=500000					f_{17} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.8e1	8.0e0	2.5e1	1.8e1	15	1.0e2	3.5e1	1.9e2	1.0e2
1	14	7.1e4	2.5e2	3.1e4	3.5e4	0	*29e−1*	*17e−1*	*46e−1*	2.0e6
1e−1	2	3.3e6	1.6e5	9.0e6	8.4e4	.	.	.	.	.
1e−3	0	*35e−2*	*72e−3*	*68e−2*	4.5e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{18} in 5-D, N=15, mFE=500000					f_{18} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	4.0e2	4.8e1	9.8e2	4.0e2	11	7.5e5	4.8e2	2.0e6	2.3e4
1	5	1.1e6	2.9e4	2.9e6	9.2e4	0	*81e−1*	*54e−1*	*18e+0*	2.0e6
1e−1	1	7.4e6	8.7e5	1.6e7	3.7e5	.	.	.	.	.
1e−3	0	*12e−1*	*12e−2*	*38e−1*	4.5e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{19} in 5-D, N=15, mFE=500000					f_{19} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.8e1	5.5e0	5.1e1	3.8e1	15	2.1e2	1.0e2	3.2e2	2.1e2
1	15	1.7e4	2.5e2	4.8e4	1.7e4	0	*19e−1*	*11e−1*	*38e−1*	1.9e6
1e−1	1	7.1e6	5.7e5	1.5e7	6.9e4	.	.	.	.	.
1e−3	0	*25e−2*	*17e−2*	*57e−2*	4.9e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{20} in 5-D, N=15, mFE=500000					f_{20} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	7.8e1	1.5e1	2.0e2	7.8e1	15	6.7e2	3.7e2	9.9e2	6.7e2
1	15	4.2e4	3.0e2	4.5e5	4.2e4	15	1.4e4	6.4e3	3.8e4	1.4e4
1e−1	3	2.2e6	2.0e5	4.9e6	1.8e5	0	*40e−2*	*30e−2*	*51e−2*	2.0e6
1e−3	3	2.2e6	1.8e3	4.4e6	1.8e5	.	.	.	.	.
1e−5	3	2.2e6	2.0e5	4.9e6	1.8e5	.	.	.	.	.
1e−8	3	2.2e6	2.1e5	5.3e6	1.9e5	.	.	.	.	.

Δf	f_{21} in 5-D, N=15, mFE=500000					f_{21} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	7.9e1	1.5e1	1.6e2	7.9e1	9	1.3e6	4.7e2	4.0e6	1.8e3
1	9	3.5e5	4.1e2	1.0e6	1.8e4	3	8.0e6	3.5e3	1.6e7	4.0e3
1e−1	4	1.4e6	1.4e3	3.0e6	5.8e3	2	1.3e7	1.0e4	3.5e7	7.9e3
1e−3	4	1.4e6	5.8e3	3.0e6	9.8e3	2	1.3e7	4.0e4	3.7e7	3.0e4
1e−5	4	1.4e6	9.6e3	4.0e6	1.5e4	2	1.3e7	9.2e4	2.6e7	7.2e4
1e−8	4	1.4e6	1.4e4	3.5e6	1.8e4	2	1.3e7	1.2e5	3.3e7	9.7e4

Δf	f_{22} in 5-D, N=15, mFE=500000					f_{22} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.9e4	7.2e1	2.2e5	2.9e4	6	3.0e6	4.2e2	8.0e6	7.8e2
1	12	2.3e5	8.9e2	5.1e5	1.1e5	1	2.8e7	2.0e6	5.5e7	2.6e3
1e−1	6	7.9e5	8.8e3	3.0e6	4.1e4	0	*11e+0*	*20e−1*	*22e+0*	1.6e6
1e−3	6	8.2e5	2.4e4	2.5e6	6.5e4	.	.	.	.	.
1e−5	6	8.3e5	3.8e4	2.1e6	8.1e4	.	.	.	.	.
1e−8	6	8.4e5	4.8e4	2.0e6	9.2e4	.	.	.	.	.

Δf	f_{23} in 5-D, N=15, mFE=500000					f_{23} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	4.3e0	1.0e0	7.0e0	4.3e0	15	6.9e0	1.0e0	9.0e0	6.9e0
1	15	3.5e4	3.8e2	2.2e5	3.5e4	13	4.5e5	2.7e4	2.1e6	1.4e5
1e−1	2	3.5e6	4.9e5	7.3e6	2.6e5	0	*77e−2*	*43e−2*	*11e−1*	1.8e6
1e−3	0	*21e−2*	*73e−3*	*76e−2*	4.8e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{24} in 5-D, N=15, mFE=500000					f_{24} in 20-D, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	10	2.7e5	6.9e2	1.0e6	2.0e4	0	*99e+0*	*69e+0*	*13e+1*	2.0e6
1	0	*97e−1*	*73e−1*	*15e+0*	4.9e5	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 1: Shown are, for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\mathrm{opt}}+\Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value ($\mathbf{RT_{succ}}$). If $f_{\mathrm{opt}}+\Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

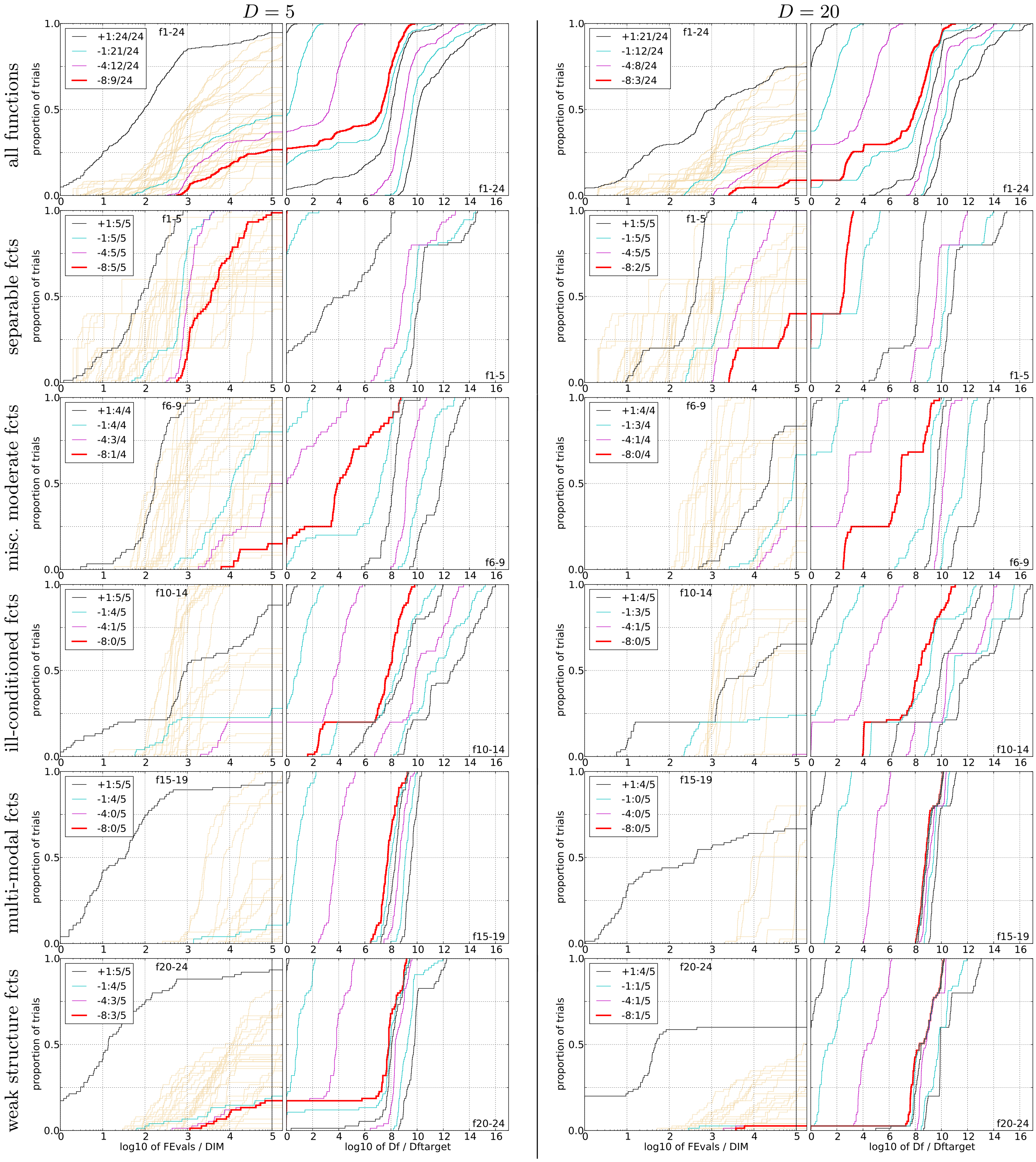

Figure 2: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10\,D, 100\,D \ldots$ function evaluations (from right to left cycling black-cyan-magenta). The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value. Light brown lines in the background show ECDFs for target value 10^{-8} of all algorithms benchmarked during BBOB-2009.

Table 2: ERT loss ratio (see Figure 3) compared to the respective best result from BBOB-2009 for budgets given in the first column. The last row $\mathbf{RL_{US}/D}$ gives the number of function evaluations in unsuccessful runs divided by dimension. Shown are the smallest, 10%-ile, 25%-ile, 50%-ile, 75%-ile and 90%-ile value (smaller values are better).

	f_1–f_{24} **in 5-D**, maxFE/D=100000					
#FEs/D	best	10%	25%	**med**	75%	90%
2	3.6	3.9	5.5	9.4	15	27
10	4.4	6.2	8.9	11	16	48
100	1.6	13	19	28	57	2.6e2
1e3	8.3	51	74	1.5e2	2.8e2	1.0e3
1e4	8.6	1.0e2	3.1e2	9.7e2	1.4e3	2.6e3
1e5	8.6	5.5e2	9.6e2	3.0e3	4.5e3	7.3e3
RL_{US}/D	1e5	1e5	1e5	1e5	1e5	1e5

	f_1–f_{24} **in 20-D**, maxFE/D=100000					
#FEs/D	best	10%	25%	**med**	75%	90%
2	2.2	4.7	6.3	35	1.1e2	1.1e2
10	2.0	4.3	8.6	12	18	1.8e2
100	2.6	16	21	39	1.2e2	7.3e2
1e3	0.82	7.6	1.1e2	1.7e2	5.9e2	6.6e3
1e4	12	71	5.3e2	1.2e3	2.4e3	7.7e3
1e5	32	7.1e2	1.5e3	3.8e3	9.2e3	2.8e4
1e6	50	4.0e3	8.0e3	1.8e4	3.4e4	9.4e4
RL_{US}/D	1e5	1e5	1e5	1e5	1e5	1e5

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