For Realz Mode!

Reversing an MBR from the CSAW CTF

(and how to write a keygen with a SMT solver)

t0x0 numinit

DC858, DC619, and V A P O R S E C

2017-10-11

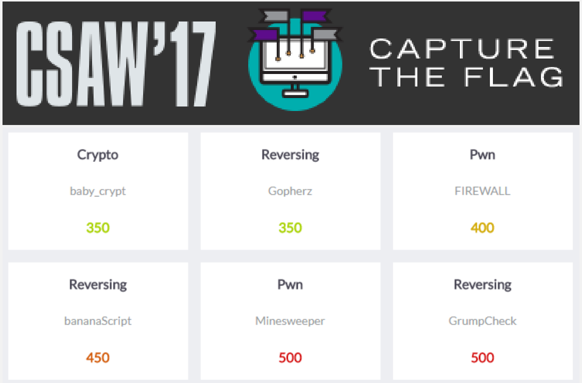
Outline

About us

- Morgan (@numinit)
  - Software developer by day
  - Tinkerer on everything by night
  - First Def Con and Toorcon experiences this year
  - Doing CSAW CTF for a while
- t0x0 (@t0x0pg)
  - Writes lots of interpreted code
  - Lives mostly in Windows world
  - Jack of all trades, master of none (so far)
  - Obsessively curious

What's CSAW?

An annual security capture the flag run by New York Polytechnic that contains challenges with a wide range of difficulties, attracting everyone from undergraduates to well-known teams



What's a security capture the flag?

- 48 hours of caffeinated reverse engineering and exploitation

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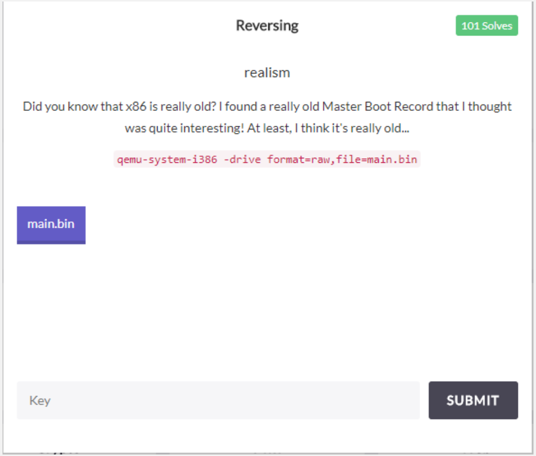
- 48 hours of caffeinated reverse engineering and exploitation
- More seriously: an event where you break a bunch of programs to retrieve hidden strings of text (the flags)
- An opportunity to learn new things - we wouldn't even be presenting if it wasn't for this CTF

Who are V A P O R S E C ?

- A new A E S T H E T I C San Diego CTF team
- Had 10 participants during the CSAW CTF
- Achieved 15th in CSAW industry professional bracket, not bad for the first time

RE400 what?...

- Challenge worth 400 points
- Reverse Engineering category
- We get some hints right away...
  - This is an MBR
  - ...from an x86 system



A place to start...

- Wikipedia, of course!

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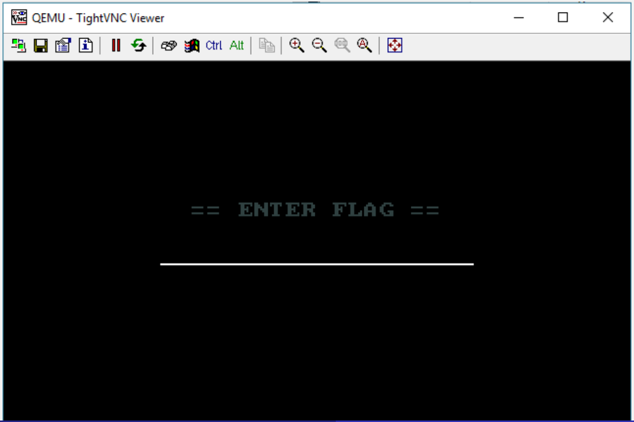
  - 512 bytes
  - MBR signature: 55 AA
  - "expected to contain real mode machine language instructions"
  - little-endian
  - loads at 0000:7C00

Tool Time!

qemu (gift wrapped)

- `-s` (gdb stub)
- `-S` (suspend)
- `-vnc:1` (enable remote VNC monitor)

```
qemu-system-i386 -s -S -vnc :1 -drive format=raw,file=main.bin
```



Tool Time!

qemu (gift wrapped)

- `-s` (gdb stub)
- `-S` (suspend)
- `-vnc:1` (enable remote VNC monitor)
- QEMU/Monitor
  - `info registers`
  - `system reset`

```
ES =9f40 0009f400 0000ffff 00009300
CS =f000 000f0000 0000ffff 00009b00
SS =0000 00000000 0000ffff 00009300
DS =0000 00000000 0000ffff 00009300
FS =0000 00000000 0000ffff 00009300
GS =0000 00000000 0000ffff 00009300
LDT=0000 00000000 0000ffff 00008200
TR =0000 00000000 0000ffff 00008b00
GDT=     000fd3a8 00000037
IDT=     00000000 000003ff
CR0=00000012 CR2=00000000 CR3=00000000 CR4=00000600
DR0=00000000 DR1=00000000 DR2=00000000 DR3=00000000
DR6=ffff0ff0 DR7=00000400
EFER=0000000000000000
FCW=037f FSW=0000 [ST=0] FTW=00 MXCSR=00001f80
FPR0=0000000000000000 0000 FPR1=0000000000000000 0000
FPR2=0000000000000000 0000 FPR3=0000000000000000 0000
FPR4=0000000000000000 0000 FPR5=0000000000000000 0000
FPR6=0000000000000000 0000 FPR7=0000000000000000 0000
XMM00=00000000000000000000000000000000 XMM01=00000000000000000000000000000000
XMM02=00000000000000000000000000000000 XMM03=00000000000000000000000000000000
XMM04=00000000000000000000000000000000 XMM05=00000000000000000000000000000000
XMM06=00000000000000000000000000000000 XMM07=00000000000000000000000000000000
(qemu)
```

- `target remote localhost:1234`
- `set architecture i8086` (bootloaders are 16 bit, right?)
- `display/i $pc` - print program counter
- `br *0xADDR` - set breakpoint
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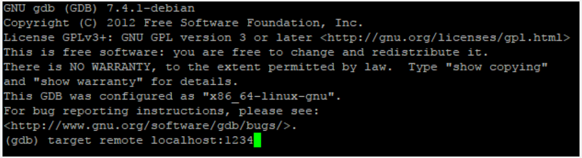

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- `info reg` and `info frame`

```
(gdb) info reg
eax            0x0      0
ecx            0x8b     139
edx            0x26183  156035
ebx            0x6ef2   28402
esp            0x6ed2   0x6ed2
ebp            0x1234   0x1234
esi            0x0      0
edi            0x0      0
eip            0xbdd2   0xbdd2
eflags         0x46     [ PF ZF ]
cs             0xf000   61440
ss             0x0      0
ds             0x0      0
es             0x9f40   40768
fs             0x0      0
gs             0x0      0
(gdb)
```

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- `br *0xADDR` - set breakpoint
- `si` - run one instruction
- `c` - continue
- `info reg` and `info frame`
- `x /CT 0xADDR` - display C units of type T
- `set {int}0xADDR = 42`
- `set {char[4]} 0xADDR = "AAA"`

```
(gdb) info frame
Stack level 0, frame at 0x123c:
 eip = 0x656a; saved eip 0x5f5f5f5f
 called by frame at 0x5f5f5f49
 Arglist at 0x1234, args:
 Locals at 0x1234, Previous frame's sp is 0x123c
 Saved registers:
  ebp at 0x1234, eip at 0x1238
```

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```
(gdb) x /8x 0x1234
0x1234: 0x5f    0x5f    0x5f    0x5f    0x5f    0x5f    0x5f    0x5f
(gdb) set {int} 0x1234 = 41
(gdb) x /8x 0x1234
0x1234: 0x29    0x00    0x00    0x00    0x5f    0x5f    0x5f    0x5f
(gdb) set {int} 0x1234 = 0x41
(gdb) x /8x 0x1234
0x1234: 0x41    0x00    0x00    0x00    0x5f    0x5f    0x5f    0x5f
(gdb) c
Continuing.
^C
Program received signal SIGINT, Interrupt.
```

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0x1234: 0x5f     0x5f     0x5f     0x5f     0x5f     0x5f     0x5f     0x5f
(gdb) set {int} 0x1234 = 41
(gdb) x /8x 0x1234
0x1234: 0x29     0x00     0x00     0x00     0x5f     0x5f     0x5f     0x5f
(gdb) set {int} 0x1234 = 0x41
(gdb) x /8x 0x1234
0x1234: 0x41     0x00     0x00     0x00     0x5f     0x5f     0x5f     0x5f
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Continuing.
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Program received signal SIGINT, Interrupt.
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```
(gdb) x /8x 0x1234
0x1234: 0x5f     0x5f     0x5f     0x41     0x5f     0x5f     0x5f     0x5f
(gdb) set {int} 0x1234 = 0x5f5f5f41
(gdb) x /8x 0x1234
0x1234: 0x41     0x5f     0x5f     0x5f     0x5f     0x5f     0x5f     0x5f
```

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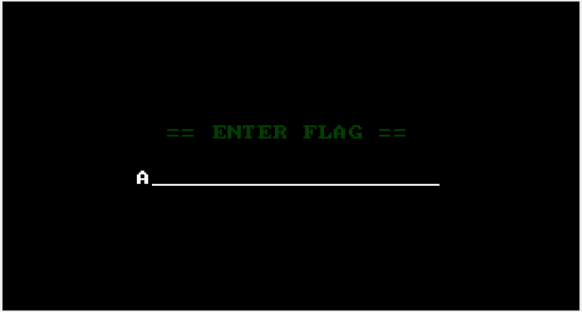
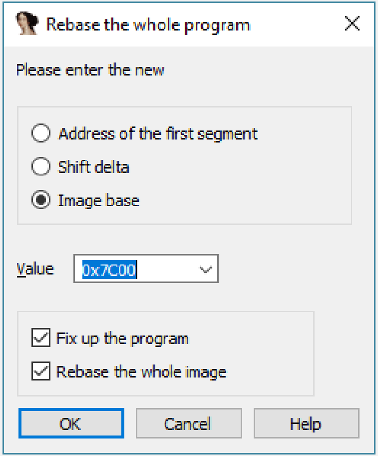


- Rebase


Rebase the whole program
Please enter the new
Address of the first segment
Shift delta
Image base
Value
0x7C00
Fix up the program
Rebase the whole image
OK
Cancel
Help

- Rebase
- Common ASM
  - `int`
  - `mov`
  - `inc`, `dec`
  - `or`, `and`, `not`, `xor`...
  - `cmp`
  - `jmp`, `jz`, `jge`, `jle`...

```
seg000:7C00                 mov     ax, 13h
seg000:7C03                 int     10h             ; - VIDEO - SET VIDEO MODE
seg000:7C03                                         ; AL = mode
seg000:7C05                 mov     eax, cr0
seg000:7C08                 and     ax, 0FFFBh
seg000:7C0B                 or      ax, 2
seg000:7C0E                 mov     cr0, eax
seg000:7C11                 mov     eax, cr4
seg000:7C14                 or      ax, 600h
seg000:7C17                 mov     cr4, eax
seg000:7C1A                 mov     word ptr ds:1266h, 0Ah
seg000:7C20                 mov     bx, 0
```

Tool Time!

IDA

- Rebase
- Common ASM
- Registers
  - E AX - Accumulator
  - E CX - Counter
  - E DX - Data
  - E BX - Base
  - E SP - Stack pointer
  - E BP - Stack base pointer
  - E SI - Source
  - E DI - Destination

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Disassembly...

- Look for hints

Disassembly...

```
seg000:7C23 loc_7C23:                              ; CODE XREF: seg000:7C2C↓j
seg000:7C23                 mov     byte ptr [bx+1234h], 5Fh ; '_'
seg000:7C28                 inc     bx
seg000:7C29                 cmp     bx, 15h
seg000:7C2C                 jle     short loc_7C23
seg000:7C2E                 mov     ds:byte 7D63+65h, 0
```

```
                mov     cr4, eax
                mov     word ptr ds:1266h, 0Ah
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  - Loops

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```

- Look for hints
  - Loops
  - Comparisons

```
seg000:7C6F                 cmp     dword ptr ds:1234h, 67616C66h
seg000:7C78                 jnz     loc_7D4D
```

Disassembly...

- Look for hints
  - Loops
  - Comparisons
  - Unknowns

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seg000:7C23 loc_7C23:                               ; CODE XREF: seg000:7C2C↓j
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seg000:7C7C                 movaps  xmm0, xmmword ptr ds:1238h
seg000:7C81                 movaps  xmm5, xmmword ptr ds:loc_7C00
seg000:7C86                 pshufd  xmm0, xmm0, 1Eh
seg000:7C8B                 mov     si, 8
seg000:7C8E
seg000:7C8E loc_7C8E:                               ; CODE XREF: seg000:7CC1↓j
seg000:7C8E                 movaps  xmm2, xmm0
seg000:7C91                 andps   xmm2, xmmword ptr [si+7D90h]
seg000:7C96                 psadbw  xmm5, xmm2
seg000:7C9A                 movaps  xmmword ptr ds:1268h, xmm5
```

Putting it all together...

- We know that 0x7C6F compares user input to "flag"

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seg000:7C6F                 cmp     dword ptr ds:1234h, 67616C66h
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- There's a bunch of complicated instructions right after that cmp

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seg000:7C8E
seg000:7C8E loc_7C8E:                               ; CODE XREF: seg000:7CC1↓j
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- We know that 0x7C6F compares user input to "flag"
- There's a bunch of complicated instructions right after that cmp
- We can use gdb and qemu/monitor to see what's happening...

```
XMM00=7d41414141414141414141414141417b XMM01=00000000000000000000000000000000
XMM02=00000000000000000000000000000000 XMM03=00000000000000000000000000000000
XMM04=00000000000000000000000000000000 XMM05=220f02c883fbe083c0200f10cd0013b8
XMM06=00000000000000000000000000000000 XMM07=00000000000000000000000000000000
```

Putting it all together...

- We know that 0x7C6F compares user input to "flag"
- There's a bunch of complicated instructions right after that cmp
- We can use gdb and qemu/monitor to see what's happening...
- Now we just have to work backwards from there.

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```
XMM00=4141417b414141417d41414141414141 XMM01=00000000000000000000000000000000
XMM02=4141417b414141007d41414141414100 XMM03=00000000000000000000000000000000
XMM04=00000000000000000000000000000000 XMM05=220f02c883fbe083c0200f10cd0013b8
```

Other useful tools...that I didn't know about

- Binary Ninja
- pwndbg
- Radare

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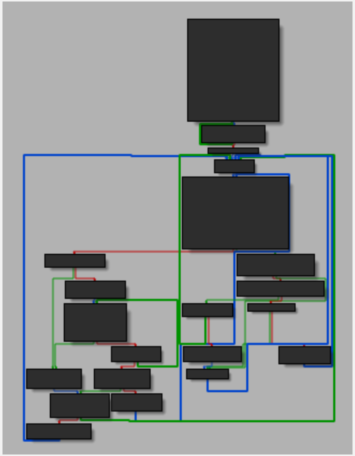
- We have an x86 boot sector that's asking us for the flag
- The flag is clearly in the boot sector SOMEWHERE...
  - ...but not in plaintext, because this is a 400 point challenge and that would be too easy
- So, where is it?

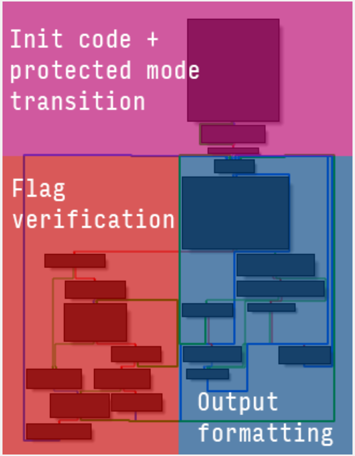
Reversing the boot sector

- Open the file in your favorite disassembler (e.g. IDA); rebase at `0x7c00`



Reversing the boot sector

- Open the file in your favorite disassembler (e.g. IDA); rebase at `0x7c00`
- We can visually pick out three sections:
  - Init code (responsible for enabling SSE instructions)
  - Display code (identifiable by a large number of `int` instructions)
  - Some other code that uses Intel SSE2 instructions



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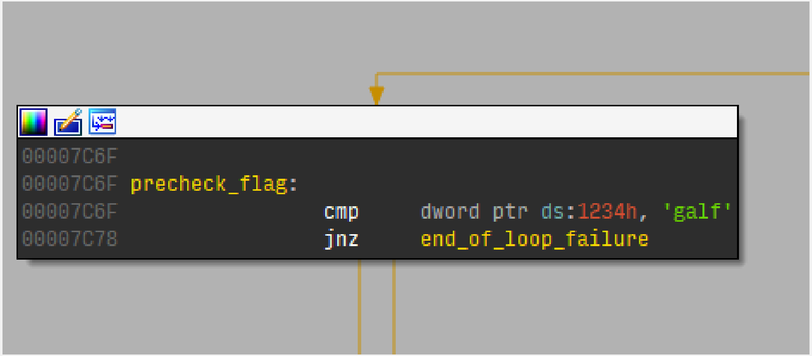

As it turns out...

After several hours of staring at x86 assembly

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As it turns out...

After several hours of staring at x86 assembly

- The flag is hashed using a custom algorithm implemented with Intel SSE instructions (and this isn't very surprising)
- *Ugh...*
- We have to find an input to the hash algorithm that hashes to the same value that's stored in the boot sector

Suddenly, SSE2 instructions

“My Intel CPU can do *that?*”

The author of the challenge decided to use a bunch of obscure x86 SSE2 instructions to force us to trawl through Intel documentation

Note

SSE2 instructions operate on XMM registers, which are 128 bits (16 bytes) wide.

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- pshufd
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- movaps: Moves to/from/between XMM registers
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- pshufd: Reorders 32-bit words in XMM registers
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Note

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Question

Why are these instructions useful for writing a hash function (even if it's a bad hash function that we can break?)

Packed Sum of Absolute Differences of Bytes in Word

That's a mouthful...

xmm0:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Packed Sum of Absolute Differences of Bytes in Word

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xmm0:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

(minus)

xmm1:	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

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xmm0:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

(minus)

xmm1:	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

$\downarrow$

xmm0:	−15	−13	−11	−9	−7	−5	−3	−1	1	3	5	7	9	11	13	15

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(minus)

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↓

xmm0:	−15	−13	−11	−9	−7	−5	−3	−1	1	3	5	7	9	11	13	15

(absolute values)

xmm0:	15	13	11	9	7	5	3	1	1	3	5	7	9	11	13	15

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(minus)

xmm1:	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

↓

xmm0:	−15	−13	−11	−9	−7	−5	−3	−1	1	3	5	7	9	11	13	15

(absolute values)

xmm0:	15	13	11	9	7	5	3	1	1	3	5	7	9	11	13	15

(16-bit sums)

xmm0:	0	0	0	0	0	0	64	0	0	0	0	0	0	0	64	0

Packed Sum of Absolute Differences of Bytes in Word

That's a mouthful...

xmm0:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	(minus)															
xmm1:	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
	↓															
xmm0:	−15	−13	−11	−9	−7	−5	−3	−1	1	3	5	7	9	11	13	15
	(absolute values)															
xmm0:	15	13	11	9	7	5	3	1	1	3	5	7	9	11	13	15
	(16-bit sums)															
xmm0:	0	0	0	0	0	0	64	0	0	0	0	0	0	0	64	0

It's hard to go back...

You have to find two sets of 8 bytes where the absolute values of their differences sum to 64

SAT solvers to the rescue!

How to master Sudoku without memorizing strategies

"I think a serious case can be made that the decline in the American economy can be blamed on the sapping of the mental energy and productivity of the American workforce that sudoku addiction alone has wrought"

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d	c	6	1	9	0	e	b	5	7	3	2	4	8	f	a
5	8	a	4	f	c	2	d	9	0	b	e	3	7	6	1
2	b	9	7	5	3	6	1	4	8	a	f	d	0	e	c
0	3	f	e	4	a	8	7	d	c	6	1	9	b	2	5
8	d	c	6	1	9	3	2	b	5	e	7	0	4	a	f
b	1	5	0	d	8	c	6	f	a	9	4	7	2	3	e
9	a	4	f	7	5	0	e	2	3	1	8	b	d	c	6
7	2	e	3	b	4	a	f	0	d	c	6	1	9	5	8
e	4	d	c	6	1	9	0	a	b	2	5	8	f	7	3
1	9	8	2	3	7	5	4	e	6	f	0	a	c	d	b
f	6	7	a	2	d	b	c	1	9	8	3	5	e	0	4
3	5	0	b	a	e	f	8	7	4	d	c	6	1	9	2
a	e	3	d	c	6	1	9	8	f	7	b	2	5	4	0
6	0	2	5	e	b	d	3	c	1	4	9	f	a	8	7
c	7	1	9	8	f	4	5	6	2	0	a	e	3	b	d
4	f	b	8	0	2	7	a	3	e	5	d	c	6	1	9

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Notice anything strange about this puzzle?
Stranger still: a SAT solver created it from thin air

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5	8	a	4	f	c	2	d	9	0	b	e	3	7	6	1
2	b	9	7	5	3	6	1	4	8	a	f	d	0	e	c
0	3	f	e	4	a	8	7	d	c	6	1	9	b	2	5
8	d	c	6	1	9	3	2	b	5	e	7	0	4	a	f
b	1	5	0	d	8	c	6	f	a	9	4	7	2	3	e
9	a	4	f	7	5	0	e	2	3	1	8	b	d	c	6
7	2	e	3	b	4	a	f	0	d	c	6	1	9	5	8
e	4	d	c	6	1	9	0	a	b	2	5	8	f	7	3
1	9	8	2	3	7	5	4	e	6	f	0	a	c	d	b
f	6	7	a	2	d	b	c	1	9	8	3	5	e	0	4
3	5	0	b	a	e	f	8	7	4	d	c	6	1	9	2
a	e	3	d	c	6	1	9	8	f	7	b	2	5	4	0
6	0	2	5	e	b	d	3	c	1	4	9	f	a	8	7
c	7	1	9	8	f	4	5	6	2	0	a	e	3	b	d
4	f	b	8	0	2	7	a	3	e	5	d	c	6	1	9

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- Symbolic execution engines
  - Convert CPU instructions into a SMT solver language

In case this all seemed too abstract...

I get by with a little help from my friends in *NP*

SMT solvers let you write a type of SQL that can be used to solve certain hard* problems

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If there's one thing you should take from this presentation: SMT solvers are tools for solving constraint problems, just like SQL is a tool for making sense of large amounts of data

Some modern SMT solvers and symbolic execution engines

- Z3 (SMT)
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We'll be mainly focusing on using Z3, since all the other tools use it under the hood (and it's awesome)

Back to our sudoku...

How do you create a hexadoku puzzle out of thin air with Z3?

Back to our sudoku...

How do you create a hexadoku puzzle out of thin air with Z3?

Creating the board

```
import z3

# Define a 16x16 Sudoku
order = 4; side = order ** 2

# Create a solver and a 16x16 board (list of lists)
smt = z3.Solver()
cells = [list() for row in range(side)]
for r in range(side):
    for c in range(side):
        cells[r].append(z3.Int('cell_%d_%d' % (r, c)))
```

Telling z3 how to play sudoku

We'll define our first rule: each cell has to be between 0 and 15

Telling z3 how to play sudoku

We'll define our first rule: each cell has to be between 0 and 15

The most basic of Sudoku rules

```
for r in range(side):
    for c in range(side):
        # Each cell must be between 0 and 15.
        # Also, this is our first constraint!
        # Add it to the solver with `smt.add`.
        smt.add(z3.And(cells[r][c] >= 0, cells[r][c] < side))
```

Telling z3 how to play sudoku

Now, the rules everyone knows

Telling z3 how to play sudoku

Now, the rules everyone knows

Rows and columns

```
# Rows must have unique values
for r in range(side):
    this_row = [cells[r][c] for c in range(side)]

    # Ensures that all values are represented only once
    smt.add(z3.Distinct(*this_row))

# Columns must have unique values
for c in range(side):
    this_col = [cells[r][c] for r in range(side)]
    smt.add(z3.Distinct(*this_col))
```

Telling z3 how to play sudoku

Mini-boxes are slightly more complicated

Telling z3 how to play sudoku

Mini-boxes are slightly more complicated

Mini-boxes

```
# Each mini-box must have unique values
for r in range(0, side, order):
    for c in range(0, side, order):
        selected = []
        for i in range(box_size):
            for j in range(box_size):
                selected.append(cells[r + i][c + j])

        smt.add(z3.Distinct(*selected))
```

Telling z3 how to play *our version of* sudoku

Now for the initial values

... dc858 isn't sudoku-able, but dc619 is

Telling z3 how to play *our version of* sudoku

Now for the initial values
... dc858 isn't sudoku-able, but dc619 is

Initial values

```
# Now, define the initial values
smt.add(cells[0][0] == 0xd)
smt.add(cells[0][1] == 0xc)
smt.add(cells[0][2] == 0x6)
smt.add(cells[0][3] == 0x1)
smt.add(cells[0][4] == 0x9)

# ... etc.
```

Telling z3 to start solving

This will take a little bit of time

Telling z3 to start solving

This will take a little bit of time

Start solving!

```
if smt.check() == z3.unsat:
    print('Sudoku is impossible to solve :(')
else:
    # Retrieve each cell from the model as a python long
    model = smt.model()
    result = [list() for row in range(side)]
    for r in range(side):
        for c in range(side):
            cell = model[r][c]
            print('%x ' % model[cell].as_long(), end='')
        print()
```

The results...

d	c	6	1	9	0	e	b	5	7	3	2	4	8	f	a
5	8	a	4	f	c	2	d	9	0	b	e	3	7	6	1
2	b	9	7	5	3	6	1	4	8	a	f	d	0	e	c
0	3	f	e	4	a	8	7	d	c	6	1	9	b	2	5
8	d	c	6	1	9	3	2	b	5	e	7	0	4	a	f
b	1	5	0	d	8	c	6	f	a	9	4	7	2	3	e
9	a	4	f	7	5	0	e	2	3	1	8	b	d	c	6
7	2	e	3	b	4	a	f	0	d	c	6	1	9	5	8
e	4	d	c	6	1	9	0	a	b	2	5	8	f	7	3
1	9	8	2	3	7	5	4	e	6	f	0	a	c	d	b
f	6	7	a	2	d	b	c	1	9	8	3	5	e	0	4
3	5	0	b	a	e	f	8	7	4	d	c	6	1	9	2
a	e	3	d	c	6	1	9	8	f	7	b	2	5	4	0
6	0	2	5	e	b	d	3	c	1	4	9	f	a	8	7
c	7	1	9	8	f	4	5	6	2	0	a	e	3	b	d
4	f	b	8	0	2	7	a	3	e	5	d	c	6	1	9

The power of SMT solvers

Maybe familiar to anyone who's tried Prolog

We only had to define the constraints of the puzzle we wanted, and Z3 found one satisfying those constraints

Constraint solving our boot sector

"Hey, Z3, I want you to find me input values that, after being put through this crazy operation (among other things), end up being equal to the ones stored in the boot sector"

xmm0:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

(minus)

xmm1:	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

↓

xmm0:	−15	−13	−11	−9	−7	−5	−3	−1	1	3	5	7	9	11	13	15

(absolute values)

xmm0:	15	13	11	9	7	5	3	1	1	3	5	7	9	11	13	15

(16-bit sums)

xmm0:	0	0	0	0	0	0	64	0	0	0	0	0	0	0	64	0

Useful primitives

- We need to implement the pshufd and psadbw x86 instructions in Z3
- Documentation at `http://x86.renejeschke.de` already provides us pseudocode for these instructions

Operation

```
Destination[0..31] = (Source >> (Order[0..1] * 32))[0..31];
Destination[32..63] = (Source >> (Order[2..3] * 32))[0..31];
Destination[64..95] = (Source >> (Order[4..5] * 32))[0..31];
Destination[96..127] = (Source >> (Order[6..7] * 32))[0..31];
```

Useful constraints

The flag looks like `flag{some_text_here}`, but the word "flag" is chopped off before it's loaded into an XMM register, so the first and last characters of the remaining 16 are `{` and `}`, respectively.

Useful constraints

The flag looks like `flag{some_text_here}`, but the word “flag” is chopped off before it’s loaded into an XMM register, so the first and last characters of the remaining 16 are { and }, respectively. This may not look like much, but we already have a few constraints...

Initial constraints

```
import z3
smt = z3.Solver()
flag = [z3.BitVec('flag_%d' % i, 8) for i in range(16)]

# Flag must start with '{' and end with '}', and the rest must be ASCII
smt.add(z3.And(flag[0] == ord('{'), flag[-1] == ord('}')))
for i in range(1, 15):
    smt.add(z3.And(flag[i] >= 32, flag[i] < 127))
```

Understanding the hash function's word shuffling

The flag is initially shuffled using the pshufd instruction; create a new constraint ("shuf") that just describes the permutation

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xmm0:	{	A	B	C	D	E	F	G	H	I	J	K	L	M	N	}

↓ `pshufd xmm0, xmm0, 0x1e`

xmm0:	H	I	J	K	L	M	N	}	D	E	F	G	{	A	B	C

Implementing the hash function's word shuffling

Implementing the word shuffling (pshufd)

```python
# Create the permuted flag, add trivially true constraints
shuf = [z3.BitVec('pshufd_%d' % i, 8) for i in range(16)]

smt.add(
    z3.And(shuf[0] == flag[8], shuf[1] == flag[9],
           shuf[2] == flag[10], shuf[3] == flag[11], ...)
)
```

Implementing the hash function’s bitmasking

For each of the hash function’s 8 rounds, a different mask is applied to the shuffled value, so create another set of intermediate values called “masked”

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Implementing the masking operation (andps)

```
# Masks for each round:
# ffffffff ffffff00 ffffffff ffffff00
# ffffffff ffff00ff ffffffff ffff00ff
# ffffffff ff00ffff ffffffff ff00ffff
# ffffffff 00ffffff ffffffff 00ffffff
# ... etc.
mask = [0x00 if i == round else 0xff for i in range(8)] * 2
masked = [z3.BitVec('masked_%d_%d' % (round, i), 8) for i in range(16)]
for i in range(16):
    smt.add(masked[i] == (shuf[i] & mask[i]))
```

Implementing the packed sum of absolute differences

A single x86 instruction turned into like 30 lines of Python...

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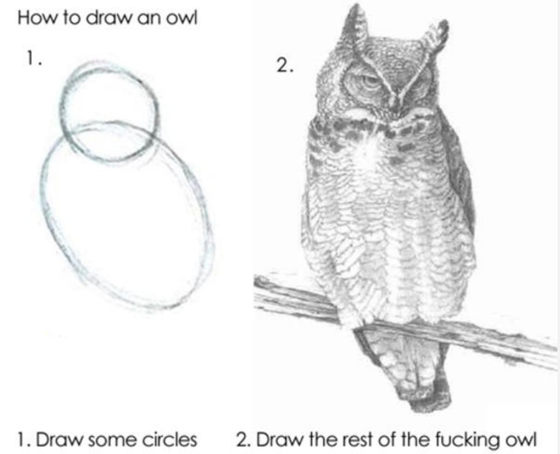
Implementing psadbw (greatly simplified; trust me, it's nasty)

```
ps_lo, ps_hi = z3.BitVec('psadbw_lo', 16), z3.BitVec('psadbw_hi', 16)

smt.add(ps_lo == (
    Abs(iv[0] - masked[0]) + Abs(iv[1] - masked[1]) +
    Abs(iv[2] - masked[2]) + Abs(iv[3] - masked[3]) + ...
)) # and repeat for ps_hi. Assumes that we have an `Abs` helper.

res = [z3.BitVec('psadbw_%d' % i, 8) for i in range(16)]
smt.add(res[0] == ps_lo & 0xff)
smt.add(res[1] == (ps_lo >> 8) & 0xff)
smt.add(res[2] == 0) # ... etc. Read Intel docs if you care.
```

And the rest of the algorithm…

AKA: continuing to give up on explaining the details

- Would be boring to present, since there were a few arbitrary things thrown into the challenge that made it deliberately more annoying to reverse-engineer
- … Like x86 tricks where they loaded a register like `eax` and used `ah` and `al` backwards
- Not to mention that the algorithm has to run for 8 rounds, so the logic repeats 8 times…



Once we've got the constraints defined...

Now, we can solve it as usual, and retrieve the flag from the model

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Solving for the flag

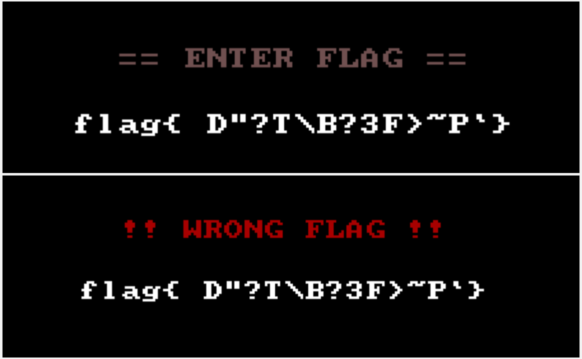
```
if smt.check() == z3.unsat:
    print('Formula was unsatisfiable :-(')
else:
    print('Formula was satisfiable.')
    model = smt.model()
    flag_str = 'flag'
    for i in range(16):
        flag_str += chr(model[flag[i]].as_long())

    print(flag_str)
```

Run it, and...

```
Formula was satisfiable.
flag{ D`?T\B?3F> P`}
```

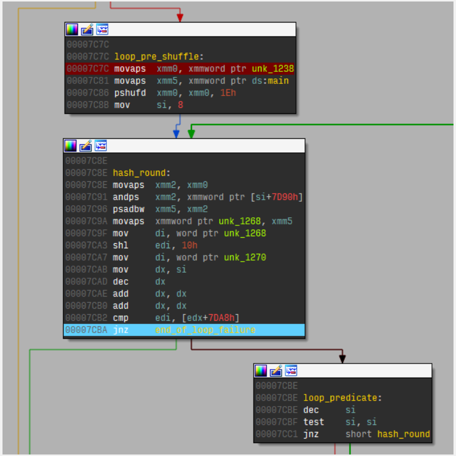
Does it work?

Well, no



Does it work?

But, using IDA's debugger and setting breakpoints, we see that it got through the first round of the hash function, but failed on the second - turns out, we forgot to solve the remaining rounds



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- If you're reimplementing some piece of code in Z3, make sure its flow matches the original's
    - Match control structures as faithfully as possible
    - Use Python functions to implement complex behaviors like Intel SSE2 instructions with the correct series of `smt.add` calls

After refactoring to support multiple rounds...

We're getting closer...

```
Using 6 rounds.
Formula was satisfiable.
flag{4e@alz_p%Z/TnW}
```

After refactoring to support multiple rounds...

We're getting closer...

```
Using 6 rounds.
Formula was satisfiable.
flag{4e@alz_p%Z/TnW}
```

And, it got through 3 out of the 8 rounds this time!

After refactoring to support multiple rounds...

We're getting closer...

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... But, we asked it to do 6, because the solver couldn't solve for 7 or 8

Something must be going wrong after round 3

Typos are your worst enemy

What about round 4?

What I typed

```
outputs = (
    0x02df028f,
    0x0290025d,
    0x02090221,
    0x027b0279,
    0x01f90233,
    0x025e0291,
    0x02290255,
    0x02110270
)
```

→

What I meant

```
outputs = (
    0x02df028f,
    0x0290025d,
    0x02090221,
    0x027b0278, # single bit typo on round 4!
    0x01f90233,
    0x025e0291,
    0x02290255,
    0x02110270
)
```

After fixing the typo...

It gets through all 8 rounds!

```
Using 8 rounds.
Formula was satisfiable.
flag{4r3alz_m0d3_y0}
```

After fixing the typo...

It gets through all 8 rounds!

```
Using 8 rounds.
Formula was satisfiable.
flag{4r3alz_m0d3_y0}
```

So, what did we learn?

Other than typos being the bane of our existence

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So, what did we learn?

Other than typos being the bane of our existence

- Emulators like QEMU can be used to help debug code for non-native architectures
- Intel x86 vector instructions are complex, but we can get past the complexity by following the documentation
- SMT solvers can be used to query constraints, similarly to how you can use SQL to query lots of data

Dennis Yurichev

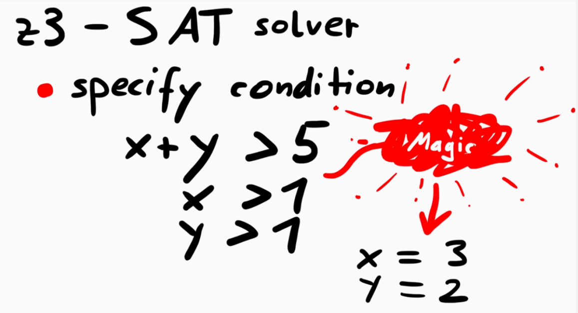
- This presentation is built on the shoulders of SMT wizards like Dennis
- Check out his fantastic writeup on SAT and SMT solvers: `https://yurichev.com/writings/SAT_SMT_draft-EN.pdf`

Quick introduction into SAT/SMT solvers and symbolic execution

Dennis Yurichev `<dennis(a)yurichev.com>`

October 3, 2017

- Published a video on Z3 two days after this presentation was delivered at the October DC858 meetup
- Is awesome and you should watch his videos



https://www.youtube.com/watch?v=TpdDq56KH1I

More shoutouts

- V A P O R S E C crew
  (ice, jkin, pixelmover, m1m, n0m4d1c, s7one ghos7, StaticFlow, & thecl0ud)
- angersock for coming up with our team name
- DC858/619 folks

The end!

- Z3 code for generating sudokus is at `https://goo.gl/ZC1546`
- Z3 code for solving the CTF challenge is available at `https://goo.gl/cJPJDN`
- Questions?