Finite difference

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# Function approximation

We consider being a function sufficiently smooth (i.e. a sufficiently high number of time derivable).

It is possible to define different approximations of this function. Also we do not have the effective formulas for the moment we will write them the following [[1](#LeV07)].

|  |  |  |
| --- | --- | --- |
| Left approximation | Centered approximation | Right approximation |
|  |  |  |

The preceding notation is for an approximation of the first order. Second and higher approximation may be indicated by an exponent. For instance the 3 order approximation of the function from the right would be indicated by the following: [[1](#LeV07)].

# Taylor expansion

## The basic Taylor formula

We will now give the basic formula of a Taylor expansion and other more convenient forms of these formulas that once assemble allow producing schemes.

For our function of a scale variable the Taylor formula sufficiently near a point may be written the following:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

This may be rewritten the following by letting:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

## Different variation on the extension

It is possible to derive a wide range of Taylor expansion depending on the point we wish to calculate it. The only thing to care of is that the variable tends to a when h tends to 0.

From (2) we can derive:

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

These expansions will serve as basis bricks to build computation schemes.

## Remark

1. These estimations depend only on and its derivatives.
2. In the case we manipulate a vector of two coordinates we should have partial derivative and the Taylor expansion is more complicated (imply mixed derivatives). But if one of the variables is considered constant while the other varies the formulas above are still valid.
3. These estimations are local. The more h is small the more they will be precise.
4. The function is the truncation error. It will give an idea of how much decimal are significant in the result. Here also if h is sufficiently small, it will increase the degree of precision of the calculation (a trade-off exists between the size of h, the memory handled and the computation time).

# Finite difference scheme

## Scheme equation

Hereunder a table that is inspired by [[2](#Cau10)]. It gives the main schemes, their names, and orders of approximation. We added the way they are derived from the above equations.

|  |  |  |  |
| --- | --- | --- | --- |
| ***Derived*** | ***Scheme*** | ***Type*** | ***Order*** |
| (2) |  | Forward |  |
| (3) |  | Backward |  |
| (2)+(3) |  | Central |  |
| (4)+(5) |  | Symmetric |  |

## Truncation errors

We can see that the error due to truncation of the higher order terms: , is always a polynomial expression of the step chosen. From [[1](#LeV07)] we can state these two equivalent forms of the error:

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

And the equivalent and more convenient to represent form:

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

# Bibliography

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|  |  |
| --- | --- |
| [1] | LeVeque R J., *Finite Difference Methods for Ordinary and Partial Differential Equations*. Philadelphia: Society for Industrial and Applied Mathematics (SIAM), 2007. |
| [2] | Causon D M. and Mingham C G., *Introductory Finite Difference Methods for PDEs*.: Causon D M.; Mingham C G. & Ventus Publishing ApS, 2010. |

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