Grade: 100

# ME6406 HW1 Thanakorn Khamvilai Report

Problem 1: Pin-Hole Optics

Solution:

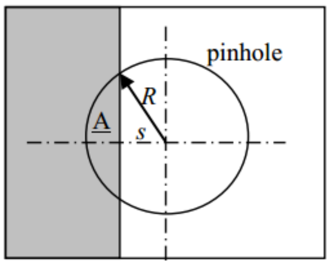
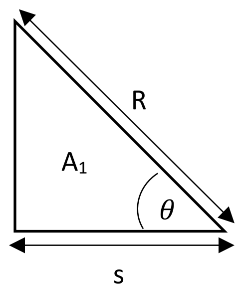



Figure 1.

Pin-hole area $\delta O = \pi R^2$

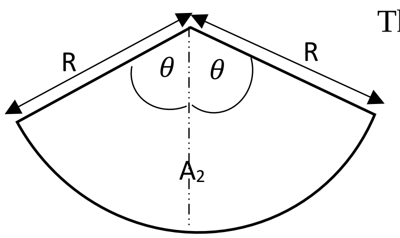
Consider the right triangle




The angle $\theta = arccos\frac{s}{R}$

and the area $A_1 = \frac{1}{2}s\sqrt{R^2 - s^2}$

Consider the Sector




The area

$$A_2 = \pi R^2 \cdot \frac{2\theta}{2\pi}$$

$$A_2 = \theta R^2$$

$$A_2 = arccos\frac{s}{R} \cdot R^2$$

Then the overlap area $\delta A = A_2 - A_1$

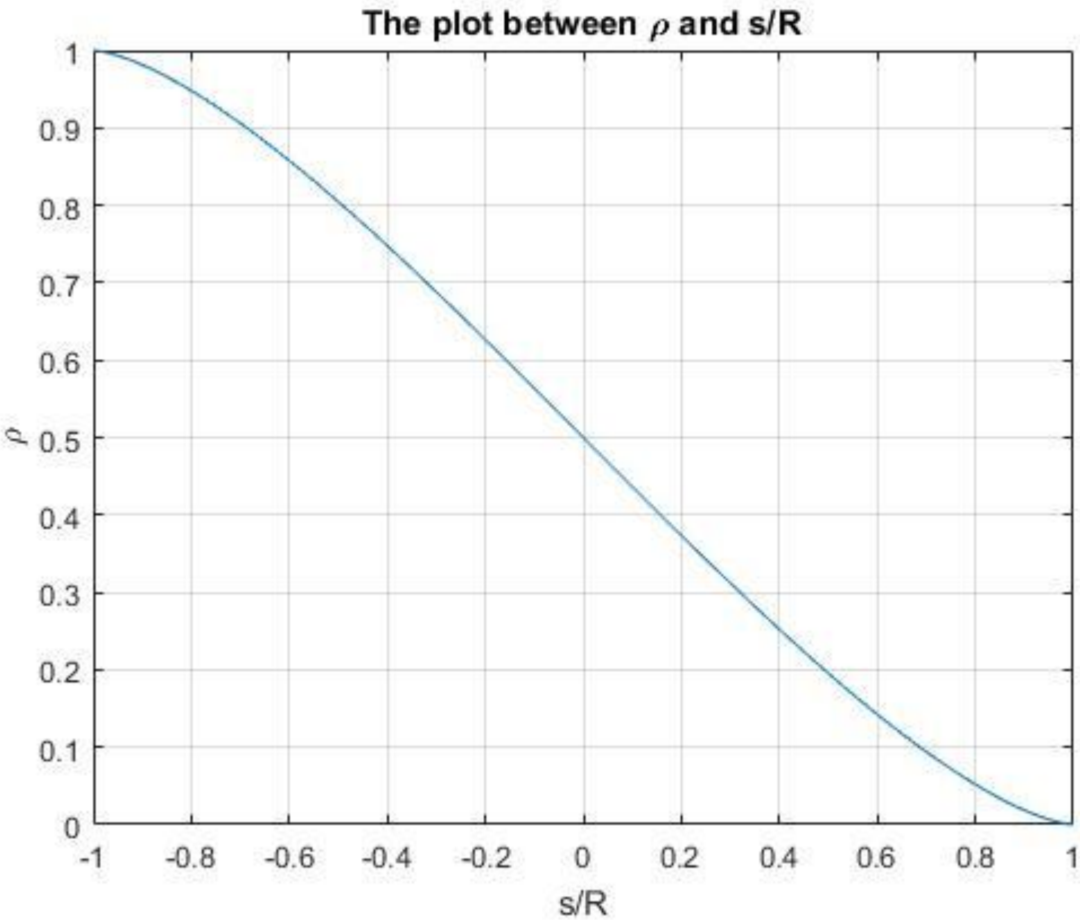
$$\delta A = arccos\frac{s}{R} \cdot R^2 - 2 \cdot \frac{1}{2}s\sqrt{R^2 - s^2}$$

$$\delta A = R^2\left[arccos\frac{s}{R} - \frac{s}{R}\sqrt{1 - \left(\frac{s}{R}\right)^2}\right]$$

Therefore, $$\rho = \frac{\delta A}{\delta O} = \frac{R^2\left[arccos\frac{s}{R} - \frac{s}{R}\sqrt{1 - \left(\frac{s}{R}\right)^2}\right]}{\pi R^2}$$

$$\rho = \frac{1}{\pi}\left[arccos\frac{s}{R} - \frac{s}{R}\sqrt{1 - \left(\frac{s}{R}\right)^2}\right]$$

Plot $\rho$ as a function of s/R using MATLAB (HW1_1.m)

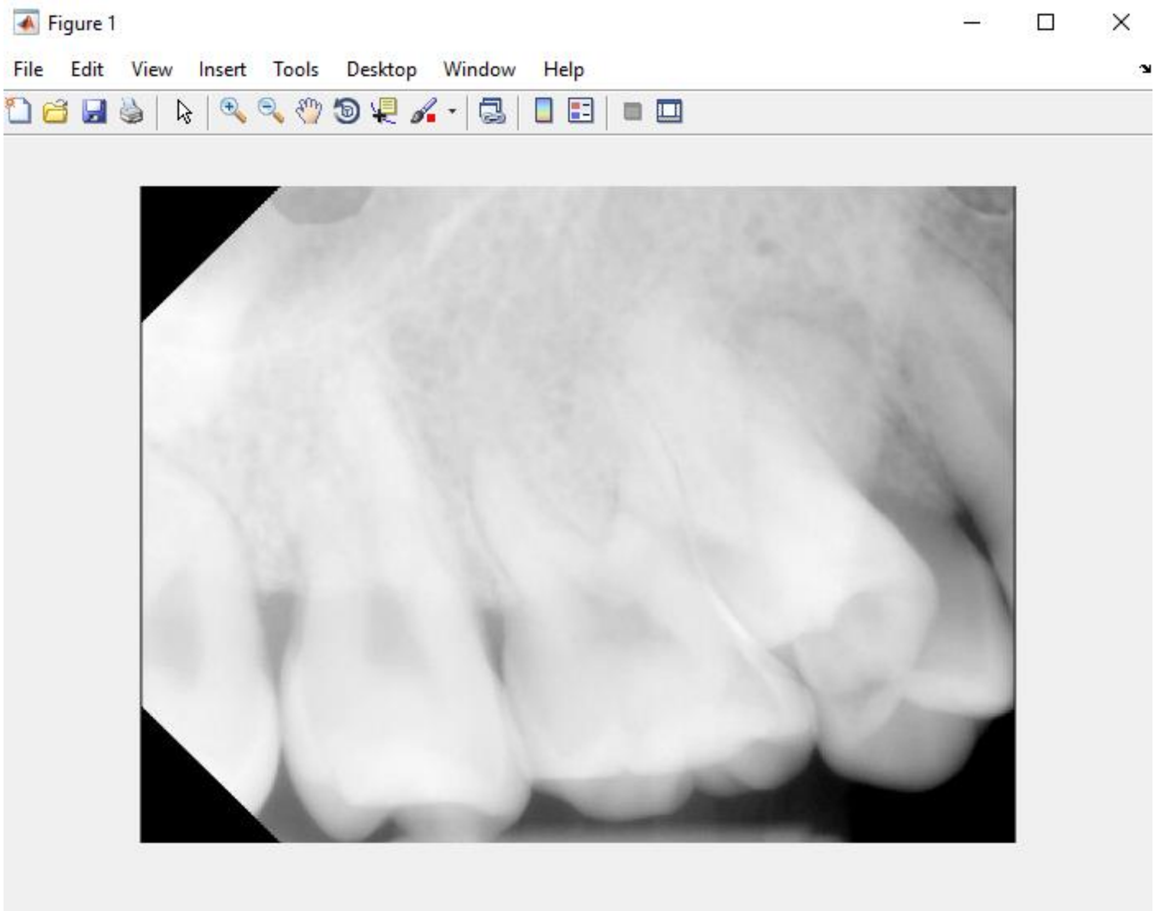


Problem 2: Histogram Equalization

2a) Solution: The table below was done using MATLAB (HW1_2a.m)

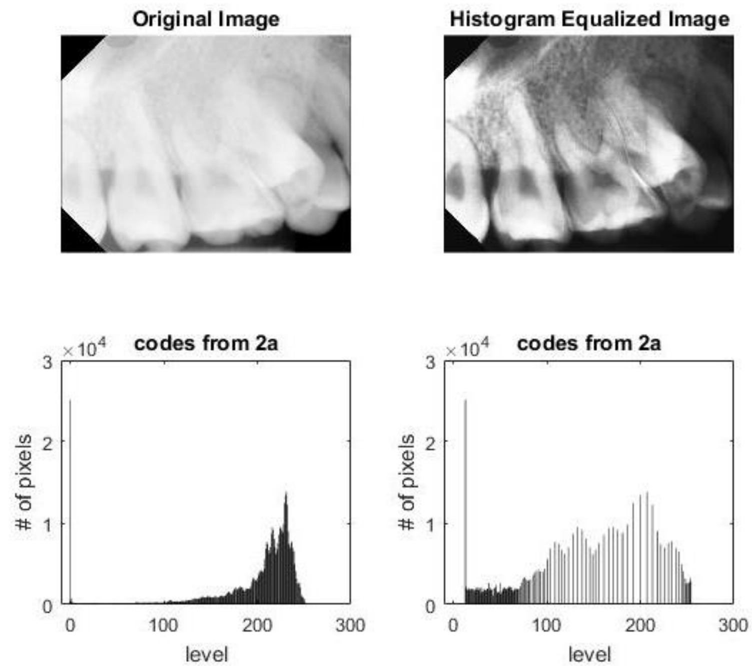
| **Gray Level** | **# of pixels** | **cdf** | **$q_k$** | **round($q_k$)** |
|---|---|---|---|---|
| 190 | 1 | 1 | 7.0833 | 7 |
| 191 | 5 | 6 | 42.5000 | 43 |
| 192 | 3 | 9 | 63.7500 | 64 |
| 193 | 5 | 14 | 99.1667 | 99 |
| 194 | 4 | 18 | 127.5000 | 128 |
| 195 | 7 | 25 | 177.0833 | 177 |
| 196 | 6 | 31 | 219.5833 | 220 |
| 197 | 2 | 33 | 233.7500 | 234 |
| 198 | 1 | 34 | 240.8333 | 241 |
| 199 | 1 | 35 | 247.9167 | 248 |
| 200 | 1 | 36 | 255.0000 | 255 |

2b) Solution: I. Read in and display the 'teeth.jpg' (HW1_2b.m)

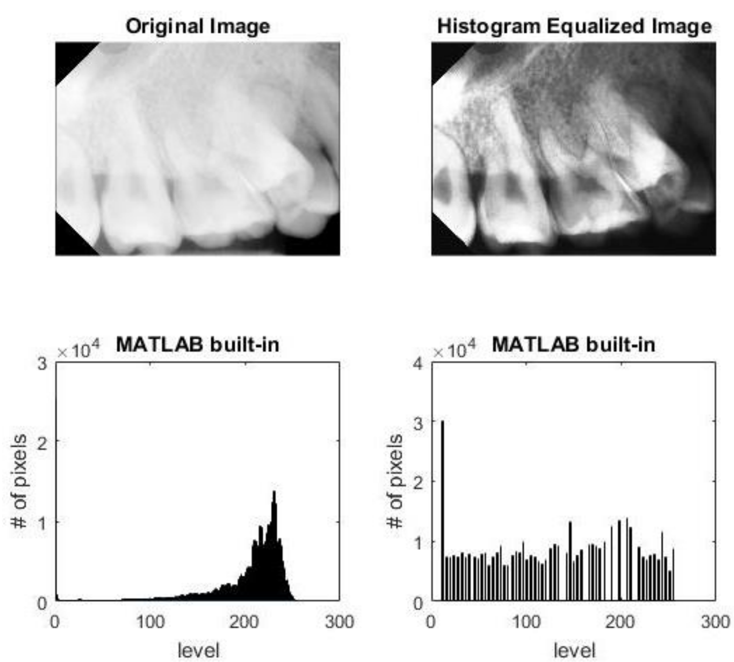


II. Compare by displaying the original and processed images and their histograms. (HW1_2b.m)

By using codes from 2a)




By using MATLAB built-in commands




There is a little difference in pixel intensity obtained from these two approaches even if both equalized images look pretty similar; thus, the plots of histogram equalization are difference.

Problem 3: Filtering Masks

3a) Solution:

$$H_x = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}, H_y = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, Z = \begin{bmatrix} 199 & 197 & 195 \\ 197 & 195 & 194 \\ 196 & 194 & 192 \end{bmatrix}$$

$$g_m = 196 + 2(194) + 192 - 199 - 2(197) - 195 = -12$$

$$g_n = 195 + 2(194) + 192 - 199 - 2(197) - 196 = -14$$

Magnitude

$$|g(m.n)| = \sqrt{g_m^2 + g_n^2} = \sqrt{(-12)^2 + (-14)^2} = 18.4391 \approx 18$$

Direction

$$\angle g(m,n) = tan^{-1}\frac{g_n}{g_m} = tan^{-1}\frac{-14}{-12} = 229.3987^o$$

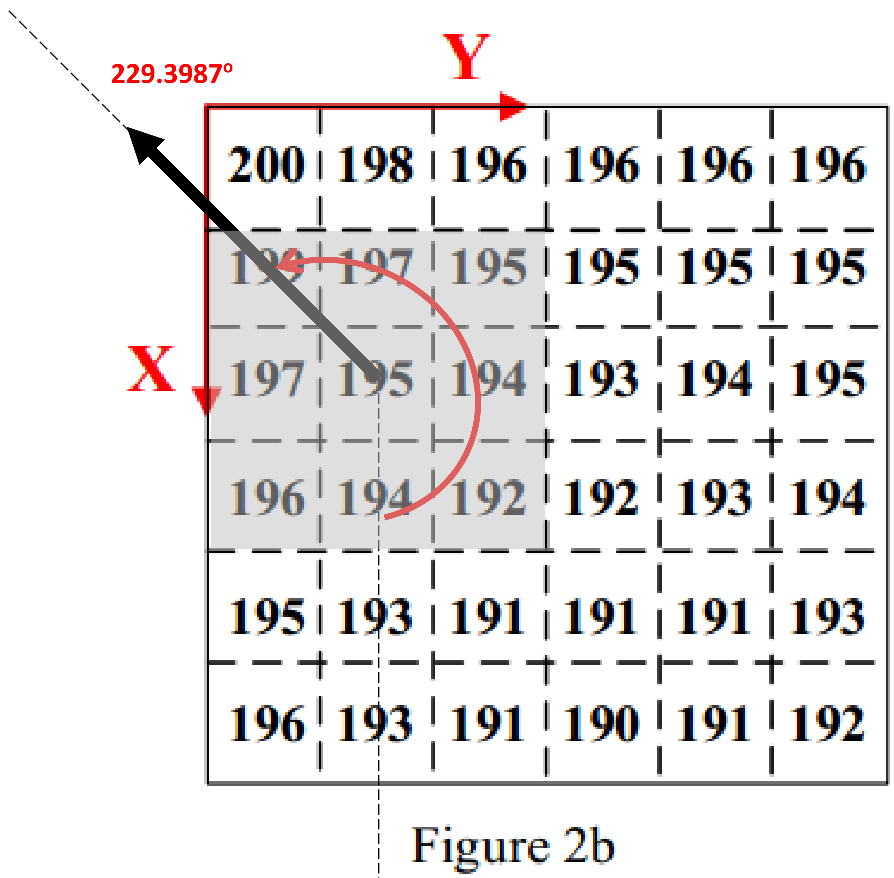
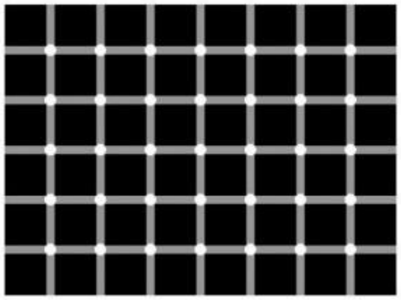



Figure 2b

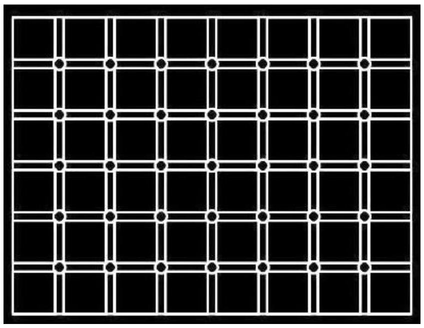
<u>3b) Solution:</u> This problem was done using MATLAB (HW1_3b.m)

Original Image

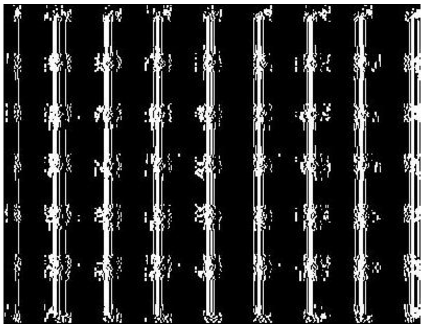


Using the same calculation as 3a)

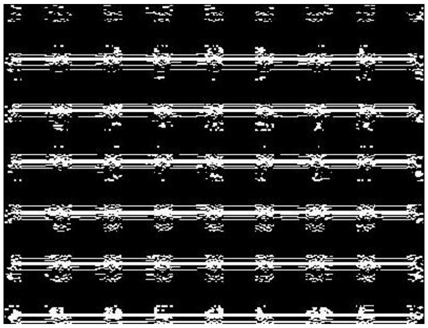
Both Direction



Vertical Direction

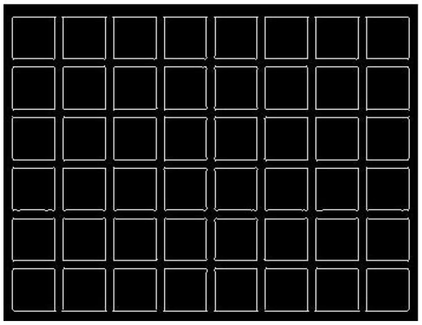


Horizontal Direction

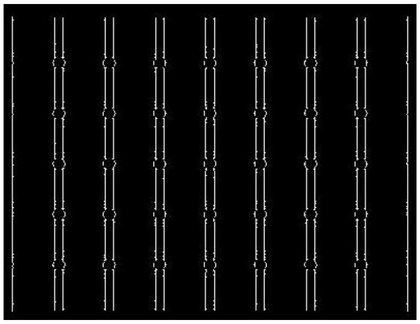


Using the MATLAB built-in command with the default threshold

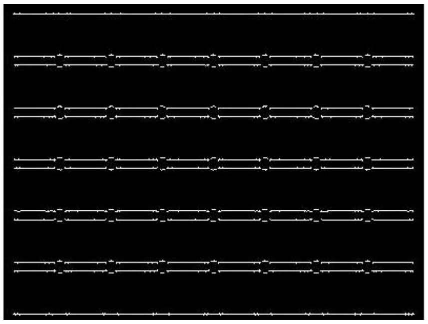
Both Direction



Vertical Direction



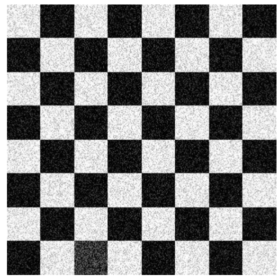
Horizontal Direction



Because of using the default threshold in built-in *edge* command, the results from these two approaches have some differences.

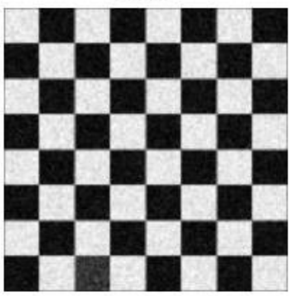
3c) Solution: This problem was done using MATLAB with 3Std Gaussian Filter i.e.7x7 for σ = 1 (3*1*2 + 1), 13x13 for σ = 2 (3*3*2 + 1), and 31x31 for σ = 5 (3*5*2 + 1) (HW1_3c.m)

Original Image

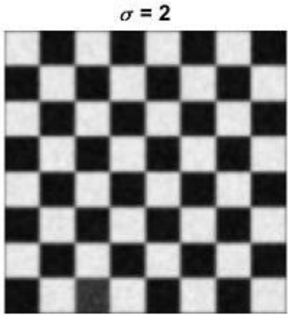


Analytical Implement

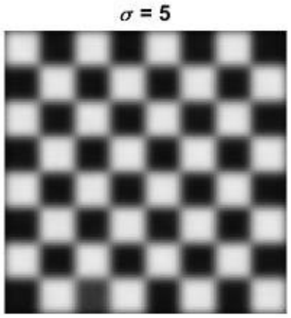
$\sigma$ = 1



$\sigma$ = 2

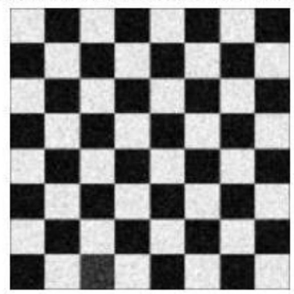


$\sigma$ = 5

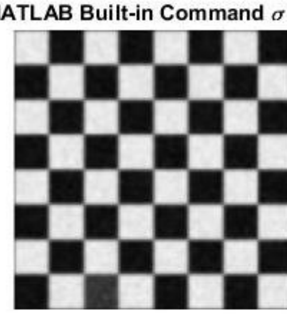


MATLAB Built-In Command

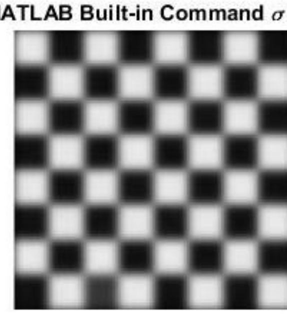
MATLAB Built-in Command $\sigma$ = 1



MATLAB Built-in Command $\sigma$ = 2



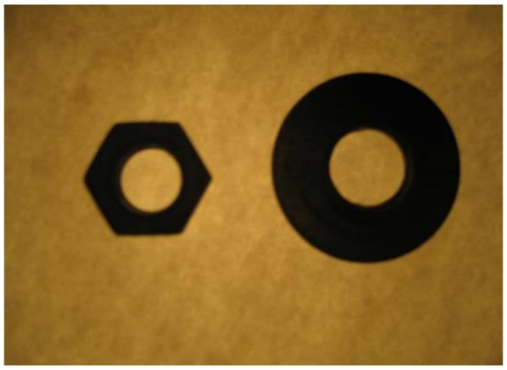
MATLAB Built-in Command $\sigma$ = 5



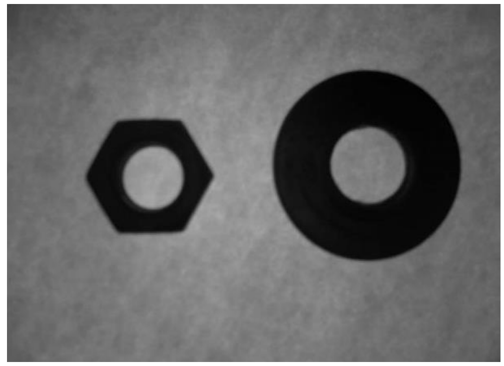
Problem 4: Low-Level Information Processing

4a) Solution: This problem was done using MATLAB (HW1_4a.m)

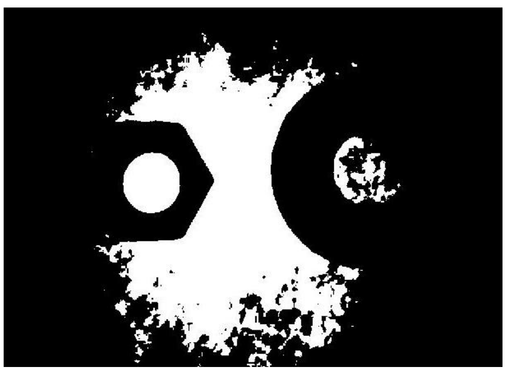
Original Image



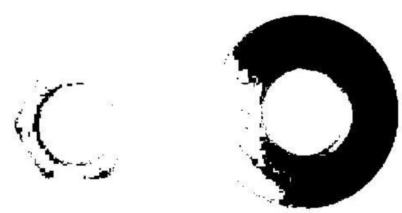
Grayscale Image



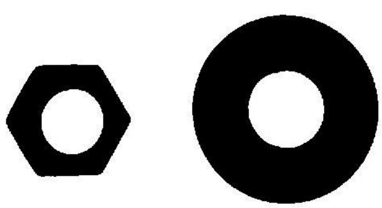
Over-Estimated Threshold



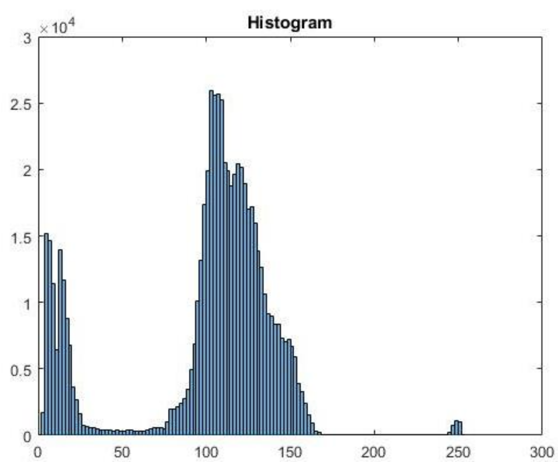
Under-Estimated Threshold



Appropriated Threshold

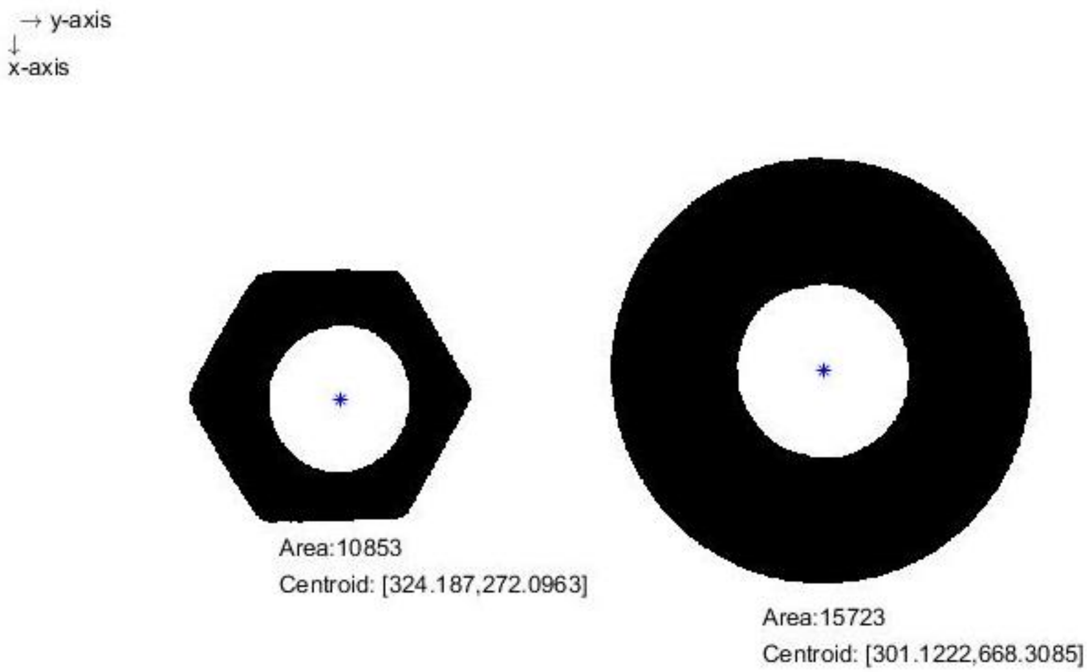


Histogram




The values of over-estimated threshold, under-estimate threshold, and appropriated threshold are chosen to be 127.5, 12.75, and 51, respectively; hence, the values 0.5 (127.5/255), 0.05 (12.75/255), and 0.2 (51/255) are used in *im2bw* function for over-estimated threshold, under-estimate threshold, and appropriated threshold, respectively.

<u>4b) Solution:</u> This problem was done using MATLAB (HW1_4b.m)




The directions of axes, the locations of centroids, and the areas of each nut are already labeled on the above figure.