Comparison of Cauchy EDA vs. G3PCX Algorithms Using BBOB 2010 Noiseless Testbed

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ABSTRACT

Generalized generation gap algorithm with parent centric crossover is compared with the estimation-of-distribution algorithm equipped with Cauchy distribution. Both algorithms were already presented at the BBOB 2009 workshop where they often showed similar performance. This paper compares them in more detail and adds to understanding of their key features and differences.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. INTRODUCTION

The generalized generation gap (G3) model was introduced by Deb in [1] and was used with the parent centric crossover operator (PCX) introduced in [2]. The performance of the G3PCX algorithm on the BBOB 2009 noiseless test suite was reported in [8]. The second algorithm in this comparison, Cauchy EDA, is an estimation of distribution algorithm with isotropic Cauchy distribution [7]. To fight the premature convergence, it uses a constant multiplier to enlarge the variance of the distribution (as suggested in [6]).

2. ALGORITHM PRESENTATION

The descriptions of the algorithms along with the paparemetr settings can be found in [8] and [7], respectively. For both algorithms, the crafting effort CrE= 0.

3. RESULTS

Results from experiments according to [4] on the benchmark functions given in [3, 5] are presented in Figures 1, 2 and 3 and in Table 1. The **expected running time (ERT)**, used in the figures and table, depends on a given target function value, $f_t = f_{opt} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t, summed over all trials and divided by the number of trials that actually reached f_t [4, 9]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t (10^{-8} in Figure 1) using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

G3PCX ouperforms Cauchy EDA on functions 1, 5, 6, 8, 9, 12, 16, 21, 22, 23, while Cauchy EDA beats G3PCX on functions 2, 7, 10, 13, 17, 18, i.e. in this small competition the G3PCX wins 10:6. (The results on the other functions are mixed, or niether algorithm solved the problem successfuly.)

In Fig. 1, we can often see a peek at the beginning of the ERT ratio lines (funtions 1, 5, 7, 11, 13, 14). The peak means that Cauchy EDA is much less efficient in the beginning of the search than the G3PCX algorithm. This is probably due to the fact that the probabilistic model used by Cauchy EDA needs some time to adapt to the fitness landscape, so that while G3PCX improves the best-so-far solution rather quickly right from the beginning, Cauchy EDA blunders. After finding the right model, the Cauchy EDA is sometimes able to close the gap and take the lead (e.g. both ellipsoid functions 2 and 10 and for discus function 11 for dimensions lower than 20, or for the sharp ridge function 13).

G3PCX failed on functions 7 (Step-ellipsoid) and on both Schaffer's functions 17 and 18. It seems that for these functions the global point of view represented by a unimodal Cauchy distribution is a better approach. Also for function 13 (sharp ridge problem), it seems that the global probabilistic model is better.

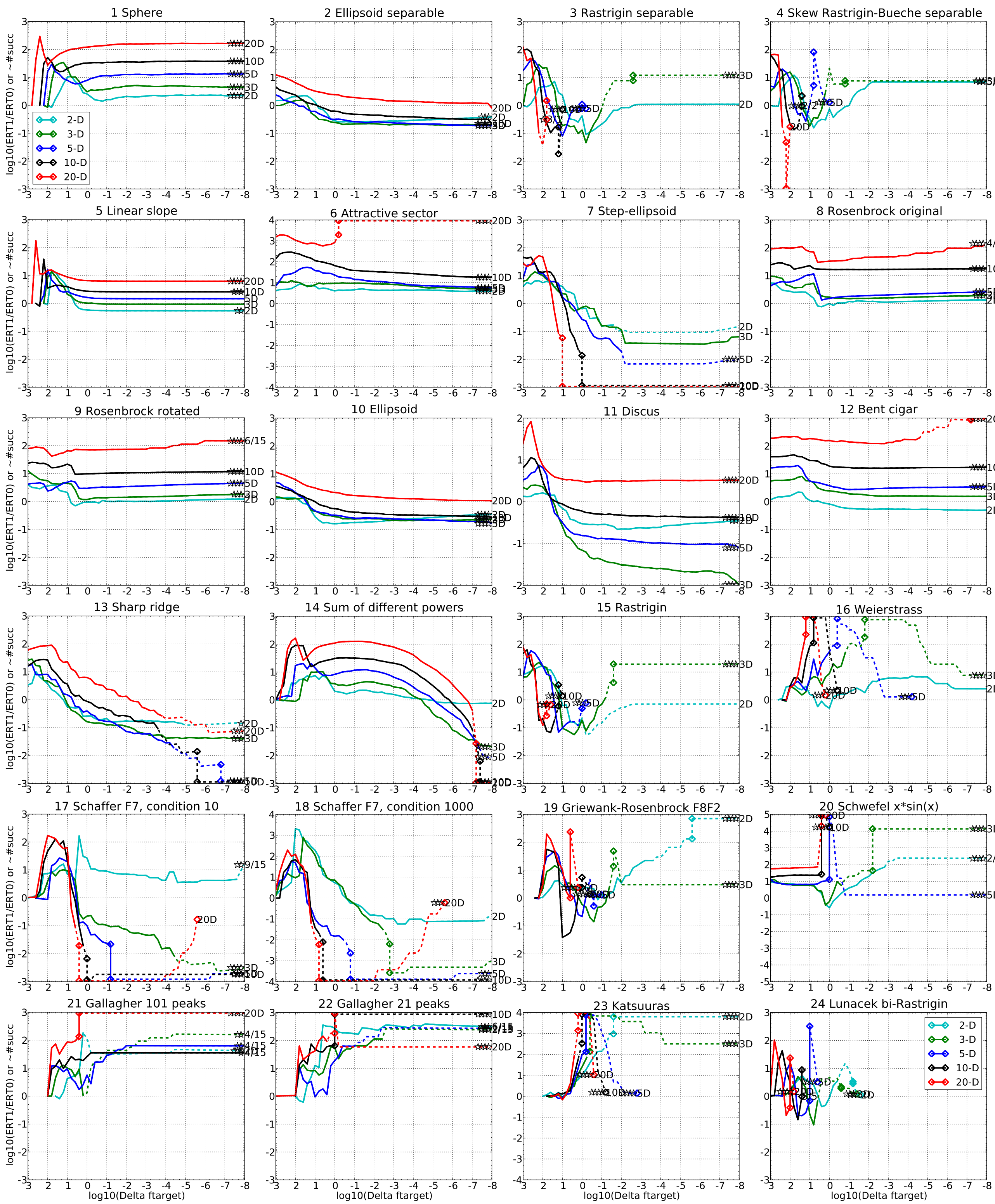

Figure 1: ERT ratio of CauchyEDA divided by G3PCX versus $\log_{10}(\Delta f)$ for f_1–f_{24} in 2, 3, 5, 10, 20, 40-D. Ratios $< 10^0$ indicate an advantage of CauchyEDA, smaller values are always better. The line gets dashed when for any algorithm the ERT exceeds thrice the median of the trial-wise overall number of f-evaluations for the same algorithm on this function. Symbols indicate the best achieved Δf-value of one algorithm (ERT gets undefined to the right). The dashed line continues as the fraction of successful trials of the other algorithm, where 0 means 0% and the y-axis limits mean 100%, values below zero for CauchyEDA. The line ends when no algorithm reaches Δf anymore. The number of successful trials is given, only if it was in $\{1 \ldots 9\}$ for CauchyEDA (1st number) and non-zero for G3PCX (2nd number). Results are significant with $p = 0.05$ for one star and $p = 10^{-\#\star}$ otherwise, with Bonferroni correction within each figure.

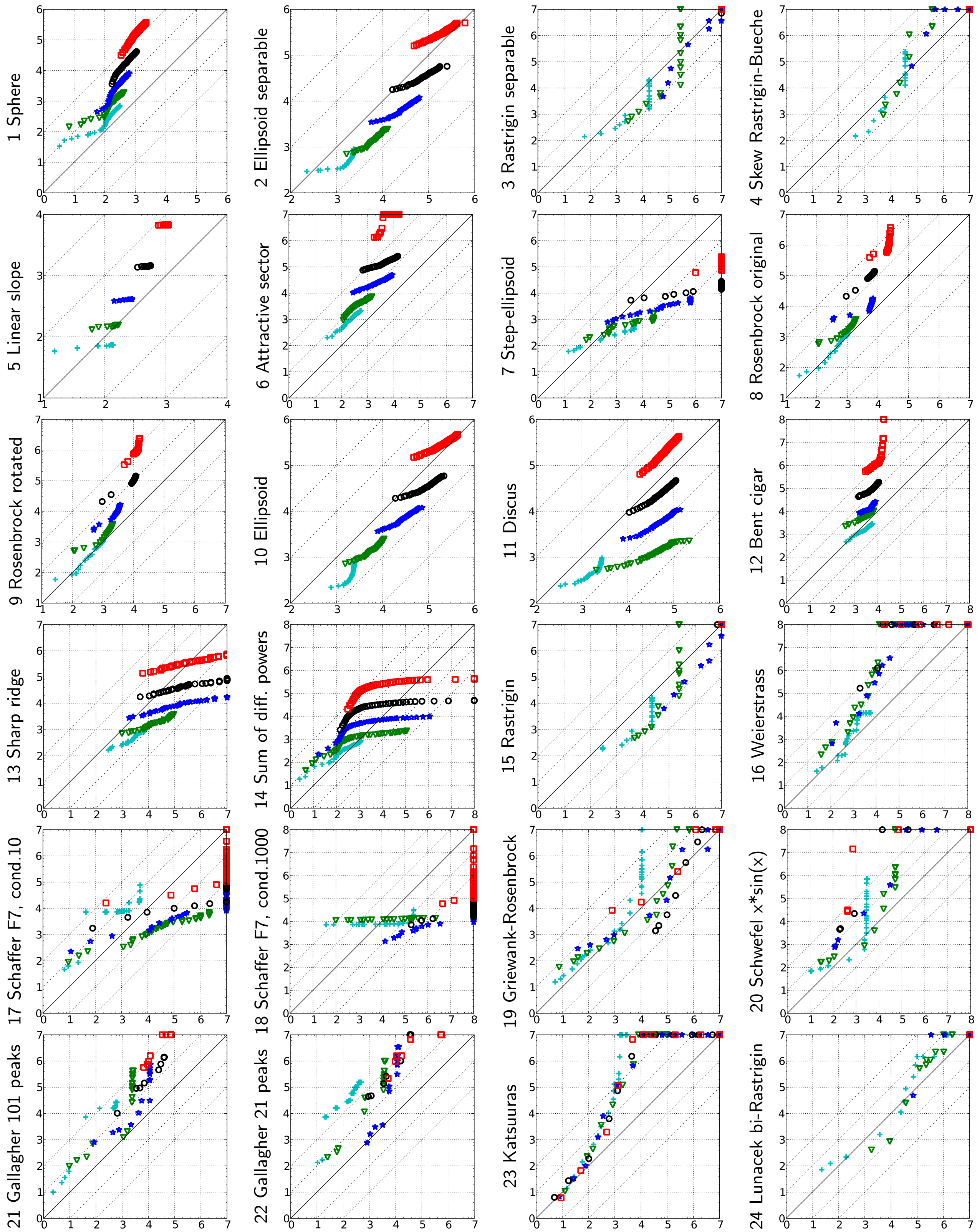

Figure 2: Expected running time (ERT in log10 of number of function evaluations) of CauchyEDA versus G3PCX for 46 target values $\Delta f \in [10^{-8}, 10]$ in each dimension for functions f_1–f_{24}. Markers on the upper or right egde indicate that the target value was never reached by CauchyEDA or G3PCX respectively. Markers represent dimension: 2:+, 3:▽, 5:⋆, 10:○, 20:□, 40:◇.

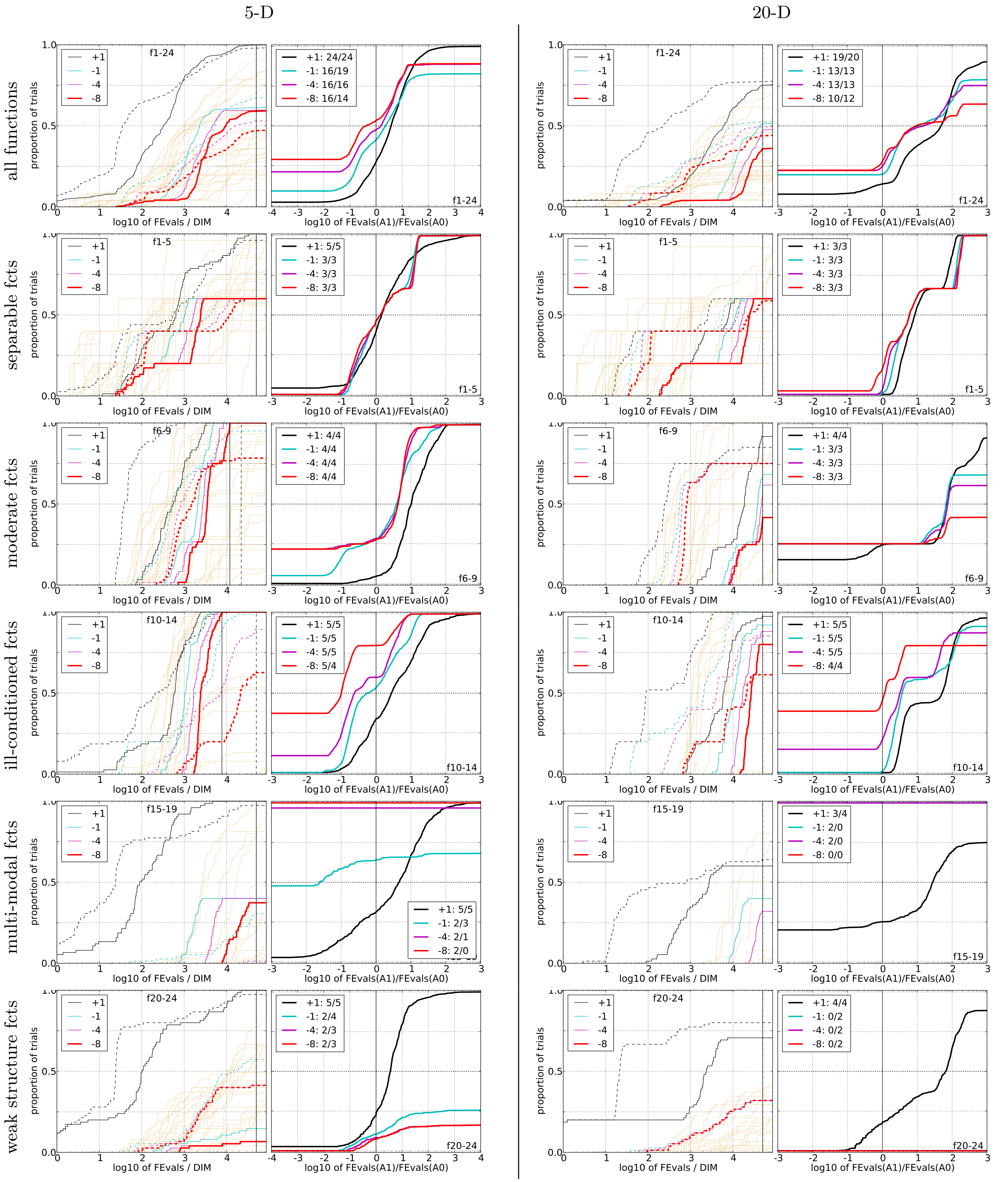

Figure 3: Empirical cumulative distributions (ECDF) of run lengths and speed-up ratios in 5-D (left) and 20-D (right). Left sub-columns: ECDF of the number of function evaluations divided by dimension D (FEvals/D) to reach a target value $f_{\rm opt} + \Delta f$ with $\Delta f = 10^k$, where $k \in \{1, -1, -4, -8\}$ is given by the first value in the legend, for CauchyEDA (solid) and G3PCX (dashed). Light beige lines show the ECDF of FEvals for target value $\Delta f = 10^{-8}$ of algorithms benchmarked during BBOB-2009. Right sub-columns: ECDF of FEval ratios of CauchyEDA divided by G3PCX, all trial pairs for each function. Pairs where both trials failed are disregarded, pairs where one trial failed are visible in the limits being > 0 or < 1. The legends indicate the number of functions that were solved in at least one trial (CauchyEDA first).

Δf (5-D)	10^1	10^0	10^{-1}	10^{-3}	10^{-5}	10^{-7}	#succ
f_1	11	12	12	12	12	12	15/15
0: G3P	5.2	$12^{\star3}$	$15^{\star3}$	$25^{\star3}$	$35^{\star3}$	$45^{\star3}$	15/15
1: Cau	$41^{\star3}$	$90^{\star3}$	$170^{\star3}$	$310^{\star3}$	$460^{\star3}$	$600^{\star3}$	15/15
f_2	83	87	88	90	92	94	15/15
0: G3P	$69^{\star3}$	$150^{\star3}$	$220^{\star3}$	$340^{\star3}$	$470^{\star3}$	$620^{\star3}$	15/15
1: Cau	$42^{\star3}$	$49^{\star3}$	$58^{\star3}$	$80^{\star3}$	$100^{\star3}$	$120^{\star3}$	15/15
f_3	720	1600	1600	1600	1700	1700	15/15
0: G3P	$84^{\star2}$	$2100^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$6.7^{\star2}$	$2200^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_4	810	1600	1700	1800	1900	1900	15/15
0: G3P	$76^{\star2}$	$2200^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$85^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_5	10	10	10	10	10	10	15/15
0: G3P	$14^{\star3}$	$25^{\star3}$	$27^{\star3}$	$28^{\star3}$	$28^{\star3}$	$28^{\star3}$	15/15
1: Cau	$39^{\star3}$	$41^{\star3}$	$41^{\star3}$	$41^{\star3}$	$41^{\star3}$	$41^{\star3}$	15/15
f_6	110	210	280	580	1000	1300	15/15
0: G3P	$2.4^{\star2}$	$3^{\star2}$	$4.8^{\star2}$	$4.7^{\star2}$	$5.1^{\star2}$	$5.5^{\star2}$	15/15
1: Cau	$92^{\star3}$	$69^{\star3}$	$68^{\star3}$	$47^{\star3}$	$35^{\star3}$	$34^{\star3}$	15/15
f_7	24	320	1200	1600	1600	1600	15/15
0: G3P	$19^{\star3}$	18	$45^{\star3}$	$410^{\star3}$	$410^{\star3}$	$410^{\star3}$	2/15
1: Cau	$33^{\star3}$	$4.9^{\star3}$	$2.4^{\star2}$	$2.9^{\star3}$	$2.9^{\star3}$	$3.4^{\star3}$	15/15
f_8	73	270	340	390	410	420	15/15
0: G3P	$4.6^{\star2}$	$20^{\star3}$	$18^{\star3}$	$17^{\star3}$	$16^{\star3}$	$16^{\star3}$	15/15
1: Cau	$49^{\star3}$	$31^{\star3}$	$33^{\star3}$	$34^{\star3}$	$37^{\star3}$	$40^{\star3}$	15/15
f_9	35	130	210	300	340	370	15/15
0: G3P	$14^{\star3}$	$18^{\star3}$	$14^{\star3}$	$11^{\star3}$	$10^{\star3}$	$9.8^{\star3}$	15/15
1: Cau	$71^{\star3}$	$54^{\star3}$	$45^{\star3}$	$41^{\star3}$	$42^{\star3}$	$43^{\star3}$	15/15
f_{10}	350	500	570	630	830	880	15/15
0: G3P	$22^{\star3}$	$27^{\star3}$	$36^{\star3}$	$49^{\star3}$	$51^{\star3}$	$64^{\star3}$	15/15
1: Cau	$11^{\star3}$	$9^{\star3}$	$9.4^{\star3}$	$12^{\star3}$	$11^{\star3}$	$13^{\star3}$	15/15
f_{11}	140	200	760	1200	1500	1700	15/15
0: G3P	$55^{\star3}$	$110^{\star3}$	$45^{\star3}$	$49^{\star3}$	$56^{\star3}$	$61^{\star3}$	14/15
1: Cau	$18^{\star3}$	$17^{\star3}$	$6^{\star3}$	$5.3^{\star3}$	$5.6^{\star3}$	$5.9^{\star3}$	15/15
f_{12}	110	270	370	460	1300	1500	15/15
0: G3P	$14^{\star3}$	$11^{\star2}$	$13^{\star3}$	$12^{\star3}$	$5.2^{\star3}$	$5.1^{\star3}$	15/15
1: Cau	$79^{\star3}$	$41^{\star3}$	$35^{\star3}$	$38^{\star3}$	$17^{\star3}$	$17^{\star3}$	15/15
f_{13}	130	190	250	1300	1800	2300	15/15
0: G3P	$14^{\star3}$	$60^{\star3}$	$150^{\star3}$	$120^{\star3}$	$590^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$21^{\star3}$	$24^{\star3}$	$25^{\star3}$	$7.4^{\star3}$	$7.3^{\star3}$	$7.3^{\star3}$	15/15
f_{14}	9.8	41	58	140	250	480	15/15
0: G3P	1.7	$3.5^{\star3}$	$3.5^{\star3}$	$5^{\star3}$	$26^{\star3}$	$390^{\star3}$	3/15
1: Cau	23	$29^{\star3}$	$40^{\star3}$	$33^{\star3}$	$28^{\star3}$	$19^{\star3}$	15/15
f_{15}	510	9300	1.9e4	2.0e4	2.1e4	2.1e4	14/15
0: G3P	$130^{\star2}$	$370^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$12^{\star3}$	$190^{\star2}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_{16}	120	610	2700	1.0e4	1.2e4	1.2e4	15/15
0: G3P	1	$22^{\star}$	44	$350^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	5.6	$1200^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_{17}	5.2	210	900	3700	6400	7900	15/15
0: G3P	2.2	130	$290^{\star2}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	44	$13^{\star3}$	$7^{\star3}$	$4.3^{\star3}$	$5.3^{\star3}$	$13^{\star3}$	14/15
f_{18}	100	380	4000	9300	1.1e4	1.2e4	15/15
0: G3P	130	$800^{\star2}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$13^{\star3}$	$12^{\star3}$	$2.4^{\star}$	$2.7^{\star3}$	$3.7^{\star3}$	$8.6^{\star3}$	14/15
f_{19}	1	1	240	1.2e5	1.2e5	1.2e5	15/15
0: G3P	39	9.5e4	$1.4e4^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	300	2.1e4	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_{20}	16	850	3.8e4	5.4e4	5.5e4	5.5e4	14/15
0: G3P	$7.4^{\star2}$	$36^{\star2}$	$88^{\star3}$	$62^{\star3}$	$61^{\star3}$	$61^{\star3}$	1/15
1: Cau	$48^{\star3}$	$460^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_{21}	41	1200	1700	1700	1700	1800	14/15
0: G3P	2.1	4.7	6.8	6.7	6.7	6.6	15/15
1: Cau	20	27	$190^{\star}$	$420^{\star2}$	$420^{\star2}$	$410^{\star2}$	4/15
f_{22}	71	390	940	1000	1000	1100	14/15
0: G3P	12	15	13	$12^{\star}$	$12^{\star2}$	$12^{\star2}$	15/15
1: Cau	11	280	$780^{\star2}$	$3500^{\star2}$	$3400^{\star2}$	$3300^{\star2}$	1/15
f_{23}	3	520	1.4e4	3.2e4	3.3e4	3.4e4	15/15
0: G3P	2.6	2.4	$8.6^{\star}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	2.2	$230^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_{24}	1600	2.2e5	6.4e6	9.6e6	1.3e7	1.3e7	3/15
0: G3P	$44^{\star2}$	∞	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$30^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15

Δf (20-D)	10^1	10^0	10^{-1}	10^{-3}	10^{-5}	10^{-7}	#succ
f_1	43	43	43	43	43	43	15/15
0: G3P	$8^{\star3}$	$13^{\star3}$	$18^{\star3}$	$27^{\star3}$	$37^{\star3}$	$48^{\star3}$	15/15
1: Cau	$730^{\star3}$	$1600^{\star3}$	$2500^{\star3}$	$4300^{\star3}$	$6100^{\star3}$	$7800^{\star3}$	15/15
f_2	380	390	390	390	390	390	15/15
0: G3P	$130^{\star3}$	$210^{\star3}$	$320^{\star3}$	$550^{\star3}$	$760^{\star3}$	$990^{\star3}$	14/15
1: Cau	$410^{\star3}$	$510^{\star3}$	$610^{\star3}$	$800^{\star3}$	$990^{\star3}$	$1200^{\star3}$	15/15
f_3	5100	7600	7600	7600	7600	7700	15/15
0: G3P	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_4	4700	7600	7700	7700	7800	1.4e5	9/15
0: G3P	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_5	41	41	41	41	41	41	15/15
0: G3P	$19^{\star3}$	$25^{\star3}$	$26^{\star3}$	$27^{\star3}$	$27^{\star3}$	$27^{\star3}$	15/15
1: Cau	$160^{\star3}$	$170^{\star3}$	$170^{\star3}$	$170^{\star3}$	$170^{\star3}$	$170^{\star3}$	15/15
f_6	1300	2300	3400	5200	6700	8400	15/15
0: G3P	1.4	1.5	1.5	$1.5^{\star2}$	$1.7^{\star3}$	$1.7^{\star3}$	15/15
1: Cau	$1000^{\star3}$	$1300^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_7	1400	4300	9500	1.7e4	1.7e4	1.7e4	15/15
0: G3P	$760^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$44^{\star3}$	$29^{\star3}$	$18^{\star3}$	$14^{\star3}$	$14^{\star3}$	$14^{\star3}$	15/15
f_8	2000	3900	4000	4200	4400	4500	15/15
0: G3P	$2.6^{\star3}$	$5.4^{\star3}$	$5.5^{\star3}$	$5.5^{\star3}$	$5.5^{\star3}$	$5.6^{\star3}$	15/15
1: Cau	$190^{\star3}$	$180^{\star3}$	$210^{\star3}$	$260^{\star3}$	$360^{\star3}$	$540^{\star3}$	4/15
f_9	1700	3100	3300	3500	3600	3700	15/15
0: G3P	$2.9^{\star2}$	$3.8^{\star3}$	$4^{\star3}$	$4.1^{\star3}$	$4.1^{\star3}$	$4.2^{\star3}$	15/15
1: Cau	$190^{\star3}$	$270^{\star3}$	$290^{\star3}$	$310^{\star3}$	$470^{\star3}$	$630^{\star3}$	6/15
f_{10}	7400	8700	1.1e4	1.5e4	1.7e4	1.7e4	15/15
0: G3P	$6.5^{\star3}$	$10^{\star3}$	$12^{\star3}$	$15^{\star3}$	$18^{\star3}$	$23^{\star3}$	15/15
1: Cau	$20^{\star3}$	$22^{\star3}$	$20^{\star3}$	$20^{\star3}$	$21^{\star3}$	$25^{\star3}$	15/15
f_{11}	1000	2200	6300	9800	1.2e4	1.5e4	15/15
0: G3P	$18^{\star3}$	$14^{\star3}$	$7^{\star3}$	$7.1^{\star3}$	$7.6^{\star3}$	$8^{\star3}$	15/15
1: Cau	$64^{\star3}$	$44^{\star3}$	$22^{\star3}$	$22^{\star3}$	$25^{\star3}$	$26^{\star3}$	15/15
f_{12}	1000	1900	2700	4100	1.2e4	1.4e4	15/15
0: G3P	2.7	2.8	3	$2.9^{\star2}$	1.2	1.3	15/15
1: Cau	$510^{\star3}$	$440^{\star3}$	$420^{\star3}$	$380^{\star3}$	$390^{\star3}$	$1100^{\star3}$	0/15
f_{13}	650	2000	2800	1.9e4	2.4e4	3.0e4	15/15
0: G3P	$9.3^{\star}$	$17^{\star2}$	$43^{\star3}$	$47^{\star3}$	$130^{\star3}$	$330^{\star3}$	1/15
1: Cau	$210^{\star3}$	$100^{\star3}$	$100^{\star3}$	$23^{\star3}$	$23^{\star3}$	$23^{\star3}$	15/15
f_{14}	75	240	300	930	1600	1.6e4	15/15
0: G3P	$4.1^{\star3}$	$2.4^{\star3}$	$2.8^{\star3}$	$2.7^{\star3}$	$13^{\star3}$	$59^{\star3}$	0/15
1: Cau	$280^{\star3}$	$270^{\star3}$	$350^{\star3}$	$210^{\star3}$	$180^{\star3}$	$25^{\star3}$	15/15
f_{15}	3.0e4	1.5e5	3.1e5	3.2e5	4.5e5	4.6e5	15/15
0: G3P	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_{16}	1400	2.7e4	7.7e4	1.9e5	2.0e5	2.2e5	15/15
0: G3P	17	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_{17}	63	1000	4000	3.1e4	5.6e4	8.0e4	15/15
0: G3P	$4^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$260^{\star3}$	$120^{\star3}$	$62^{\star3}$	$16^{\star3}$	$23^{\star3}$	$\infty^{\star3}$	0/15
f_{18}	620	4000	2.0e4	6.8e4	1.3e5	1.5e5	15/15
0: G3P	$7100^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$96^{\star3}$	$42^{\star3}$	$15^{\star3}$	$12^{\star3}$	$38^{\star3}$	$\infty^{\star3}$	0/15
f_{19}	1	1	3.4e5	6.2e6	6.7e6	6.7e6	15/15
0: G3P	800	∞	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	8400	∞	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_{20}	82	4.6e4	3.1e6	5.5e6	5.6e6	5.6e6	14/15
0: G3P	$5^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$340^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_{21}	560	6500	1.4e4	1.5e4	1.6e4	1.8e4	15/15
0: G3P	12	7.2	5.1	4.9	4.6	4.1	15/15
1: Cau	$1000^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_{22}	470	5600	2.3e4	2.5e4	2.7e4	1.3e5	12/15
0: G3P	$11^{\star}$	6.6	23	22	20	4	9/15
1: Cau	$470^{\star3}$	$1200^{\star2}$	$\infty^{\star}$	$\infty^{\star}$	$\infty^{\star}$	$\infty^{\star3}$	0/15
f_{23}	3.2	1600	6.7e4	4.9e5	8.1e5	8.4e5	15/15
0: G3P	2.8	$7.8^{\star2}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	1.9	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
f_{24}	1.3e6	7.5e6	5.2e7	5.2e7	5.2e7	5.2e7	3/15
0: G3P	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15
1: Cau	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	$\infty^{\star3}$	0/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values for functions f_1–f_{24}. The median number of conducted function evaluations is additionally given in *italics*, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\mathrm{opt}} + 10^{-8}$. 0: G3P is G3PCX and 1: Cau is CauchyEDA. Bold entries are statistically significantly better compared to the other algorithm, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the $\star$ symbol, with Bonferroni correction of 48.

Table 2: The average time demands per function evaluation (in microseconds) of the two compared algorithms.

Dim	2	3	5	10	20	40
G3PCX	410	420	440	470	540	750
CauchyEDA	51	17	9	9	11	NA

Interesting results may be found for function 14. G3PCX algorithm is orders of magnitude faster than Cauchy EDA for a broad range of target levels. But for target levels at about 10^{-5} and tighter, Cauchy EDA takes over and its results are much better. It seems that G3PCX is not even able to find some of the tighter target levels. One explanation for that might be that the stopping criterion for G3PCX is not set properly and actually prevents the algorithm from finding these target levels.

Another note can be made on the variance enlargment constant used by Cauchy EDA. It was set to be approximately optimal for the Rosenbrock's function. However, such setting may be too large for other functions. The ERT ratio for sphere function shows that with increasing problem dimensionality the gap between the algorithms gets larger. Also the results for function 21 and 22 (and possibly for functions 16 and 23) suggest, that the slow convergence of Cauchy EDA prevents it to be restarted more often which is the key to solve these problems; on the contrary, G3PCX converges probably much faster and is thus restarted more often which gives it a chance to have higher success rate.

Looking at Fig. 3, it can be stated that G3PCX beats CauchyEDA mainly on the separable functions (where for 20D, we can expect the G3PCX to be faster than Cauchy EDA at least 80% if time regardless of the target level), on moderate functions (where G3PCX would be winner about 75% of time), and on weak-structure functions (where Cauchy EDA almost does not work at all). On the other hand, Cauchy EDA has higher success rates on ill-conditioned and multi-modal functions, but compared to G3PCX and other algorithms it is orders of magnitude slower.

4. CPU TIMING EXPERIMENTS

The time requirements of both algorithms are taken from the respective articles, [8] and [7]. The multistart algorithm was run with the maximal number of evaluations set to 10^5, the basic algorithm was restarted for at least 30 seconds. The experiment was conducted on Intel Core 2 CPU, T5600, 1.83 GHz, 1 GB RAM with Windows XP SP3 in MATLAB R2007b. The comparison of the average time demands per function evaluation are shown in Table 2.

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