ME6406 HW3

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Grade: 95

1 Camera Model and Calibration

1.1 a) Camera Model

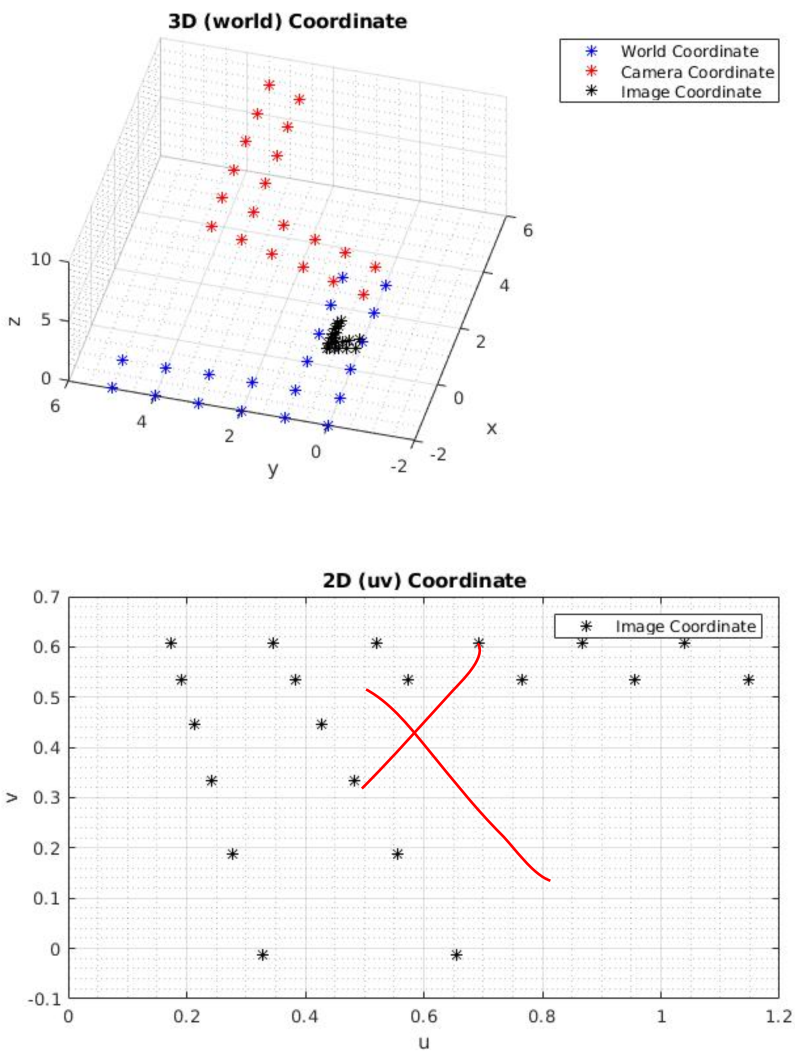
CameraCalibration.m

The 2D image coordinates are shown in Table 1

Table 1: 2D image coordinates (uv)

u coordinate	*v coordinate*
0.1733	0.6067
0.3467	0.6067
0.5200	0.6067
0.6933	0.6067
0.8667	0.6067
1.0400	0.6067
0.1914	0.5345
0.3828	0.5345
0.5741	0.5345
0.7655	0.5345
0.9569	0.5345
1.1483	0.5345
0.4272	0.4456
0.2136	0.4456
0.4834	0.3332
0.2417	0.3332
0.5566	0.1869
0.2783	0.1869
0.6558	−0.0117
0.3279	−0.0117

and the plots are illustrated in Fig 1



-3 points

Figure 1: Camera model and calibration

1.2 b) Camera Calibration

CameraModel.m

Step 1) load 'camera_calibration_data.mat' and construct [**A**], and **b**

$$A = \begin{bmatrix} X_1 v_{d1} & Y_1 v_{d1} & -X_1 u_{d1} & -Y_1 u_{d1} & v_{d1} \\ & & . & & \\ & & . & & \\ & & . & & \\ X_n v_{dn} & Y_n v_{dn} & -X_n u_{dn} & -Y_n u_{dn} & v_{dn} \end{bmatrix}$$

$$b = \begin{bmatrix} u_{d1} \\ . \\ . \\ . \\ u_{dn} \end{bmatrix}$$

$$A\mu = b$$

where μ can be solved using Pseudo-Inverse method and defined by

$$\mu = \begin{bmatrix} r_{11}/T_y \\ r_{12}/T_y \\ r_{21}/T_y \\ r_{22}/T_y \\ T_x/T_y \end{bmatrix}$$

Note that r_{ij} is the element of [**R**], and T_x, T_y are the x, and y element of **T**, respectively

With the loaded data

$$A = \begin{bmatrix} -1.2133 & 0 & 0.3467 & 0 & 0.6067 \\ -0.6067 & 0 & 0.3467 & 0 & 0.6067 \\ 0 & 0 & 0 & 0 & 0.6067 \\ 0.6067 & 0 & -0.6933 & 0 & 0.6067 \\ 1.2133 & 0 & -1.7333 & 0 & 0.6067 \\ 1.8200 & 0 & -3.1200 & 0 & 0.6067 \\ -1.0690 & 0.5345 & 0.3828 & -0.1914 & 0.5345 \\ -0.5345 & 0.5345 & 0.3828 & -0.3828 & 0.5345 \\ 0 & 0.5345 & 0 & -0.5741 & 0.5345 \\ 0.5345 & 0.5345 & -0.7655 & -0.7655 & 0.5345 \\ 1.0690 & 0.5345 & -1.9138 & -0.9569 & 0.5345 \\ 1.6035 & 0.5345 & -3.4448 & -1.1483 & 0.5345 \\ -0.4456 & 0.8911 & 0.4272 & -0.8544 & 0.4456 \\ -0.8911 & 0.8911 & 0.4272 & -0.4272 & 0.4456 \\ -0.3332 & 0.9997 & 0.4834 & -1.4502 & 0.3332 \\ -0.6664 & 0.9997 & 0.4834 & -0.7251 & 0.3332 \\ -0.1869 & 0.7475 & 0.5566 & -2.2262 & 0.1869 \\ -0.3738 & 0.7475 & 0.5566 & -1.1131 & 0.1869 \\ 0.0117 & -0.0583 & 0.6558 & -3.2791 & -0.0117 \\ 0.0233 & -0.0583 & 0.6558 & -1.6396 & -0.0117 \end{bmatrix}$$

$$b = \begin{bmatrix} 0.1733 \\ 0.3467 \\ 0.5200 \\ 0.6933 \\ 0.8667 \\ 1.0400 \\ 0.1914 \\ 0.3828 \\ 0.5741 \\ 0.7655 \\ 0.9569 \\ 1.1483 \\ 0.4272 \\ 0.2136 \\ 0.4834 \\ 0.2417 \\ 0.5566 \\ 0.2783 \\ 0.6558 \\ 0.3279 \end{bmatrix}$$

Therefore, μ is

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} = \begin{bmatrix} 0.2857 \\ 0.0000 \\ 0.0000 \\ -0.2020 \\ 0.8571 \end{bmatrix}$$

Since, $\mu_1\mu_4 \neq \mu_2\mu_3, then\ T_y^2$ can be calculated from

$$T_y^2 = \frac{U - [U^2 - 4(\mu_1\mu_4 - \mu_2\mu_3)^2]^{1/2}}{2(\mu_1\mu_4 - \mu_2\mu_3)^2}$$

where

$$U = \sum_{j=1}^{4} \mu_j^2$$

Therefore,

$$T_y = 12.25$$

Then each element of $[\mathbf{R}]$ can be determined

$$r_{ij} = \mu_{ij}T_y \ (i, j = 1, 2)$$
$$r_{13}^2 = 1 - T_y^2(\mu_1^2 + \mu_2^2)$$
$$r_{23}^2 = 1 - T_y^2(\mu_3^2 + \mu_4^2)$$
$$\begin{bmatrix} r_{31} \\ r_{32} \\ r_{33} \end{bmatrix} = \begin{bmatrix} r_{11} \\ r_{12} \\ r_{13} \end{bmatrix} \times \begin{bmatrix} r_{21} \\ r_{22} \\ r_{23} \end{bmatrix}$$

There is a step to check the sign of T_y; however, in this problem the sign of T_y is already correct (positive).

Since, the sign of $r_{11}r_{21} + r_{12}r22]$ is not negative, the sign of r_{23} must be flipped.

The matrix $[\mathbf{R}]$, and the vector $\mathbf{T}$ can be constructed as

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.7071 & 0.7071 \\ 0 & -0.7071 & -0.7071 \end{bmatrix}$$

$$T = \begin{bmatrix} 3.0 \\ 3.5 \\ T_z \end{bmatrix}$$

Step 2) Construct [**A'**], **x'**, and **b'** in the similar way as [**A**], μ, and **b**

$$A'x' = b'$$

where

$$A' = \begin{bmatrix} x_1 & r_{d1}^2 x_1 & -u_{d1} \\ & . & \\ & . & \\ & . & \\ x_n & r_{dn}^2 x_n & -u_{dn} \end{bmatrix}$$

$$x' = \begin{bmatrix} f \\ fk_1 \\ T_z \end{bmatrix}$$

$$b' = \begin{bmatrix} (r_{31}X_1 + r_{32}Y_1)u_{d1} \\ . \\ . \\ . \\ (r_{31}X_n + r_{32}Y_n)u_{dn} \end{bmatrix}$$

$$x_i = r_{11}X_i + r_{12}Y_i + T_x$$

Therefore, x' can be calculated from Pseudo-Inverse Method

$$x' = \begin{bmatrix} 1.3 \\ 0 \\ 7.5 \end{bmatrix}$$

Hence, the value of $f, [\mathbf{R}], \mathbf{T}$ is computed and shown in the following equations.

$$f = 1.3$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.7071 & 0.7071 \\ 0 & \cancel{-}0.7071 & -0.7071 \end{bmatrix}$$

-2 points

$$T = \begin{bmatrix} 3.0 \\ 3.5 \\ 7.5 \end{bmatrix}$$

2 Robot Eye-on-Hand Calibration

2.1 Compute ([Rc12], Tc12) and ([Rc23], Tc23)

HW3_2.m

Since H_{c1}, H_{c2}, and H_{c3} are given, H_{c12}, and H_{c23} can be calculated from the following equation.

$$H_{cij} = H_{cj} H_{ci}^{-1}$$

Then, $R_{c12}, T_{c12}, R_{c23}$, and T_{c23} can be extracted from H_{c12}, and H_{c23} using the following equation.

$$H_{ij} = \begin{bmatrix} & R_{ij} & & T_{ij} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, $R_{c12}, T_{c12}, R_{g23}$, and T_{g23} are

$$R_{c12} = \begin{bmatrix} -0.0718 & 0.8417 & -0.5351 \\ -0.7548 & -0.3965 & -0.5225 \\ -0.6520 & 0.3664 & 0.6638 \end{bmatrix}$$

$$T_{c12} = \begin{bmatrix} 0.1319 \\ 5.2006 \\ -2.8082 \end{bmatrix}$$

$$R_{c23} = \begin{bmatrix} -0.1863 & -0.0898 & 0.9784 \\ 0.4973 & 0.8502 & 0.1727 \\ -0.8473 & 0.5187 & -0.1138 \end{bmatrix}$$

$$T_{c23} = \begin{bmatrix} -4.3792 \\ -1.2780 \\ 5.4939 \end{bmatrix}$$

2.2 Obtain the equivalent angle-axis representation (n, θ) for each of the rotation matrices; [Rc12], [Rc23], [Rg12] and [Rg23].

HW3_2.m

Since H_{g12}, and H_{g23} are given, R_{g12}, T_{g12}, R_{g23}, and T_{g23} can be extracted in the same way as in 2.1.

(n, θ) can be calculated from the following equations.

$$\theta = arccos\left(\frac{R_{11} + R_{22} + R_{33} - 1}{2}\right)$$
$$n = \frac{sin(\theta)}{2}\begin{bmatrix} R_{32} - R_{23} \\ R_{13} - R_{31} \\ R_{21} - R_{12} \end{bmatrix}$$

Note that

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

Therefore, (n, θ) for each of the rotation matrices are

$$n_{c12} = \begin{bmatrix} 0.4855 \\ 0.0639 \\ -0.8719 \end{bmatrix}$$
$$\theta_{c12} = 113.7181$$
$$n_{c23} = \begin{bmatrix} 0.1776 \\ 0.9369 \\ 0.3013 \end{bmatrix}$$
$$\theta_{c23} = 102.9986$$
$$n_{g12} = \begin{bmatrix} -0.0639 \\ 0.4856 \\ -0.8718 \end{bmatrix}$$
$$\theta_{g12} = 113.7127$$
$$n_{g23} = \begin{bmatrix} -0.9369 \\ 0.1773 \\ 0.3014 \end{bmatrix}$$
$$\theta_{g23} = 102.9843$$

2.3 Compute Pc12, Pc23, Pg12, and Pg23. Check the solutions

HW3_2.m

P can be calculated from (n, θ) using the following equation.

$$P = 2sin\left(\frac{\theta}{2}\right)n$$

Therefore, $P_{c12}, P_{c23}, P_{g12}$, and P_{g23} are

$$P_{c12} = \begin{bmatrix} 0.8130 \\ 0.1069 \\ -1.4602 \end{bmatrix}$$

$$P_{c23} = \begin{bmatrix} 0.2779 \\ 1.4664 \\ 0.4715 \end{bmatrix}$$

$$P_{g12} = \begin{bmatrix} -0.1070 \\ 0.8132 \\ -1.4600 \end{bmatrix}$$

$$P_{g23} = \begin{bmatrix} -1.4663 \\ 0.2774 \\ 0.4716 \end{bmatrix}$$

To check the solution R can be backwardly calculated from (n, θ) and P from the following equations.

$$R = \begin{bmatrix} n_1^2 + (1-n_1^2)cos(\theta) & n_1n_2(1-cos(\theta)) - n_3sin(\theta) & n_1n_3(1-cos(\theta)) + n_2sin(\theta) \\ n_1n_2(1-cos(\theta)) + n_3sin(\theta) & n_2^2 + (1-n_2^2)cos(\theta) & n_2n_3(1-cos(\theta)) - n_1sin(\theta) \\ n_1n_3(1-cos(\theta)) - n_2sin(\theta) & n_2n_3(1-cos(\theta)) + n_1sin(\theta) & n_3^2 + (1-n_3^2)cos(\theta) \end{bmatrix}$$

$$R = (1 - \frac{|P|^2}{2})I + \frac{1}{2}(PP^T + \sqrt{4-|P|^2}skew(P))$$

Note that

$$n = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$skew(P) = \begin{bmatrix} 0 & -P_3 & P_2 \\ P_3 & 0 & -P_1 \\ -P_2 & P_1 & 0 \end{bmatrix}$$

The solutions checked using MATLAB are the same as given in 2.1

2.4 Compute Pcg, [Rcg] and Tcg

HW3_2.m

P_{cg} can be calculated by solving the linear system from the following equation and two pairs of station.

$$P_{cg} = \frac{2P_{cg}^{'}}{\sqrt{1 + |P_{cg}^{'}|^2}}$$

where $P_{cg}^{'}$ satisfies

$$skew(P_{gij} + P_{cij})P_{cg}^{'} = P_{cij} - P_{gij}$$

Therefore,

$$P_{cg} = \begin{bmatrix} 0.0001 \\ -0.0001 \\ 1.4144 \end{bmatrix} \approx \begin{bmatrix} 0 \\ 0 \\ 1.414 \end{bmatrix}$$

$[R_{cg}]$ can also be calculated in the same manner as in 2.3; hence,

$$R_{cg} = \begin{bmatrix} -0.0002 & -1.0000 & -0.0000 \\ 1.0000 & -0.0002 & -0.0002 \\ 0.0002 & -0.0000 & 1.0000 \end{bmatrix} \approx \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

T_{cg} can be calculate by solving the linear system from the following equation and two pairs of station.

$$(R_{gij} - I)T_{cg} = R_{cg}T_{cij} - Tgij$$

Therefore,

$$T_{cg} = \begin{bmatrix} -0.4011 \\ -0.5975 \\ -0.3487 \end{bmatrix}$$

3 Ellipse-Circle Correspondence

HW3_3.m

Given the ellipse equation and focal length

$$(x-1)^2/4 + (y-1)^2/16 = 1$$
$$f = 0.1$$

The ellipse equation can be rewritten into a standard conic function

$$Au^2 + 2Buv + Cv^2 + 2Du + 2Ev + F = 0$$

as

$$4u^2 + v^2 - 8u - 2v - 11 = 0$$

Therefore,

$$\begin{aligned} A &= 4 \\ B &= 0 \\ C &= 1 \\ D &= -4 \\ E &= -1 \\ F &= -11 \end{aligned}$$

We can relate the camera frame (x_c, y_c, z_c), its projection (u, v), and the plane through the following equations

$$\begin{aligned} u &= f\frac{x_c}{z_c} \\ v &= f\frac{y_c}{z_c} \\ z_c &= \alpha x_c + \beta y_c + \gamma \end{aligned}$$

Substituting, these equation into a standard conic function, obtained

$$(A + 2\frac{D}{f}\alpha + \frac{F}{f^2}\alpha^2)x_c^2 + (C + 2\frac{B}{f}\beta + \frac{F}{f^2}\beta^2)y_c^2 + 2(B + \frac{D}{f}\beta + \frac{E}{f}\alpha + \frac{F}{f^2}\alpha\beta)x_c y_c$$
$$+2(\frac{D}{f}\gamma + \frac{F}{f^2}\alpha\gamma)x_c + 2(\frac{E}{f}\gamma + \frac{F}{f^2}\alpha\beta)y_c + \frac{F}{f^2}\gamma^2 = 0$$

Since, this is the circle equation, we can equate the coefficient of x_c^2 and y_c^2 term, and set the coefficient of $x_c y_c$ term to zero.

$$(A + 2\frac{D}{f}\alpha + \frac{F}{f^2}\alpha^2) = (C + 2\frac{B}{f}\beta + \frac{F}{f^2}\beta^2)$$
$$2(B + \frac{D}{f}\beta + \frac{E}{f}\alpha + \frac{F}{f^2}\alpha\beta) = 0$$

With these two equations, we can solve for two unknowns α, and β

There will be four pairs of α, and β; however, two of them are complex number, so we can ignore them.

$$\alpha = -0.0996 \; and \; \beta = -0.0143$$
$$\alpha = 0.0268 \; and \; \beta = -0.0039$$

Note that, in my MATLAB code, four of them will be shown and used for all calculation steps after that.

Plug α, and β back into the equation.

For $\alpha = -0.0996 \; and \; \beta = -0.0143$

$$\begin{aligned}
(1.0608)x_c^2 + (1.0608)y_c^2 + 2(69.5205)\gamma x_c + 2(5.7537)\gamma y_c - 1100\gamma^2 &= 0 \\
x_c^2 + y_c^2 + 2(65.5351\gamma)x_c + 2(5.4239\gamma)y_c - 1036.9\gamma^2 &= 0 \\
(x_c + 65.5351\gamma)^2 + (y_c + 5.4239\gamma)^2 - 1036.9\gamma^2 &= (65.5351\gamma)^2 + (5.4239\gamma)^2 \\
(x_c + 65.5351\gamma)^2 + (y_c + 5.4239\gamma)^2 &= 5361\gamma^2
\end{aligned}$$

For $\alpha = 0.0268 \; and \; \beta = -0.0039$

$$\begin{aligned}
(1.0608)x_c^2 + (1.0608)y_c^2 + 2(-69.5205\gamma)x_c + 2(-5.7537\gamma)y_c - 1100\gamma^2 &= 0 \\
x_c^2 + y_c^2 + 2(-65.5351\gamma)x_c + 2(-5.4239\gamma)y_c - 1036.9\gamma^2 &= 0 \\
(x_c - 65.5351\gamma)^2 + (y_c - 5.4239\gamma)^2 - 1036.9\gamma^2 &= (-65.5351\gamma)^2 + (-5.4239\gamma)^2 \\
(x_c - 65.5351\gamma)^2 + (y_c - 5.4239\gamma)^2 &= 5361\gamma^2
\end{aligned}$$

Since, we know that γ must be positive (object must be in front of camera), we can ignore the case where $\alpha = -0.0996 \; and \; \beta = -0.0143$ because it will give us the negative value of circle's center.

Since, the radius of the circle is known, we can compare it with the obtained equation.

$$\begin{aligned}
5361\gamma^2 &= 1^2 \\
\gamma &= \sqrt{1/5361} \\
\gamma &= 0.0137
\end{aligned}$$

Therefore, the plane equation that contains the circle is

$$\begin{aligned}
z_c &= \alpha x_c + \beta y_c + \gamma \\
z_c &= 0.0268x_c - 0.0039y_c + 0.0137
\end{aligned}$$

To find the circle equation , we substitute γ back into the equation

$$\begin{aligned}
(x_c - 65.5351\gamma)^2 + (y_c - 5.4239\gamma)^2 &= 5361\gamma^2 \\
(x_c - 0.8951)^2 + (y_c - 0.0741)^2 &= 1
\end{aligned}$$

Therefore, the center of the circle O_x, O_y, and the distance away from the camera frame O_z are

$$
\begin{aligned}
O_x &= 0.8951 \\
O_y &= 0.0741 \\
O_z &= 0.0374
\end{aligned}
$$

Note that O_z can be obtained from the plane equation by substituting the center of the circle.