

Let $D_k : C_k \rightarrow C_{k-1}$ denote the boundary operator between k and $(k-1)$ chains. Let it also denote the matrix wrt the simplex basis. The dimension of the nullspace of the symmetrix matrix

$$\begin{pmatrix} D_k^* D_k & D_{k+1} \\ D_{k+1}^* & -I \end{pmatrix} \tag{1}$$

is the dimension of $H_k = \text{ns } D_k / \text{im } D_{k+1}$, i.e. the Betti number b_k .

Can this be used to estimate b_k ?