ME6406 HW1 Thanakorn Khamvilai Report

Problem 1: Pin-Hole Optics

Solution:

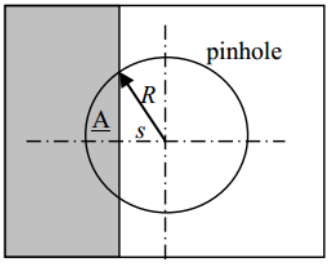
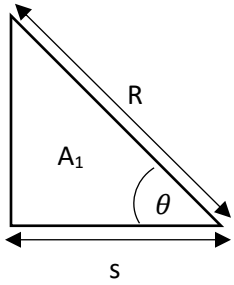


Figure 1.

Pin-hole area $\delta O = \pi R^2$

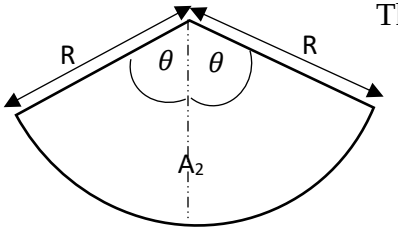
Consider the right triangle



The angle $\theta = arccos\frac{s}{R}$

and the area $A_1 = \frac{1}{2}s\sqrt{R^2 - s^2}$

Consider the Sector



The area $A_2 = \pi R^2 \cdot \frac{2\theta}{2\pi}$

$A_2 = \theta R^2$

$A_2 = arccos\frac{s}{R} \cdot R^2$

Then the overlap area $\delta A = A_2 - A_1$

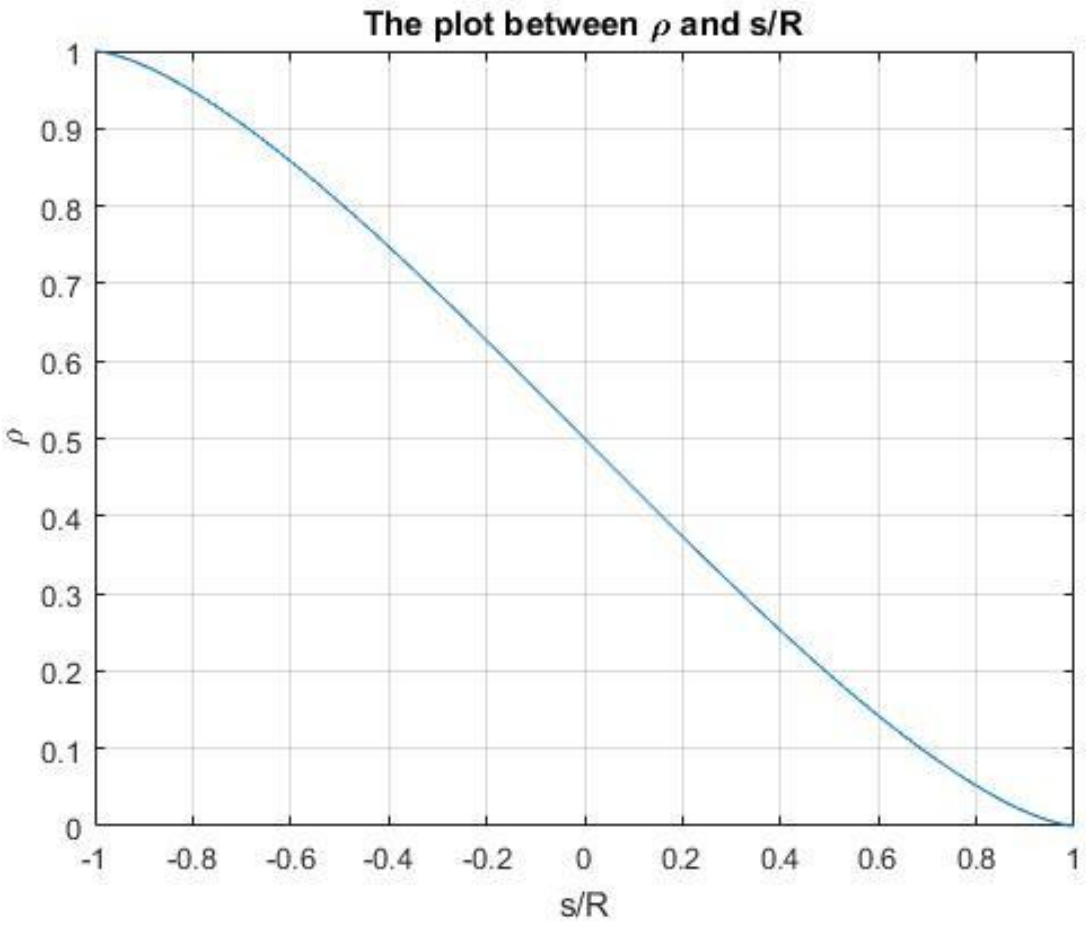
$$\delta A = arccos\frac{s}{R} \cdot R^2 - 2 \cdot \frac{1}{2}s\sqrt{R^2 - s^2}$$

$$\delta A = R^2\left[arccos\frac{s}{R} - \frac{s}{R}\sqrt{1 - \left(\frac{s}{R}\right)^2}\right]$$

Therefore, $\rho = \frac{\delta A}{\delta O} = \frac{R^2\left[arccos\frac{s}{R} - \frac{s}{R}\sqrt{1-\left(\frac{s}{R}\right)^2}\right]}{\pi R^2}$

$$\rho = \frac{1}{\pi}\left[arccos\frac{s}{R} - \frac{s}{R}\sqrt{1 - \left(\frac{s}{R}\right)^2}\right]$$

Plot ρ as a function of s/R using MATLAB (HW1_1.m)

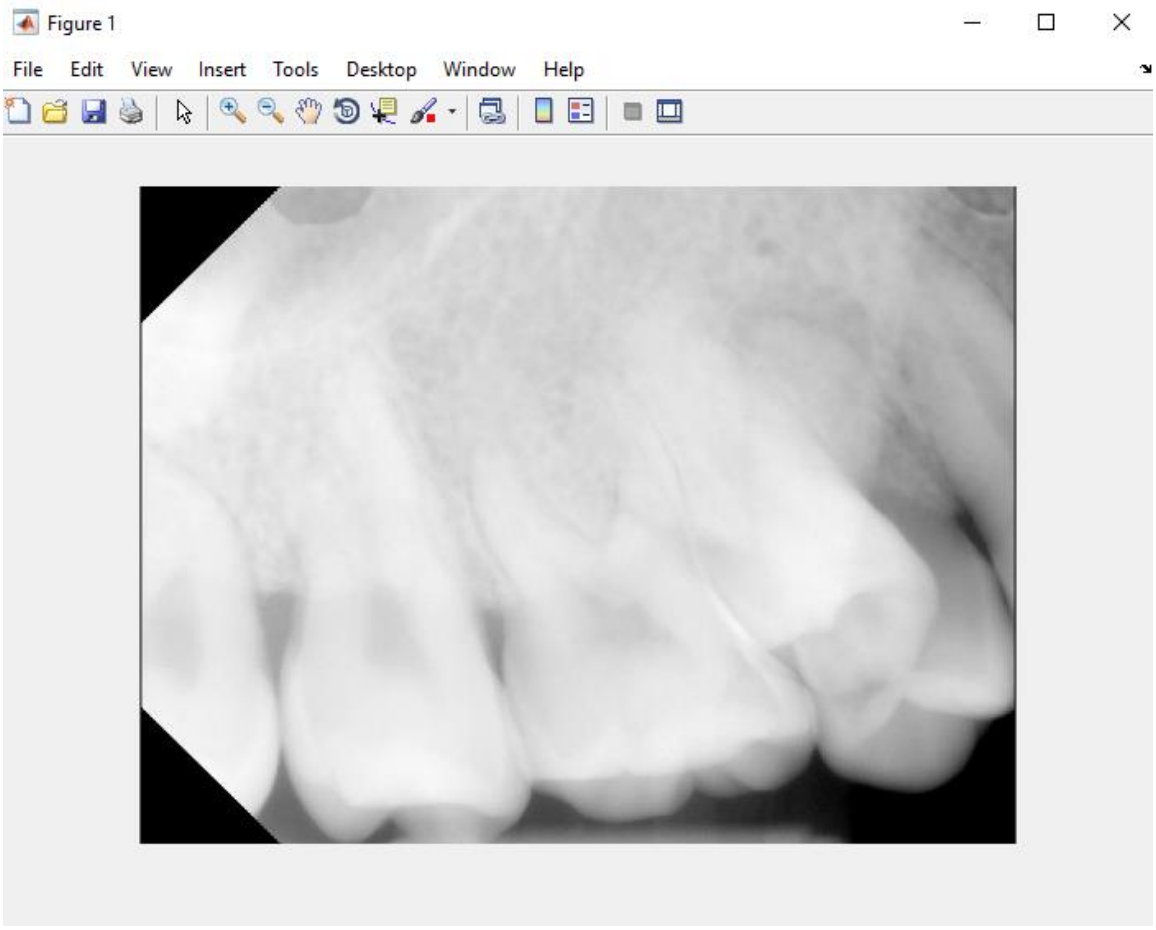


Problem 2: Histogram Equalization

2a) Solution: The table below was done using MATLAB (HW1_2a.m)

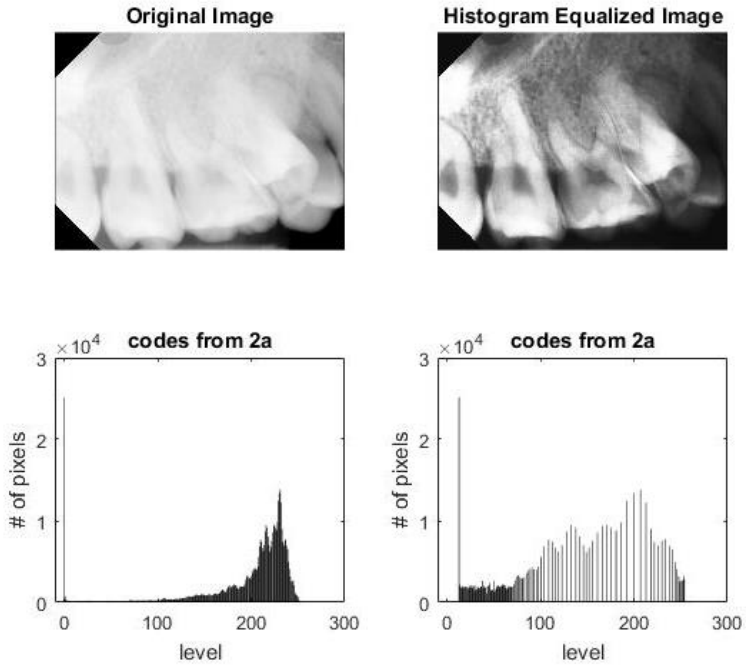
Gray Level	**# of pixels**	**cdf**	**q_k**	**round(q_k)**
190	1	1	7.0833	7
191	5	6	42.5000	43
192	3	9	63.7500	64
193	5	14	99.1667	99
194	4	18	127.5000	128
195	7	25	177.0833	177
196	6	31	219.5833	220
197	2	33	233.7500	234
198	1	34	240.8333	241
199	1	35	247.9167	248
200	1	36	255.0000	255

2b) Solution: I. Read in and display the ‘teeth.jpg’ (HW1_2b.m)

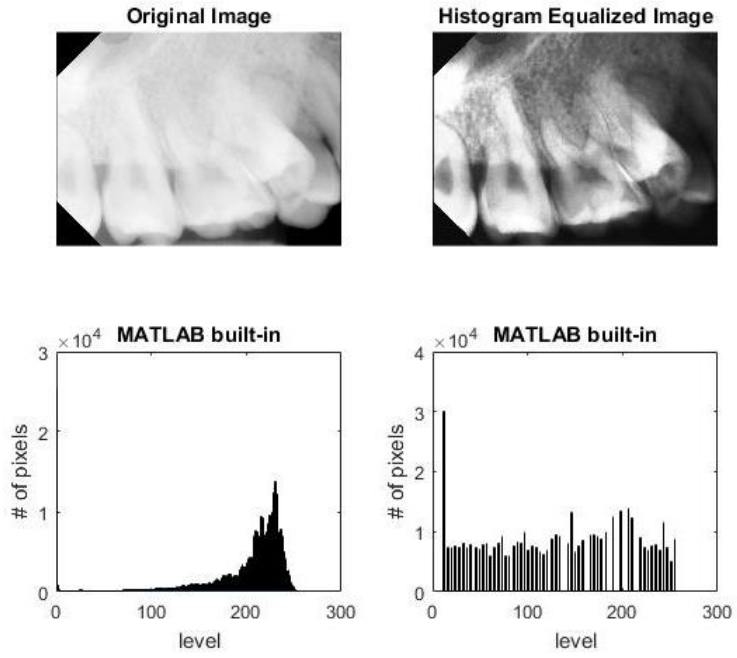


II. Compare by displaying the original and processed images and their histograms. (HW1_2b.m)

By using codes from 2a)



By using MATLAB built-in commands



There is a little difference in pixel intensity obtained from these two approaches even if both equalized images look pretty similar; thus, the plots of histogram equalization are difference.

Problem 3: Filtering Masks

3a) Solution:

$$H_x = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}, H_y = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, Z = \begin{bmatrix} 199 & 197 & 195 \\ 197 & 195 & 194 \\ 196 & 194 & 192 \end{bmatrix}$$

$g_m = 196 + 2(194) + 192 - 199 - 2(197) - 195 = -12$

$g_n = 195 + 2(194) + 192 - 199 - 2(197) - 196 = -14$

Magnitude

$|g(m.n)| = \sqrt{g_m^2 + g_n^2} = \sqrt{(-12)^2 + (-14)^2} = 18.4391 \approx 18$

Direction

$\angle g(m,n) = tan^{-1}\frac{g_n}{g_m} = tan^{-1}\frac{-14}{-12} = 229.3987^o$

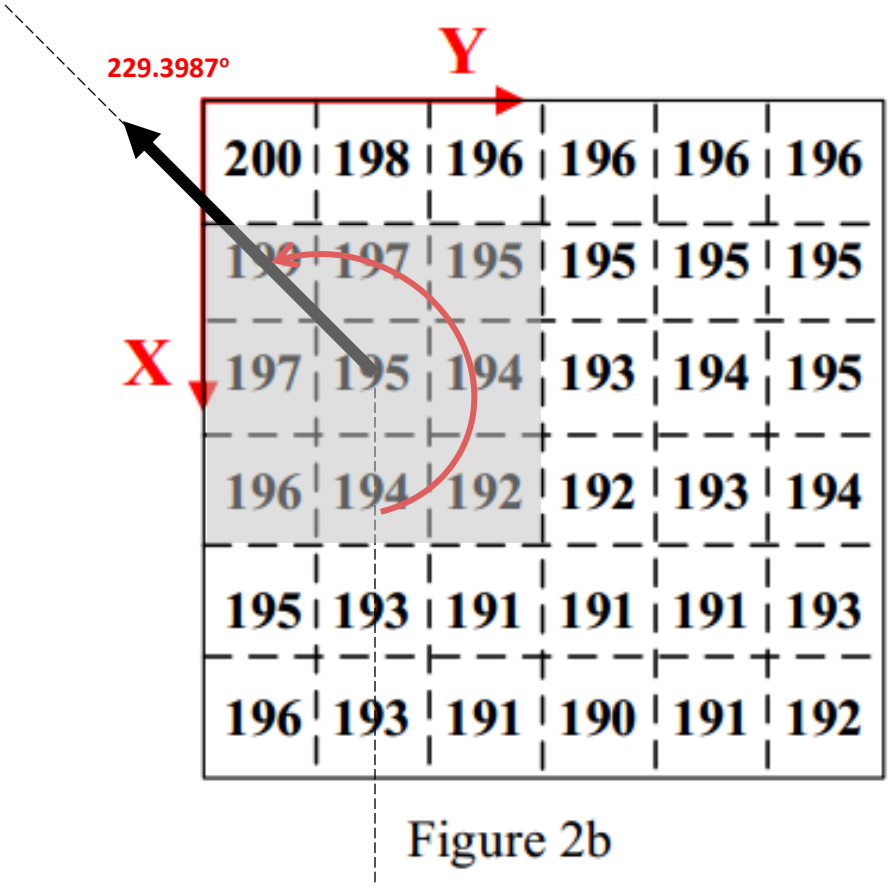
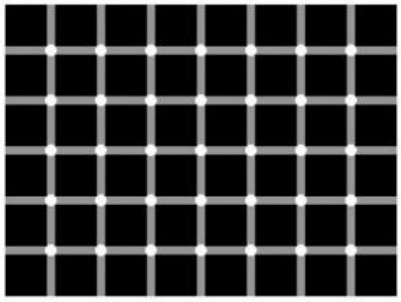


Figure 2b

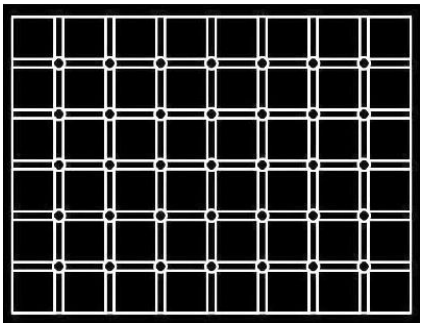
3b) Solution: This problem was done using MATLAB (HW1_3b.m)

Original Image

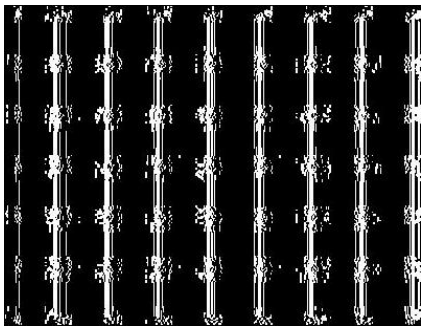


Using the same calculation as 3a)

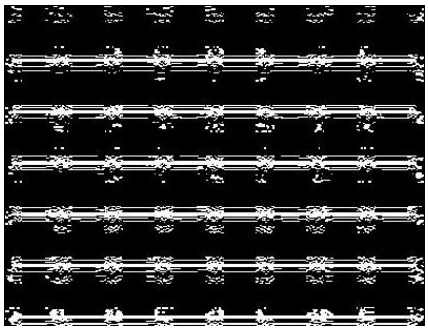
Both Direction



Vertical Direction

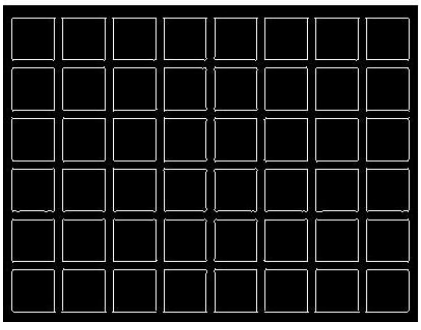


Horizontal Direction

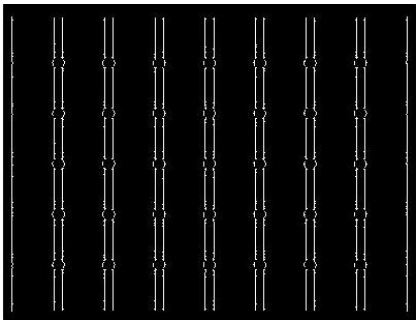


Using the MATLAB built-in command with the default threshold

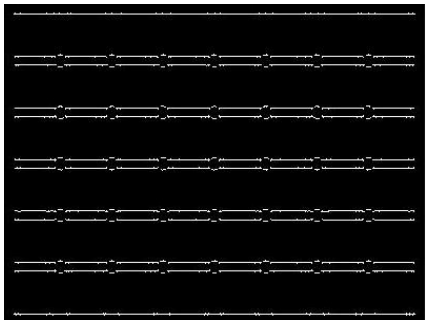
Both Direction



Vertical Direction



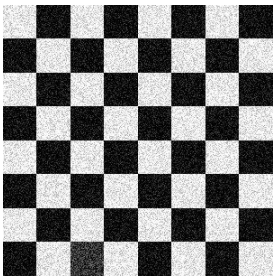
Horizontal Direction



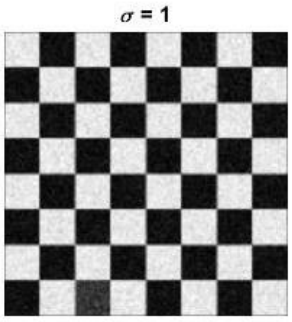
Because of using the default threshold in built-in *edge* command, the results from these two approaches have some differences.

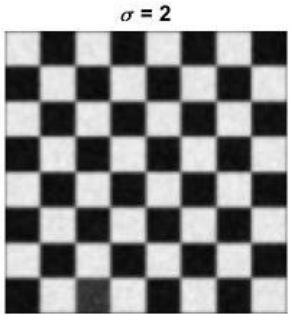
3c) Solution: This problem was done using MATLAB with 3Std Gaussian Filter i.e.7x7 for σ = 1 (3*1*2 + 1), 13x13 for σ = 2 (3*3*2 + 1), and 31x31 for σ = 5 (3*5*2 + 1) (HW1_3c.m)

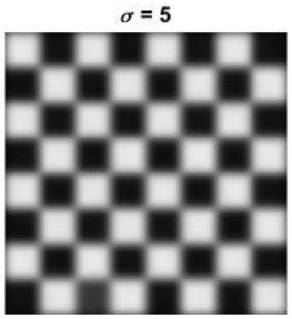
Original Image



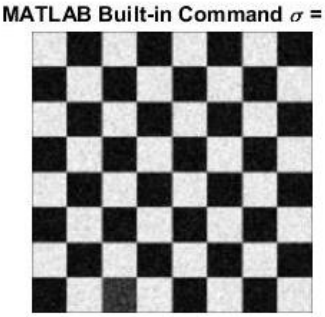
Analytical Implement

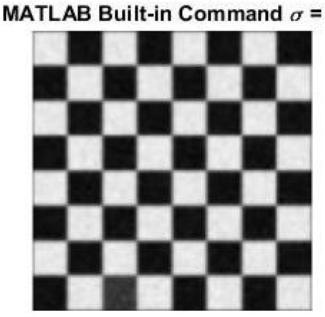


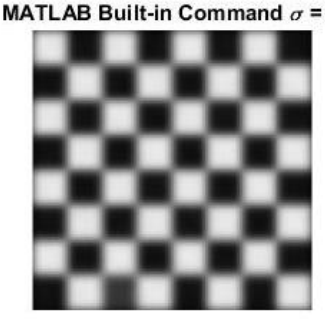




MATLAB Built-In Command



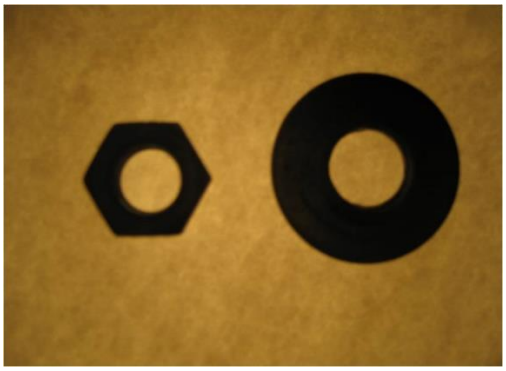




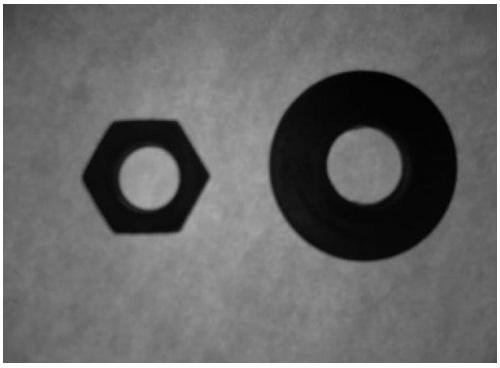
Problem 4: Low-Level Information Processing

4a) Solution: This problem was done using MATLAB (HW1_4a.m)

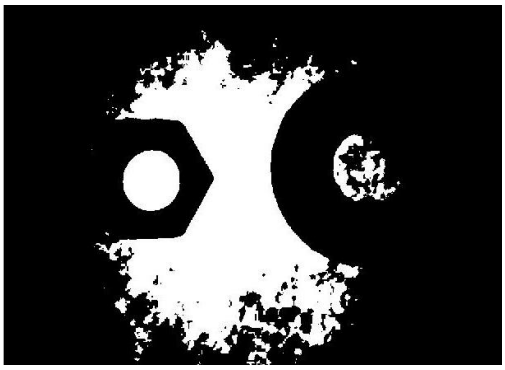
Original Image



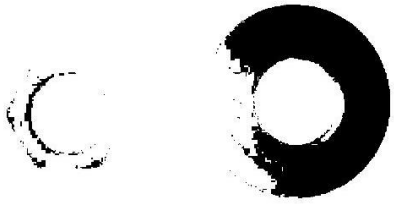
Grayscale Image



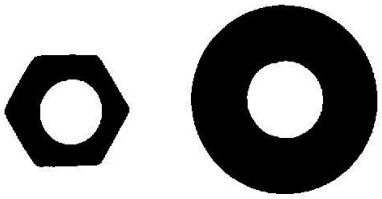
Over-Estimated Threshold



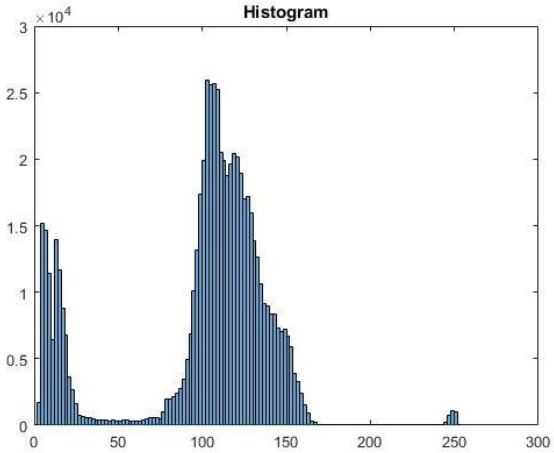
Under-Estimated Threshold



Appropriated Threshold

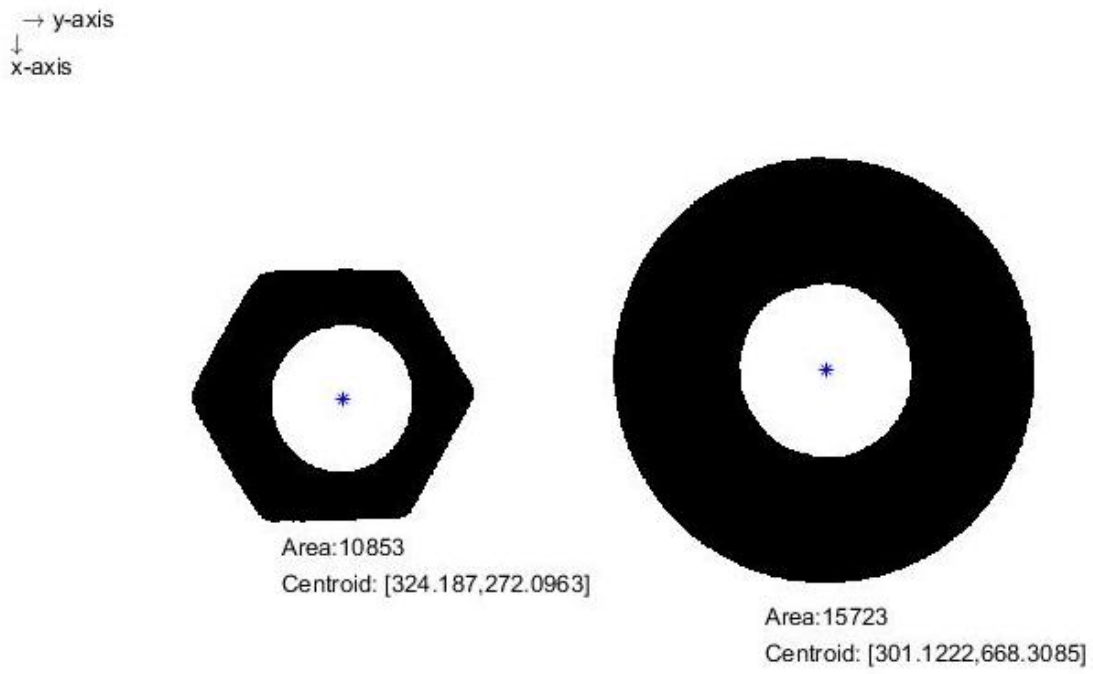


Histogram



The values of over-estimated threshold, under-estimate threshold, and appropriated threshold are chosen to be 127.5, 12.75, and 51, respectively; hence, the values 0.5 (127.5/255), 0.05 (12.75/255), and 0.2 (51/255) are used in *im2bw* function for over-estimated threshold, under-estimate threshold, and appropriated threshold, respectively.

4b) Solution: This problem was done using MATLAB (HW1_4b.m)



The directions of axes, the locations of centroids, and the areas of each nut are already labeled on the above figure.