Black-Box Optimization Benchmarking the IPOP-CMA-ES on the Noiseless Testbed

Comparison to the BIPOP-CMA-ES

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ABSTRACT

We benchmark the Covariance Matrix Adaptation-Evolution Strategy (CMA-ES) algorithm with an Increasing POPulation size (IPOP) restart policy on the BBOB noiseless testbed. The IPOP-CMA-ES is compared to the BIPOP-CMA-ES and is shown to perform at best two times faster on multi-modal functions f_{15} to f_{19} whereas it does not solve weakly structured functions f_{22}, f_{23} and f_{24}.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Evolution strategy

1. ALGORITHM PRESENTATION

The algorithm Covariance Matrix Adaptation-Evolution Strategy (CMA-ES) [9] is a stochastic search method based on a population. We choose to apply the $(\mu/\mu_w, \lambda)$-CMA-ES [3, 7, 8] in this paper. The Increasing POPulation-size (IPOP) restart policy was proposed for the CMA-ES in [1]. The resulting IPOP-CMA-ES algorithm uses a population doubling in size at each restarts.

We compare the performances of the IPOP-CMA-ES to those of the BIPOP-CMA-ES [4] which was proposed to the BBOB 2009 workshop. The BIPOP-CMA-ES distributes the allocated budget —number of function evaluations— between a doubling population size and a small population size policy. The BIPOP-CMA-ES showed good performances on the function testbeds of the BBOB 2009 workshop [4].

The implementation of the IPOP-CMA-ES that we benchmark is the version 3.40beta of the Matlab code available at `http://www.lri.fr/~hansen/cmaesintro.html`. We use the parameter settings described in [4] for the BIPOP-CMA-ES. Therefore, the only difference between BIPOP-CMA-ES and IPOP-CMA-ES is that all the allocated budget is assigned to the doubling population size restart policy.

No additional parameter tuning has been done, the crafting effort [5] of IPOP-CMA-ES computes to CrE = 0, which was also the case for the BIPOP-CMA-ES.

2. CPU TIMING EXPERIMENT

The complete algorithms were run on f_8 for at least 30 seconds. Results for the IPOP-CMA-ES are 1.8; 1.5; 1.3; 1.1; 1.1; 1.5 and 3.4 $\times 10^{-4}$ seconds per function evaluation for dimension 2; 3; 5; 10; 20; 40 and 80. These figures were obtained on a Intel Core 2 6700 processor (2.66 GHz) with Linux 2.6.28-18 and Matlab R2008a.

3. RESULTS

The data for BIPOP-CMA-ES were obtained using the BBOB 2009 experimental set-up which differ from that of BBOB 2010 only in the number of test function instances considered (respectively 1 to 5 for BBOB 2009 and 1 to 15 for BBOB 2010) and the number of repetitions on each of these function instances (resp. 3 for BBOB 2009 and 1 for BBOB 2010).

Results from experiments according to [5] on the benchmark functions given in [2, 6] are presented in Figures 1, 2, 3 and 4 and in Tables 1 and 2. The **expected running time (ERT)**, used in the figures and tables, depends on a given target function value, $f_t = f_{opt} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t, summed over all trials and divided by the number of trials that actually reached f_t [5, 10]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t (10^{-8} in Figure 1) using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

Figure 3 shows that the proportion of functions solved by

BIPOP-CMA-ES is larger than IPOP-CMA-ES. The most prominent differences between the performances of the two algorithms are in the group of the multi-modal functions (f_{15} to f_{19}) and that of the weakly structured multi-modal functions (f_{20} to f_{24}).

The IPOP-CMA-ES is shown to perform faster on functions f_7, f_{13}, f_{15}, f_{16}, f_{17}, f_{18}, f_{19} by a factor of around two at most when the dimension of the search space is larger than 10. The IPOP-CMA-ES solves function f_{19} but is slower than the BIPOP-CMA-ES in dimension smaller than 5. The IPOP-CMA-ES does not solve functions f_{22} f_{23} and f_{24} when the dimension is larger than 10, whereas the BIPOP-CMA-ES does. The fact that BIPOP-CMA-ES can solve f_{23} and f_{24} can be attributed to the small population size management of BIPOP-CMA-ES. Finally, neither the IPOP-CMA-ES nor the BIPOP-CMA-ES solve functions f_3 when the dimension of the search space is larger than 10, f_4 when the dimension is larger than 3 and f_{20} when the dimension is larger than 40.

4. REFERENCES

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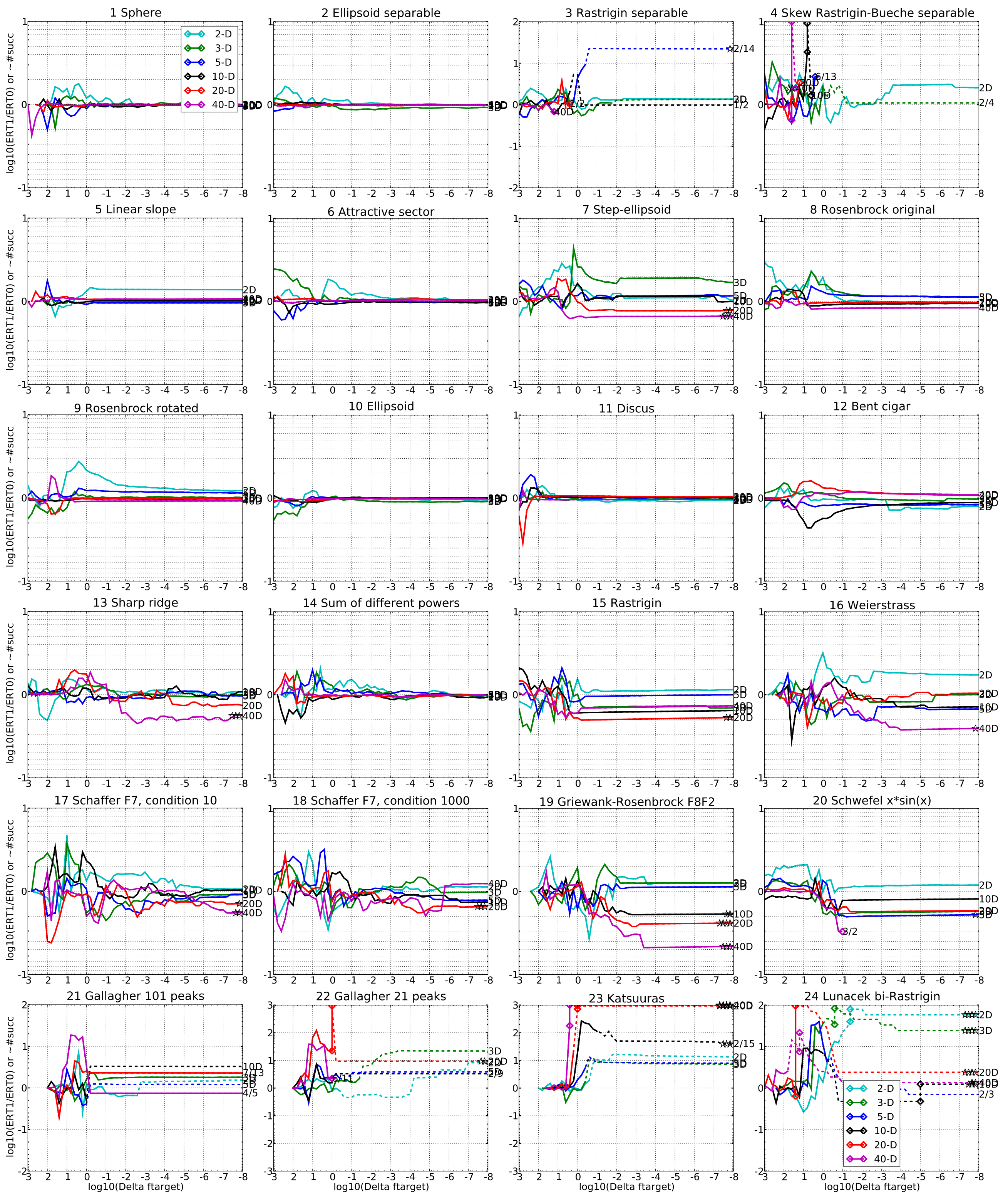

Figure 1: ERT ratio of IPOP-CMA divided by BIPOP-CMA versus $\log_{10}(\Delta f)$ for f_1–f_{24} in 2, 3, 5, 10, 20, 40-D. Ratios $< 10^0$ indicate an advantage of IPOP-CMA, smaller values are always better. The line gets dashed when for any algorithm the ERT exceeds thrice the median of the trial-wise overall number of f-evaluations for the same algorithm on this function. Symbols indicate the best achieved Δf-value of one algorithm (ERT gets undefined to the right). The dashed line continues as the fraction of successful trials of the other algorithm, where 0 means 0% and the y-axis limits mean 100%, values below zero for IPOP-CMA. The line ends when no algorithm reaches Δf anymore. The number of successful trials is given, only if it was in $\{1 \dots 9\}$ for IPOP-CMA (1st number) and non-zero for BIPOP-CMA (2nd number). Results are significant with $p = 0.05$ for one star and $p = 10^{-\#\star}$ otherwise, with Bonferroni correction within each figure.

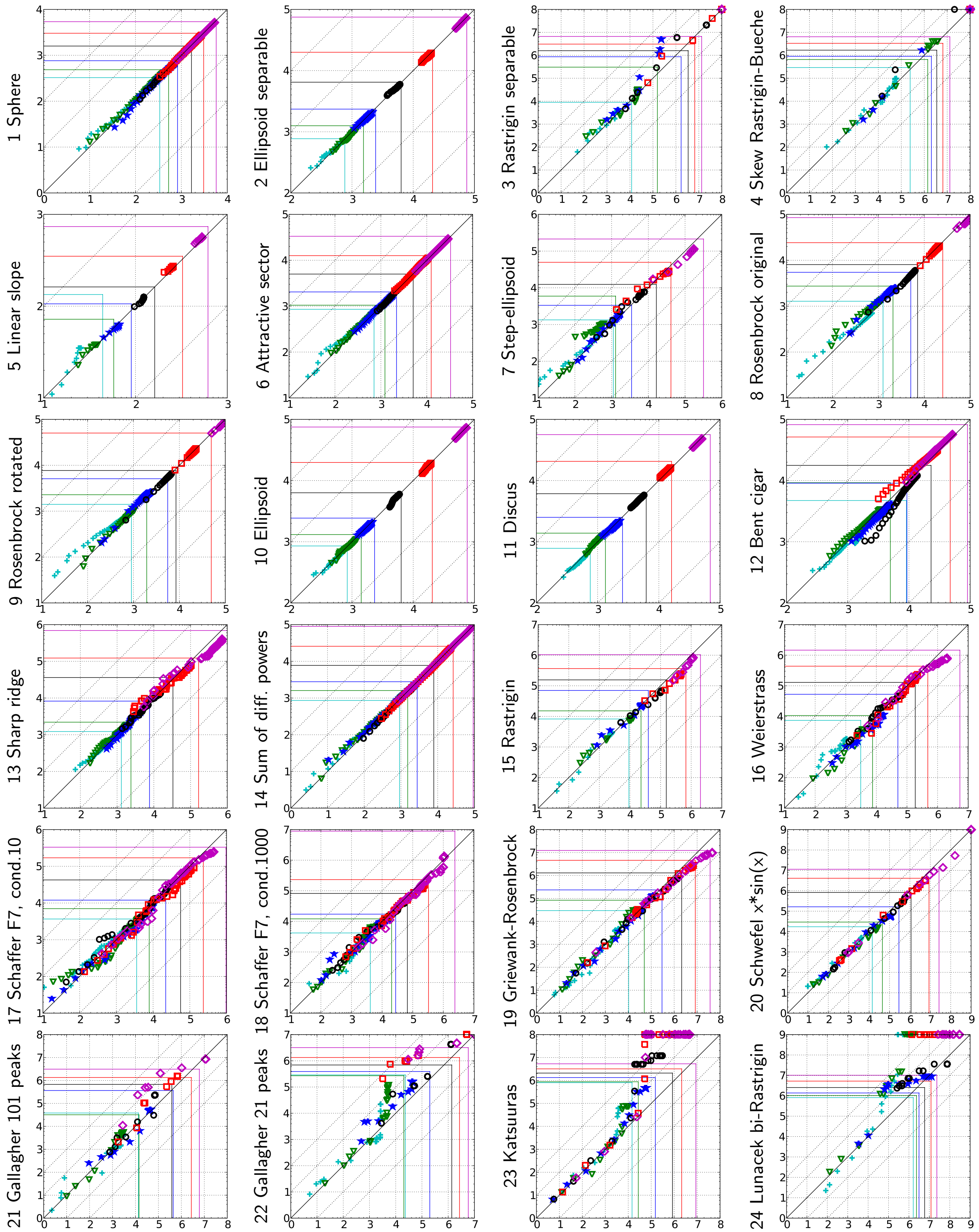

Figure 2: Expected running time (ERT in log10 of number of function evaluations) of IPOP-CMA versus BIPOP-CMA for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension for functions f_1–f_{24}. Markers on the upper or right egde indicate that the target value was never reached by IPOP-CMA or BIPOP-CMA respectively. Markers represent dimension: 2:+, 3:▽, 5:⋆, 10:○, 20:□, 40:◇.

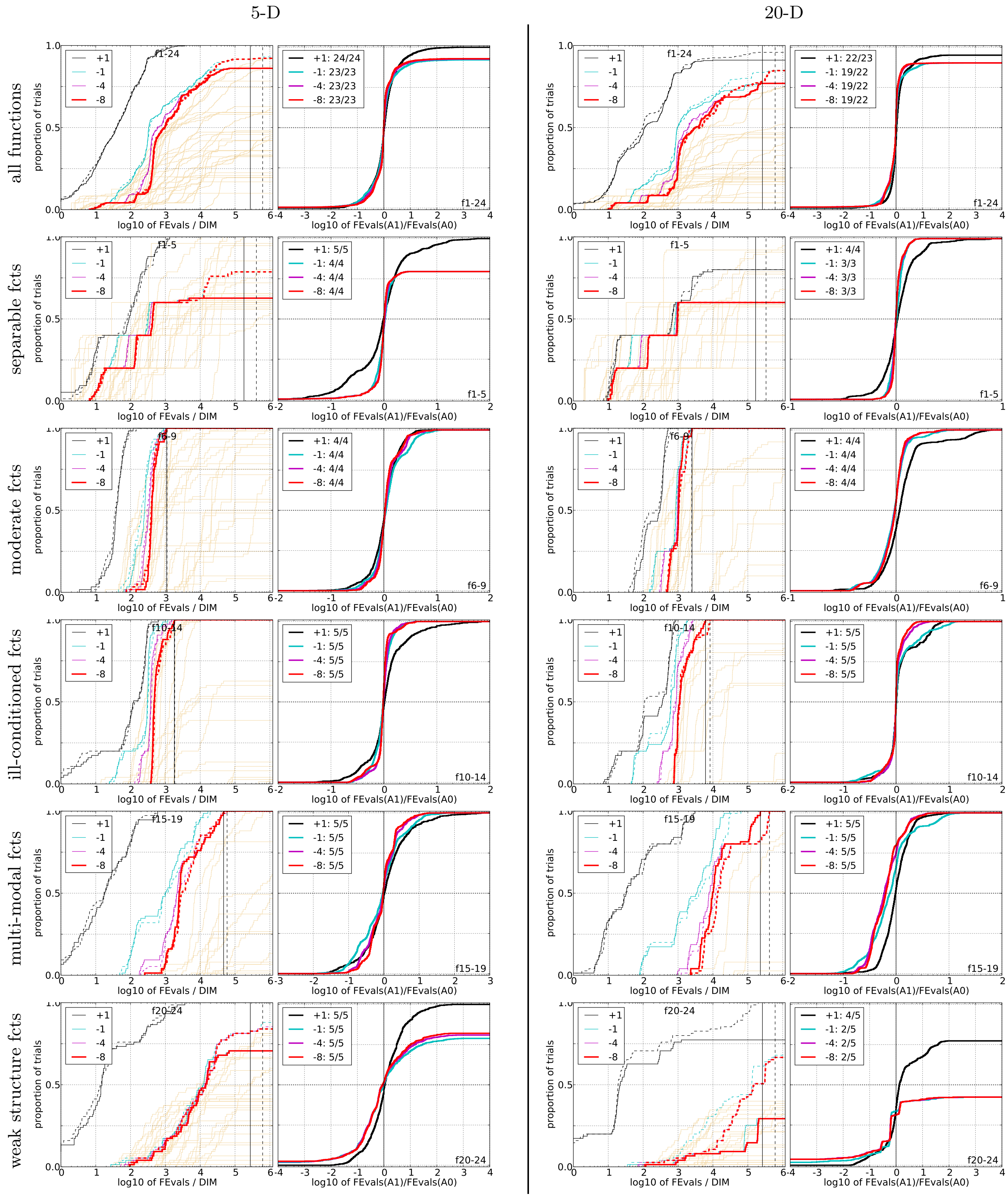

Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\rm opt} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for IPOP-CMA (solid) and BIPOP-CMA (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of IPOP-CMA divided by BIPOP-CMA, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (IPOP-CMA first).

5-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
$\mathbf{f_1}$	11	12	12	12	12	12	15/15
0: BIP	3.2	9.0	15	27	40	53	15/15
1: IPO	2.5	8.0	14	27	39	51	15/15
$\mathbf{f_2}$	83	87	88	90	92	94	15/15
0: BIP	13	16	18	20	21	22	15/15
1: IPO	14	16	18	19	21	22	15/15
$\mathbf{f_3}$	716	1622	1637	1646	1650	1654	15/15
0: BIP	1.4	16	139	139	139	140	14/15
1: IPO	2.2	70	3130	3113	3106	3099	2/15
$\mathbf{f_4}$	809	1633	1688	1817	1886	1903	15/15
0: BIP	2.7	∞	∞	∞	∞	∞ *1.8e6*	0/15
1: IPO	2.0	∞	∞	∞	∞	∞ *8.5e5*	0/15
$\mathbf{f_5}$	10	10	10	10	10	10	15/15
0: BIP	4.5	6.5	6.6	6.6	6.6	6.6	15/15
1: IPO	4.6	6.0	6.3	6.3	6.3	6.3	15/15
$\mathbf{f_6}$	114	214	281	580	1038	1332	15/15
0: BIP	2.3	2.1	2.2	1.7	1.3	1.3	15/15
1: IPO	2.5	2.1	2.2	1.7	1.3	1.2	15/15
$\mathbf{f_7}$	24	324	1171	1572	1572	1597	15/15
0: BIP	5.0	1.5	1	1	1	1	15/15
1: IPO	4.4	1.7	1.2	1.2	1.2	1.2	15/15
$\mathbf{f_8}$	73	273	336	391	410	422	15/15
0: BIP	3.2	3.7	4.5	4.8	5.1	5.4	15/15
1: IPO	3.5	4.8	5.3	5.6	5.8	6.1	15/15
$\mathbf{f_9}$	35	127	214	300	335	369	15/15
0: BIP	5.8	8.7	7.2	6.4	6.3	6.2	15/15
1: IPO	6.0	11	8.7	7.5	7.3	7.2	15/15
$\mathbf{f_{10}}$	349	500	574	626	829	880	15/15
0: BIP	3.5	2.9	2.7	2.8	2.3	2.4	15/15
1: IPO	3.6	2.9	2.7	2.8	2.3	2.3	15/15
$\mathbf{f_{11}}$	143	202	763	1177	1467	1673	15/15
0: BIP	8.4	7.2	2.2	1.6	1.4	1.3	15/15
1: IPO	8.6	7.3	2.1	1.6	1.4	1.3	15/15
$\mathbf{f_{12}}$	108	268	371	461	1303	1494	15/15
0: BIP	11	7.4	7.4	7.7	3.3	3.3	15/15
1: IPO	9.4	6.1	6.2	6.3	2.8	2.8	15/15
$\mathbf{f_{13}}$	132	195	250	1310	1752	2255	15/15
0: BIP	3.9	5.4	5.9	1.6	1.5	1.7	15/15
1: IPO	3.1	5.0	5.3	1.4	1.6	1.6	15/15
$\mathbf{f_{14}}$	10	41	58	139	251	476	15/15
0: BIP	1.1	2.8	3.7	4.6	5.4	4.5	15/15
1: IPO	2.2	2.9	3.8	4.7	5.4	4.4	15/15
$\mathbf{f_{15}}$	511	9310	19369	20073	20769	21359	14/15
0: BIP	1.6	1.5	1.2	1.2	1.2	1.2	15/15
1: IPO	2.3	1.3	1.2	1.2	1.2	1.2	15/15
$\mathbf{f_{16}}$	120	612	2662	10449	11644	12095	15/15
0: BIP	3.0	3.6	2.6	1.3	1.4	1.4	15/15
1: IPO	2.5	2.3	1.7	0.96	0.94	0.95	15/15
$\mathbf{f_{17}}$	5.2	215	899	3669	6351	7934	15/15
0: BIP	3.4	1	1	1	1	1.2	15/15
1: IPO	4.8	1.1	0.97	0.77	0.81	1.0	15/15
$\mathbf{f_{18}}$	103	378	3968	9280	10905	12469	15/15
0: BIP	1	3.4	1	1	1.2	1.3	15/15
1: IPO	1.2	2.7	0.87	1.0	1.0	0.99	15/15
$\mathbf{f_{19}}$	1	1	242	1.20e5	1.21e5	1.22e5	15/15
0: BIP	20	2801	161	1	1	1	15/15
1: IPO	21	1720	125	1.1	1.1	1.1	15/15
$\mathbf{f_{20}}$	16	851	38111	54470	54861	55313	14/15
0: BIP	3.3	8.2	2.8	2.1	2.2	2.2	15/15
1: IPO	3.9	11	1.4	1.1	1.1	1.1	15/15
$\mathbf{f_{21}}$	41	1157	1674	1705	1729	1757	14/15
0: BIP	2.3	14	24	25	25	25	15/15
1: IPO	6.3	5.6	30	31	31	31	14/15
$\mathbf{f_{22}}$	71	386	938	1008	1040	1068	14/15
0: BIP	6.9	20	45	42	41	40	15/15
1: IPO	12	48	166	161	158	155	11/15
$\mathbf{f_{23}}$	3.0	518	14249	31654	33030	34256	15/15
0: BIP	1.7	13	3.7	1.8	1.8	1.8	15/15
1: IPO	2.2	26	33	15	14	14	11/15
$\mathbf{f_{24}}$	1622	2.16e5	6.36e6	9.62e6	1.28e7	1.28e7	3/15
0: BIP	2.1	1.6	1	1	1	1	3/15
1: IPO	2.9	18	1.4	0.94	0.70	0.70	2/15

20-D

Δf	1e+1	1e+0	1e-1	1e-3	1e-5	1e-7	#succ
$\mathbf{f_1}$	43	43	43	43	43	43	15/15
0: BIP	7.9	14	20	33	45	57	15/15
1: IPO	8.0	14	20	33	46	58	15/15
$\mathbf{f_2}$	385	386	387	390	391	393	15/15
0: BIP	35	40	44	47	48	50	15/15
1: IPO	35	41	43	45	47	48	15/15
$\mathbf{f_3}$	5066	7626	7635	7643	7646	7651	15/15
0: BIP	12	∞	∞	∞	∞	∞ *5.7e6*	0/15
1: IPO	13	∞	∞	∞	∞	∞ *2.9e6*	0/15
$\mathbf{f_4}$	4722	7628	7666	7700	7758	1.41e5	9/15
0: BIP	∞	∞	∞	∞	∞	∞ *5.5e6*	0/15
1: IPO	∞	∞	∞	∞	∞	∞ *2.8e6*	0/15
$\mathbf{f_5}$	41	41	41	41	41	41	15/15
0: BIP	5.1	6.2	6.3	6.3	6.3	6.3	15/15
1: IPO	5.8	6.5	6.7	6.7	6.7	6.7	15/15
$\mathbf{f_6}$	1296	2343	3413	5220	6728	8409	15/15
0: BIP	1.5	1.3	1.2	1.1	1.2	1.2	15/15
1: IPO	1.7	1.3	1.2	1.2	1.2	1.2	15/15
$\mathbf{f_7}$	1351	4274	9503	16524	16524	16969	15/15
0: BIP	1	4.9	3.5	2.2	2.2	2.1	15/15
1: IPO	1.9	4.8	2.7	$\mathbf{1.7}^{\star}$	$\mathbf{1.7}^{\star}$	$\mathbf{1.6}^{\star}$	15/15
$\mathbf{f_8}$	2039	3871	4040	4219	4371	4484	15/15
0: BIP	4.0	4.0	4.3	4.5	4.6	4.6	15/15
1: IPO	3.7	3.9	4.2	4.4	4.4	4.5	15/15
$\mathbf{f_9}$	1716	3102	3277	3455	3594	3727	15/15
0: BIP	4.7	5.7	6.0	6.1	6.1	6.1	15/15
1: IPO	4.6	5.7	6.0	6.1	6.1	6.1	15/15
$\mathbf{f_{10}}$	7413	8661	10735	14920	17073	17476	15/15
0: BIP	1.9	1.8	1.6	1.2	1.1	1.1	15/15
1: IPO	1.8	1.8	1.5	1.2	1.1	1.1	15/15
$\mathbf{f_{11}}$	1002	2228	6278	9762	12285	14831	15/15
0: BIP	10	5.1	1.9	1.4	1.2	1.0	15/15
1: IPO	11	5.4	2.1	1.4	1.2	1.1	15/15
$\mathbf{f_{12}}$	1042	1938	2740	4140	12407	13827	15/15
0: BIP	3.0	4.0	4.5	4.5	1.9	2.0	15/15
1: IPO	4.8	5.3	5.5	5.1	2.1	2.2	15/15
$\mathbf{f_{13}}$	652	2021	2751	18749	24455	30201	15/15
0: BIP	4.3	2.7	5.1	1.5	2.3	3.0	15/15
1: IPO	6.5	4.8	6.2	1.4	1.7	2.3	15/15
$\mathbf{f_{14}}$	75	239	304	932	1648	15661	15/15
0: BIP	3.9	2.9	3.7	4.1	6.2	1.2	15/15
1: IPO	3.7	2.8	3.6	3.9	6.0	1.2	15/15
$\mathbf{f_{15}}$	30378	1.47e5	3.12e5	3.20e5	4.49e5	4.59e5	15/15
0: BIP	1	2.0	1.4	1.4	1	1	15/15
1: IPO	1.1	$\mathbf{1.1}^{\star}$	$\mathbf{0.69}^{\star}$	$\mathbf{0.70}^{\star}$	$\mathbf{0.52}^{\star\downarrow}$	$\mathbf{0.53}^{\star\downarrow}$	15/15
$\mathbf{f_{16}}$	1384	27265	77015	1.88e5	1.98e5	2.20e5	15/15
0: BIP	1.7	1.0	1.2	1	1	1	15/15
1: IPO	1.7	0.81	0.92	0.84	1.1	1.0	15/15
$\mathbf{f_{17}}$	63	1030	4005	30677	56288	80472	15/15
0: BIP	2.2	1	1	1.2	1.3	1.4	15/15
1: IPO	2.1	0.94	1.2	0.76	0.99	1.0	15/15
$\mathbf{f_{18}}$	621	3972	19561	67569	1.31e5	1.47e5	15/15
0: BIP	1.0	2.4	1.2	1.1	1.7	1.6	15/15
1: IPO	1.1	1.8	1.1	0.97	$\mathbf{1.0}^{\star}$	$\mathbf{1.1}^{\star 2}$	15/15
$\mathbf{f_{19}}$	1	1	3.43e5	6.22e6	6.69e6	6.74e6	15/15
0: BIP	169	23770	1.2	1	1	1	15/15
1: IPO	161	27333	0.71	$\mathbf{0.38}^{\star 3\downarrow 4}$	$\mathbf{0.41}^{\star 3\downarrow 3}$	$\mathbf{0.41}^{\star 3\downarrow 3}$	15/15
$\mathbf{f_{20}}$	82	46150	3.10e6	5.54e6	5.59e6	5.64e6	14/15
0: BIP	4.3	9.2	1	1	1	1	14/15
1: IPO	4.6	6.4	0.65	0.57	0.58	0.58	15/15
$\mathbf{f_{21}}$	561	6541	14103	14643	15567	17589	15/15
0: BIP	3.2	55	48	46	43	39	13/15
1: IPO	3.7	139	110	106	100	88	7/15
$\mathbf{f_{22}}$	467	5580	23491	24948	26847	1.35e5	12/15
0: BIP	6.8	13	$\mathbf{215}^{\star}$	$\mathbf{202}^{\star}$	$\mathbf{188}^{\star}$	$\mathbf{37}^{\star}$	5/15
1: IPO	445	287	∞	∞	∞	∞ *1.3e6*	0/15
$\mathbf{f_{23}}$	3.2	1614	67457	4.89e5	8.11e5	8.38e5	15/15
0: BIP	4.3	$\mathbf{32}^{\star 3}$	$\mathbf{1}^{\star 3}$	$\mathbf{2.0}^{\star 3}$	$\mathbf{1.2}^{\star 3}$	$\mathbf{1.2}^{\star 3}$	15/15
1: IPO	4.3	23082	∞	∞	∞	∞ *2.5e6*	0/15
$\mathbf{f_{24}}$	1.34e6	7.48e6	5.19e7	5.20e7	5.20e7	5.20e7	3/15
0: BIP	$\mathbf{1}^{\star 3}$	$\mathbf{1}^{\star 3}$	$\mathbf{1}^{\star 3}$	$\mathbf{1}^{\star 3}$	$\mathbf{1}^{\star 3}$	$\mathbf{1}^{\star 3}$	3/15
1: IPO	∞	∞	∞	∞	∞	∞ *5.1e6*	0/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\mathrm{opt}} + 10^{-8}$. 0: BIP is BIPOP-CMA and 1: IPO is IPOP-CMA. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

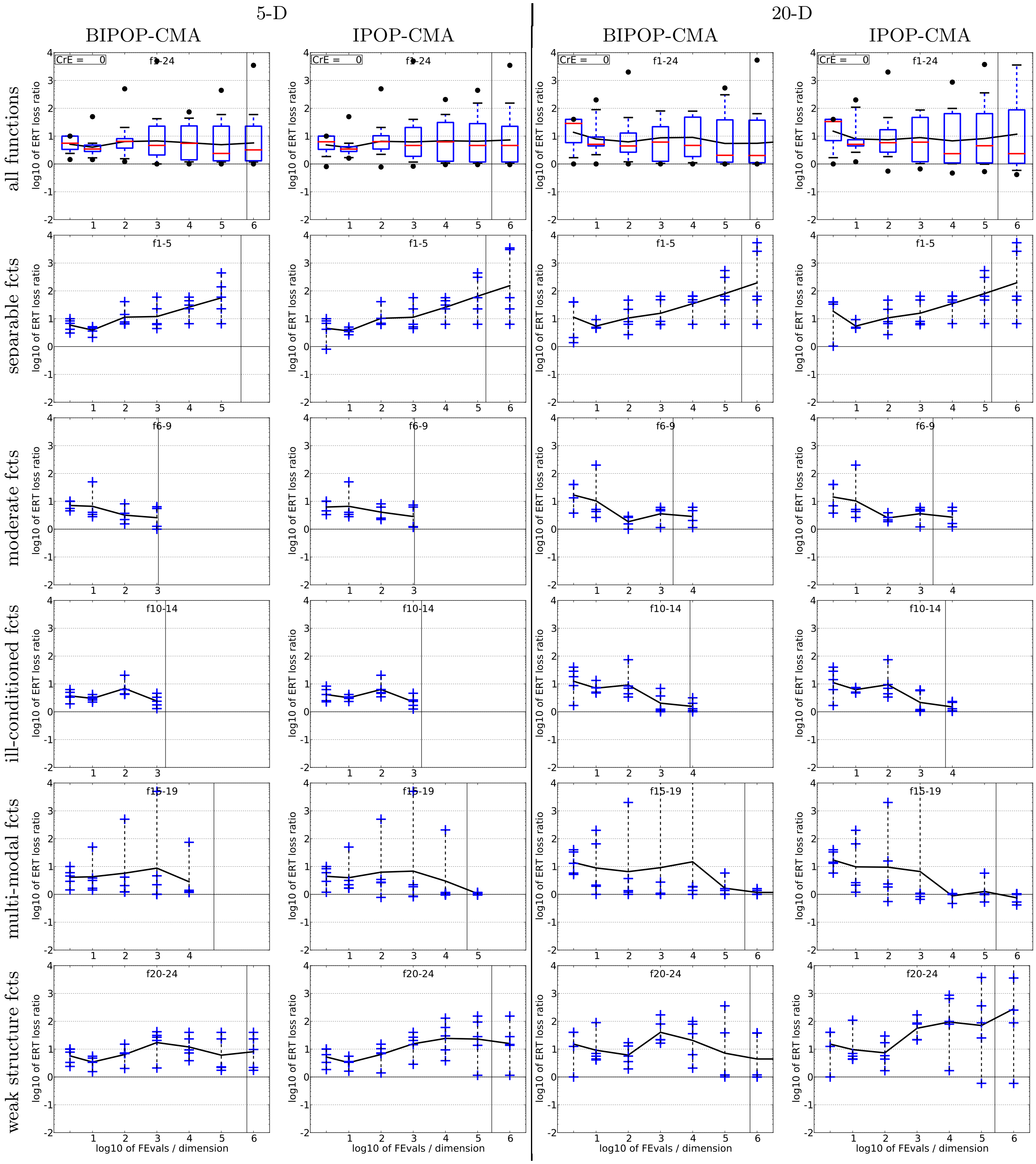

Figure 4: ERT loss ratio versus given budget FEvals. The target value f_t for ERT is the smallest (best) recorded function value such that $\mathrm{ERT}(f_t) \leq \mathrm{FEvals}$ for the presented algorithm. Shown is FEvals divided by the respective best $\mathrm{ERT}(f_t)$ from BBOB-2009 for functions f_1–f_{24} in 5-D and 20-D. Each ERT is multiplied by $\exp(\mathrm{CrE})$ correcting for the parameter crafting effort. Line: geometric mean. Box-Whisker error bar: 25-75%-ile with median (box), 10-90%-ile (caps), and minimum and maximum ERT loss ratio (points). The vertical line gives the maximal number of function evaluations in this function subset.

Table 2: ERT **loss ratio (see Figure 4) compared to the respective best result from BBOB-2009 for budgets given in the first column. The last row $\mathbf{RL_{US}/D}$ gives the number of function evaluations in unsuccessful runs divided by dimension. Shown are the smallest, 10%-ile, 25%-ile, 50%-ile, 75%-ile and 90%-ile value (smaller values are better). ERT Loss ratio is equal to zero if the algorithm considered outperformed all algorithms from BBOB-2009.**

BIPOP-CMA

	f_1–f_{24} **in 5-D**, maxFE/D=622854					
#FEs/D	best	10%	25%	**med**	75%	90%
2	1.4	2.3	3.3	5.3	9.2	10
10	1.4	1.6	2.7	3.4	4.6	10
100	1.2	1.5	3.0	6.4	7.9	23
1e3	1.0	1.0	1.9	4.6	22	44
1e4	1.0	1.2	1.4	5.1	23	46
1e5	1.0	1.2	1.3	2.3	15	68
1e6	1.0	1.2	1.3	2.8	16	68
RL_{US}/D	3e5	3e5	4e5	4e5	6e5	6e5

IPOP-CMA

	f_1–f_{24} **in 5-D**, maxFE/D=274645					
#FEs/D	best	10%	25%	**med**	75%	90%
2	0.80	1.8	3.1	6.1	9.2	10
10	1.6	1.7	2.7	3.4	4.1	10
100	0.78	2.1	3.1	6.3	9.2	23
1e3	0.83	1.1	1.8	4.5	17	42
1e4	0.94	1.1	1.2	5.4	30	67
1e5	0.94	1.1	1.2	3.6	25	1.7e2
1e6	0.94	1.1	1.2	3.6	19	4.4e2
RL_{US}/D	7e4	1e5	2e5	2e5	3e5	3e5

BIPOP-CMA

	f_1–f_{24} **in 20-D**, maxFE/D=605134					
#FEs/D	best	10%	25%	**med**	75%	90%
2	1.0	1.7	5.5	23	40	40
10	1.0	2.1	4.4	5.0	8.3	1.0e2
100	1.0	1.2	2.3	4.1	11	49
1e3	1.0	1.0	1.2	6.1	22	89
1e4	1.0	1.1	1.6	3.9	44	81
1e5	1.0	1.0	1.1	2.0	22	3.1e2
1e6	1.0	1.0	1.1	1.8	22	3.2e2
1e7	1.0	1.0	1.1	1.8	22	2.7e3
RL_{US}/D	1e5	1e5	3e5	3e5	3e5	5e5

IPOP-CMA

	f_1–f_{24} **in 20-D**, maxFE/D=261553					
#FEs/D	best	10%	25%	**med**	75%	90%
2	1.0	1.6	6.6	31	40	40
10	1.2	2.6	4.4	5.0	7.4	1.2e2
100	0.55	1.8	2.5	5.1	16	49
1e3	0.66	1.0	1.2	6.0	34	95
1e4	0.47	1.0	1.1	2.3	56	1.5e2
1e5	0.53	0.94	1.1	3.4	56	3.8e2
1e6	0.42	0.58	1.0	2.3	76	3.8e3
RL_{US}/D	6e4	6e4	7e4	1e5	1e5	3e5