# Unbounded Population MO-CMA-ES for the Bi-Objective BBOB Test Suite

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#### UP-MO-CMA-ES in a nutshell

• Population S of Individuals:  $(x_i, \sigma_i, C_i)$  ,  $i = 1, \dots$ 

```
while Stopping criterion is not met do
```

```
Select parent from S based on Hypervolume Contribution;
Sample Offspring with Crossover;
if Offspring is non-dominated in S then
Adapt (\sigma, C) of parent and offspring;
Add offspring to S;
end
else
```

Adapt  $\sigma$  of parent;

end

end



#### Parent Selection

- Select parent based on Hypervolume Contribution
- Select extremum points with probability  $p_{\text{extreme}}$
- Otherwise select parent i with probability

$$p_i = rac{\delta ext{Vol}_S(f(x_i))^{lpha}}{\sum_j \delta ext{Vol}_S(f(x_j))^{lpha}} \ .$$



### Crossover

- C<sub>i</sub> Covariance matrix of parent
- i-1, i+1 neighbours of the parent on the front in f-value
- Covariance matrix of offspring

$$C = (1 - c_r)C_i + \frac{c_r}{2} \left(\frac{x_{i-1} - x_i}{\sigma_i}\right) \left(\frac{x_{i-1} - x_i}{\sigma_i}\right)^{\mathsf{T}} + \frac{c_r}{2} \left(\frac{x_{i+1} - x_i}{\sigma_i}\right) \left(\frac{x_{i+1} - x_i}{\sigma_i}\right)^{\mathsf{T}}$$



## Covariance-Matrix-Adaptation

- Parent  $(C_i, \sigma_i, x_i)$ , Offspring  $(C, \sigma, x)$
- Adapt Covariance matrix of offspring by

$$C \leftarrow (1 - c_{\mathsf{cov}})C + c_{\mathsf{cov}}\left(\frac{x - x_i}{\sigma_i}\right)\left(\frac{x - x_i}{\sigma_i}\right)^{\mathsf{T}}$$
.

Same for parent



## Step Size adaptation

- Success based as in MO-CMA-ES
- Running estimate of success rate
- Adjust  $\sigma$  until success rate 1/2

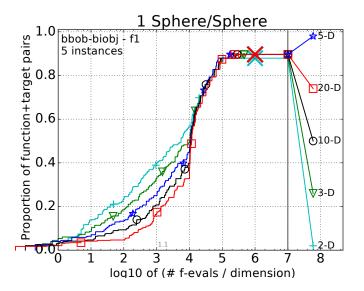


## Multi-Objective Exploration

- Dominance-based selection gets stuck in local optima
- Run k = 100 instances in round robin fashion
- D initial points per instance
- Merge fronts after 10<sup>4</sup>D iterations
- Run single front until budget exhausted

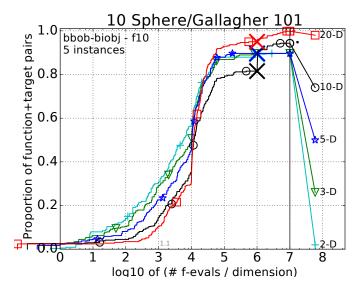


## Results on Sphere/Sphere



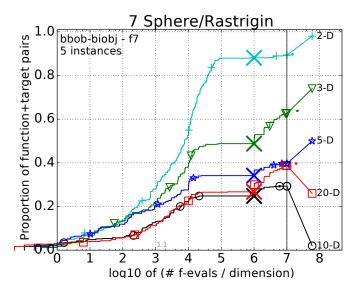


# Results on Sphere/Rastrigin



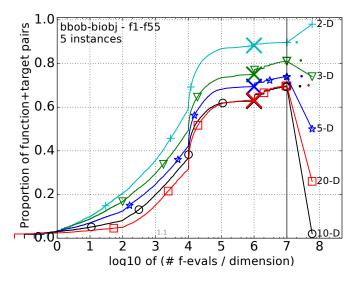


# Results on Sphere/Rastrigin





## Overall Results





#### **Thanks**

See you at BBComp Session!

