Black-Box Optimization Benchmarking for Noiseless Function Testbed using Artificial Bee Colony Algorithm

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ABSTRACT

This paper benchmarks the Artificial Bee Colony (ABC) algorithm using the noise-free BBOB 2010 testbed. The results show how this algorithm is highly successful in the separable and weak structured functions.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. ARTIFICIAL BEE COLONY

The ABC algorithm was first proposed in [7]. The algorithms was inspired by the method adopted by a swarm of honey bees to locate food sources. There are two different honey bee groups that share knowledge in order to successfully locate such sources. First, there are the *employed bees* that are currently exploiting a food source. Second, there are the *onlookers* that wait at the nest and establish communication with the employed bees.

In ABC, the swarm is divided into employed bees, scouts and onlookers. S_n solutions to the problem are randomly initialized in the function domain and referred to as food sources. A number of employed bees, set as the number of the food sources (half the colony size), are used to find new food sources using the following equation:

$$v_{ij} = x_{ij} + \phi_{ij} \times (x_{ij} - x_{kj}), \tag{1}$$

for $j \in \{1 \dots D\}$ where d is the number of dimensions, ϕ_{ij} is a random number uniformly distributed in the range [-1,1], k is the index of a randomly chosen solution, $\mathbf{x}_i$ is the current food source exploited by employed bee i, and $\mathbf{v}_i$ is the new food source to be exploited. Both $\mathbf{v}_i$ and $\mathbf{x}_i$ are then compared against each other and the employed bee exploits the better food source.

Next, each onlooker bee randomly selects a food source to exploit according to the probability given in equation 2:

$$p_i = \frac{fit_i}{\sum_{j=1}^{S_n} fit_j}, \tag{2}$$

where fit_i is the fitness of the i^{th} food source. Then, each onlooker bee tries to find a better food source around the selected one using equation 1.

If a food source cannot be improved for a predetermined number of cycles, referred to as *limit*, this food source is abandoned. The employed bee that was exploiting this food source becomes a scout and randomly selects another food source in the domain according to:

$$x_{ij} = x_j^{min} + rand \times (x_j^{max} - x_j^{min}), \tag{3}$$

where x_j^{min} and x_j^{max} are the minimum and maximum domain bounds.

The ABC algorithm is shown in Algorithm 1.

Algorithm 1 The ABC algorithm

Require: Max_Cycles, ColonySize, *limit*.
Initialize the food sources
Evaluate the food sources
Cycle=1
while $Cycle \leq Max\_Cycles$ **do**
  for each employed bee i **do**
    Produce a new solution
    Evaluate the new solution
    Apply Greedy selection choosing the better solution
  end for
  for each onlooker bee i **do**
    Probabilistically choose a solution according to p_i
    Produce a new solution
    Evaluate the new solution
    Apply Greedy selection choosing the better solution
  end for
  Re-initialize solutions not improved for *limit* cycles
  Memorize the best solution
  Cycle = Cycle + 1
end while
return best solution

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This algorithm was applied to multidimensional and multimodal function optimization in [7, 2, 11]. Previous studies performed to assess the performance ABC included the work in [12] showing that the ABC algorithm performs better than Particle Swarm Optimization (PSO), an Evolutionary Algorithm (EA) and Differential Evolution (DE) on a small suite of classical benchmark functions.

Another study was carried in [10] that compared ABC against PSO, a Genetic Algorithm (GA), DE and an Evolutionary Strategy (ES) algorithm on a larger number of functions. It was shown that the performance of ABC is better than or at least similar to those algorithms while having a smaller number of parameters to tune.

The work in [9] compared ABC to Harmony Search (HS) and the Bees Algorithm (BA) proposed in [13]. The comparison was based on a small set of classical functions and the ABC showed superior performance over both algorithms while producing reasonable results for higher dimensions.

2. PARAMETER TUNING

For ABC, the work in [1] indicated that the there is no need to have a huge colony size in order to provide good results. We use 40 bees as our previous experiments conducted on the CEC05 benchmarks [14] and repeated using populations of 20, 40 and 100 bees for different problem sizes showed that this setting provided the best results on average.

The recommendations in [10] were followed by setting the *limit* parameters to $S_n \times D$, although recent research [1] indicated that lower values might be needed for more difficult functions. The same parameter values are used for all functions, hence the crafting error is zero.

3. CPU TIMING EXPERIMENT

For the timing experiment, ABC was run on f8 and restarted until at least 30 seconds had passed (according to Figure 2 in [5]).

The experiments have been conducted with an Intel Core 2 Quad 2.4 GHz under Windows Vista using the MATLAB-code provided. The results were 2.0×10^{-4} seconds per function evaluation in dimensions 2 up to 20. A dependency of CPU time on the search space dimensionality is not visible.

4. RESULTS

Results from experiments according to [4] on the benchmark functions given in [3, 6] are presented in Figures 1, 2 and 3 and in Tables 1 and 2.

Experiments use the ABC code available at [8]. The ABC algorithm is allowed to perform a maximum of $10^5 \times D$ function evaluations for all test functions and no restart mechanism was used.

Results show that the performance is very good for separable functions f1-f5 where the intended target is reached for all dimensions. It's also worth a note that the slope function is successfully solved without forcing the boundary conditions on the bees' movement. The algorithm also has an acceptable performance for the multi-modal functions with the global weak structure f20, f21 and f22 specially in the low dimensionality.

Figure 2 also indicates that the ABC algorithm is very successful (in comparison with the BBOB2009 algorithms) in reaching a target value of 10^{-8} in functions f3 and f4 for all dimensions.

Table 2: ERT loss ratio (see Figure 3) compared to the respective best result from BBOB-2009 for budgets given in the first column. The last row $\mathbf{RL_{US}/D}$ gives the number of function evaluations in unsuccessful runs divided by dimension. Shown are the smallest, 10%-ile, 25%-ile, 50%-ile, 75%-ile and 90%-ile value (smaller values are better).

	$f1$–$f24$ in 5-D, maxFE/D=100011					
#FEs/D	best	10%	25%	**med**	75%	90%
2	1.6	1.9	2.5	4.0	5.8	10
10	2.3	3.3	3.8	5.1	8.3	50
100	2.4	5.4	6.9	10	15	42
1e3	1.8	3.0	13	23	52	90
1e4	0.89	4.4	39	63	2.7e2	4.0e2
1e5	1.1	4.7	80	3.2e2	1.8e3	2.8e3
RL_{US}/D	1e5	1e5	1e5	1e5	1e5	1e5
	$f1$–$f24$ in 20-D, maxFE/D=100002					
#FEs/D	best	10%	25%	**med**	75%	90%
2	1.0	5.4	13	31	40	40
10	4.6	7.0	23	1.7e2	2.0e2	2.0e2
100	0.73	7.7	11	18	49	2.0e3
1e3	0.12	2.6	20	46	95	4.2e2
1e4	0.20	4.8	59	1.2e2	4.0e2	8.1e2
1e5	0.47	4.8	1.0e2	7.7e2	2.3e3	4.8e3
1e6	0.47	9.0	2.7e2	5.3e3	1.6e4	4.3e4
RL_{US}/D	1e5	1e5	1e5	1e5	1e5	1e5

5. CONCLUSION

This paper benchmarks the artificial bee colony (ABC) algorithm using the noise-free BBOB 2010 testbed. The better performance of the ABC algorithm was depicted in the separable and weak structured functions.

6. REFERENCES

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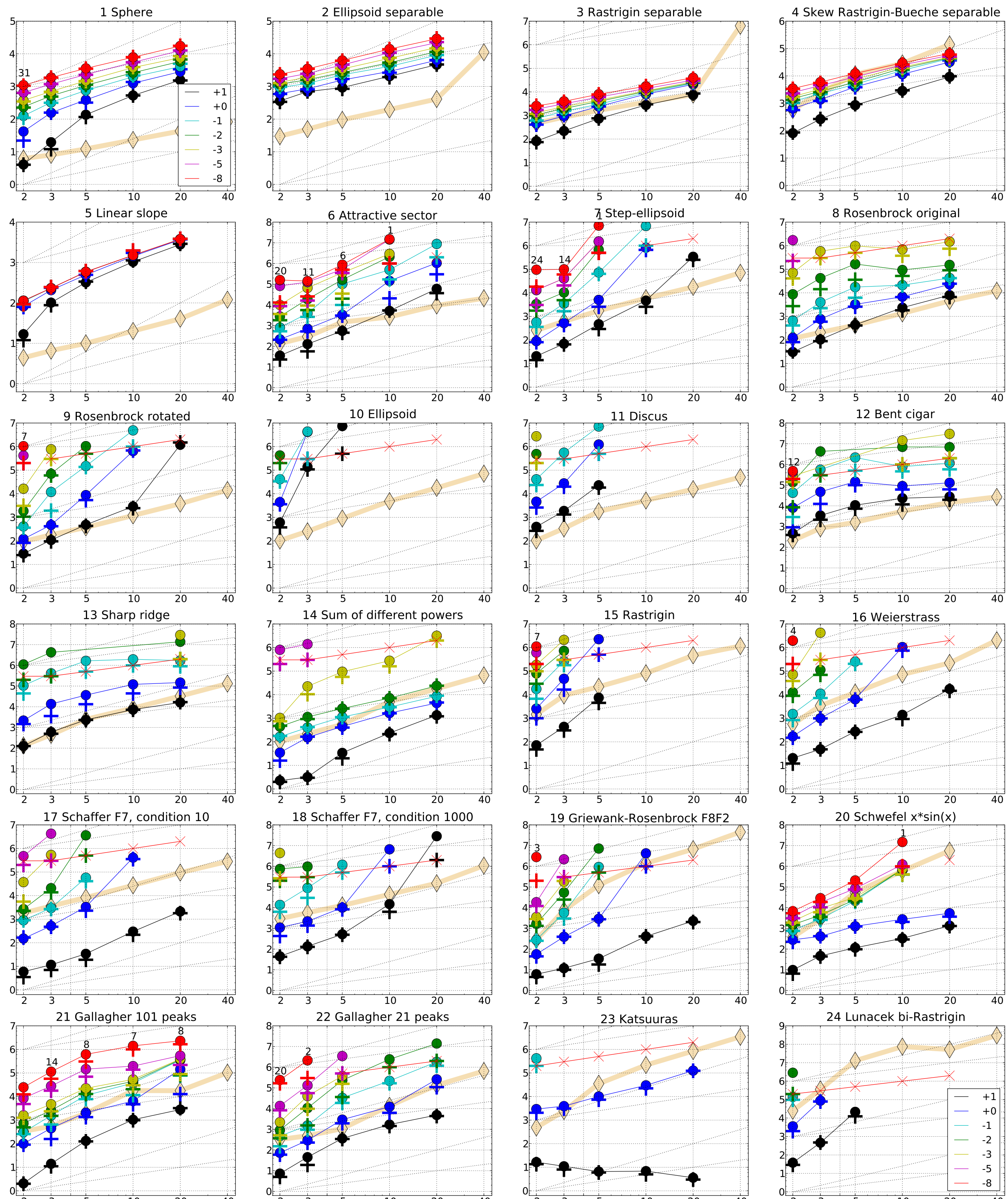

Figure 1: Expected Running Time (ERT, ●) to reach $f_{\rm opt} + \Delta f$ and median number of f-evaluations from successful trials (+), for $\Delta f = 10^{\{+1,0,-1,-2,-3,-5,-8\}}$ (the exponent is given in the legend of f_1 and f_{24}) versus dimension in log-log presentation. For each function and dimension, $\mathrm{ERT}(\Delta f)$ equals to $\#\mathrm{FEs}(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\rm opt} + \Delta f$ was surpassed. The $\#\mathrm{FEs}(\Delta f)$ are the total number (sum) of f-evaluations while $f_{\rm opt} + \Delta f$ was not surpassed in the trial, from all (successful and unsuccessful) trials, and $f_{\rm opt}$ is the optimal function value. Crosses (×) indicate the total number of f-evaluations, $\#\mathrm{FEs}(-\infty)$, divided by the number of trials. Numbers above ERT-symbols indicate the number of successful trials. Y-axis annotations are decimal logarithms. The thick light line with diamonds shows the single best results from BBOB-2009 for $\Delta f = 10^{-8}$. Additional grid lines show linear and quadratic scaling.

Δf	**f_1 in 5-D**, N=15, mFE=3980					**f_1 in 20-D**, N=15, mFE=18380				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.4e2	1.2e1	3.1e2	1.4e2	15	1.6e3	6.5e2	2.7e3	1.6e3
1	15	3.9e2	1.7e2	7.5e2	3.9e2	15	2.8e3	1.1e3	4.6e3	2.8e3
1e−1	15	7.6e2	2.8e2	1.0e3	7.6e2	15	4.1e3	1.4e3	6.2e3	4.1e3
1e−3	15	1.5e3	1.2e3	1.8e3	1.5e3	15	7.6e3	3.8e3	9.7e3	7.6e3
1e−5	15	2.3e3	2.2e3	2.6e3	2.3e3	15	1.3e4	1.2e4	1.3e4	1.3e4
1e−8	15	3.6e3	3.4e3	3.9e3	3.6e3	15	1.7e4	1.6e4	1.8e4	1.7e4

Δf	**f_2 in 5-D**, N=15, mFE=7022					**f_2 in 20-D**, N=15, mFE=30621				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	9.3e2	5.7e2	1.5e3	9.3e2	15	4.6e3	2.8e3	6.4e3	4.6e3
1	15	1.6e3	7.4e2	2.6e3	1.6e3	15	6.4e3	4.9e3	8.2e3	6.4e3
1e−1	15	2.3e3	1.6e3	3.4e3	2.3e3	15	9.1e3	7.0e3	1.2e4	9.1e3
1e−3	15	3.4e3	2.0e3	4.4e3	3.4e3	15	1.6e4	1.3e4	1.9e4	1.6e4
1e−5	15	4.6e3	3.7e3	5.5e3	4.6e3	15	2.2e4	2.0e4	2.4e4	2.2e4
1e−8	15	6.3e3	5.8e3	6.8e3	6.3e3	15	2.9e4	2.7e4	3.1e4	2.9e4

Δf	**f_3 in 5-D**, N=15, mFE=9620					**f_3 in 20-D**, N=15, mFE=73315				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	7.4e2	4.1e2	1.2e3	7.4e2	15	7.4e3	2.9e3	1.1e4	7.4e3
1	15	2.4e3	1.3e3	3.3e3	2.4e3	15	2.1e4	6.9e3	2.9e4	2.1e4
1e−1	15	3.0e3	2.0e3	4.0e3	3.0e3	15	2.3e4	8.5e3	3.2e4	2.3e4
1e−3	15	4.5e3	3.7e3	5.3e3	4.5e3	15	2.7e4	1.3e4	3.7e4	2.7e4
1e−5	15	6.0e3	5.3e3	6.9e3	6.0e3	15	3.2e4	2.2e4	3.9e4	3.2e4
1e−8	15	8.0e3	7.1e3	9.4e3	8.0e3	15	4.0e4	3.5e4	4.5e4	4.0e4

Δf	**f_4 in 5-D**, N=15, mFE=21552					**f_4 in 20-D**, N=15, mFE=121198				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	8.6e2	3.6e2	1.3e3	8.6e2	15	9.3e3	3.8e3	1.5e4	9.3e3
1	15	4.0e3	2.7e3	6.0e3	4.0e3	15	3.2e4	1.5e4	4.3e4	3.2e4
1e−1	15	5.0e3	3.2e3	6.6e3	5.0e3	15	4.3e4	1.5e4	8.1e4	4.3e4
1e−3	15	7.8e3	5.7e3	1.3e4	7.8e3	15	4.9e4	2.9e4	8.4e4	4.9e4
1e−5	15	9.2e3	6.9e3	1.4e4	9.2e3	15	5.7e4	3.9e4	1.0e5	5.7e4
1e−8	15	1.2e4	9.5e3	1.6e4	1.2e4	15	6.6e4	5.3e4	1.1e5	6.6e4

Δf	**f_5 in 5-D**, N=15, mFE=980					**f_5 in 20-D**, N=15, mFE=5940				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.2e2	1.2e2	4.5e2	3.2e2	15	2.8e3	1.9e3	3.7e3	2.8e3
1	15	4.9e2	2.1e2	7.6e2	4.9e2	15	3.7e3	2.0e3	5.0e3	3.7e3
1e−1	15	5.8e2	2.1e2	8.9e2	5.8e2	15	3.8e3	2.1e3	5.2e3	3.8e3
1e−3	15	5.9e2	2.1e2	9.1e2	5.9e2	15	3.8e3	2.3e3	5.2e3	3.8e3
1e−5	15	5.9e2	2.1e2	9.1e2	5.9e2	15	3.8e3	2.3e3	5.2e3	3.8e3
1e−8	15	5.9e2	2.1e2	9.1e2	5.9e2	15	3.8e3	2.3e3	5.2e3	3.8e3

Δf	**f_6 in 5-D**, N=15, mFE=500058					**f_6 in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	5.6e2	2.1e2	9.4e2	5.6e2	15	6.0e4	9.7e3	1.5e5	6.0e4
1	15	3.3e3	1.0e3	5.5e3	3.3e3	11	1.1e6	1.1e5	2.9e6	3.3e5
1e−1	13	1.0e5	5.6e3	5.1e5	2.6e4	3	8.8e6	8.7e5	2.1e7	8.3e5
1e−3	9	3.6e5	1.3e4	1.0e6	2.6e4	0	*54e-2*	*76e-3*	*14e-1*	1.1e6
1e−5	8	5.2e5	2.1e4	1.5e6	7.9e4	.	.	.	.	.
1e−8	6	8.8e5	5.4e4	2.1e6	1.3e5	.	.	.	.	.

Δf	**f_7 in 5-D**, N=15, mFE=500055					**f_7 in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	4.7e2	6.1e1	1.4e3	4.7e2	15	3.4e5	5.3e4	9.4e5	3.4e5
1	15	5.0e3	1.1e3	1.0e4	5.0e3	0	*60e-1*	*37e-1*	*83e-1*	5.8e5
1e−1	15	7.3e4	1.5e4	1.5e5	7.3e4	.	.	.	.	.
1e−3	4	1.5e6	2.5e4	3.6e6	1.3e5	.	.	.	.	.
1e−5	4	1.5e6	2.5e4	3.6e6	1.3e5	.	.	.	.	.
1e−8	1	7.1e6	5.9e5	1.7e7	9.0e4	.	.	.	.	.

Δf	**f_8 in 5-D**, N=15, mFE=500057					**f_8 in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	4.4e2	1.8e2	8.1e2	4.4e2	15	8.0e3	4.6e3	1.4e4	8.0e3
1	15	3.2e3	9.7e2	6.6e3	3.2e3	15	2.3e4	1.3e4	3.1e4	2.3e4
1e−1	15	1.8e4	3.6e3	1.5e4	1.8e4	15	4.2e4	3.3e4	5.4e4	4.2e4
1e−3	6	9.8e5	4.7e4	2.1e6	2.3e5	11	1.5e6	1.6e5	3.4e6	7.6e5
1e−5	0	*10e-4*	*25e-5*	*18e-3*	1.2e5	0	*24e-5*	*47e-6*	*29e-4*	8.0e5
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_9 in 5-D**, N=15, mFE=500056					**f_9 in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	4.8e2	2.1e2	8.1e2	4.8e2	13	1.2e6	9.5e4	2.2e6	8.9e5
1	15	8.8e3	2.0e3	3.1e4	8.8e3	0	*92e-1*	*67e-1*	*10e+0*	2.0e6
1e−1	15	1.5e5	8.0e3	3.5e5	1.5e5	.	.	.	.	.
1e−3	0	*13e-3*	*18e-4*	*49e-3*	3.1e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{10} in 5-D**, N=15, mFE=500059					**f_{10} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	1	7.4e6	9.2e5	1.6e7	4.2e5	0	*50e+2*	*34e+2*	*79e+2*	1.3e6
1	0	*55e+0*	*15e+0*	*17e+1*	2.6e5	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{11} in 5-D**, N=15, mFE=500059					**f_{11} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.3e4	7.4e2	6.9e4	2.3e4	0	*92e+0*	*65e+0*	*10e+1*	5.2e5
1	5	1.2e6	1.5e5	2.8e6	2.3e5	.	.	.	.	.
1e−1	1	7.1e6	6.2e5	1.7e7	1.2e5	.	.	.	.	.
1e−3	0	*15e-1*	*18e-2*	*28e-1*	1.8e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{12} in 5-D**, N=15, mFE=500059					**f_{12} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.1e4	4.7e3	2.1e4	1.1e4	15	2.8e4	1.5e4	2.6e4	2.8e4
1	14	1.5e5	2.5e4	2.8e5	1.1e5	15	1.3e5	2.6e4	2.2e5	1.3e5
1e−1	3	2.2e6	1.7e5	5.2e6	1.7e5	12	1.2e6	2.3e5	2.6e6	6.8e5
1e−3	0	*29e-2*	*54e-3*	*49e-2*	2.8e5	1	3.0e7	4.0e6	6.8e7	2.0e6
1e−5	.	.	.	.	.	0	*18e-3*	*40e-4*	*23e-2*	1.0e6
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{13} in 5-D**, N=15, mFE=500059					**f_{13} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.3e3	4.1e2	4.5e3	2.3e3	15	1.6e4	8.6e3	2.3e4	1.6e4
1	15	3.6e4	5.7e3	7.8e4	3.6e4	15	1.5e5	4.0e4	3.7e5	1.5e5
1e−1	4	1.7e6	2.2e5	3.7e6	2.8e5	10	1.7e6	4.5e5	4.2e6	7.2e5
1e−3	0	*15e-2*	*71e-3*	*62e-2*	1.7e5	1	2.9e7	3.2e6	6.7e7	1.2e6
1e−5	.	.	.	.	.	0	*60e-3*	*93e-4*	*15e-2*	1.3e6
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{14} in 5-D**, N=15, mFE=500057					**f_{14} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.4e1	1.1e1	7.3e1	3.4e1	15	1.3e3	4.4e2	2.8e3	1.3e3
1	15	4.5e2	1.7e2	8.3e2	4.5e2	15	4.4e3	1.5e3	6.8e3	4.4e3
1e−1	15	1.1e3	7.1e2	1.5e3	1.1e3	15	8.6e3	6.4e3	1.1e4	8.6e3
1e−3	15	9.4e4	8.0e3	2.3e5	9.4e4	8	3.1e6	1.0e6	6.7e6	1.4e6
1e−5	0	*27e-5*	*11e-5*	*89e-5*	1.8e5	0	*10e-4*	*82e-5*	*13e-4*	1.8e6
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{15} in 5-D**, N=15, mFE=500055					**f_{15} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	7.5e3	1.4e3	8.3e3	7.5e3	0	*84e+0*	*69e+0*	*99e+0*	1.1e6
1	3	2.3e6	2.7e5	5.3e6	2.6e5	.	.	.	.	.
1e−1	0	*11e-1*	*15e-2*	*20e-1*	2.7e5	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{16} in 5-D**, N=15, mFE=500057					**f_{16} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.7e2	1.3e2	4.1e2	2.7e2	15	1.7e4	2.5e3	4.1e4	1.7e4
1	15	6.4e3	2.7e3	1.0e4	6.4e3	0	*39e-1*	*28e-1*	*45e-1*	1.2e6
1e−1	13	2.5e5	4.3e4	5.9e5	1.8e5	.	.	.	.	.
1e−3	0	*48e-3*	*17e-3*	*11e-2*	2.9e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{17} in 5-D**, N=15, mFE=500053					**f_{17} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.4e1	6.0e0	8.0e1	3.4e1	15	2.1e3	5.6e2	3.9e3	2.1e3
1	15	3.3e3	9.9e2	7.5e3	3.3e3	0	*40e-1*	*27e-1*	*44e-1*	1.2e6
1e−1	15	5.8e4	1.9e4	1.1e5	5.8e4	.	.	.	.	.
1e−3	0	*31e-3*	*10e-3*	*54e-3*	2.6e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{18} in 5-D**, N=15, mFE=500059					**f_{18} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	5.2e2	1.6e2	1.1e3	5.2e2	1	2.8e7	2.4e6	6.6e7	4.2e5
1	15	1.0e4	2.7e3	2.2e4	1.0e4	0	*13e+0*	*10e+0*	*15e+0*	8.9e5
1e−1	5	1.2e6	1.0e5	2.7e6	1.9e5	.	.	.	.	.
1e−3	0	*14e-2*	*53e-3*	*24e-2*	2.6e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{19} in 5-D**, N=15, mFE=500057					**f_{19} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.4e1	5.0e0	9.7e1	3.4e1	15	2.3e3	1.2e3	3.5e3	2.3e3
1	15	2.9e3	9.5e2	5.0e3	2.9e3	0	*37e-1*	*34e-1*	*40e-1*	1.5e6
1e−1	6	9.3e5	7.5e4	2.2e6	1.8e5	.	.	.	.	.
1e−3	0	*12e-2*	*34e-3*	*20e-2*	3.1e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{20} in 5-D**, N=15, mFE=466095					**f_{20} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.2e2	5.7e1	1.8e2	1.2e2	15	1.3e3	1.0e3	1.8e3	1.3e3
1	15	1.3e3	4.6e2	2.2e3	1.3e3	15	5.4e3	2.4e3	1.0e4	5.4e3
1e−1	15	2.1e4	4.1e3	4.4e4	2.1e4	0	*23e-2*	*18e-2*	*28e-2*	1.7e6
1e−3	15	3.2e4	1.1e4	5.4e4	3.2e4	.	.	.	.	.
1e−5	15	8.4e4	2.3e4	1.6e5	8.4e4	.	.	.	.	.
1e−8	15	2.1e5	4.6e4	4.5e5	2.1e5	.	.	.	.	.

Δf	**f_{21} in 5-D**, N=15, mFE=500057					**f_{21} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.3e2	6.3e1	2.2e2	1.3e2	15	2.8e3	9.8e2	4.4e3	2.8e3
1	15	2.1e3	5.8e2	5.5e3	2.1e3	15	1.4e5	4.7e3	6.6e5	1.4e5
1e−1	15	1.1e4	7.3e2	2.7e4	1.1e4	15	3.5e5	1.6e4	8.5e5	3.5e5
1e−3	15	2.2e4	2.7e3	4.8e4	2.2e4	15	3.9e5	4.4e4	9.1e5	3.9e5
1e−5	14	1.5e5	5.9e3	3.8e5	1.1e5	15	5.4e5	1.0e5	1.3e6	5.4e5
1e−8	8	6.2e5	6.2e4	1.6e6	1.8e5	8	2.3e6	2.7e5	6.2e6	5.6e5

Δf	**f_{22} in 5-D**, N=15, mFE=500058					**f_{22} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.6e2	4.9e1	8.3e2	3.6e2	15	4.9e3	2.6e3	8.0e3	4.9e3
1	15	2.9e3	2.6e2	8.9e3	2.9e3	15	2.6e5	6.0e3	7.6e5	2.6e5
1e−1	15	3.3e4	1.4e3	8.5e4	3.3e4	10	1.8e6	2.4e5	4.3e6	8.2e5
1e−3	10	3.8e5	3.2e4	1.0e6	1.3e5	0	*44e-3*	*56e-4*	*69e-2*	1.0e6
1e−5	2	3.4e6	2.9e5	8.3e6	1.9e5	.	.	.	.	.
1e−8	0	*45e-5*	*64e-7*	*17e-3*	1.9e5	.	.	.	.	.

Δf	**f_{23} in 5-D**, N=15, mFE=500054					**f_{23} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	6.5e0	1.0e0	1.4e1	6.5e0	15	3.6e0	2.0e0	5.0e0	3.6e0
1	15	1.0e4	2.0e3	1.8e4	1.0e4	15	1.3e5	2.5e4	2.4e5	1.3e5
1e−1	0	*37e-2*	*27e-2*	*42e-2*	1.5e5	0	*64e-2*	*54e-2*	*79e-2*	9.6e5
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{24} in 5-D**, N=15, mFE=500059					**f_{24} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.2e4	9.1e3	6.7e4	2.2e4	0	*12e+1*	*92e+0*	*14e+1*	1.3e6
1	0	*42e-1*	*25e-1*	*56e-1*	2.4e5	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 1: Shown are, for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\mathrm{opt}}+\Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value ($\mathrm{RT_{succ}}$). If $f_{\mathrm{opt}}+\Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

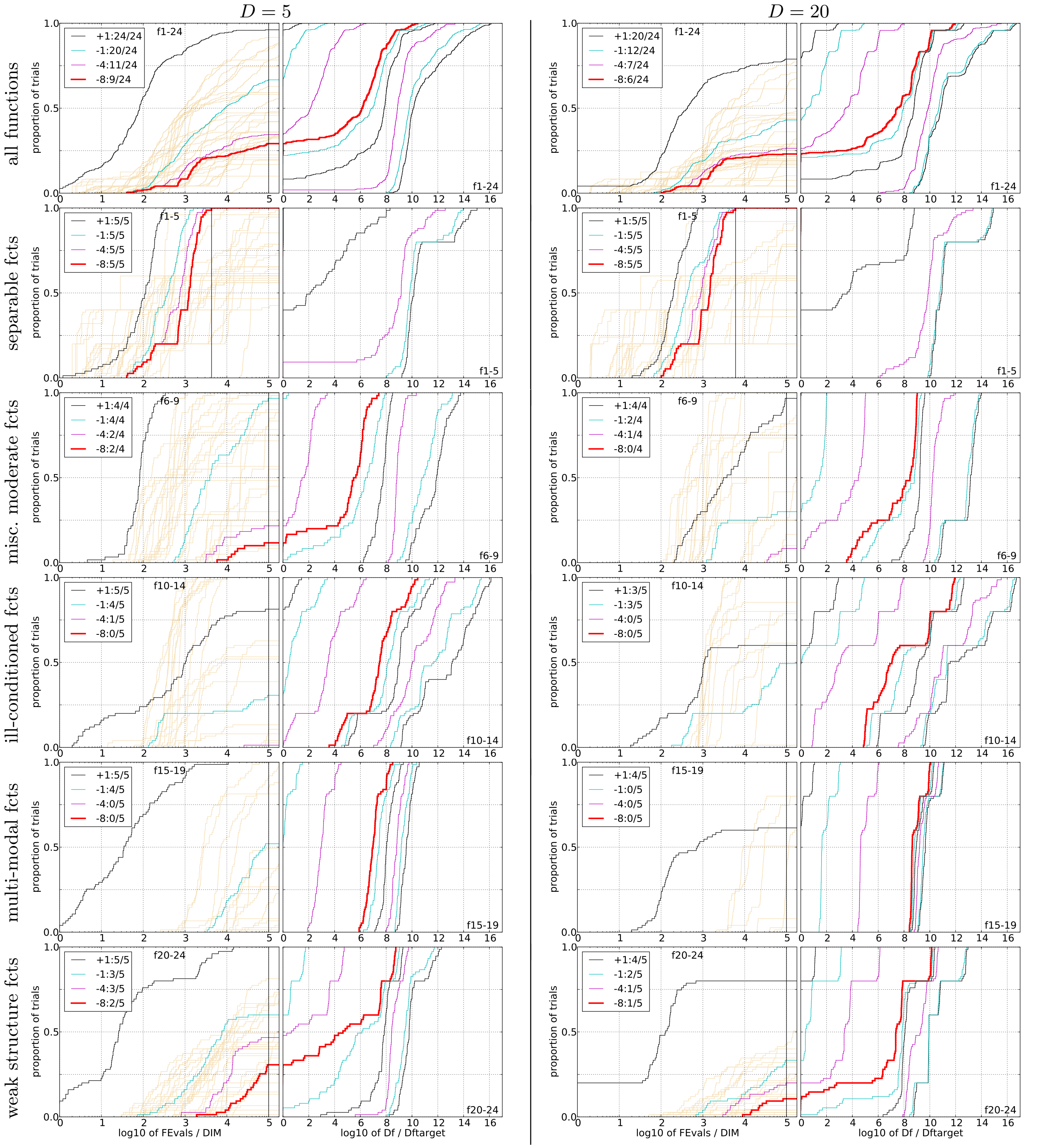

Figure 2: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\rm opt} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10\,D, 100\,D \ldots$ function evaluations (from right to left cycling black-cyan-magenta). The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value. Light brown lines in the background show ECDFs for target value 10^{-8} of all algorithms benchmarked during BBOB-2009.

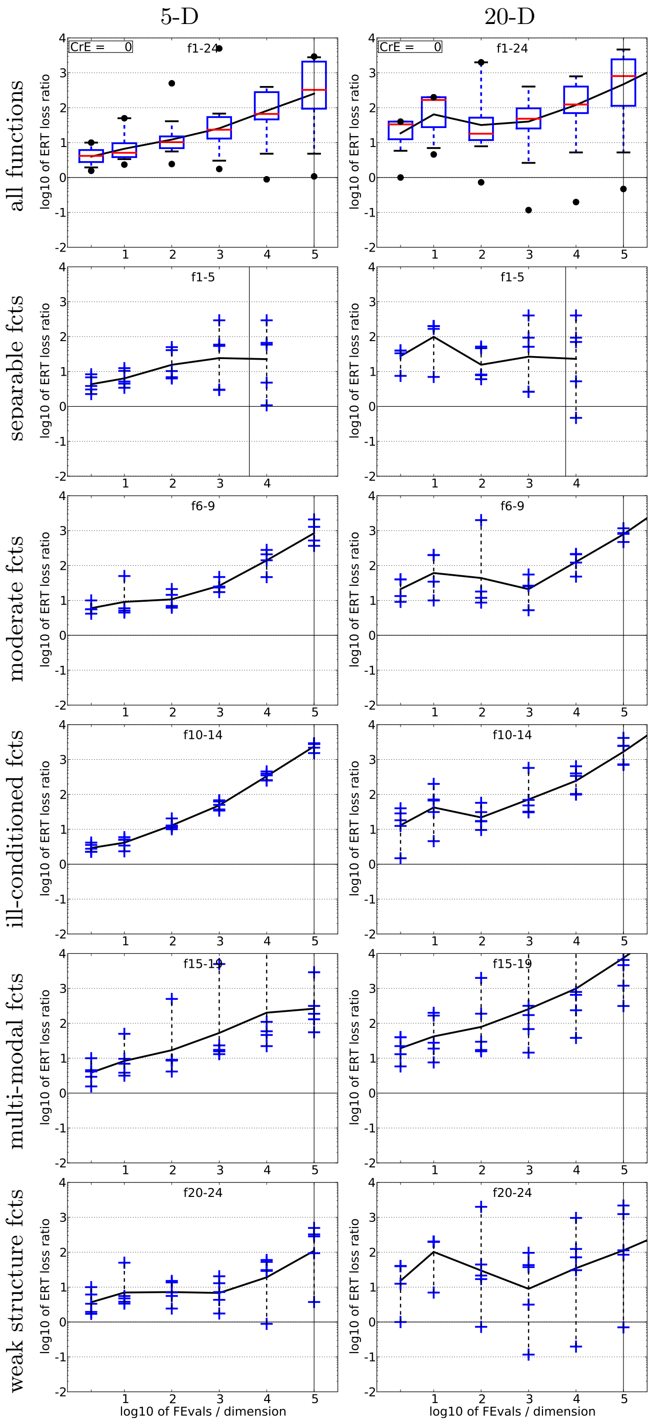

Figure 3: ERT loss ratio versus given budget FEvals. The target value $f_{\rm t}$ for ERT (see Figure 1) is the smallest (best) recorded function value such that $\mathrm{ERT}(f_{\rm t}) \leq \mathrm{FEvals}$ for the presented algorithm. Shown is FEvals divided by the respective best $\mathrm{ERT}(f_{\rm t})$ from BBOB-2009 for functions f_1–f_{24} in 5-D and 20-D. Each ERT is multiplied by $\exp(\mathrm{CrE})$ correcting for the parameter crafting effort. Line: geometric mean. Box-Whisker error bar: 25-75%-ile with median (box), 10-90%-ile (caps), and minimum and maximum ERT loss ratio (points). The vertical line gives the maximal number of function evaluations in this function subset.

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