مثال أول (

$$y(x_{i+1}) = y_{i+1} = y_i + hy_i' + \frac{h^2}{2!} y_i'' + \frac{h^3}{3!} y_i'''$$

$$\Delta \quad O(h^3)$$

$$y' = x^2 y - 1 \Big|_o = -1$$

$$y'' = 2xy + x^2 y' = 2xy + x^2(x^2y - 1) = 2xy + x^4 y - x^2 \Big|_o = 0$$

$$y''' = 2y + 2xy' + 2xy' + x^2 y''$$

$$= 2y + 2x(x^2 y - 1) + 2xy' + x^2(2xy + x^4 y - x^2)$$

$$= \boxed{2y} + 2x^3 y - 2x + 2xy + 2x^3 y + x^6 y - x^4 \Big|_o = 2$$

$$y_{i+1}^{\cdot} = y_{0,1} = y_0 + 0.1(y_0') + \frac{(0.1)^2}{2}(y_0'') + \frac{(0.1)^3}{6}(y_0''')$$

$$= 1 + 0.1(-1) + \frac{(0.1)^2}{2}(0) + \frac{(0.1)^3}{6}(2)$$

$$= 1 - 0.1 + \frac{(0.1)^3}{3}$$

تعريف

$y = 1 \implies y_{i+1} = a y_i \implies 1 = a(1) \implies \underline{a = 1}$

خطأ

$\left.\begin{array}{l} y = x \\ y' = 1 \end{array}\right\} \implies y_{i+1}^{(1)} = a y_i^{(1)} + bh + ch^{(1)}$

$\qquad x + h = ax + bh + ch$

$\left.\begin{array}{l} \implies \end{array}\right\} \implies \begin{array}{l} a = 1 \\ b + c = 1 \end{array}$

تربيعي

$\left.\begin{array}{l} y = x^2 \\ y' = 2x \end{array}\right\} \implies y_{i+1} = a y_i + bh(2x) + ch(2(x+h))$

$(x+h)^2 = ax^2 + 2bhx + 2chx + 2ch^2$

$x^2 + 2xh + h^2 = ax^2 + (2b + 2c)hx + 2ch^2$

$$\Downarrow$$

$$\underline{\begin{array}{l} a = 1 \\ b + c = 1 \\ 2c = 1 \implies c = \tfrac{1}{2} \end{array}} \left.\begin{array}{l} \\ \\ \end{array}\right\} \underline{b = \tfrac{1}{2}}$$