Example paper: Black-Box Optimization Benchmarking Template for the Comparison of More than Two Algorithms on the Noiseless Testbed

Draft version *

BBOBies

ABSTRACT

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—global optimization, unconstrained optimization; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization

1. RESULTS

Results from experiments according to [?] on the benchmark functions given in [?, ?] are presented in Figures 1, 2 and 3 and in Tables 1 and 2. The expected running time (ERT), used in the figures and table, depends on a given target function value, $f_{\rm t} = f_{\rm opt} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach $f_{\rm t}$, summed over all trials and divided by the number of trials that actually reached f_t [?, ?]. Statistical signifi**cance** is tested with the rank-sum test for a given target Δf_{t} $(10^{-8} \text{ as in Figure 1}) \text{ using, for each trial, either the number}$ of needed function evaluations to reach $\Delta f_{\rm t}$ (inverted and multiplied by -1), or, if the target was not reached, the best Δf -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

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^{*}Submission deadline: March 28th.

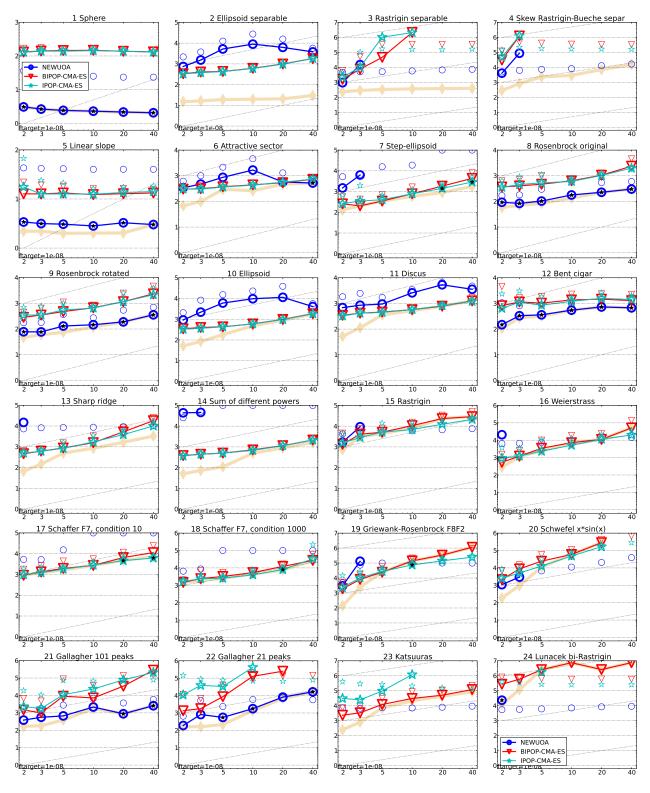


Figure 1: Expected running time (ERT in number of f-evaluations) divided by dimension for target function value 10^{-8} as \log_{10} values versus dimension. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24} . Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Horizontal lines give linear scaling, slanted dotted lines give quadratic scaling. Black stars indicate statistically better result compared to all other algorithms with p < 0.01 and Bonferroni correction number of dimensions (six). Legend: \circ : NEWUOA, ∇ : BIPOP-CMA-ES, \star : IPOP-CMA-ES.

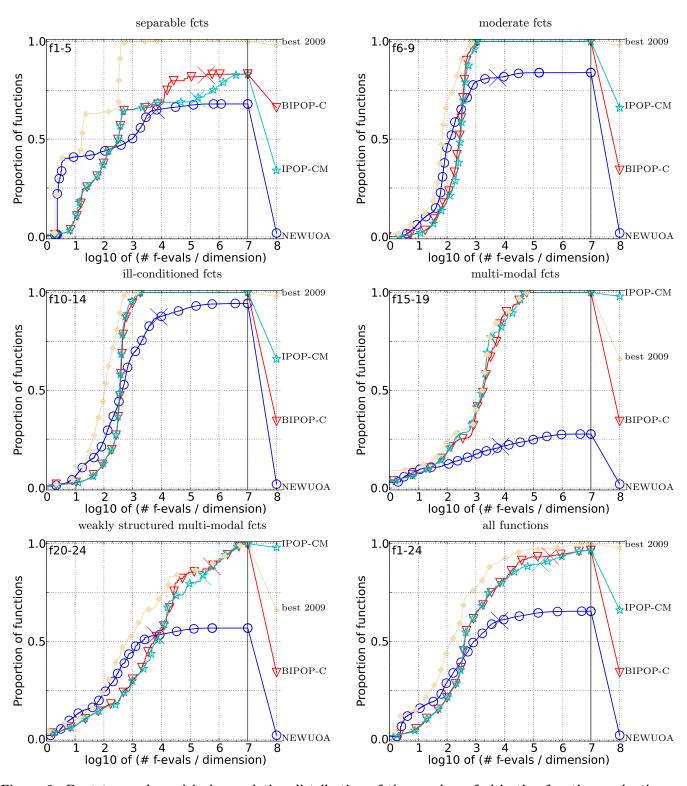


Figure 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

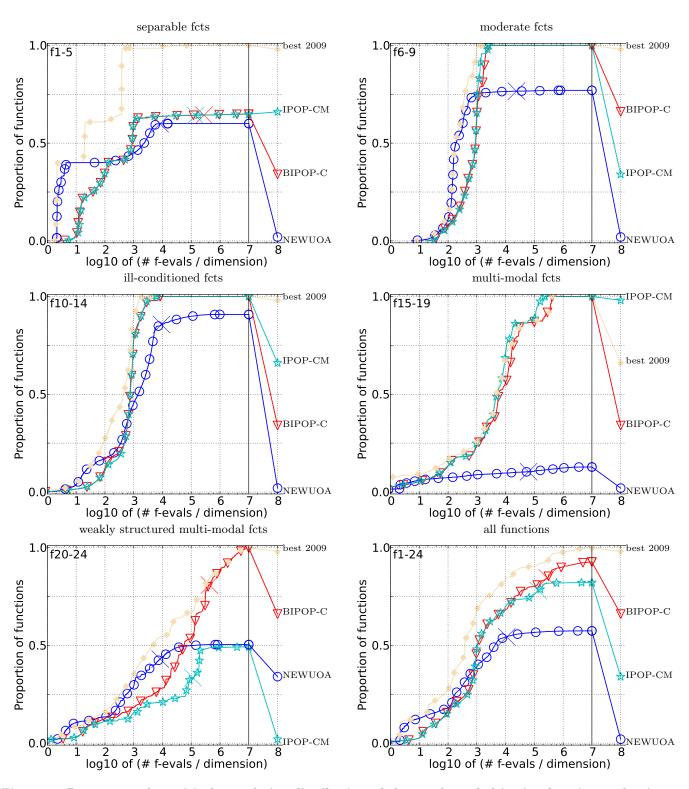


Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt} 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\# \operatorname{succ} \Delta f_{\operatorname{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
$\frac{\Delta f_{\mathrm{opt}}}{\mathbf{f1}}$ 1e1	12	12	12	12	12	15/15 f13	132	195	250	1310	1752	2255	15/15
NEWUOA 1.1(0)	1(0)*4	1(0)*4	1(0)*4	1(0)*4	1(0)*4	15/1NEWUOA		9.3(12)	35(36)	54(54)	335(373)	∞ 4e4	0/15
BIPOP-C 3.2(2)	9.0(4)	15(4)	27(5)	40(4)	53(6)	15/1BIPOP-C		5.4(3)	5.9(3)	1.6(0.3)	1.5(0.2)	1.7(0.8)	15/15
IPOP-CM $ 2.5(2)$	8.0(3)	14(3)	27(3)	39(3)	51(3)	15/1POP-CM		5.0(2)	5.3 (3)	1.4(0.6)	1.6(0.3)	1.6(0.6)	15/15
$\frac{\Delta f_{\text{opt}}}{\mathbf{f2}}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\# \operatorname{succ} \Delta f_{\operatorname{opt}}$		1e0	1e-1			e-7	#succ
	87	88	90	92	94	15/15 f14	10	41	58			76	15/15
NEWUOA 5.7(4)*		45(30)	85(32)	129(33)	166(54)	15/1NEWUOA		1(0.4)*3	1(0.3)*4	1.2(0.3)*4		525(2961)	0/15
BIPOP-C 13(4) IPOP-CM 14(4)	16(3) 16(4)	18(2) 18(2)	20(2) 19(2)	21(2) 21(2)	22(2) 22 (2)	15/18IPOP-C 15/18POP-CM		2.8(1) 2.9(2)	3.7(0.9) $3.8(1)$		5.4 (0.5) 5.4(0.7)	4.5(0.3) $4.4(0.5)$	$\frac{15}{15}$
	10(4) 1e0	1e-1	1e-3	1e-5	1e-7	1 /	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
$\frac{\Delta f_{\text{opt}}}{\mathbf{f3}}$ 1e1	1622	1637	1646	1650	1654	15/15 f15	511	9310	19369	20073	20769	21359	14/15
NEWUOA 6.1(8)	229(246)	∞	∞	∞	∞ 3e4	0/18EWUOA		41(45)	∞	∞	∞	∞ 3e4	0/15
BIPOP-C 1.4(1)	16(17)	139(107)	139(107)	139(107)	140 (107)	14/1BIPOP-C		1.5(1)	1.2(0.7)	1.2(0.7)	1.2(0.7)	1.2(0.7)	15/15
IPOP-CM $2.2(4)$	70(138)	3130(3633)	3113(3795)	3106(4028)	3099(3614)	2/1POP-CM		1.3(0.8)	1.2(1)	1.2(1)	1.2(1)	1.2(1)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7		1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f4 809	1633	1688	1817	1886	1903	15/15 f16	120	612 29(23)	2662	10449	11644	12095	15/15 0/15
NEWUOA 27(25) BIPOP-C 2.7(3)	305(33		∞	∞	∞ 3e4 ∞ 2e6	0/1NEWUOA 0/1BIPOP-C		3.6(3)	$_{2.6(1)}^{\infty}$	∞ $1.3(2)$	∞ 1.4(2)	$\infty 4e4$ 1.4(2)	15/15
IPOP-CM 2.0 (3)	∞ ∞	∞	∞ ∞	∞ ∞	∞ 2e0 ∞ 9e5	0/19POP-CM		2.3(3)	1.7(1)	0.96(0.9)		0.95(0.7)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\#\operatorname{succ}\Delta f_{\mathrm{opt}}$		1e0	1e-1	1e-3	1e-5	1e-7	#succ
f5 10	10	10	10	10	10	15/15 f17	5.2	215	899	3669	6351	7934	15/15
NEWUOA 1.3(0.1)						15/1NEWUOA	2.3(2)	40(47)	617(689)	∞	∞	∞ 3e4	0/15
BIPOP-C 4.5(2)	6.5(3)	6.6(2)	6.6(2)	6.6(2)	6.6(2)	15/1BIPOP-C	3.4(3)	1(0.2)	1(2)	1(0.7)	1(0.5)	1.2(0.5)	15/15
IPOP-CM $4.6(2)$	6.0(3)	6.3(3)	6.3(3)	6.3(3)	6.3(3)	15/19 OP-CM	4.8(6)	1.1(0.5)	0.97(2)	0.77(1)	0.81 (0.7)	1.0(0.4)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\# \operatorname{succ} \Delta f_{\operatorname{opt}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f6 114	214	281	580	1038	1332	15/15 f18	103	378	3968	9280	10905	12469	15/15
NEWUOA 1.7(2)	2.4(1)	3.6(2)	3.3(2)	2.7(1)	2.9(1)	I _{15/1} NEWUOA	31(28)	1351(1817		∞	∞	∞ 9e4	0/15
BIPOP-C 2.3(1)	2.1(0.6)	2.2(0.6)	1.7(0.2)	1.3(0.3)	1.3(0.2)	15/18IPOP-C 15/18POP-CM	1 2(0.7)	3.4(5) 2.7(6)	1(1) 0.87 (1	1(0.3) 1.0(0.4)	1.2(0.7) 1.0(0.3)	1.3(0.6) 0.99 (0.1)	$\frac{15}{15}$
IPOP-CM 2.5(0.9)	2.1(0.6)	2.2 (0.4)	1.7(0.2)	1.3(0.2)	1.2(0.1)			1e0	1e-1	1.0(0.4)	1e-5	1e-7	#succ
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\# \operatorname{succ}^{\Delta f_{\text{opt}}}$ 15/15 f19	1e1	1	242	1.2e5		1.2e5	15/15
f7 24 NEWUOA 10(15)	324 13(18)	1171	1572	1572	1597 ∞ 3e4	15/15 119 0/19EWUOA		2.7e4(2e4			1.2es ∞	1.2e3 ∞ 5e5	0/15
BIPOP-C 5.0(5)	1.5(18)	60(59) 1(1)	∞ 1(0.9)	1 (0.9)	∞ 3e4 1(0.9)	15/18IPOP-C	20(16)	2801(507				1(0.7)	15/15
IPOP-CM 4.4(3)	1.7(1)	1.2(0.9)		1.2(0.6)	1.2(0.6)	15/14POP-CM	21(25)	1720 (176)	0) 125 (94	1.1(0	.7) 1.1(0.7)	1.1(0.7)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	I#succ \Delta font	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f8 73	273	336	391	410	422	15/15 f20	16	851	38111	54470	54861	55313	14/15
NEWUOA1(0.9)*3	1.1(0.8)	*2 1.2(0.5)*		4 1.2 (0.4)*4	1.2(0.4)*4	15/1NEWUOA	1(0.2)*2	3.3(4)	∞	∞	∞	∞ 3e4	0/15
BIPOP-C 3.2(1)	3.7(2)	4.5(2)	4.8(2)	5.1(2)	5.4(2)	15/1BIPOP-C	3.3(3)	8.2(10)	2.8(1)	2.1(0.8)	2.2(0.8)	2.2(0.8)	15/15
IPOP-CM $3.5(2)$	4.8(5)	5.3(4)	5.6(4)	5.8(3)	6.1(3)	15/1FOP-CM		11(11)	1.4 (0.9)	1.1(0.7)	1.1(0.7)	1.1(0.7)	15/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	$\# \operatorname{succ}^{\Delta f_{\mathrm{opt}}}$	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f9 35	127	214	300	335	369	15/15 f21	41	1157 2.2(3)	1674	1705	1729	1757 1.9(2)	14/15 15/15
NEWUOA 1.8(0.7)		$2.5(2)^{*2}$	1.9(1)*3	$1.9(1)^{*3}$	1.7(0.9)*4	15/15/EWUOA	2 3(2)	2.2(3) 14(9)	1.8(2) 24(35)	1.8(2) 25(36)	1.8(2) 25(36)	25(36)	15/15
BIPOP-C 5.8(2)	8.7(3)	7.2(2)	6.4(2)	6.3(1)	6.2(1)	15/15 POP-CM	6.3(17)	5.6(6)	30(34)	31(38)	31(41)	31(40)	14/15
IPOP-CM 6.0(2)	11(10)	8.7(6)	7.5(5)	7.3(4)	7.2(4)		1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ - 70pt		386	938	1008	1040	1068	14/15
f10 349 NEWUOA3.1(3)	500 5.5(4)	574 8.1(7)	626 14(9)	829 16(8)	880 21(8)	110/12	0 4 (0)	2.1(2)	2.0(3)*	2.1(3)*	2.3(3)	2.4(3)	15/15
BIPOP-C 3.5(0.8)	2.9(0.4)	2.7(0.4)	2.8(0.2)	2.3(0.2)	2.4(0.1)	15/18IPOP-C	6.9(11)	20(14)	45(94)	42(88)	41(85)	40(83)	15/15
IPOP-CM 3.6(1.0)	2.9(0.5)	2.7(0.3)	2.8(0.3)	2.3(0.2)	2.3 (0.2)	15/15 OP-CM	12(26)	48(77)	166(219)	161(220)	158(211)	155(196)	11/15
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7		1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f11 143	202	763	1177	1467	1673	15/15 f23 NEWUOA 15/15 DOD C	3.0	518	14249	31654	33030	34256	15/15
NEWUOA 3.5(2)*3	4.7(2)*	1.8(0.6)	1.8(0.4)	2.0(0.3)	2.2(0.4)	NEWUOA 15/15IPOP-C	0.2(4)	2.4(2) 13(15)	7.1(8) $3.7(4)$	$\infty \\ 1.8(2)$	∞ 1.8(2)	$\infty 3e4$ 1.8(2)	0/15 15/15
BIPOP-C 8.4(3)	7.2(2)	2.2(0.3)	1.6(0.2)	1.4(0.1)	1.3(0.1)		2.2(2)	26(32)	33(47)	15(21)	14(20)	14(20)	11/15
IPOP-CM 8.6(2)	7.3(1)	2.1(0.2)	1.6(0.1)	1.4(0.1)	1.3(0.1)	$ 15/15 $ $\# \operatorname{suec} \Delta f_{\text{opt}}$	() 1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
$\Delta f_{ m opt}$ 1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ Dopt f24	1622	2.2e5	6.4e6	9.6e6	1.3e7	1.3e7	3/15
f12 108	268	371	461	1303	1494	15/15 EWILOA	20(2)	2.1(2)	∞	∞	∞	∞ 3e4	0/15
		a = (a) ±	0 0 (0) +	(-)+								C 504	
NEWUOA 3.5(3)	2.6 (3)	2.5(2)*	2.6(2)*	1.1(1)*	1.1(1.0)	Fo FRIDOR-C	19 1 (2)	1.6(3)	1 (1.0)	1(1.0)	1(1)	1(1)	3/15
		2.5(2)* 7.4(6) 6.2(4)	2.6(2)* 7.7(5) 6.3(4)	1.1(1)* 3.3(2) 2.8(2)	1.1(1.0) 3.3(2) 2.8(2)	15/15IPOP-C 15/17POP-CM 15/15	19 1 (2)						

Table 1: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 5. The inter-80%tile range divided by two is given in braces. The median number of conducted function evaluations is additionally given in italics, if $\mathrm{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\mathrm{opt}} + 10^{-8}$. Best results are printed in bold.

$ \begin{array}{c c} \Delta f_{\text{opt}} & \text{1e1} \\ \hline \mathbf{f1} & 43 \end{array} $	1e0	1e-1	1e-3	1e-5	1e-7	$\#\operatorname{succ}\Delta f_{\operatorname{opt}}$		1e0	1e-1	1e-3	1e-5	1e-7	#succ
	43	43	43	43	43	15/15 f13	652	2021	2751	18749	24455	30201	15/15
NEWUOA 1.0(0.0)*4								3.0(5)	9.3(12)	19(21)	∞	∞ 2e5	0/15
BIPOP-C 7.9(2) IPOP-CM 8.0(1)	14(3) 14(2)	20(2) 20(2)	33(4) 33(1)	45(3) 46(2)	57(3) 58(2)	15/1 B IPOP-C 15/1 B POP-CM		2.7(2) 4.8(5)	5.1 (6) 6.2(5)	1.5(0.8) 1.4(0.8)	2.3(2) 1.7(0.8)	3.0(2) 2.3(1)	$\frac{15}{15}$ $\frac{15}{15}$
	1e0		1e-3	1e-5	1e-7			1e0	1e-1	1.4(0.8) 1e-3	1e-5	1e-7	#succ
$\frac{\Delta f_{\mathrm{opt}}}{\mathbf{f2}}$ 1e1	386		390	391	393	$\# \operatorname{succ} \Delta f_{\text{opt}}$ 15/15 f14	75	239	304	932	1648	15661	15/15
NEWUOA18(8)*3	42(21)		125(43)	174(51)	219(67)	15/18EWUOA			1(0.3)*4	1(0.2)*4		43(32)	0/15
BIPOP-C 35(7)	40(4)	44(4)	47(2)	48(2)	50(2)	15/1BIPOP-C	3.9(1)	2.9(0.4)	3.7(0.4)	4.1(0.3)	6.2(0.5)	1.2(0.1)	15/15
IPOP-CM 35(4)	41(4)	43(3)	45(3)	47 (2)	48(2)	15/1 B POP-CM	3.7(2)	2.8(0.5)	3.6(0.5)	3.9(0.5)	6.0(0.6)	1.2(0.1)	15/15
$\frac{\Delta f_{\text{opt}}}{\mathbf{f3}}$ 1e1 5066	1e0	1e-1	1e-3	1e-5	1e-7	$\# \operatorname{succ} \Delta f_{\operatorname{opt}}$		1e0	1e-1	1e-3	1e-5	1e-7	#succ
f3 5066 NEWUOA ∞	7626 ∞	7635 ∞	7643 ∞	7646 ∞	7651 ∞ 1e5	15/15 f15 0/18EWUOA	30378	1.5e5 ∞	3.1e5 ∞	3.2e5 ∞	4.5e5 ∞	4.6e5 ∞ 1e5	15/15 0/15
BIPOP-C 12(7)	∞	∞	∞	∞	∞ 1e5 ∞ 6e6	0/1BIPOP-C		2.0(0.8)	1.4(0.5)	1.4(0.5)	1(0.3)	1(0.3)	15/15
IPOP-CM 13(6)	∞	∞	∞	∞	∞ 3e6	0/15POP-CM		1.1(0.4)	0.69(0.4)	0.70 (0.4)		20.53(0.3) ₁	
$ \begin{array}{c c} \Delta f_{\text{opt}} & 1 \text{e} 1 \\ \hline \mathbf{f4} & 4722 \end{array} $	1e0	1e-1	1e-3	1e-5	1e-7	$\# \operatorname{succ} \Delta f_{\mathrm{opt}}$ 9/15 f16	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f4 4722 NEWUOA ∞	7628	7666	7700	7758	1.4e5	9/15 f16	1384	27265	77015	1.9e5	2.0e5	2.2e5	15/15
NEWUOA ∞ BIPOP-C ∞	∞ ∞	∞	∞ ∞	∞	∞ 2e5 ∞ 6e6	0/15NEWUOA 0/15BIPOP-C	16(17)	∞	∞ 1.0(0.7)	∞ 1 (0.57)	∞ 1(0.7)	∞ 2e5	0/15
IPOP-CM ∞	∞	∞	∞	∞	∞ 3e6	0/15IPOP-CM		1.0(0.7) 0.81 (0.6)	1.2(0.7) 0.92 (0.6)	1(0.7) 0.84(0.4)	1(0.7) 1.1(0.7)	1(0.7) 1.0(0.6)	$\frac{15}{15}$
	1e0	1e-1	1e-3	1e-5	1e-7	$\#_{\text{succ}}\Delta f_{\text{opt}}$		1e0	1e-1	1e-3	1e-5	1e-7	#succ
$\frac{\Delta f_{\text{opt}}}{\text{f5}}$ 1e1	41	41	41	41	41	15/15 f17	63	1030	4005	30677	56288	80472	15/15
NEWUOA 1.2(0.1)*4	1.5(0.4)*4	$1.6(0.5)^{*4}$	1.6(0.5)*4	1.6(0.5)*4	$1.6(0.5)^{*4}$	15/1NEWUOA		∞	∞	∞	∞	∞ 2e6	0/15
BIPOP-C 5.1(0.8)	6.2(1)	6.3(1)	6.3(1)	6.3(1)	6.3(1)	15/1BIPOP-C		1(0.3)	1 (1)	1.2(1)	1.3(0.6)	1.4(0.7)	15/15
IPOP-CM $ 5.8(1)$	6.5(1)	6.7(1.0)	6.7(1.0)	6.7(1.0)	6.7(1.0)	15/1 B POP-CM	2.1(1)	0.94 (0.3)	1.2(2)	0.76 (0.6)	0.99 (0.3)	1.0(0.7)	15/15
$\frac{\Delta f_{ m opt}}{{ m f6}} \frac{1 { m e1}}{1296}$	1e0	1e-1	1e-3	1e-5	1e-7		1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
	2343	3413	5220	6728	8409	15/15 f18	621	3972	19561	67569	1.3e5	1.5e5	15/15
NEWUOA 1 (0.3)*2 BIPOP-C 1.5 (0.4)	1(0.4)	1(0.5)	1.1(0.5)	1.3(0.8)	1.3(0.7) 1.2(0.1)	15/1SEWUOA 15/1BIPOP-C		0.00 0.00	∞ 1.2(0.9)	∞ 1.1(0.6)	∞ 1.7(0.7)	$\infty 2e6$ 1.6(0.6)	0/15 15/15
IPOP-CM 1.7(0.3)	1.3(0.2) 1.3(0.1)	1.2(0.2) 1.2(0.1)	1.1(0.2) 1.2(0.1)	1.2(0.1) 1.2(0.1)	1.2(0.1) $1.2(0.1)$	15/15POP-CM		1.8(2)	1.1(0.6)	0.97 (0.7)	1.0(0.4)*2	1.1(0.4)*2	15/15
	1e0	1e-1	1e-3	1e-5	1e-7	1 /	! ` ′	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f7 1351						#succ∆f _{opt}							
$\frac{\Delta f_{ m opt}}{{ m f7}} \begin{array}{ c c } 1{ m e}1 \\ \hline 1351 \\ { m NEWUOA} \\ \infty \end{array}$	4274 ∞	9503 ∞	16524 ∞	16524 ∞	16969 ∞ 5e5	15/15 f19	1	1 4.3e6(5e	3.4e5	6.2e6 ∞	6.7e6 ∞	6.7e6 ∞ 2e6	15/15 0/15
NEWUOA ∞ BIPOP-C $1(0.5)$	4274 ∞ $4.9(2)$	9503 ∞ $3.5(0.6)$	16524 ∞ $2.2(0.3)$	16524 ∞ $2.2(0.3)$	16969 $\infty 5e5$ $2.1(0.3)$	15/15 f19 0/18EWUOA 15/18IPOP-C	1 76 (50)* 169(74)	1 4.3e6(5e 2.4e4 (1e	$3.4e5$ $6) \infty$ $e4)1.2(0.6)$	6.2e6 ∞ 1(0.3)	6.7e6 ∞ 1(0.3)	$6.7e6$ $\infty 2e6$ $1(0.3)$	15/15 0/15 15/15
NEWUOA ∞ BIPOP-C $1(0.5)$ IPOP-CM $1.9(2)$	4274 ∞ $4.9(2)$ $4.8(2)$	9503 ∞ $3.5(0.6)$ 2.7 (2)	16524 ∞ $2.2(0.3)$ $1.7(1.0)^*$	16524 ∞ $2.2(0.3)$ $1.7(1.0)*$	16969 $\infty 5e5$ $2.1(0.3)$ $1.6(0.9)^*$	15/15 f19 0/18EWUOA	1 76 (50)* 169(74)	1 4.3e6(5e 2.4e4 (1e	3.4e5 6) ∞	6.2e6 ∞ 1(0.3)	6.7e6 ∞	$6.7e6$ $\infty 2e6$ $1(0.3)$	15/15 0/15 15/15
NEWUOA ∞ BIPOP-C $1(0.5)$ IPOP-CM $1.9(2)$	4274 ∞ $4.9(2)$ $4.8(2)$ $1e0$	9503 ∞ $3.5(0.6)$ 2.7 (2) 1e-1	16524 ∞ $2.2(0.3)$ $1.7(1.0)^{\star}$ $1e-3$	16524 ∞ $2.2(0.3)$ $1.7(1.0)^{\star}$ $1e-5$	16969 $\infty 5e5$ $2.1(0.3)$ $1.6(0.9)$ * $1e-7$	15/15 f19 0/1 S EWUOA 15/1 B IPOP-C 15/1 B POP-CM	1 76(50)* 169(74) 161(86)	1 4.3e6(5e 2.4e4 (1e 2.7e4(2e	$3.4e5$ $6) \infty$ $e4)1.2(0.6)$ $4)0.71(0.5)$ $1e-1$	6.2e6 ∞ 1(0.3) 0.38 (0.1) 1e-3	6.7e6 ∞ 1(0.3)	$6.7e6$ $\infty 2e6$ $1(0.3)$	15/15 0/15 15/15
$\begin{array}{c c} \text{NEWUOA} & \infty \\ \text{BIPOP-C} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline & \Delta f_{\text{opt}} & 1 \mathbf{e} 1 \\ \hline & \mathbf{f8} & 2039 \\ \end{array}$	4274 ∞ $4.9(2)$ $4.8(2)$ $1e0$ 3871	9503 ∞ $3.5(0.6)$ $2.7(2)$ $1e-1$ 4040	16524 ∞ $2.2(0.3)$ $1.7(1.0)$ * $1e-3$ 4219	16524 ∞ $2.2(0.3)$ $1.7(1.0)$ * $1e-5$ 4371	16969 $\infty 5e5$ $2.1(0.3)$ $1.6(0.9)$ * $1e-7$ 4484	$15/15$ f19 0/18EWUOA 15/18IPOP-C 15/18POP-CM #succ Δf_{opt} 15/15 f20	1 76(50)* 169(74) 161(86) 1e1	1 4.3e6(5e 2.4e4 (1e 2.7e4(2e 1e0 46150	$3.4e5$ $6) \infty$ $e4)1.2(0.6)$ $4)0.71(0.5)$ $1e-1$ $3.1e6$	$6.2e6$ ∞ $1(0.3)$ $0.38(0.1)$	$6.7e6$ ∞ $1(0.3)$ $\downarrow 40.41(0.2)$	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 40.41(0.2) \\ \downarrow \\ 1e-7 \\ 5.6e6 \end{array}$	15/15 0/15 15/15 15/15 415/15 #succ 14/15
$ \begin{array}{c c} {\rm NEWUOA} & \infty \\ {\rm BIPOP-C} & 1 (0.5) \\ {\rm IPOP-CM} & 1.9 (2) \\ \hline & \Delta f_{\rm opt} & 1e1 \\ \hline & {\rm f8} & 2039 \\ {\rm NEWUOA} & 1 (0.3)^{\star 4} \\ \end{array} $	4274 ∞ $4.9(2)$ $4.8(2)$ $1e0$ 3871 $1(0.6)^{*4}$	9503 ∞ $3.5(0.6)$ 2.7 (2) $1e-1$ 4040 $1(0.6)^{*4}$	16524 ∞ $2.2(0.3)$ $1.7(1.0)^*$ $1e-3$ 4219 $1(0.5)^{*4}$	16524 ∞ $2.2(0.3)$ $1.7(1.0)^*$ $1e-5$ 4371 $1(0.5)^{*4}$	16969 $\infty 5e5$ $2.1(0.3)$ $1.6(0.9)^*$ $1e-7$ 4484 $1(0.5)^*4$	15/15 f19 0/INSEWUOA 15/IBIPOP-C 15/IEPOP-CM #succ Δ fopt 15/15 f20 15/1NEWUOA	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4	1 4.3e6(5e 2.4e4 (1e 2.7e4(2e 1e0 46150 15(17)	$3.4e5$ $6) \infty$ $e4)1.2(0.6)$ $4)0.71(0.5)$ $1e-1$ $3.1e6$ ∞	6.2e6 ∞ 1(0.3) 0.38(0.1) 1e-3 5.5e6 ∞	$ \begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ \downarrow 0.41(0.2) \\ 1e-5 \\ \hline 5.6e6 \\ \infty \end{array} $	$6.7e6$ $\infty 2e6$ $1(0.3)$ $40.41(0.2)$ $1e-7$ $5.6e6$ $\infty 4e5$	15/15 0/15 15/15 415/15 #succ 14/15 0/15
$\begin{array}{l} {\rm NEWUOA} \\ {\rm BIPOP-C} \ 1 (0.5) \\ {\rm IPOP-CM} \ 1.9 (2) \\ \hline \Delta f_{\rm opt} & 1 {\rm e} 1 \\ \hline {\rm f8} & 2039 \\ {\rm NEWUOA} 1 (0.3)^{*4} \\ {\rm BIPOP-C} \ 4.0 (1) \\ \end{array}$	4274 ∞ $4.9(2)$ $4.8(2)$ $1e0$ 3871 $1(0.6)^{*4}$ $4.0(0.7)$	9503 ∞ $3.5(0.6)$ $2.7(2)$ $1e-1$ 4040 $1(0.6)^{*4}$ $4.3(0.6)$	16524 ∞ $2.2(0.3)$ $1.7(1.0)^*$ $1e-3$ 4219 $1(0.5)^{*4}$ $4.5(0.6)$	16524 ∞ $2.2(0.3)$ $1.7(1.0)^*$ $1e-5$ 4371 $1(0.5)^*4$ $4.6(0.6)$	16969 $\infty 5e5$ 2.1(0.3) $1.6(0.9)^*$ 1e-7 4484 $1(0.5)^{*4}$ 4.6(0.6)	$\begin{array}{cccc} 15/15 & \mathbf{f19} \\ 0/18 & \mathrm{EWUOA} \\ 15/18 & \mathrm{IPOP-CM} \\ 15/18 & \mathrm{OP-CM} \\ \# & \mathrm{succ} \Delta f_{\mathrm{opt}} \\ 15/15 & \mathbf{f20} \\ 15/18 & \mathrm{EWUOA} \\ 15/18 & \mathrm{IPOP-C} \end{array}$	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1)	1 4.3e6(5e 2.4e4 (16 2.7e4(2e 1e0 46150 15(17) 9.2(4)	$3.4e5$ $6) \infty$ $e4)1.2(0.6)$ $4)0.71(0.5)$ $1e-1$ $3.1e6$ ∞ $1(0.5)$	$6.2e6$ ∞ $1(0.3)$ $0.38(0.1)$ $1e-3$ $5.5e6$ ∞ $1(0.3)$	$ \begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ \bullet .41(0.2) \\ 1e-5 \\ \hline 5.6e6 \\ \infty \\ 1(0.3) \end{array} $	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 40.41(0.2) \downarrow \\ 1e-7 \\ \hline 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \end{array}$	15/15 0/15 15/15 15/15 415/15 #succ 14/15 0/15 14/15
$ \begin{array}{lll} {\rm NEWUOA} & \\ {\rm NEWUOA} & \\ {\rm BIPOP-C} & 1(0.5) \\ {\rm IPOP-CM} & 1.9(2) \\ \\ \hline & \mathbf{f8} & 2039 \\ {\rm NEWUOAI} & (0.3)^{*4} \\ {\rm BIPOP-C} & 4.0(1) \\ {\rm IPOP-CM} & 3.7(0.9) \\ \end{array} $	4274 ∞ $4.9(2)$ $4.8(2)$ $1e0$ 3871 $1(0.6)^{*4}$ $4.0(0.7)$ $3.9(0.5)$	$\begin{array}{c} 9503 \\ \infty \\ 3.5(0.6) \\ \textbf{2.7}(2) \\ 1\text{e-1} \\ 4040 \\ \textbf{1}(0.6)^{\star 4} \\ 4.3(0.6) \\ 4.2(0.5) \end{array}$	16524 ∞ $2.2(0.3)$ $1.7(1.0)^*$ $1e-3$ 4219 $1(0.5)^*4$ $4.5(0.6)$ $4.4(0.4)$	$ \begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e-5 \\ 4371 \\ 1(0.5)^{*4} \\ 4.6(0.6) \\ 4.4(0.4) \end{array} $	16969 $\infty 5e5$ $2.1(0.3)$ $1.6(0.9)^*$ $1e-7$ 4484 $1(0.5)^*4$ $4.6(0.6)$ $4.5(0.4)$	15/15 f19 0/INEWUOA 15/IBIPOP-CM 15/IBPOP-CM #succ Δ f _{opt} 15/15 f20 15/INEWUOA 15/INIPOP-C 15/INPOP-CM	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 4.6(1)	1 4.3e6(5e 2.4e4 (1e 2.7e4(2e 1e0 46150 15(17) 9.2(4) 6.4 (2)	$3.4e5$ $6) \infty$ $e4)1.2(0.6)$ $4)0.71(0.5)$ $1e-1$ $3.1e6$ ∞ $1(0.5)$ $0.65(0.3)$	$6.2e6$ ∞ $1(0.3)$ $0.38(0.1)$ $1e-3$ $5.5e6$ ∞ $1(0.3)$ $0.57(0.2)$	$ \begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ \bullet 0.41(0.2) \\ 1e-5 \\ \hline 5.6e6 \\ \infty \\ 1(0.3) \\ 0.58(0.2) \end{array} $	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 40.41(0.2) \downarrow \\ 1e-7 \\ \hline 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \end{array}$	15/15 0/15 15/15 15/15 415/15 #succ 14/15 0/15 14/15 15/15
$ \begin{array}{lll} \text{NEWUOA} & \\ \text{NEWUOA} & \\ \text{BIPOP-CD} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \hline f8 & 2039 \\ \text{NEWUOA1} & (0.3)^{*4} \\ \text{BIPOP-C} & 4.0(1) \\ \text{IPOP-CM} & 3.7(0.9) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \end{array} $	4274 ∞ $4.9(2)$ $4.8(2)$ $1e0$ 3871 $1(0.6)*4$ $4.0(0.7)$ $3.9(0.5)$ $1e0$	9503 ∞ $3.5(0.6)$ $2.7(2)$ $1e-1$ 4040 $1(0.6)^{*4}$ $4.3(0.6)$ $4.2(0.5)$ $1e-1$	16524 ∞ $2.2(0.3)$ $1.7(1.0)^*$ $1e-3$ 4219 $1(0.5)^*4$ $4.5(0.6)$ $4.4(0.4)$ $1e-3$	16524 ∞ $2.2(0.3)$ $1.7(1.0)^*$ $1e-5$ 4371 $1(0.5)^*4$ $4.6(0.6)$ $4.4(0.4)$ $1e-5$	16969 $\infty 5e5$ $2.1(0.3)$ $1.6(0.9)^*$ $1e-7$ 4484 $1(0.5)^*4$ $4.6(0.6)$ $4.5(0.4)$ $1e-7$	15/15 f19 0/IISEWUOA 15/IISIPOP-C 15/IIPOP-CM #succ Δf_{opt} 15/IIS f20 15/IISIPOP-C 15/IIPOP-CM #succ Δf #s	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 4.6(1) 1e1	1 4.3e6(5e 2.4e4(1e 2.7e4(2e 1e0 46150 15(17) 9.2(4) 6.4(2) 1e0	$3.4e5$ $6) \infty$ $=4)1.2(0.6)$ $4)0.71(0.5)$ $1e-1$ $3.1e6$ ∞ $1(0.5)$ $0.65(0.3)$	$6.2e6$ ∞ $1(0.3)$ $0.38(0.1)$ $1e-3$ $5.5e6$ ∞ $1(0.3)$ $0.57(0.2)$	$6.7e6$ ∞ $1(0.3)$ $40.41(0.2)$ $1e-5$ $5.6e6$ ∞ $1(0.3)$ $0.58(0.2)$ $1e-5$	$\begin{array}{c} \textbf{6.7e6} \\ \infty \ 2e6 \\ \textbf{1(0.3)} \\ \textbf{40.41(0.2)} \downarrow \\ \textbf{1e-7} \\ \textbf{5.6e6} \\ \infty \ 4e5 \\ \textbf{1(0.3)} \\ \textbf{0.58(0.2)} \\ \textbf{1e-7} \end{array}$	15/15 0/15 15/15 15/15 415/15 #succ 14/15 0/15 14/15 15/15 #succ
$ \begin{array}{c c} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-CD} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline \frac{\Delta f_{\text{opt}}}{\text{f8}} & 2039 \\ \text{NEWUOA} & 1(0.3) *^4 \\ \text{BIPOP-CD} & 3.7(0.9) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline \text{f9} & 1716 \\ \hline \end{array} $	4274 ∞ $4.9(2)$ $4.8(2)$ $1e0$ 3871 $1(0.6)^{*4}$ $4.0(0.7)$ $3.9(0.5)$ $1e0$ 3102	$\begin{array}{c} 9503 \\ \infty \\ 3.5(0.6) \\ \mathbf{2.7(2)} \\ 1e\text{-}1 \\ 4040 \\ 1(0.6)^{\star 4} \\ 4.3(0.6) \\ 4.2(0.5) \\ 1e\text{-}1 \\ 3277 \end{array}$	16524 ∞ 2.2(0.3) 1.7(1.0)* 1e-3 4219 1(0.5)* ⁴ 4.5(0.6) 4.4(0.4) 1e-3 3455	16524 ∞ $2.2(0.3)$ $1.7(1.0)^*$ $1e-5$ 4371 $1(0.5)^*4$ $4.6(0.6)$ $4.4(0.4)$ $1e-5$ 3594	16969 $\infty 5e5$ $2.1(0.3)$ $1.6(0.9)*$ $1e-7$ 4484 $1(0.5)*^4$ $4.6(0.6)$ $4.5(0.4)$ $1e-7$ 3727	15/15 f19 0/INEWUOA 15/IBIPOP-C 15/IBPOP-CM #succ Δ fopt 15/IB F20 15/IB F20 15/IB F20 15/IB F20 15/IPOP-CM #succ Δ fopt 15/IB F20 15/IB F20 15/IB F20 15/IB F21	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 1e1 561	1 4.3e6(5e 2.4e4(1e 2.7e4(2e 1e0 46150 15(17) 9.2(4) 6.4(2) 1e0 6541	$3.4e5$ $6) \infty$ $e4)1.2(0.6)$ $4)0.71(0.5)$ $1e-1$ $3.1e6$ ∞ $1(0.5)$ $0.65(0.3)$ $1e-1$ 14103	6.2e6 ∞ 1(0.3) 0.38(0.1) 1e-3 5.5e6 ∞ 1(0.3) 0.57(0.2) 1e-3 14643	$ \begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ \downarrow \mathbf{p}.41(0.2) \\ 1e-5 \\ 5.6e6 \\ \infty \\ 1(0.3) \\ 0.58(0.2) \\ 1e-5 \\ 15567 \end{array} $	$6.7e6$ $\infty 2e6$ $1(0.3)$ $40.41(0.2)$ $1e-7$ $5.6e6$ $\infty 4e5$ $1(0.3)$ $0.58(0.2)$ $1e-7$ 17589	15/15 0/15 15/15 15/15 #succ 14/15 0/15 14/15 15/15 #succ 15/15
$\begin{array}{lll} \text{NEWUOA} & \\ \text{NEWUOA} & \\ \text{BIPOP-CD} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline & \Delta f_{\text{opt}} & \text{le1} \\ \hline & \mathbf{f8} & 2039 \\ \text{NEWUOA1} & (0.3) *^{4} \\ \text{BIPOP-C} & 4.0(1) \\ \text{IPOP-CM} & 3.7(0.9) \\ \hline & \Delta f_{\text{opt}} & \text{le1} \\ \hline & \mathbf{f9} & 1716 \\ \text{NEWUOA1} & 0.0(0.2) *^{4} \\ \text{BIPOP-C} & 4.7(2) \\ \hline \end{array}$	4274 ∞ $4.9(2)$ $4.8(2)$ $1e0$ 3871 $1(0.6) *4$ $4.0(0.7)$ $3.9(0.5)$ $1e0$ 3102	9503 ∞ $3.5(0.6)$ $2.7(2)$ $1e-1$ 4040 $1(0.6)^{*4}$ $4.3(0.6)$ $4.2(0.5)$ $1e-1$	16524 ∞ $2.2(0.3)$ $1.7(1.0)^*$ $1e-3$ 4219 $1(0.5)^*4$ $4.5(0.6)$ $4.4(0.4)$ $1e-3$	16524 ∞ $2.2(0.3)$ $1.7(1.0)^*$ $1e-5$ 4371 $1(0.5)^*4$ $4.6(0.6)$ $4.4(0.4)$ $1e-5$	16969 $\infty 5e5$ $2.1(0.3)$ $1.6(0.9)^*$ $1e-7$ 4484 $1(0.5)^*4$ $4.6(0.6)$ $4.5(0.4)$ $1e-7$	15/15 f19 0/18EWUOA 15/18IPOP-C 15/18POP-CM #suc $\Delta f_{\rm opt}$ 15/18 EWUOA 15/18IPOP-C #suc $\Delta f_{\rm opt}$ 15/15 f21 15/18 EWUOA 15/18IPOP-C 15/18POP-C 15/18POP-C 15/18POP-C 15/18IPOP-C 15/18IPOP-	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 1e1 561 1.7(3) 3.2(6)	1 4.3e6(5e 2.4e4(1e 2.7e4(2e 1e0 46150 15(17) 9.2(4) 6.4(2) 1e0	$3.4e5$ $6) \infty$ $=4)1.2(0.6)$ $4)0.71(0.5)$ $1e-1$ $3.1e6$ ∞ $1(0.5)$ $0.65(0.3)$	$6.2e6$ ∞ $1(0.3)$ $0.38(0.1)$ $1e-3$ $5.5e6$ ∞ $1(0.3)$ $0.57(0.2)$	$6.7e6$ ∞ $1(0.3)$ $40.41(0.2)$ $1e-5$ $5.6e6$ ∞ $1(0.3)$ $0.58(0.2)$ $1e-5$	$\begin{array}{c} \textbf{6.7e6} \\ \infty \ 2e6 \\ \textbf{1(0.3)} \\ \textbf{40.41(0.2)} \downarrow \\ \textbf{1e-7} \\ \textbf{5.6e6} \\ \infty \ 4e5 \\ \textbf{1(0.3)} \\ \textbf{0.58(0.2)} \\ \textbf{1e-7} \end{array}$	15/15 0/15 15/15 415/15 415/15 #succ 14/15 0/15 14/15 15/15 #succ 15/15 15/15 15/15 13/15
$\begin{array}{c c} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-C} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline \text{f8} & 2039 \\ \text{NEWUOA} & 1(0.3)^{*4} \\ \text{BIPOP-C} & 4.0(1) \\ \text{IPOP-CM} & 3.7(0.9) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline \text{f9} & 1716 \\ \text{NEWUOA} & 1.0(0.2)^{*4} \\ \end{array}$	4274 0 $4.9(2)$ $4.8(2)$ $1e0$ 3871 $1(0.6)^{*4}$ $4.0(0.7)$ $3.9(0.5)$ $1e0$ 3102 $1(0.6)^{*4}$	$\begin{array}{c} 9503 \\ \infty \\ 3.5(0.6) \\ \textbf{2.7}(2) \\ 1\text{e-1} \\ 4040 \\ \textbf{1}(0.6)^{\star 4} \\ 4.3(0.6) \\ 4.2(0.5) \\ 1\text{e-1} \\ 3277 \\ \textbf{1}(0.6)^{\star 4} \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ \textbf{1.7}(1.0)^{\star} \\ \textbf{1e-3} \\ 4219 \\ \textbf{1}(0.5)^{\star 4} \\ 4.5(0.6) \\ 4.4(0.4) \\ \textbf{1e-3} \\ 3455 \\ \textbf{1}(0.5)^{\star 4} \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ \textbf{1.7}(1.0)^{\star} \\ 1e\text{-}5 \\ 4371 \\ \textbf{1}(0.5)^{\star}4 \\ 4.6(0.6) \\ 4.4(0.4) \\ 1e\text{-}5 \\ 3594 \\ \textbf{1}(0.5)^{\star}4 \end{array}$	16969 ∞ $5e5$ 2.1(0.3) $1.6(0.9)^*$ 1e-7 4484 $1(0.5)^{*4}$ 4.6(0.6) 4.5(0.4) 1e-7 3727 $1(0.5)^{*4}$	15/15 f19	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 1e1 561 1.7(3) 3.2(6)	1 4.3e6(5e 2.4e4(1e 2.7e4(2e 1e0 46150 15(17) 9.2(4) 6.4(2) 1e0 6541 2.2(2) 55(48)	$3.4e5$ $6) \infty$ $e431.2(0.6)$ $4)0.71(0.5)$ $1e-1$ $3.1e6$ ∞ $1(0.5)$ $0.65(0.3)$ $1e-1$ 14103 $1.2(2)$	6.2e6 ∞ 1(0.3) 0.38(0.1) 1e-3 5.5e6 ∞ 1(0.3) 0.57(0.2) 1e-3 14643 1.2(2)	$ \begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ \downarrow 0.41(0.2) \\ 1e-5 \\ \hline 5.6e6 \\ \infty \\ 1(0.3) \\ 0.58(0.2) \\ 1e-5 \\ \hline 15567 \\ 1.1(2) \end{array} $	$6.7e6$ $\infty 2e6$ $1(0.3)$ \mathcal{A} $0.41(0.2)$ $0.58(0.2)$ $0.58(0.2)$ $0.58(0.2)$ 0.589 0.10	15/15 0/15 15/15 15/15 #succ 14/15 0/15 14/15 15/15 #succ 15/15 15/15
$\begin{array}{c c} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-C} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \hline \textbf{f8} & 2039 \\ \text{NEWUOA} & 1(0.3)^{*4} \\ \text{BIPOP-C} & 4.0(1) \\ \text{IPOP-CM} & 3.7(0.9) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \hline \textbf{f9} & 1716 \\ \text{NEWUOA} & 1.0(0.2)^{*4} \\ \text{BIPOP-C} & 4.7(2) \\ \text{IPOP-CM} & 4.6(0.8) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \end{array}$	$4274 \\ \infty \\ 4.9(2) \\ 4.8(2) \\ 1e0 \\ 3871 \\ 1(0.6)^{*4} \\ 4.0(0.7) \\ 3.9(0.5) \\ 1e0 \\ 3102 \\ 1(0.6)^{*4} \\ 5.7(1) \\ 5.7(0.5) \\ 1e0$	$\begin{array}{c} 9503 \\ \infty \\ 3.5(0.6) \\ 2.7(2) \\ 1e-1 \\ 4040 \\ 1.(0.6)^{*4} \\ 4.3(0.6) \\ 4.2(0.5) \\ 1e-1 \\ 3277 \\ 1(0.6)^{*4} \\ 6.0(1) \\ 6.0(0.4) \\ 1e-1 \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ \textbf{1.7}(1.0)^{\star} \\ \textbf{1e-3} \\ 4219 \\ 4.5(0.6) \\ 4.4(0.4) \\ \textbf{1e-3} \\ 3455 \\ \textbf{1}(0.5)^{\star 4} \\ 6.1(1) \\ 6.1(0.4) \\ \textbf{1e-3} \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e-5 \\ 4371 \\ 1(0.5)^{*4} \\ 4.6(0.6) \\ 4.4(0.4) \\ 1e-5 \\ 3594 \\ 1(0.5)^{*4} \\ 6.1(1.0) \\ 6.1(0.4) \\ 1e-5 \end{array}$	16969 ∞ $5e5$ 2.1(0.3) $1.6(0.9)^*$ 1e-7 4484 4.6(0.6) 4.5(0.4) 1e-7 3727 $1(0.5)^*4$ 6.1(0.9) 6.1(0.9) 6.1(0.3) 1e-7	15/15 f19 0/18 EWUOA 15/18 F19 POP-C 15/18 POP-C 15/18 F20 15/18 F20 15/18 F20 POP-C 15/18 POP-C	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 4.6(1) 1e1 561 1.7(3) 3.2(6) 3.7(5) 1e1	1 4.366(5e 2.4e4(1t 2.7e4(2e 1e0 46150 15(17) 9.2(4) 6.4(2) 1e0 65541 2.2(2) 55(48) 139(202) 1e0	$\begin{array}{c} 3.4e5 \\ 6) \infty \\ 4)1.2(0.6) \\ 4)0.71(0.5) \\ 1e-1 \\ 3.1e6 \\ \infty \\ 1(0.5) \\ 0.65(0.3) \\ 1e-1 \\ 14103 \\ 1.2(2) \\ 48(95) \\ 110(140) \\ 1e-1 \end{array}$	$\begin{array}{c} \textbf{6.2e6} \\ \infty \\ \textbf{1}(0.3) \\ \textbf{0.38}(0.1) \\ \textbf{1e-3} \\ \textbf{5.5e6} \\ \infty \\ \textbf{1}(0.3) \\ \textbf{0.57}(0.2) \\ \textbf{1e-3} \\ \textbf{12}(2) \\ \textbf{46}(93) \\ \textbf{106}(136) \\ \textbf{1e-3} \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ \downarrow \textbf{0.41}(0.2) \\ 1e-5 \\ 5.6e6 \\ \infty \\ 1(0.3) \\ \textbf{0.58}(0.2) \\ 1e-5 \\ 15567 \\ \textbf{1.1}(2) \\ 43(85) \\ 100(127) \\ 1e-5 \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 40.41(0.2) \downarrow \\ 1e-7 \\ 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \\ 1e-7 \\ 17589 \\ 1(1) \\ 39(74) \\ 88(111) \\ 1e-7 \end{array}$	15/15 0/15 15/15 15/15 #succ 14/15 15/15 #succ 15/15 #succ 15/15 13/15 7/15 #succ
$ \begin{array}{c c} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-CD} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f8 & 2039 \\ \text{NEWUOA1}(0.3)^{*4} \\ \text{BIPOP-C} & 4.0(1) \\ \text{IPOP-CM} & 3.7(0.9) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f9 & 1716 \\ \text{NEWUOA1}.0(0.2)^{*4} \\ \text{BIPOP-C} & 4.7(2) \\ \text{IPOP-CM} & 4.6(0.8) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f10 & 7413 \\ \hline \end{array} $	4274 ∞ $4.9(2)$ $4.8(2)$ $1e0$ 3871 $1(0.6)^{*4}$ $4.0(0.7)$ $3.9(0.5)$ $1e0$ 3102 $1(0.6)^{*4}$ $5.7(1)$ $5.7(0.5)$ $1e0$ 8661	$\begin{array}{c} 9503 \\ \infty \\ 3.5(0.6) \\ \textbf{2.7}(2) \\ 1e-1 \\ 4040 \\ \textbf{1}(0.6)^{\star 4} \\ 4.3(0.6) \\ 4.2(0.5) \\ 1e-1 \\ 3277 \\ \textbf{1}(0.6)^{\star 4} \\ 6.0(1) \\ 6.0(0.4) \\ 1e-1 \\ 10735 \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-3} \\ 4219 \\ 1(0.5)^{*4} \\ 4.5(0.6) \\ 4.4(0.4) \\ 1e^{-3} \\ 3455 \\ 1(0.5)^{*4} \\ 6.1(1) \\ 6.1(0.4) \\ 1e^{-3} \\ 14920 \\ \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-5} \\ 4371 \\ 1(0.5)^{*4} \\ 4.4(0.4) \\ 1e^{-5} \\ 3594 \\ 1(0.5)^{*4} \\ 6.1(1.0) \\ 6.1(0.4) \\ 1e^{-5} \\ 17073 \\ \end{array}$	$\begin{array}{c} 16969 \\ \infty \ 5e5 \\ 2.1(0.3) \\ 1.6(0.9)^{\star} \\ 1e-7 \\ 4484 \\ 1(0.5)^{\star 4} \\ 4.6(0.6) \\ 4.5(0.4) \\ 1e-7 \\ 3727 \\ 1(0.5)^{\star 4} \\ 6.1(0.9) \\ 6.1(0.3) \\ 1e-7 \\ 17476 \end{array}$	15/15 f19 0/18EWUOA 15/18IPOP-C 15/18POP-CM #succ $\Delta f_{\rm opt}$ 15/18 f20 15/18 F20 15/18 F20 15/18 F21 15/18 F21 15/18 EWUOA 15/18 EWUOA 15/18 EWUOA 15/18 EWUOA 15/18 EWUOA 15/18 EWOP-CM #succ $\Delta f_{\rm opt}$ #succ $\Delta f_{\rm opt}$ 15/15 f21 15/18 EWUOA 15/18 EWOP-CM #succ $\Delta f_{\rm opt}$ 15/15 f22	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 4.6(1) 1e1 561 1.7(3) 3.2(6) 3.7(5) 1e1 467	1 4.366(5e 2.464(1e 2.764(2e 1e0 46150 15(17) 9.2(4) 6.4(2) 1e0 6541 2.2(2) 555(48) 139(202) 1e0 5580	$3.4e5$ $6) \infty$ $401.2(0.6)$ $401.71(0.5)$ $1e-1$ $3.1e6$ ∞ $1(0.5)$ $0.65(0.3)$ $1e-1$ 14103 $1.2(2)$ $48(95)$ $110(140)$ $1e-1$ 23491	$\begin{array}{c} \textbf{6.2e6} \\ \infty \\ \textbf{1}(0.3) \\ \textbf{0.38}(0.1) \\ \textbf{1e-3} \\ \textbf{5.5e6} \\ \infty \\ \textbf{1}(0.3) \\ \textbf{0.57}(0.2) \\ \textbf{1e-3} \\ \textbf{14643} \\ \textbf{1.2}(2) \\ \textbf{46}(93) \\ \textbf{106}(136) \\ \textbf{1e-3} \\ \textbf{24948} \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ 1(0.3) \\ 40.41(0.2) \\ \hline 1e-5 \\ 5.6e6 \\ \infty \\ 1(0.3) \\ 0.58(0.2) \\ \hline 1e-5 \\ 15567 \\ 1.1(2) \\ 43(85) \\ 100(127) \\ 1e-5 \\ 26847 \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 10.3) \\ 40.41(0.2) \downarrow \\ 1e-7 \\ 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \\ 1e-7 \\ 17589 \\ 1(1) \\ 39(74) \\ 88(111) \\ 1e-7 \\ 1.3e5 \\ \end{array}$	15/15 0/15 15/15 15/15 #succ 14/15 0/15 14/15 15/15 15/15 15/15 13/15 7/15 #succ 12/15
$ \begin{array}{c c} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-CD} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline & \textbf{f8} & 2039 \\ \text{NEWUOA1} & (0.3) *^4 \\ \text{BIPOP-C} & 4.0(1) \\ \text{IPOP-CM} & 3.7(0.9) \\ \hline & \Delta f_{\text{opt}} & 1e1 \\ \hline & \textbf{f9} & 1716 \\ \text{NEWUOA1} & 0.0(0.2) *^4 \\ \text{BIPOP-CC} & 4.7(2) \\ \text{IPOP-CM} & 4.6(0.8) \\ \hline & \Delta f_{\text{opt}} & 1e1 \\ \hline & \textbf{f10} & 7413 \\ \text{NEWUOA1.7} & (0.5) \\ \end{array} $	$\begin{array}{c} 4274 \\ \infty \\ 4.9(2) \\ 4.8(2) \\ 1e0 \\ 3871 \\ 1(0.6)^{\star 4} \\ 4.0(0.7) \\ 3.9(0.5) \\ 1e0 \\ 3102 \\ 1(0.6)^{\star 4} \\ 5.7(1) \\ 5.7(0.5) \\ 1e0 \\ 8661 \\ 2.6(0.8) \end{array}$	$\begin{array}{c} 9503 \\ \infty \\ 3.5(0.6) \\ \textbf{2.7}(2) \\ 1e-1 \\ 4040 \\ 4.3(0.6) \\ 4.2(0.5) \\ 1e-1 \\ 3277 \\ 1(0.6) \\ ^{\star}4 \\ 6.0(1) \\ 6.0(0.4) \\ 1e-1 \\ 10735 \\ 3.3(1) \\ \end{array}$	16524 \times 2.2(0.3) 1.7(1.0)* 1e-3 4219 4.5(0.6) 4.4(0.4) 1e-3 3455 1(0.5)*4 6.1(1) 6.1(0.4) 1e-3 14920 4.0(0.8)	16524 0 2.2(0.3) 1.7(1.0)* 1e-5 4371 1(0.5)*4 4.6(0.6) 4.4(0.4) 1e-5 3594 1(0.5)*4 6.1(1.0) 6.1(0.4) 1e-5 17073 4.7(0.8)	16969 ∞ 5e5 2.1(0.3) 1.6(0.9)* 1e-7 4484 1(0.5)*4 4.6(0.6) 4.5(0.4) 1e-7 1(0.5)*4 6.1(0.9) 6.1(0.3) 1e-7 17476 5.8(1)	15/15 f19 0/18 EWUOA 15/18 IPOP-C 15/18 POP-CM 15/15 f20 15/18 POP-CM 15/18 POP-CM $\# succ \Delta f_{opt}$ $15/15 f21$ $15/18 EWUOA 15/18 IPOP-CM 15/18 IPOP-CM 15/18 IPOP-CM 15/18 IPOP-CM 15/15 f21 15/18 IPOP-CM 15/15 IPOP-CM 15/15 f22 15/18 IPOP-CM 15/15 f22 15/18 EWUOA 15/18 IPOP-CM 15/15 IPOP-CM 15/15 f22 15/18 EWUOA 15/18 EWUOA$	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 4.6(1) 1e1 561 1.7(3) 3.2(6) 3.7(5) 1e1 467 1(1)*	1 4.3e6(5e 2.4e4(1t 2.7e4(2e 1e0 15(17) 9.2(4) 6.4(2) 1e0 6.55(48) 13.9(202) 1e0 5580 4.9(6)	$\begin{array}{c} 3.4e5 \\ 6) \infty \\ 4)1.2(0.6) \\ 4)0.71(0.5) \\ 1e-1 \\ 3.1e6 \\ \infty \\ 1(0.5) \\ 0.65(0.3) \\ 1e-1 \\ 14103 \\ 1.2(2) \\ 48(95) \\ 110(140) \\ 1e-1 \\ 23491 \\ 6.8(8) \end{array}$	6.2e6 \times (10.3) 0.38(0.1) 1e-3 5.5e6 \times (10.3) 0.57(0.2) 1e-3 14643 1.2(2) 46(93) 106(136) 1e-3 24948 6.4(8)	6.7e6 ∞ 1(0.3) ↓0.41(0.2), 1e-5 5.6e6 ∞ 1(0.3) 0.58(0.2) 1e-5 15567 1.1(2) 43(85) 100(127) 1e-5 26847 6.0(7)	$6.7e6$ $\infty 2e6$ $1(0.3)$ $49.41(0.2)$ $1e-7$ $5.6e6$ $\infty 4e5$ $1(0.3)$ $0.58(0.2)$ $1e-7$ 175.89 $1(1)$ $39(74)$ $88(111)$ $1e-7$ $1.3e5$ $1.2(1)$	15/15 0/15 15/15 15/15 15/15 14/15 0/15 14/15 15/15 #succ 15/15 15/15 15/15 13/15 7/15 #succ 15/15 15/15 15/15 13/15 7/15
$\begin{array}{c c} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-CD} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f8 & 2039 \\ \text{NEWUOA1} & (0.3)^{*4} \\ \text{BIPOP-C} & 4.0(1) \\ \text{IPOP-CM} & 3.7(0.9) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f9 & 1716 \\ \text{NEWUOA1}.0(0.2)^{*4} \\ \text{BIPOP-C} & 4.7(2) \\ \text{IPOP-CM} & 4.6(0.8) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f10 & 7413 \\ \text{NEWUOA1}.7(0.5) \\ \text{BIPOP-C} & 1.9(0.2) \\ \end{array}$	$\begin{array}{c} 4274 \\ \infty \\ 4.9(2) \\ 4.8(2) \\ 160 \\ 3871 \\ 1(0.6)^{\star 4} \\ 4.0(0.7) \\ 3.9(0.5) \\ 1e0 \\ 3102 \\ \vdots \\ 1(0.6)^{\star 4} \\ 5.7(1) \\ 5.7(0.5) \\ 1e0 \\ 8661 \\ 2.6(0.8) \\ 1.8(0.2) \end{array}$	$\begin{array}{c} 9503 \\ \infty \\ 3.5(0.6) \\ \textbf{2.7}(2) \\ 1e-1 \\ 4040 \\ \textbf{1}(0.6)^{\star 4} \\ 4.3(0.6) \\ 4.2(0.5) \\ 1e-1 \\ 3277 \\ \textbf{1}(0.6)^{\star 4} \\ 6.0(1) \\ 6.0(0.4) \\ 1e-1 \\ 10735 \\ 3.3(1) \\ 1.6(0.1) \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-3} \\ 4219 \\ 1(0.5)^{*4} \\ 4.5(0.6) \\ 4.4(0.4) \\ 1e^{-3} \\ 3455 \\ 1(0.5)^{*4} \\ 6.1(1) \\ 6.1(0.4) \\ 1e^{-3} \\ 14920 \\ 4.0(0.8) \\ 1.2(0.0) \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-5} \\ 4371 \\ 1(0.5)^{*4} \\ 4.4(0.4) \\ 1e^{-5} \\ 3594 \\ 1(0.5)^{*4} \\ 6.1(1.0) \\ 6.1(0.4) \\ 1e^{-5} \\ 17073 \\ 4.7(0.8) \\ 1.1(0.0) \end{array}$	16969 ∞ 5e5 2.1(0.3) 1.6(0.9)* 1e-7 4484 1(0.5)*4 4.6(0.6) 4.5(0.4) 1e-7 3727 1(0.5)*4 6.1(0.9) 6.1(0.3) 1e-7 17476 5.8(1) 1.1(0.0)	15/15 f19 0/18 EWUOA 15/18 IPOP-C 15/18 EWUOA 15/18 EWUOA 15/18 IPOP-C 15/18 IPOP	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 4.6(1) 1e1 561 1.7(3) 3.2(6) 3.7(5) 1e1 467 1(1)* 6.8(13)	1 4.3e6(5e 2.4e4(1e 2.7e4(2e 1e0 16) 15(17) 9.2(4) 6.4(2) 1e0 6541 2.2(2) 55(48) 139(202) 1e0 5580 4.9(6) 13(21)	$\begin{array}{c} 3.4e5 \\ 6) \infty \\ 6) \infty \\ 4)0.12(0.6) \\ 4)0.71(0.5) \\ 1e-1 \\ 3.1e6 \\ \infty \\ 1(0.5) \\ 0.65(0.3) \\ 1e-1 \\ 14103 \\ 1.2(2) \\ 48(95) \\ 110(140) \\ 1e-1 \\ 23491 \\ 6.8(8) \\ 215(260) \end{array}$	6.2e6 0.38(0.1) 1(0.3) 0.38(0.1) 1e-3 5.5e6 0.57(0.2) 1e-3 14643 1.2(2) 46(93) 106(136) 1e-3 24948 6.4(8) 202(244)	6.7e6 0.5e6 1(0.3) 1e-5 5.6e6 0.58(0.2) 1e-5 15567 1.1(2) 43(85) 100(127) 1e-5 26847 6.0(7) 188(232)	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 2.66 \\ 1(0.41(0.2)_{\downarrow} \\ 40.41(0.2)_{\downarrow} \\ 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \\ 1e-7 \\ 17589 \\ 1(1) \\ 39(74) \\ 88(111) \\ 1e-7 \\ 1.3e5 \\ 1.2(1) \\ 37(46) \end{array}$	15/15 0/15 15/15 15/15 14/15 14/15 0/15 14/15 14/15 15/15 15/15 15/15 17/15 #succ 12/15 7/15
$\begin{array}{lll} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-CD} & 1 (0.5) \\ \text{IPOP-CM} & 1.9 (2) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f8 & 2039 \\ \text{NEWUOA1} & (0.3)^{*4} \\ \text{BIPOP-C} & 4.0 (1) \\ \text{IPOP-CM} & 3.7 (0.9) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f9 & 1716 \\ \text{NEWUOA1} & 1.0 (0.2)^{*4} \\ \text{BIPOP-C} & 4.7 (2) \\ \text{IPOP-CM} & 4.6 (0.8) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f10 & 7413 \\ \text{NEWUOA1}.7 (0.5) \\ \text{BIPOP-C} & 1.9 (0.2) \\ \text{IPOP-CM} & 1.8 (0.2) \\ \hline \end{array}$	$\begin{array}{c} 4274 \\ \infty \\ 4.9(2) \\ 4.8(2) \\ 1e0 \\ 3871 \\ 1(0.6)^{\star 4} \\ 4.0(0.7) \\ 3.9(0.5) \\ 1e0 \\ 3102 \\ 1(0.6)^{\star 4} \\ 5.7(1) \\ 5.7(0.5) \\ 1e0 \\ 8661 \\ 2.6(0.8) \end{array}$	$\begin{array}{c} 9503 \\ \infty \\ 3.5(0.6) \\ \textbf{2.7}(2) \\ 1e-1 \\ 4040 \\ 4.3(0.6) \\ 4.2(0.5) \\ 1e-1 \\ 3277 \\ 1(0.6) \\ ^{\star}4 \\ 6.0(1) \\ 6.0(0.4) \\ 1e-1 \\ 10735 \\ 3.3(1) \\ \end{array}$	16524 \times 2.2(0.3) 1.7(1.0)* 1e-3 4219 4.5(0.6) 4.4(0.4) 1e-3 3455 1(0.5)*4 6.1(1) 6.1(0.4) 1e-3 14920 4.0(0.8)	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^* \\ 1e^{-5} \\ 4371 \\ 1(0.5)^{*4} \\ 4.6(0.6) \\ 4.4(0.4) \\ 1e^{-5} \\ 3594 \\ 1(0.5)^{*4} \\ 6.1(1.0) \\ 6.1(0.4) \\ 1e^{-5} \\ 17073 \\ 4.7(0.8) \\ 1.1(0.0) \\ 1.1(0.0) \end{array}$	$\begin{array}{c} 16969\\ \infty \ 5e5\\ 2.1(0.3)\\ 1.6(0.9)^{\star}\\ 1e-7\\ 4484\\ 1(0.5)^{\star 4}\\ 4.6(0.6)\\ 4.5(0.4)\\ 1e-7\\ 3727\\ 1(0.5)^{\star 4}\\ 6.1(0.9)\\ 6.1(0.3)\\ 1e-7\\ 17476\\ 5.8(1)\\ 1.1(0.0)\\ 1.1(0.0)\\ \end{array}$	15/15 f19 0/18EWUOA 15/18IPOP-C 15/18POP-CM #suc $\Delta f_{\rm opt}$ 15/18 f20 15/18 f20 15/18 f20 15/18 f21 15/15 f22 15/18 f20 f21 15/18 f22 15/18 f20 f21 15/18 f22 15/18 f20 f21 15/18 f	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 4.6(1) 1e1 561 1.7(3) 3.2(6) 3.7(5) 1e1 467 1(1)* 6.8(13) 445(1389)	1 4.3e6(5e 2.4e4(1t 2.7e4(2e 1e0 15(17) 9.2(4) 6.4(2) 1e0 6.55(48) 13.9(202) 1e0 5580 4.9(6)	$\begin{array}{c} 3.4e5 \\ 6) \infty \\ 6) \infty \\ 4)1.2(0.6) \\ 4)0.71(0.5) \\ 1e-1 \\ 3.1e6 \\ \infty \\ 1(0.5) \\ 0.65(0.3) \\ 1e-1 \\ 14103 \\ 1.2(2) \\ 48(95) \\ 110(140) \\ 1e-1 \\ 23491 \\ 6.8(8) \\ 215(260) \\ \infty \end{array}$	$\begin{array}{c} \textbf{6.2e6} \\ \infty \\ \textbf{1}(0.3) \\ \textbf{0.38}(0.1) \\ \textbf{1e-3} \\ \textbf{5.5e6} \\ \infty \\ \textbf{1}(0.3) \\ \textbf{0.57}(0.2) \\ \textbf{1e-3} \\ \textbf{14643} \\ \textbf{1.2}(2) \\ \textbf{46}(93) \\ \textbf{106}(136) \\ \textbf{1e-3} \\ \textbf{24948} \\ \textbf{6.4}(8) \\ \textbf{202}(244) \\ \infty \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ 1(0.3) \\ 40.41(0.2) \\ \hline 1e-5 \\ 5.6e6 \\ \infty \\ 1(0.3) \\ 0.58(0.2) \\ \hline 1e-5 \\ 15567 \\ 1.1(2) \\ 43(85) \\ 100(127) \\ 1e-5 \\ 26847 \\ 6.0(7) \\ 188(232) \\ \infty \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 2.66 \\ 1(0.3) \\ 40.41(0.2) \downarrow \\ 1e-7 \\ 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \\ 1e-7 \\ 17589 \\ 1(1) \\ 39(74) \\ 88(111) \\ 1e-7 \\ 1.3e5 \\ 1.2(1) \\ 37(46) \\ \infty \ 1e6 \\ \end{array}$	15/15 0/15 15/15 45/15 45/15 #succ 14/15 0/15 15/15 15/15 15/15 13/15 7/15 #succ 12/15 7/15 5/15 0/15
$\begin{array}{c c} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-CD} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f8 & 2039 \\ \text{NEWUOA1} & (0.3)^{*4} \\ \text{BIPOP-C} & 4.0(1) \\ \text{IPOP-CM} & 3.7(0.9) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f9 & 1716 \\ \text{NEWUOA1}.0(0.2)^{*4} \\ \text{BIPOP-C} & 4.7(2) \\ \text{IPOP-CM} & 4.6(0.8) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f10 & 7413 \\ \text{NEWUOA1}.7(0.5) \\ \text{BIPOP-C} & 1.9(0.2) \\ \end{array}$	$\begin{array}{c} 4274 \\ \infty \\ 4.9(2) \\ 4.8(2) \\ 1e0 \\ 3871 \\ 1(0.6)^{*4} \\ 4.0(0.7) \\ 3.9(0.5) \\ 1e0 \\ 3102 \\ 1(0.6)^{*4} \\ 5.7(1) \\ 5.7(0.5) \\ 1e0 \\ 8661 \\ 2.6(0.8) \\ 1.8(0.2) \\ 1.8(0.2) \\ 1.8(0.2) \end{array}$	$\begin{array}{c} 9503 \\ \infty \\ 3.5(0.6) \\ \textbf{2.7}(2) \\ 1e-1 \\ 4040 \\ \textbf{1}(0.6)^{\star 4} \\ 4.3(0.6) \\ 4.2(0.5) \\ 1e-1 \\ 3277 \\ \textbf{1}(0.6)^{\star 4} \\ 6.0(0.4) \\ 1e-1 \\ 10735 \\ 3.3(1) \\ 1.6(0.1) \\ \textbf{1.5}(0.1) \\ \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-3} \\ 4219 \\ 1(0.5)^{*4} \\ 4.5(0.6) \\ 4.4(0.4) \\ 1e^{-3} \\ 3455 \\ 1(0.5)^{*4} \\ 6.1(1) \\ 6.1(0.4) \\ 1e^{-3} \\ 14920 \\ 4.0(0.8) \\ 1.2(0.0) \\ 1.2(0.1) \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-5} \\ 4371 \\ 1(0.5)^{*4} \\ 4.4(0.4) \\ 1e^{-5} \\ 3594 \\ 1(0.5)^{*4} \\ 6.1(1.0) \\ 6.1(0.4) \\ 1e^{-5} \\ 17073 \\ 4.7(0.8) \\ 1.1(0.0) \end{array}$	16969 ∞ 5e5 2.1(0.3) 1.6(0.9)* 1e-7 4484 1(0.5)*4 4.6(0.6) 4.5(0.4) 1e-7 3727 1(0.5)*4 6.1(0.9) 6.1(0.3) 1e-7 17476 5.8(1) 1.1(0.0)	15/15 f19 0/18 EWUOA 15/18 IPOP-C 15/18 EWUOA 15/18 EWUOA 15/18 IPOP-C 15/18 IPOP	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 4.6(1) 1e1 561 1.7(3) 3.2(6) 3.7(5) 1e1 467 1(1)* 6.8(13) 445(1389)	1 4.366(5e 2.464(1e 2.764(2e 1.60) 15(17) 9.2(4) 6.4(2) 1e0 6541 2.2(2) 55(48) 39(202) 1e0 5580 4.9(6) 13(21) 287(349) 1e0 1614	$\begin{array}{c} 3.4e5 \\ 6) \infty \\ 6) \infty \\ 4)0.12(0.6) \\ 4)0.71(0.5) \\ 1e-1 \\ 3.1e6 \\ \infty \\ 1(0.5) \\ 0.65(0.3) \\ 1e-1 \\ 14103 \\ 1.2(2) \\ 48(95) \\ 110(140) \\ 1e-1 \\ 23491 \\ 6.8(8) \\ 215(260) \end{array}$	6.2e6 0.38(0.1) 1(0.3) 0.38(0.1) 1e-3 5.5e6 0.57(0.2) 1e-3 14643 1.2(2) 46(93) 106(136) 1e-3 24948 6.4(8) 202(244)	6.7e6 0.5e6 1(0.3) 1e-5 5.6e6 0.58(0.2) 1e-5 15567 1.1(2) 43(85) 100(127) 1e-5 26847 6.0(7) 188(232)	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 2.66 \\ 1(0.41(0.2)_{\downarrow} \\ 40.41(0.2)_{\downarrow} \\ 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \\ 1e-7 \\ 17589 \\ 1(1) \\ 39(74) \\ 88(111) \\ 1e-7 \\ 1.3e5 \\ 1.2(1) \\ 37(46) \end{array}$	15/15 0/15 15/15 15/15 14/15 14/15 0/15 14/15 14/15 15/15 15/15 15/15 17/15 #succ 12/15 7/15
$ \begin{array}{c c} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-CD} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f8 & 2039 \\ \text{NEWUOA1} & (0.3) *4 \\ \text{BIPOP-C} & 4.0(1) \\ \text{IPOP-CM} & 3.7(0.9) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f9 & 1716 \\ \text{NEWUOA1} & 0(0.2) *4 \\ \text{BIPOP-C} & 4.7(2) \\ \text{IPOP-CM} & 4.6(0.8) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f10 & 7413 \\ \text{NEWUOA1} & 1.7(0.5) \\ \text{BIPOP-C} & 1.9(0.2) \\ \text{IPOP-CM} & 1.8(0.2) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f11 & 1002 \\ \text{NEWUOA1} & 5(2) \\ \hline \end{array} $	4274 ∞ $4.9(2)$ $4.8(2)$ $1e0$ 3871 $1(0.6)^{*4}$ $4.0(0.7)$ $3.9(0.5)$ $1e0$ 3102 $1(0.6)^{*4}$ $5.7(0.5)$ $1e0$ 8661 $2.6(0.8)$ $1.8(0.2)$ $1.8(0.2)$ $1e0$ 2228 $13(2)$	$\begin{array}{c} 9503 \\ \infty \\ 3.5(0.6) \\ \textbf{2.7}(2) \\ 1e-1 \\ 4040 \\ \textbf{1}(0.6)^{\star 4} \\ 4.3(0.6) \\ 4.2(0.5) \\ 1e-1 \\ 3277 \\ \textbf{1}(0.6)^{\star 4} \\ 6.0(0.4) \\ 1e-1 \\ 10735 \\ 3.3(1) \\ 1.6(0.1) \\ 1.5(0.1) \\ 1e-1 \\ 6278 \\ 5.8(0.6) \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-3} \\ 4219 \\ 4.5(0.6) \\ 4.4(0.4) \\ 1e^{-3} \\ 3455 \\ 1(0.5)^{*4} \\ 6.1(1) \\ 6.1(0.4) \\ 1e^{-3} \\ 14920 \\ 4.0(0.8) \\ 1.2(0.0) \\ 1.2(0.1) \\ 1e^{-3} \\ 9762 \\ 6.1(0.5)^{*} \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-5} \\ 4371 \\ 1(0.5)^{*4} \\ 4.6(0.6) \\ 4.4(0.4) \\ 1e^{-5} \\ 3594 \\ 1(0.5)^{*4} \\ 6.1(1.0) \\ 6.1(0.4) \\ 1e^{-5} \\ 17073 \\ 4.7(0.8) \\ 1.1(0.0) \\ 1.1(0.0) \\ 1e^{-5} \\ 12285 \\ 6.6(0.3) \end{array}$	$\begin{array}{c} 16969\\ \infty \ 5e5\\ 2.1(0.3)\\ 1.6(0.9)^{\star}\\ 1e-7\\ 4484\\ 1(0.5)^{\star 4}\\ 4.6(0.6)\\ 4.5(0.4)\\ 1e-7\\ 3727\\ 1(0.5)^{\star 4}\\ 6.1(0.9)\\ 6.1(0.3)\\ 1e-7\\ 17476\\ 5.8(1)\\ 1.1(0.0)\\ 1.1(0.0)\\ 1e-7\\ 14831\\ 6.5(0.3)\\ \end{array}$	15/15 f19 0/18 EWUOA 15/18 IPOP-C 15/18 IPOP-C 15/18 EWUOA 15/18 IPOP-C 15/18 EWUOA 15/18 IPOP-C 15/18 EWUOA 15/18 IPOP-C 15/18 EWUOA 15/18 IPOP-C	1	1 4.366(5e 2.464(1t 2.764(2e 1e0 46150 15(17) 9.2(4) 6.4(2) 1e0 6541 2.2(2) 55(48) 13(9(202) 1e0 5580 4.9(6) 13(21) 287(349) 1e0	$\begin{array}{c} 3.4e5 \\ 6) \infty \\ 6) \infty \\ 34)1.2(0.6) \\ 4)0.71(0.5) \\ 1e-1 \\ 3.1e6 \\ \infty \\ 1(0.5) \\ 0.65(0.3) \\ 1e-1 \\ 14103 \\ 1.2(2) \\ 48(95) \\ 110(140) \\ 1e-1 \\ 23491 \\ 6.8(8) \\ 215(260) \\ \infty \\ 1e-1 \\ 1e-1$	$\begin{array}{c} 6.2e6\\ \infty\\ 1(0.3)\\ 1(0.3)\\ \textbf{0.38}(0.1)\\ 1e-3\\ \hline 5.5e6\\ \infty\\ 1(0.3)\\ \textbf{0.57}(0.2)\\ 1e-3\\ 14643\\ 1.2(2)\\ 46(93)\\ 106(136)\\ 1e-3\\ 24948\\ \textbf{6.4}(8)\\ 202(244)\\ \infty\\ 1e-3\\ \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ 1(0.3) \\ 40.41(0.2), \\ 1e-5 \\ 5.6e6 \\ \infty \\ 1(0.3) \\ 0.58(0.2) \\ 1e-5 \\ 15567 \\ 1.1(2) \\ 43(85) \\ 100(127) \\ 1e-5 \\ 26847 \\ 6.0(7) \\ 188(232) \\ \infty \\ 1e-5 \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 40.41(0.2) \downarrow \\ 1e-7 \\ 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \\ 1e-7 \\ 17589 \\ 1(1) \\ 39(74) \\ 88(111) \\ 1e-7 \\ 1.3e5 \\ 1.2(1) \\ 37(46) \\ \infty \ 1e6 \\ 1e-7 \\ \end{array}$	15/15 0/15 15/15 415/15 415/15 415/15 14/15 114/15 115/15 48succ 15/15 13/15 7/15 48succ 12/15 7/15 48succ 12/15 7/15 48succ 12/15 7/15 48succ 12/15 12/15 5/15 13/15 48succ 48succ 12/15 12/15 5/15 48succ 48succ 12/15 12/15 5/15 48succ 48succ 12/15 5/15 5/15 5/15 48succ 48suc
$\begin{array}{c c} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-C} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \hline 8 & 2039 \\ \text{NEWUOA} & 1(0.3)^{*4} \\ \text{BIPOP-C} & 4.0(1) \\ \text{IPOP-CM} & 3.7(0.9) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \hline f9 & 1716 \\ \text{NEWUOA} & 1.0(0.2)^{*4} \\ \text{BIPOP-C} & 4.7(2) \\ \text{IPOP-CM} & 4.6(0.8) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \hline f10 & 7413 \\ \text{NEWUOA} & 1.7(0.5) \\ \text{BIPOP-C} & 1.9(0.2) \\ \text{IPOP-CM} & 1.8(0.2) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \hline f11 & 1002 \\ \text{NEWUOA} & 15(2) \\ \text{BIPOP-C} & 10(0.5) \\ \end{array}$	$\begin{array}{c} 4274 \\ \infty \\ 4.9(2) \\ 4.8(2) \\ 160 \\ 3871 \\ 1(0.6)^{\star 4} \\ 4.0(0.7) \\ 3.9(0.5) \\ 1e0 \\ 3102 \\ 1(0.6)^{\star 4} \\ 5.7(1) \\ 5.7(0.5) \\ 1e0 \\ 8661 \\ 2.6(0.8) \\ 1.8(0.2) \\ 1.8(0.2) \\ 1.8(0.2) \\ 160 \\ 2228 \\ 13(2) \\ 5.1(0.3) \end{array}$	$\begin{array}{c} 9503 \\ \infty \\ \infty \\ 3.5(0.6) \\ 2.7(2) \\ \mathbf{1c-1} \\ 4040 \\ 1(0.6)^{\star 4} \\ 4.3(0.6) \\ 4.2(0.5) \\ \mathbf{1c-1} \\ 3277 \\ 1(0.6)^{\star 4} \\ 6.0(1) \\ 6.0(0.4) \\ \mathbf{1c-1} \\ 10735 \\ 3.3(1) \\ 1.5(0.1) \\ 1.5(0.1) \\ \mathbf{1c-1} \\ 6278 \\ 5.8(0.6) \\ 1.9(0.1) \\ 1.9(0.1) \\ \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^* \\ 1e^{-3} \\ 4219 \\ 1(0.5)^*4 \\ 4.5(0.6) \\ 4.4(0.4) \\ 1e^{-3} \\ 3455 \\ 1(0.5)^*4 \\ 6.1(1) \\ 6.1(0.4) \\ 1e^{-3} \\ 14920 \\ 4.0(0.8) \\ 1.2(0.0) \\ 1.2(0.1) \\ 1e^{-3} \\ 9762 \\ 6.1(0.5) \\ 1.4(0.0) \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-5} \\ 4371 \\ 1(0.5)^{*4} \\ 4.6(0.6) \\ 4.4(0.4) \\ 1e^{-5} \\ 3594 \\ 1(0.5)^{*4} \\ 6.1(1.0) \\ 6.1(0.4) \\ 1e^{-5} \\ 17073 \\ 4.7(0.8) \\ 1.1(0.0) \\ 1.1(0.0) \\ 1e^{-5} \\ 12285 \\ 6.6(0.3) \\ 1.2(0.0) \end{array}$	$\begin{array}{c} 16969\\ \infty \ 5e5\\ 2.1(0.3)\\ \textbf{1.6}(0.9)^{\star}\\ \textbf{1e-7}\\ 4484\\ \textbf{1}(0.5)^{\star}4\\ 4.6(0.6)\\ 4.5(0.4)\\ \textbf{1e-7}\\ 3727\\ \textbf{1}(0.5)^{\star}4\\ 6.1(0.9)\\ 6.1(0.9)\\ 6.1(0.3)\\ \textbf{1e-7}\\ 17476\\ 5.8(1)\\ \textbf{1.1}(0.0)\\ \textbf{1.1}(0.0)\\ \textbf{1.1}(0.0)\\ \textbf{1.2}\\ \textbf{1.3}\\ \textbf{1.3}\\ \textbf{1.3}\\ \textbf{1.4}\\ \textbf{1.4}\\ \textbf{1.4}\\ \textbf{1.5}\\ 1.5$	15/15 f19 0/18 EWUOA 15/18 IPOP-C 15/18 POP-CM 15/18 PO	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 4.6(1) 1e1 561 1.7(3) 3.2(6) 3.7(5) 1e1 467(1)* 6.8(13) 445(1389) 1e1 3.2 12(8) 4.3(5)	1 4.366(5e 2.4e4(1t 2.7e4(2e 1e0 46150 15(17) 9.2(4) 6.4(2) 1e0 6541 2.2(2) 55(48) 13(21) 287(349) 1e0 1614 3.5(3)*3 32(33)	$\begin{array}{c} 3.4e5 \\ 6) \infty \\ 6) \infty \\ 4)0.71(0.5) \\ 1e-1 \\ 3.1e6 \\ \infty \\ 1(0.5) \\ 0.65(0.3) \\ 1e-1 \\ 14103 \\ 1.2(2) \\ 48(95) \\ 110(140) \\ 1e-1 \\ 23491 \\ 6.8(8) \\ 215(260) \\ \infty \\ 1e-1 \\ 67457 \\ 32(37) \\ 1(0.8)^*3 \end{array}$	$\begin{array}{c} \textbf{6.2e6} \\ \infty \\ \textbf{1}(0.3) \\ \textbf{0.38}(0.1) \\ \textbf{1e-3} \\ \textbf{5.5e6} \\ \infty \\ \textbf{1}(0.3) \\ \textbf{0.57}(0.2) \\ \textbf{1e-3} \\ \textbf{14643} \\ \textbf{1.2}(2) \\ \textbf{46}(93) \\ \textbf{106}(136) \\ \textbf{1e-3} \\ \textbf{24948} \\ \textbf{6.4}(8) \\ \textbf{202}(244) \\ \infty \\ \textbf{1e-3} \\ \textbf{4.9e5} \\ \infty \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ 1(0.3) \\ 40.41(0.2) \\ 1e-5 \\ 5.6e6 \\ \infty \\ 1(0.3) \\ 0.58(0.2) \\ 1e-5 \\ 15567 \\ 1.1(2) \\ 43(85) \\ 100(127) \\ 1e-5 \\ 26847 \\ 6.0(7) \\ 188(232) \\ \infty \\ 1e-5 \\ 8.1e5 \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 1(0.3) \\ 40.41(0.2) \\ 1e-7 \\ 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \\ 1e-7 \\ 17589 \\ 1(1) \\ 39(74) \\ 88(111) \\ 1e-7 \\ 1.3e5 \\ 1.2(1) \\ 37(4e) \\ \infty \ 1e6 \\ 1e-7 \\ 8.4e5 \\ \end{array}$	15/15 0/15 15/15 15/15 15/15 15/15 14/15 11/15 15/15 15/15 15/15 13/15 7/15 #succ 12/15 12/15 5/15 0/15 #succ 15/15 15/15 15/15 15/15 15/15 15/15 15/15
$ \begin{array}{c c} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-CD} & 1 (0.5) \\ \text{IPOP-CM} & 1.9 (2) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \hline f8 & 2039 \\ \text{NEWUOA1} & (0.3)^{*4} \\ \text{BIPOP-C} & 4.0 (1) \\ \text{IPOP-CM} & 3.7 (0.9) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \hline f9 & 1716 \\ \text{NEWUOA1.0} & (0.2)^{*4} \\ \text{BIPOP-C} & 4.7 (2) \\ \text{IPOP-CM} & 4.6 (0.8) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \hline f10 & 7413 \\ \text{NEWUOA1.7} & (0.5) \\ \text{BIPOP-C} & 1.9 (0.2) \\ \text{IPOP-CM} & 1.8 (0.2) \\ \hline \Delta f_{\text{opt}} & \text{le1} \\ \hline f11 & 1002 \\ \text{NEWUOA1.5} & (2) \\ \text{BIPOP-C} & 10 (0.5) \\ \text{BIPOP-CD} & 10 (0.5) \\ \text{IPOP-CM} & 11 (2) \\ \end{array} $	$\begin{array}{c} 4274 \\ \infty \\ 4.9(2) \\ 4.8(2) \\ 160 \\ 3871 \\ 1(0.6)^{\star 4} \\ 4.0(0.7) \\ 3.9(0.5) \\ 160 \\ 3102 \\ 1(0.6)^{\star 4} \\ 5.7(1) \\ 5.7(0.5) \\ 1e0 \\ 8661 \\ 2.6(0.8) \\ 1.8(0.2) \\ 1.8(0.2) \\ 1.8(0.2) \\ 1e0 \\ 2228 \\ 13(2) \\ 5.1(0.3) \\ 5.4(0.9) \end{array}$	$\begin{array}{c} 9503 \\ \infty \\ 3.5(0.6) \\ \textbf{2.7}(2) \\ \textbf{1e-1} \\ 4040 \\ \textbf{1}(0.6)^{\star 4} \\ 4.3(0.6) \\ 4.2(0.5) \\ \textbf{1e-1} \\ 3277 \\ \textbf{1}(0.6)^{\star 4} \\ 6.0(1) \\ 6.0(0.4) \\ \textbf{1e-1} \\ \textbf{10735} \\ 3.3(1) \\ \textbf{1.5}(0.1) \\ \textbf{1.5}(0.1) \\ \textbf{1.5}(0.1) \\ \textbf{2.1}(0.3) \\ \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-3} \\ 4219 \\ 1(0.5)^{*4} \\ 4.5(0.6) \\ 4.4(0.4) \\ 1e^{-3} \\ 3455 \\ 1(0.5)^{*4} \\ 6.1(1) \\ 6.1(0.4) \\ 1e^{-3} \\ 14920 \\ 4.0(0.8) \\ 1.2(0.0) \\ 1.2(0.1) \\ 1e^{-3} \\ 9762 \\ 6.1(0.5) \\ 1.4(0.0) \\ 1.4(0.0) \\ 1.4(0.0) \\ 1.4(0.0) \\ \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-5} \\ 4371 \\ 1(0.5)^{*4} \\ 4.4(0.4) \\ 1e^{-5} \\ 3594 \\ 1(0.5)^{*4} \\ 6.1(1.0) \\ 6.1(0.4) \\ 1e^{-5} \\ 17073 \\ 4.7(0.8) \\ 1.1(0.0) \\ 1.1(0.0) \\ 1.2285 \\ 6.6(0.3) \\ 1.2(0.0) \\ 1.2(0.2) \\ \end{array}$	$\begin{array}{c} 16969\\ \infty \ 5e5\\ 2.1(0.3)\\ \textbf{1.6}(0.9)^{\star}\\ \textbf{1e-7}\\ 4484\\ \textbf{1}(0.5)^{\star4}\\ 4.6(0.6)\\ 4.5(0.4)\\ \textbf{1e-7}\\ 3727\\ \textbf{1}(0.5)^{\star4}\\ 6.1(0.9)\\ 6.1(0.3)\\ \textbf{1e-7}\\ 17476\\ 5.8(1)\\ \textbf{1.1}(0.0)\\ \textbf{1.1}(0.0)\\ \textbf{1.1}(0.0)\\ \textbf{1.6}-7\\ 14831\\ 6.5(0.3)\\ \textbf{1.0}(0.0)\\ \textbf{1.1}(0.1)\\ \end{array}$	15/15 f19 0/18 EWUOA 15/18 F19 POP-CM 15/18 POP-CM 15/18 F20 15/1	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 4.6(1) 1e1 561 1.7(3) 3.2(6) 3.7(5) 1e1 467 1(1)* 6.8(13) 445(1389) 1e1 3.2 12(8) 4.3(5) 4.3(6)	1 4.366(5e 2.4e4(1e 2.7e4(2e 1e0 46150) 15(17) 9.2(4) 6.4(2) 1e0 6541 2.2(2) 55(48) 139(202) 1e0 5580 4.9(6) 13(21) 287(349) 1e0 1614 3.5(3)*3	$\begin{array}{c} 3.4e5 \\ 6) \infty \\ 6) \infty \\ 4)0.71(0.5) \\ 1e-1 \\ 3.1e6 \\ \infty \\ 1(0.5) \\ 0.65(0.3) \\ 1e-1 \\ 14103 \\ 1.2(2) \\ 48(95) \\ 110(140) \\ 1e-1 \\ 23491 \\ 6.8(8) \\ 215(260) \\ \infty \\ 1e-1 \\ 67457 \\ 32(37) \\ 1(0.8)^*3 \end{array}$	$\begin{array}{c} 6.2e6\\ \infty\\ 1(0.3)\\ 1(0.3)\\ \textbf{0.38}(0.1)\\ 1e-3\\ 5.5e6\\ \infty\\ 1(0.3)\\ \textbf{0.57}(0.2)\\ 1e-3\\ 14643\\ \textbf{1.2}(2)\\ 46(93)\\ 106(136)\\ 1e-3\\ 24948\\ \textbf{6.4}(8)\\ 202(244)\\ \infty\\ 1e-3\\ 4.9e5\\ \infty \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ \downarrow 40.41(0.2) \\ \hline 1e-5 \\ 5.6e6 \\ \infty \\ 1(0.3) \\ 0.58(0.2) \\ 1e-5 \\ 15567 \\ 1.1(2) \\ 43(85) \\ 100(127) \\ 1e-5 \\ 26847 \\ 6.0(7) \\ 188(232) \\ \infty \\ 1e-5 \\ 8.1e5 \\ \infty \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 2.64 \\ 1(0.3) \\ 40.41(0.2) \downarrow \\ 1e-7 \\ 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \\ 1e-7 \\ 17589 \\ 1(1) \\ 39(74) \\ 88(111) \\ 1e-7 \\ 1.3e5 \\ 1.2(1) \\ 37(46) \\ \infty \ 1e6 \\ 1e-7 \\ 8.4e5 \\ \infty \ 2e5 \end{array}$	15/15 0/15 15/15 15/15 15/15 14/15 14/15 11/15 1
$\begin{array}{lll} \text{NEWUOA} & \infty \\ \text{BIPOP-C} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f8 & 2039 \\ \text{NEWUOA1} & (0.3) *^4 \\ \text{BIPOP-C} & 4.0(1) \\ \text{IPOP-CM} & 3.7(0.9) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f9 & 1716 \\ \text{NEWUOA1} & 0.0.2) *^4 \\ \text{BIPOP-C} & 4.7(2) \\ \text{IPOP-CM} & 4.6(0.8) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f10 & 7413 \\ \text{NEWUOA1} & 1.7(0.5) \\ \text{BIPOP-C} & 1.9(0.2) \\ \text{IPOP-CM} & 1.8(0.2) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f11 & 1002 \\ \text{NEWUOA1} & 52 \\ \text{SIPOP-C} & 10(0.5) \\ \text{IPOP-CM} & 1.9(0.5) \\ \text{IPOP-CM} $	$\begin{array}{c} 4274 \\ \infty \\ 4.9(2) \\ 4.8(2) \\ 160 \\ 3871 \\ 1(0.6)^{*4} \\ 4.0(0.7) \\ 3.9(0.5) \\ 1e0 \\ 3102 \\ 1.0(6)^{*4} \\ 5.7(1) \\ 5.7(0.5) \\ 1e0 \\ 2.6(0.8) \\ 1.8(0.2) \\ 1.8(0.2) \\ 1e0 \\ 2228 \\ 13(2) \\ 5.1(0.3) \\ 5.4(0.9) \\ 1e0 \end{array}$	$\begin{array}{c} 9503 \\ \infty \\ \infty \\ 3.5(0.6) \\ \textbf{2.7}(2) \\ 1e-1 \\ 4040 \\ \textbf{1}(0.6)^{\star 4} \\ 4.3(0.6) \\ 4.2(0.5) \\ 1e-1 \\ 3277 \\ \textbf{1}(0.6)^{\star 4} \\ 6.0(0.4) \\ 1e-1 \\ 10735 \\ 3.3(1) \\ 1.6(0.1) \\ 1.5(0.1) \\ 1e-1 \\ 6278 \\ 5.8(0.6) \\ 1.9(0.1) \\ 1.9(0.1) \\ 1.9(0.1) \\ 1.10(3) \\ 1e-1 \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-3} \\ 4219 \\ 1(0.5)^{*4} \\ 4.5(0.6) \\ 4.4(0.4) \\ 1e^{-3} \\ 3455 \\ 1(0.5)^{*4} \\ 6.1(1) \\ 6.1(0.4) \\ 1e^{-3} \\ 14920 \\ 4.0(0.8) \\ 1.2(0.0) \\ 1.2(0.1) \\ 1e^{-3} \\ 9762 \\ 6.1(0.5) \\ 1.4(0.0) \\ 1.4(0.0) \\ 1e^{-3} \\ 1.4(0.0) \\ 1.4(0.2) \\ 1e^{-3} \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^* \\ 1e^{-5} \\ 4371 \\ 1(0.5)^{*4} \\ 4.6(0.6) \\ 4.4(0.4) \\ 1e^{-5} \\ 3594 \\ 1(0.5)^{*4} \\ 6.1(1.0) \\ 6.1(0.4) \\ 1e^{-5} \\ 17073 \\ 4.7(0.8) \\ 1.1(0.0) \\ 1e^{-5} \\ 12285 \\ 6.6(0.3) \\ 1.2(0.0) \\ 1.2(0.2) \\ 1e^{-5} \\ \end{array}$	$\begin{array}{c} 16969\\ \infty \ 5e5\\ 2.1(0.3)\\ 1.6(0.9)^{\star}\\ 1e\text{-}7\\ 4484\\ 1(0.5)^{\star 4}\\ 4.6(0.6)\\ 4.5(0.4)\\ 1e\text{-}7\\ 3727\\ 1(0.5)^{\star 4}\\ 6.1(0.9)\\ 6.1(0.9)\\ 6.1(0.9)\\ 1e\text{-}7\\ 17476\\ 5.8(1)\\ 1.1(0.0)\\ 1e\text{-}7\\ 14831\\ 6.5(0.3)\\ 1.0(0.0)\\ 1.0(0.0)\\ 1.1(0.1)\\ 1e\text{-}7\\ \end{array}$	15/15 f19 0/18 EWUOA 15/18 IPOP-C 15/18 POP-CM #succ $\Delta f_{\rm opt}$ 15/18 POP-CM 15/18 POP-CM #succ $\Delta f_{\rm opt}$ 15/18 EWUOA 15/18 IPOP-C 15/18 POP-CM #succ $\Delta f_{\rm opt}$ 15/18 EWUOA 15/18 IPOP-C 15/18 POP-CM #succ $\Delta f_{\rm opt}$ 15/18 EWUOA 15/18 IPOP-C 15/18 POP-CM #succ $\Delta f_{\rm opt}$ 15/18 EWUOA 15/18 IPOP-C 15/18 POP-CM #succ $\Delta f_{\rm opt}$ 15/18 EWUOA 15/18 IPOP-C 15/18 POP-C 15/18 POP-C 15/18 POP-C 15/18 IPOP-C 15/1	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 4.6(1) 1e1 561 1.7(3) 3.2(6) 3.7(5) 1e1 467 1(1)* 6.8(13) 445(1389) 1e1 3.2 12(8) 4.3(5) 4.3(6) 1e1	1 4.366(5e 2.4e4(1e 2.7e4(2e 1e0 46150 15(17) 9.2(4) 6.4(2) 1e0 6541 2.2(2) 55(48) 3.9(202) 1e0 5580 4.9(6) 13(21) 287(349) 1e0 1614 3.5(3)*3 32(33) 2.3e4(2e4 1e0	$\begin{array}{c} 3.4e5 \\ 6) \infty \\ 6) \infty \\ 4)11.2(0.6) \\ 4)0.71(0.5) \\ 1e-1 \\ 3.1e6 \\ \infty \\ 1(0.5) \\ 0.655(0.3) \\ 1e-1 \\ 14103 \\ 1.2(2) \\ 48(95) \\ 110(140) \\ 1e-1 \\ 23491 \\ 6.8(8) \\ 215(260) \\ \infty \\ 1e-1 \\ 67457 \\ 32(37) \\ 1(0.8) *3 \\) \infty \\ 1e-1 \end{array}$	$\begin{array}{c} 6.2e6\\ \infty\\ 1(0.3)\\ 1(0.3)\\ \textbf{0.38}(0.1)\\ 1e-3\\ 5.5e6\\ \infty\\ 1(0.3)\\ \textbf{0.57}(0.2)\\ 1e-3\\ 14643\\ \textbf{1.2}(2)\\ 46(93)\\ 106(136)\\ 1e-3\\ 24948\\ \textbf{6.4}(8)\\ 202(244)\\ \infty\\ 1e-3\\ 4.9e5\\ \infty\\ 2.0(1)\\ \infty\\ 1e-3\\ 1.9e5\\ \infty\\ 1e-3\\ 1.9e5\\ \infty\\ 1.9e5\\ 1.9$	$\begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ 1(0.3) \\ 40.41(0.2) \\ \hline 1e-5 \\ 5.6e6 \\ \infty \\ 1(0.3) \\ 0.58(0.2) \\ 1e-5 \\ 15567 \\ 1.1(2) \\ 43(85) \\ 100(127) \\ 1e-5 \\ 26847 \\ 6.0(7) \\ 188(232) \\ \infty \\ 1e-5 \\ 8.1e5 \\ \infty \\ 1e-5 \\ 1.2(0.9) \\ \infty \\ 1e-5 \\ \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 1(0.3) \\ 40.41(0.2) \downarrow \\ \hline 1e-7 \\ 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \\ \hline 1e-7 \\ 17589 \\ 1(1) \\ 39(74) \\ 88(111) \\ 1e-7 \\ 1.3e5 \\ 1.2(1) \\ 37(46) \\ \infty \ 1e6 \\ \hline 1e-7 \\ 8.4e5 \\ \infty \ 2e5 \\ 1.2(0.9) \\ \infty \ 3e6 \\ \hline 1e-7 \\ \end{array}$	15/15 0/15 15/15 15/15 15/15 14/15 14/15 114/15 115/15 13/15 13/15 7/15 7/15 7/15 0/15 15/15 0/15 15/15 0/15 15/15 0/15
$ \begin{array}{c c} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-C} & 1 (0.5) \\ \text{IPOP-CM} & 1.9 (2) \\ \hline \Delta f_{\text{opt}} & 1 \text{te1} \\ \hline \textbf{f8} & 2039 \\ \text{NEWUOA} & 1 (0.3)^{*4} \\ \text{BIPOP-C} & 4.0 (1) \\ \text{IPOP-CM} & 3.7 (0.9) \\ \hline \Delta f_{\text{opt}} & 1 \text{e1} \\ \hline \textbf{f9} & 1716 \\ \text{NEWUOA} & 1.0 (0.2)^{*4} \\ \text{BIPOP-C} & 4.7 (2) \\ \text{IPOP-CM} & 4.6 (0.8) \\ \hline \Delta f_{\text{opt}} & 1 \text{e1} \\ \hline \textbf{f10} & 7413 \\ \text{NEWUOA} & 1.7 (0.5) \\ \text{BIPOP-C} & 1.9 (0.2) \\ \text{IPOP-CM} & 1.8 (0.2) \\ \hline \Delta f_{\text{opt}} & 1 \text{e1} \\ \hline \textbf{f11} & 1002 \\ \text{NEWUOA} & 1.5 (2) \\ \text{SIPOP-C} & 10 (0.5) \\ \text{IPOP-CM} & 11 (2) \\ \hline \Delta f_{\text{opt}} & 1 \text{e1} \\ \hline \textbf{f12} & 1042 \\ \hline \end{array} $	$\begin{array}{c} 4274 \\ \infty \\ 4.9(2) \\ 4.8(2) \\ 1e0 \\ 3871 \\ 1(0.6)^{*4} \\ 4.0(0.7) \\ 3.9(0.5) \\ 1e0 \\ 3102 \\ 1(0.6)^{*4} \\ 5.7(1) \\ 5.7(0.5) \\ 1e0 \\ 8661 \\ 2.6(0.8) \\ 1.8(0.2) \\ 1.8(0.2) \\ 1.8(0.2) \\ 1.9(0.3) \\ 1.8(0.2) \\ 1.9(0.3) $	$\begin{array}{c} 9503 \\ \infty \\ \infty \\ 3.5(0.6) \\ 2.7(2) \\ 1e\text{-}1 \\ 4040 \\ 1(0.6)^{*4} \\ 4.3(0.6) \\ 4.2(0.5) \\ 1e\text{-}1 \\ 3277 \\ 1(0.6)^{*4} \\ 6.0(1) \\ 6.0(0.4) \\ 1e\text{-}1 \\ 10735 \\ 3.3(1) \\ 1.6(0.1) \\ 1.5(0.1) \\ 1.5(0.1) \\ 1e\text{-}1 \\ 6278 \\ 5.8(0.6) \\ 1.9(0.1) \\ 2.1(0.3) \\ 1e\text{-}1 \\ 2740 \\ \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-3} \\ 4219 \\ 1(0.5)^{*4} \\ 4.5(0.6) \\ 4.4(0.4) \\ 1e^{-3} \\ 3455 \\ 1(0.5)^{*4} \\ 6.1(1) \\ 6.1(0.4) \\ 1e^{-3} \\ 14920 \\ 4.0(0.8) \\ 1.2(0.0) \\ 1.2(0.1) \\ 1e^{-3} \\ 9762 \\ 6.1(0.5) \\ 1.4(0.0) \\ 1.4(0.2) \\ 1e^{-3} \\ 4140 \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^* \\ 1e-5 \\ 4371 \\ 1(0.5)^{*4} \\ 4.4(0.4) \\ 1e-5 \\ 3594 \\ 1(0.5)^{*4} \\ 6.1(1.0) \\ 6.1(0.4) \\ 1e-5 \\ 17073 \\ 4.7(0.8) \\ 1.1(0.0) \\ 1.1(0.0) \\ 1.1(0.0) \\ 1.2285 \\ 6.6(0.3) \\ 1.2(0.0) \\ 1.2(0.2) \\ 1e-5 \\ 12407 \end{array}$	$\begin{array}{c} 16969 \\ \infty \ 5e5 \\ 2.1 (0.3) \\ \textbf{1.6} (0.9)^{\star} \\ \textbf{1e-7} \\ 4484 \\ \textbf{1} (0.5)^{\star} 4 \\ 4.6 (0.6) \\ 4.5 (0.4) \\ \textbf{1e-7} \\ 3727 \\ \textbf{1} (0.5)^{\star} 4 \\ 6.1 (0.9) \\ 6.1 (0.3) \\ \textbf{1e-7} \\ 17476 \\ 5.8 (1) \\ 1.1 (0.0) \\ \textbf{1.1} (0.1) \\ \textbf{1.1} (0.0) \\ \textbf{1.1} (0.1) \\ \textbf{1.1} (0.0) \\ \textbf{1.1} (0.1) \\ $	$\begin{array}{lll} 15/15 & f19 \\ 0/18 EWUOA \\ 15/18 IPOP-C \\ 15/18 IPOP-C \\ 15/18 EWUOA \\ 15/15 & f20 \\ 15/18 EWUOA \\ 15/18 IPOP-C \\ 15/18 EWUOA \\ 15/18 IPOP-C \\ 15/18 EWUOA \\ 15/18 IPOP-C \\ 15/18$	1 76(50)* 169(74) 161(86) 1e1 82 1(0.5)*4 4.3(1) 4.6(1) 1e1 561 1.7(3) 3.2(6) 3.7(5) 1e1 467 45(1389) 1e1 3.2 12(8) 4.3(5) 4.3(6) 1e1 1.3(6) 1e1 1.2(6) 1.3(6	1 4.366(5e 2.4e4(1t 2.7e4(2e 1e0 46150 15(17) 9.2(4) 6.4(2) 1e0 6541 2.2(2) 55(48) 13(21) 287(349) 1e0 1614 3.5(3)*3 32(33) 2.3e4(2e4 1e0 7.5e6	$\begin{array}{c} 3.4e5 \\ 6) \infty \\ 6) \infty \\ 3.1e6 \\ 20 \\ 3.1e0 \\ 3.1$	$\begin{array}{c} 6.2e6\\ \infty\\ 1(0.3)\\ 0.38(0.1)\\ 1e^{-3}\\ 5.5e6\\ \infty\\ 1(0.3)\\ 0.57(0.2)\\ 1e^{-3}\\ 14643\\ 1.2(2)\\ 46(93)\\ 106(136)\\ 1e^{-3}\\ 24948\\ 6.4(8)\\ 202(244)\\ \infty\\ 1e^{-3}\\ 4.9e5\\ \infty\\ 2.0(1)\\ \infty\\ 1e^{-3}\\ 5.2e7\\ \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ 1(0.3) \\ 40.41(0.2), \\ 1e-5 \\ 5.6e6 \\ \infty \\ 1(0.3) \\ 0.58(0.2) \\ 1e-5 \\ 15567 \\ 1.1(2) \\ 43(85) \\ 100(127) \\ 1e-5 \\ 26847 \\ 6.0(7) \\ 188(232) \\ \infty \\ 1e-5 \\ 8.1e5 \\ \infty \\ 1.2(0.9) \\ \infty \\ 1e-5 \\ 5.2e7 \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 2.64 \\ 1(0.3) \\ 40.41(0.2) \\ 1e-7 \\ 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \\ 1e-7 \\ 17589 \\ 1(1) \\ 39(74) \\ 88(111) \\ 1e-7 \\ 1.3e5 \\ 1.2(1) \\ 37(46) \\ \infty \ 1e6 \\ 1e-7 \\ 8.4e5 \\ \infty \ 2e5 \\ 1.2(0.9) \\ \infty \ 3e6 \\ 1e-7 \\ 5.2e7 \\ \end{array}$	15/15 0/15 15/15 15/15 14/15 0/15 14/15 15/15 15/15 15/15 13/15 17/15 5/15 0/15 5/15 0/15 5/15 0/15 0/15 0
$\begin{array}{c c} \text{NEWUOA} & \infty \\ \text{BIPOP-C} & 1(0.5) \\ \text{IPOP-CM} & 1.9(2) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f8 & 2039 \\ \text{NEWUOA1} & (0.3)^{*4} \\ \text{BIPOP-C} & 4.0(1) \\ \text{IPOP-CM} & 3.7(0.9) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f9 & 1716 \\ \text{NEWUOA1} & 1.0(0.2)^{*4} \\ \text{BIPOP-C} & 4.7(2) \\ \text{IPOP-CM} & 4.6(0.8) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f10 & 7413 \\ \text{NEWUOA1} & 1.7(0.5) \\ \text{BIPOP-C} & 1.9(0.2) \\ \text{IPOP-CM} & 1.8(0.2) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f11 & 1002 \\ \text{NEWUOA1} & 15(2) \\ \text{BIPOP-C} & 10(0.5) \\ \text{IPOP-CM} & 11(2) \\ \hline \Delta f_{\text{opt}} & 1e1 \\ \hline f12 & 1042 \\ \text{NEWUOA3.0} & 3.0(3) \\ \end{array}$	$\begin{array}{c} 4274 \\ \infty \\ 4.9(2) \\ 4.8(2) \\ 160 \\ 3871 \\ 1(0.6)^{\star 4} \\ 4.0(0.7) \\ 3.9(0.5) \\ 160 \\ 3102 \\ 1(0.6)^{\star 4} \\ 5.7(1) \\ 5.7(0.5) \\ 1e0 \\ 8661 \\ 2.6(0.8) \\ 1.8(0.2) \\ 1.8(0.2) \\ 1.8(0.2) \\ 1e0 \\ 2228 \\ 13(2) \\ 5.1(0.3) \\ 5.4(0.9) \\ 1e0 \\ 1938 \\ 3.0(2) \end{array}$	$\begin{array}{c} 9503 \\ \infty \\ 3.5(0.6) \\ 2.7(2) \\ 1e-1 \\ 4040 \\ 1(0.6)^{*4} \\ 4.3(0.6) \\ 4.2(0.5) \\ 1e-1 \\ 3277 \\ 1(0.6)^{*4} \\ 6.0(1) \\ 6.0(0.4) \\ 1e-1 \\ 10735 \\ 3.3(1) \\ 1.6(0.1) \\ 1.5(0.1) \\ 1e-1 \\ 6278 \\ 5.8(0.6) \\ 1.9(0.1) \\ 2.1(0.3) \\ 1e-1 \\ 3.0(2) \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^* \\ 1e^{-3} \\ 4219 \\ 1(0.5)^{*4} \\ 4.5(0.6) \\ 4.4(0.4) \\ 1e^{-3} \\ 3455 \\ 1(0.5)^{*4} \\ 6.1(1) \\ 6.1(0.4) \\ 1e^{-3} \\ 14920 \\ 4.0(0.8) \\ 1.2(0.0) \\ 1.2(0.1) \\ 1e^{-3} \\ 9762 \\ 6.1(0.5) \\ 1.4(0.0) \\ 1.4(0.2) \\ 1e^{-3} \\ 4140 \\ 2.5(1) \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^* \\ 1e^{-5} \\ 4371 \\ 1(0.5)^{*4} \\ 4.6(0.6) \\ 4.4(0.4) \\ 1e^{-5} \\ 3594 \\ 1(0.5)^{*4} \\ 6.1(1.0) \\ 6.1(0.4) \\ 1e^{-5} \\ 17073 \\ 4.7(0.8) \\ 1.1(0.0) \\ 1.1(0.0) \\ 1e^{-5} \\ 12285 \\ 6.6(0.3) \\ 1.2(0.0) \\ 1.2(0.2) \\ 1e^{-5} \\ 12407 \\ 1(0.4)^{*2} \end{array}$	$\begin{array}{c} 16969\\ \infty \ 5e5\\ 2.1(0.3)\\ \textbf{1.6}(0.9)^{\star}\\ \textbf{1e-7}\\ 4484\\ \textbf{1}(0.5)^{\star 4}\\ 4.6(0.6)\\ 4.5(0.4)\\ \textbf{1e-7}\\ 3727\\ \textbf{1}(0.5)^{\star 4}\\ 6.1(0.9)\\ 6.1(0.3)\\ \textbf{1e-7}\\ 17476\\ 5.8(1)\\ \textbf{1.1}(0.0)\\ \textbf{1.1}(0.0)\\ \textbf{1.1}(0.0)\\ \textbf{1.1}(0.0)\\ \textbf{1.1}(0.0)\\ \textbf{1.1}(0.1)\\ \textbf{1e-7}\\ 13827\\ \textbf{1}(0.4)^{\star 3}\\ \end{array}$	15/15 f19 0/18EWUOA 15/18IPOP-C 15/18POP-CM #succΔf _{opt} 15/18 f20 15/18IPOP-CM 15/18IPOP-CM 15/18IPOP-CM #succΔf _{opt} 15/15 f21 15/18EWUOA 15/18IPOP-CM #succΔf _{opt} 15/15 f22 15/18EWUOA 15/18IPOP-CM #succΔf _{opt} 15/15 f22 15/18EWUOA 15/18IPOP-CM 15/18IPOP-CM 15/18IPOP-CM 15/18IPOP-CM 15/15 f23 15/18EWUOA 15/18IPOP-CM #succΔf _{opt} 15/15 f23 15/18EWUOA	$\begin{array}{c} 1\\ 76(50)^{\star}\\ 169(74)\\ 161(86)\\ \\ 1e1\\ 82\\ 1(0.5)^{\star}4\\ 4.3(1)\\ 4.6(1)\\ \\ 1e1\\ \\ 561\\ 1.7(3)\\ 3.2(6)\\ 3.7(5)\\ \\ 1e1\\ 467\\ 1(1)^{\star}\\ 6.8(13)\\ 445(1389)\\ \\ 1e1\\ 3.2\\ 12(8)\\ 4.3(5)\\ 4.3(6)\\ \\ 1e1\\ \\ 1.3e6\\ \\ \infty \end{array}$	$\begin{array}{c} 1\\ 4.3e6(5e\\ \mathbf{2.4e4}(1e\\ 2.7e4(2e\\ 1e0\\ 46150\\ 15(17)\\ 9.2(4)\\ 6.4(2)\\ 1e0\\ 6541\\ 2.2(2)\\ 55(48)\\ 139(202)\\ 1e0\\ 5580\\ 4.9(6)\\ 13(21)\\ 287(349)\\ 1e0\\ 1614\\ 3.5(3)^{*3}\\ 32(33)\\ 2.3e4(2e4\\ 1e0\\ 7.5e6\\ \infty \end{array}$	$\begin{array}{c} 3.4e5 \\ 6) \infty \\ 6) \infty \\ 4)0.71(0.6) \\ 1e-1 \\ 3.1e6 \\ \infty \\ 1(0.5) \\ 0.65(0.3) \\ 1e-1 \\ 14103 \\ 1.2(2) \\ 48(95) \\ 110(140) \\ 1e-1 \\ 23491 \\ 6.8(8) \\ 215(260) \\ \infty \\ 1e-1 \\ 67457 \\ 32(37) \\ 1(0.8) ^{*3}) \\ 0 \\ \infty \\ 1e-1 \\ 5.2e7 \\ \infty \end{array}$	6.2e6 ∞ 1(0.3) 0.38(0.1) 1e-3 5.5e6 ∞ 1(0.3) 0.57(0.2) 1e-3 14643 1.2(2) 46(93) 106(136) 1e-3 24948 6.4(8) 202(244) ∞ 1e-3 4.9e5 ∞ 2.0(1) ∞ 5.2e7 ∞	$\begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ 1(0.3) \\ 40.41(0.2), \\ 1e-5 \\ 5.6e6 \\ \infty \\ 1(0.3) \\ 0.58(0.2) \\ 1e-5 \\ 15567 \\ 1.1(2) \\ 43(85) \\ 100(127) \\ 1e-5 \\ 26847 \\ 6.0(7) \\ 188(232) \\ \infty \\ 1e-5 \\ 8.1e5 \\ \infty \\ 1.2(0.9) \\ \infty \\ 1e-5 \\ 5.2e7 \\ \infty \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ (10.3) \\ 40.41(0.2) \\ 1e-7 \\ 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \\ 1e-7 \\ 17589 \\ 1(1) \\ 39(74) \\ 88(111) \\ 1e-7 \\ 1.3e5 \\ 1.2(1) \\ 37(46) \\ \infty \ 1e6 \\ 1e-7 \\ 8.4e5 \\ \infty \ 2e5 \\ 1.2(0.9) \\ \infty \ 3e6 \\ 1e-7 \\ 5.2e7 \\ \infty \ 2e5 \\ \end{array}$	15/15 0/15 15/15 15/15 #succ 114/15 14/15 14/15 115/15 13/15 17/15 13/15 7/15 5/15 0/15 #succ 12/15 7/15 0/15 #succ 12/15 7/15 0/15 #succ 12/15 7/15 0/15 #succ 12/15 7/15 0/15 #succ
$ \begin{array}{c c} \text{NEWUOA} \\ \text{NEWUOA} \\ \text{BIPOP-C} & 1 (0.5) \\ \text{IPOP-CM} & 1.9 (2) \\ \hline \Delta f_{\text{opt}} & 1 \text{te1} \\ \hline \textbf{f8} & 2039 \\ \text{NEWUOA} & 1 (0.3)^{*4} \\ \text{BIPOP-C} & 4.0 (1) \\ \text{IPOP-CM} & 3.7 (0.9) \\ \hline \Delta f_{\text{opt}} & 1 \text{e1} \\ \hline \textbf{f9} & 1716 \\ \text{NEWUOA} & 1.0 (0.2)^{*4} \\ \text{BIPOP-C} & 4.7 (2) \\ \text{IPOP-CM} & 4.6 (0.8) \\ \hline \Delta f_{\text{opt}} & 1 \text{e1} \\ \hline \textbf{f10} & 7413 \\ \text{NEWUOA} & 1.7 (0.5) \\ \text{BIPOP-C} & 1.9 (0.2) \\ \text{IPOP-CM} & 1.8 (0.2) \\ \hline \Delta f_{\text{opt}} & 1 \text{e1} \\ \hline \textbf{f11} & 1002 \\ \text{NEWUOA} & 1.5 (2) \\ \text{SIPOP-C} & 10 (0.5) \\ \text{IPOP-CM} & 11 (2) \\ \hline \Delta f_{\text{opt}} & 1 \text{e1} \\ \hline \textbf{f12} & 1042 \\ \hline \end{array} $	$\begin{array}{c} 4274 \\ \infty \\ 4.9(2) \\ 4.8(2) \\ 1e0 \\ 3871 \\ 1(0.6)^{*4} \\ 4.0(0.7) \\ 3.9(0.5) \\ 1e0 \\ 3102 \\ 1(0.6)^{*4} \\ 5.7(1) \\ 5.7(0.5) \\ 1e0 \\ 8661 \\ 2.6(0.8) \\ 1.8(0.2) \\ 1.8(0.2) \\ 1.8(0.2) \\ 1.9(0.3) \\ 1.8(0.2) \\ 1.9(0.3) $	$\begin{array}{c} 9503 \\ \infty \\ \infty \\ 3.5(0.6) \\ 2.7(2) \\ 1e\text{-}1 \\ 4040 \\ 1(0.6)^{*4} \\ 4.3(0.6) \\ 4.2(0.5) \\ 1e\text{-}1 \\ 3277 \\ 1(0.6)^{*4} \\ 6.0(1) \\ 6.0(0.4) \\ 1e\text{-}1 \\ 10735 \\ 3.3(1) \\ 1.6(0.1) \\ 1.5(0.1) \\ 1.5(0.1) \\ 1e\text{-}1 \\ 6278 \\ 5.8(0.6) \\ 1.9(0.1) \\ 2.1(0.3) \\ 1e\text{-}1 \\ 2740 \\ \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^{*} \\ 1e^{-3} \\ 4219 \\ 1(0.5)^{*4} \\ 4.5(0.6) \\ 4.4(0.4) \\ 1e^{-3} \\ 3455 \\ 1(0.5)^{*4} \\ 6.1(1) \\ 6.1(0.4) \\ 1e^{-3} \\ 14920 \\ 4.0(0.8) \\ 1.2(0.0) \\ 1.2(0.1) \\ 1e^{-3} \\ 9762 \\ 6.1(0.5) \\ 1.4(0.0) \\ 1.4(0.2) \\ 1e^{-3} \\ 4140 \end{array}$	$\begin{array}{c} 16524 \\ \infty \\ 2.2(0.3) \\ 1.7(1.0)^* \\ 1e-5 \\ 4371 \\ 1(0.5)^{*4} \\ 4.4(0.4) \\ 1e-5 \\ 3594 \\ 1(0.5)^{*4} \\ 6.1(1.0) \\ 6.1(0.4) \\ 1e-5 \\ 17073 \\ 4.7(0.8) \\ 1.1(0.0) \\ 1.1(0.0) \\ 1.1(0.0) \\ 1.2285 \\ 6.6(0.3) \\ 1.2(0.0) \\ 1.2(0.2) \\ 1e-5 \\ 12407 \end{array}$	$\begin{array}{c} 16969 \\ \infty \ 5e5 \\ 2.1 (0.3) \\ \textbf{1.6} (0.9)^{\star} \\ \textbf{1e-7} \\ 4484 \\ \textbf{1} (0.5)^{\star} 4 \\ 4.6 (0.6) \\ 4.5 (0.4) \\ \textbf{1e-7} \\ 3727 \\ \textbf{1} (0.5)^{\star} 4 \\ 6.1 (0.9) \\ 6.1 (0.3) \\ \textbf{1e-7} \\ 17476 \\ 5.8 (1) \\ 1.1 (0.0) \\ \textbf{1.1} (0.1) \\ \textbf{1.1} (0.0) \\ \textbf{1.1} (0.1) \\ \textbf{1.1} (0.0) \\ \textbf{1.1} (0.1) \\ $	$\begin{array}{lll} 15/15 & f19 \\ 0/18 EWUOA \\ 15/18 IPOP-C \\ 15/18 IPOP-C \\ 15/18 EWUOA \\ 15/15 & f20 \\ 15/18 EWUOA \\ 15/18 IPOP-C \\ 15/18 EWUOA \\ 15/18 IPOP-C \\ 15/18 EWUOA \\ 15/18 IPOP-C \\ 15/18$	$\begin{array}{c} 1\\ 76(50)^{\star}\\ 169(74)\\ 161(86)\\ \end{array}$ $\begin{array}{c} 161(86)\\ 161(86)\\ \end{array}$ $\begin{array}{c} 1e1\\ 82\\ 1(0.5)^{\star}4\\ 4.3(1)\\ 4.6(1)\\ \end{array}$ $\begin{array}{c} 1e1\\ 561\\ 1.7(3)\\ 3.2(6)\\ 3.7(5)\\ \end{array}$ $\begin{array}{c} 1.7(3)\\ 3.2(6)\\ 3.7(5)\\ \end{array}$ $\begin{array}{c} 1e1\\ 467\\ 6.8(13)\\ 445(1389)\\ \end{array}$ $\begin{array}{c} 1e1\\ 3.2\\ 12(8)\\ 4.3(5)\\ 4.3(6)\\ \end{array}$ $\begin{array}{c} 1e1\\ 3.2\\ 1.2(8)\\ 4.3(6)\\ \end{array}$ $\begin{array}{c} 1.366\\ \infty\\ 1.366\\ \infty\\ 1.0.9)\\ \end{array}$	1 4.366(5e 2.4e4(1t 2.7e4(2e 1e0 46150 15(17) 9.2(4) 6.4(2) 1e0 6541 2.2(2) 55(48) 13(21) 287(349) 1e0 1614 3.5(3)*3 32(33) 2.3e4(2e4 1e0 7.5e6	$\begin{array}{c} 3.4e5 \\ 6) \infty \\ 6) \infty \\ 3.1e6 \\ 20 \\ 3.1e0 \\ 3.1$	$\begin{array}{c} 6.2e6\\ \infty\\ 1(0.3)\\ 0.38(0.1)\\ 1e^{-3}\\ 5.5e6\\ \infty\\ 1(0.3)\\ 0.57(0.2)\\ 1e^{-3}\\ 14643\\ 1.2(2)\\ 46(93)\\ 106(136)\\ 1e^{-3}\\ 24948\\ 6.4(8)\\ 202(244)\\ \infty\\ 1e^{-3}\\ 4.9e5\\ \infty\\ 2.0(1)\\ \infty\\ 1e^{-3}\\ 5.2e7\\ \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \\ 1(0.3) \\ 1(0.3) \\ 40.41(0.2), \\ 1e-5 \\ 5.6e6 \\ \infty \\ 1(0.3) \\ 0.58(0.2) \\ 1e-5 \\ 15567 \\ 1.1(2) \\ 43(85) \\ 100(127) \\ 1e-5 \\ 26847 \\ 6.0(7) \\ 188(232) \\ \infty \\ 1e-5 \\ 8.1e5 \\ \infty \\ 1.2(0.9) \\ \infty \\ 1e-5 \\ 5.2e7 \end{array}$	$\begin{array}{c} 6.7e6 \\ \infty \ 2e6 \\ 1(0.3) \\ 2.64 \\ 1(0.3) \\ 40.41(0.2) \\ 1e-7 \\ 5.6e6 \\ \infty \ 4e5 \\ 1(0.3) \\ 0.58(0.2) \\ 1e-7 \\ 17589 \\ 1(1) \\ 39(74) \\ 88(111) \\ 1e-7 \\ 1.3e5 \\ 1.2(1) \\ 37(46) \\ \infty \ 1e6 \\ 1e-7 \\ 8.4e5 \\ \infty \ 2e5 \\ 1.2(0.9) \\ \infty \ 3e6 \\ 1e-7 \\ 5.2e7 \\ \end{array}$	15/15 0/15 15/15 15/15 14/15 0/15 14/15 15/15 15/15 15/15 13/15 17/15 5/15 0/15 5/15 0/15 5/15 0/15 0/15 0

Table 2: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 20. The inter-80%tile range divided by two is given in braces. The median number of conducted function evaluations is additionally given in italics, if $\text{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\text{opt}} + 10^{-8}$. Best results are printed in bold.