DRAFT: NCT

RA

2021-04-09

1 NCT models

Abbreviations: FG-nups = FG-nucleoporins; NCT = nucleocytoplasmic transport; NPC = nuclear pore complex; ODE = ordinary differential equations.

1.1 GSR'03 model of NCT

Ran gradient. First we implement the "minimal Ran gradient system" from [GSR03]. The equations are recapitulated in §2.1 and the constants are collected in Table 2. Following [GSR03], the "dynamic capacity" Ex is an optional maximal steady-state (positive) flux of nuclear $\mathsf{Ran\cdot GTP}$ to cytoplasmic $\mathsf{Ran\cdot GDP}$, which we determine using the additional equation (19). The fluxes are in units of concentration/time (µM/s). The ones across the nuclear boundary have positive sign when exiting the nucleus and are normalized to the nuclear volume. Thus, the *amount* exiting the nucleus per unit of time is $\mathsf{flux}\times V_{\mathrm{nuc}}$.

Simulating the ODE across the scenarios of [GSR03] we obtain results that are sufficiently close to the original, see Table 3. Importantly, an order of 1000-fold nuclear enrichment of $\mathsf{Ran\cdot GTP}$ is sustained in steady-state.

Code #1.

Coupling to Impβ-mediated transport. A coupling of the Ran gradient to importin–cargo transport was proposed in [GSR03, Fig. 6A]. We now formulate a version of it. The following equations comprise the handling of cargo by $\mathsf{Imp}\beta$ in the cytoplasm,

$$\frac{\mathrm{d}}{\mathrm{d}t}[\mathsf{Imp}\beta\cdot\mathsf{Ran\cdot GTP}]_{\mathrm{cyt}} = -\mathsf{R}_{\mathsf{cyt}} + \mathsf{F}_{\mathsf{Imp}\beta\cdot\mathsf{Ran\cdot GTP}}\frac{V_{\mathrm{nuc}}}{V_{\mathrm{cyt}}} - \mathsf{GAP}_{\mathsf{Imp}\beta} + \mathsf{Knockoff}_{\mathsf{cyt}} \tag{1a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}[\mathsf{Imp}\beta]_{\mathrm{cyt}} = +\mathsf{R}_{\mathsf{cyt}} + \mathsf{C}_{\mathsf{cyt}} + \mathsf{F}_{\mathsf{Imp}\beta}\frac{V_{\mathrm{nuc}}}{V_{\mathrm{cyt}}} + \mathsf{GAP}_{\mathsf{Imp}\beta} \tag{1b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}[\mathsf{Imp}\beta\cdot\mathsf{Cargo}]_{\mathrm{cyt}} = -\mathsf{C}_{\mathsf{cyt}} + \mathsf{F}_{\mathsf{Imp}\beta\cdot\mathsf{Cargo}}\frac{V_{\mathrm{nuc}}}{V_{\mathrm{cyt}}} - \mathsf{Knockoff}_{\mathsf{cyt}} \tag{1c}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}[\mathsf{Cargo}]_{\mathrm{cyt}} = +\mathsf{C}_{\mathsf{cyt}} + \mathsf{F}_{\mathsf{Cargo}}\frac{V_{\mathrm{nuc}}}{V_{\mathrm{cyt}}} + \mathsf{Knockoff}_{\mathsf{cyt}} \tag{1d}$$

with the fluxes

$$\mathsf{R}_{\mathsf{cyt}} := -k^{\mathrm{R}}_{\mathrm{on}}[\mathsf{Imp}\beta][\mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{cyt}} + k^{\mathrm{R}}_{\mathrm{off}}[\mathsf{Imp}\beta\cdot\ \mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{cyt}} \tag{2a}$$

$$\mathsf{C}_{\mathsf{cyt}} := -k^{\mathrm{C}}_{\mathrm{on}}[\mathsf{Imp}\beta][\mathsf{Cargo}]_{\mathrm{cyt}} + k^{\mathrm{C}}_{\mathrm{off}}[\mathsf{Imp}\beta\cdot\ \mathsf{Cargo}]_{\mathrm{cyt}}. \tag{2b}$$

The forward flux of the reaction

$$\mathsf{Imp}\beta\cdot\ \mathsf{Cargo} + \mathsf{Ran}\cdot\mathsf{GTP} \rightleftharpoons \mathsf{Imp}\beta\cdot\ \mathsf{Ran}\cdot\mathsf{GTP} + \mathsf{Cargo} \tag{3}$$

is called Knockoff. It is modeled as a one-way reaction with forward rate k_{knockoff}. The GSR equations are modified accordingly:

$$\tfrac{\mathrm{d}}{\mathrm{d}t}[\mathsf{Ran}\cdot\mathsf{GDP}]_{\mathrm{cyt}} = (13\mathrm{a}) + \mathsf{GAP}_{\mathsf{Imp}\beta} \tag{13a$'$}$$

$$\tfrac{\mathrm{d}}{\mathrm{d}t}[\mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{cyt}} = (13\mathrm{b}) + \mathsf{R}_{\mathsf{cyt}} - \mathsf{Knockoff}_{\mathsf{cyt}} \tag{13b$'$}$$

Analogous nuclear equations (without GAP) are implemented but are omitted here. Analogously to (17a)/(17b) we have the additional nuclear-to-cytoplasmic diffusion fluxes

$$\mathsf{F}_{\mathsf{Imp}\beta\cdot\mathsf{Ran}\cdot\mathsf{GTP}},\quad \mathsf{F}_{\mathsf{Imp}\beta},\quad \mathsf{F}_{\mathsf{Imp}\beta\cdot\mathsf{Cargo}},\quad \mathsf{F}_{\mathsf{Cargo}} \tag{4}$$

with the permeability constants given in Table 1.

SPR experiments of [Cat+01] indicated that the IBB domain of importin-α binds importin-β and undergoes a conformational change,

$$\mathsf{A} + \mathsf{B} \rightleftharpoons \mathsf{AB} \rightleftharpoons \mathsf{A}^*\mathsf{B}. \tag{5}$$

We therefore assume the analogous reaction

$$\mathsf{Cargo} + \mathsf{Imp}\beta \underset{\mathsf{k}_{d1}}{\overset{\mathsf{k}_{a1}}{\rightleftharpoons}} \mathsf{Cargo}\cdot\mathsf{Imp}\beta \underset{\mathsf{k}_{d2}}{\overset{\mathsf{k}_{a2}}{\rightleftharpoons}} \mathsf{Cargo}^*\cdot\mathsf{Imp}\beta. \tag{6}$$

Examples of the kinetic constants are available in [Cat+01, Table I], e.g.,

$$k_{a1} = 0.11\,\mu\mathrm{M}^{-1}\,\mathrm{s}^{-1},\quad k_{d1} = 0.024\,\mathrm{s}^{-1},\qquad k_{a2} = 0.024\,\mathrm{s}^{-1},\quad k_{d2} = 7.4\times10^{-4}\,\mathrm{s}^{-1}, \tag{7}$$

for an IBB domain binding to $\mathsf{Imp}\beta$. The intermediate state in (5) is transient on a moderately relevant time-scale, see Fig. 1 (code #2). Therefore, in the present model we lump the complexed states together and take $k^{\mathrm{C}}_{\mathrm{on}} := k_{a1}$ and $k^{\mathrm{C}}_{\mathrm{off}} := k_{d1}\frac{k_{d2}}{k_{a2}+k_{d2}}$ as the effective kinetic constants for (2b), cf. Table 1.

With the constants from Table 1, the steady-state of the model (reached after some $10^4\,\mathrm{s}$) is reported in Fig. 2. Nuclear accumulation of free cargo is 37-fold. Sensitivity analysis shows that, in relative terms, the final nuclear concentration of free cargo depends most strongly on k_{knockoff}. Doubling k_{knockoff} almost doubles the nuclear concentration.

Code #3.

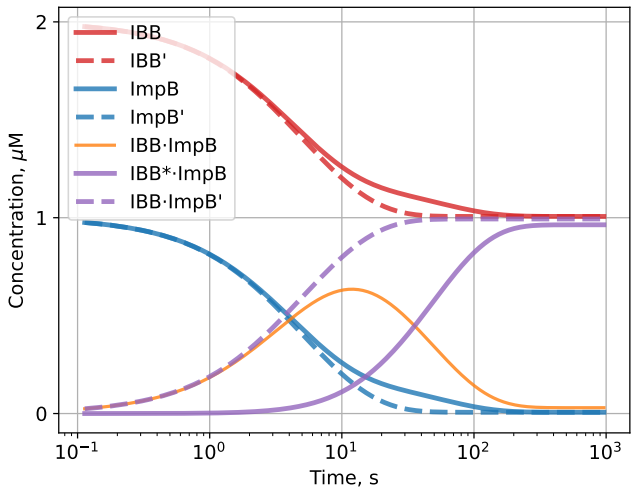

Figure 1: Stand-alone simulation of (5) starting with 2 µM IBB and 1 µM Impβ with the constants (7). The dashed counterpart is the effective system of the form $\mathsf{A}+\mathsf{B} \rightleftharpoons \mathsf{AB}$, cf. §1.1.

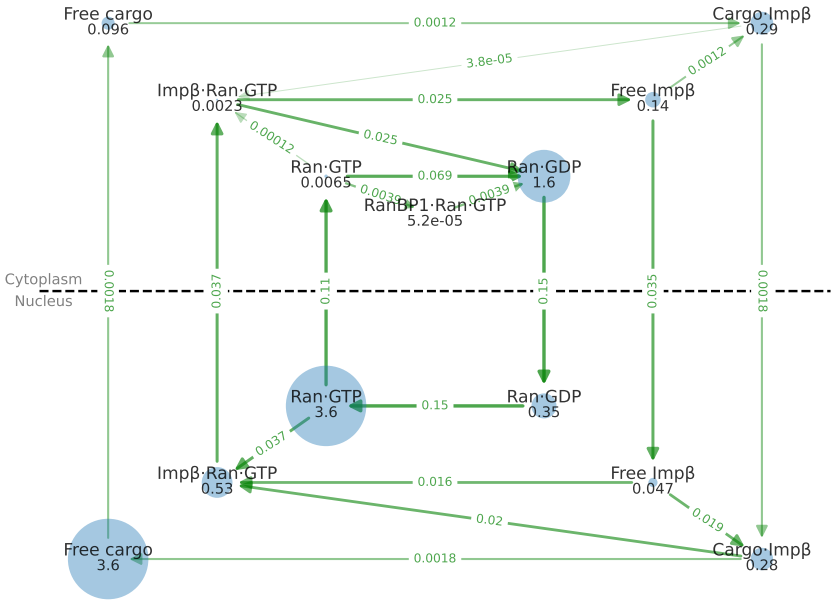

Figure 2: Steady-state of the transport system from §1.1 with conditions of Table 1. The free cargo shows 37-fold accumulation in the nucleus; total nuclear to total cytoplasmic cargo is 10-fold. Units are µM for species and $\mu\mathrm{M\,s^{-1}}$ for fluxes. Initial conditions: $[\mathsf{Ran}\cdot\mathsf{GDP}]_{\mathrm{cyt}} = 5\,\mu\mathrm{M}$, $[\mathsf{Imp}\beta]_{\mathrm{cyt}} = 1\,\mu\mathrm{M}$, $[\mathsf{Cargo}]_{\mathrm{cyt}} = 3\,\mu\mathrm{M}$, all else zero.

(2a)	$k_{\text{on}}^{\text{R}} = 0.096\,\mu\text{M}^{-1}\,\text{s}^{-1}, \quad k_{\text{off}}^{\text{R}} = 4.8 \times 10^{-6}\,\text{s}^{-1}$	[GSR03, Supp. Table A], [RM05, Table II]
(2b)	$k_{\text{on}}^{\text{C}} = 0.11\,\mu\text{M}^{-1}\,\text{s}^{-1}, \quad k_{\text{off}}^{\text{C}} = 7.2 \times 10^{-4}\,\text{s}^{-1}$	[Cat+01, Table I], [RM05, Table II]
(3)	$k_{\text{knockoff}} = 2 \times 10^{-2}\,\mu\text{M}^{-1}\text{s}^{-1}$	[RM05, Table II]
(4)	$D_{\mathsf{Imp}\beta\cdot\mathsf{Ran}\cdot\mathsf{GTP}} = 0.07\,\text{s}^{-1}, \quad D_{\mathsf{Imp}\beta} = 0.4\,\text{s}^{-1}$ $D_{\mathsf{Imp}\beta\cdot\mathsf{Cargo}} = 0.25\,\text{s}^{-1}, \quad D_{\mathsf{Cargo}} = 5 \times 10^{-4}\,\text{s}^{-1}$	[RM05, Table III]

Table 1: Constants for the $\mathsf{Imp}\beta$-mediated transport from §1.1.

1.2 NPC as compartment

It has been observed TODO(1): ref that $\mathsf{Imp}\beta$ accumulate inside the NPCs as they bind to the FG-nups, possibly shuttling the cargo across the pore without leaving themselves. To account for this we propose a three-compartment model with cytoplasm, nucleus and the nuclear envelope as the three compartments. Cytoplasmic and nuclear volume is as in Table 2. For the envelope volume (potentially including some perimembrane space) we take $10^{-3}\,\text{pL}$. The unoccupied NPC space is called $\mathsf{NPC}_{\mathsf{vacant}}$, of which we take $1\,\text{pL} \times 1\,\mu\text{M}$. The crux is now that the nuclear envelope, having small volume, has a high concentration of NPCs. At the nuclear envelope we posit the reactions

$$\mathsf{Imp}\beta_i + \mathsf{NPC}_{\mathsf{vacant}} \rightleftharpoons \mathsf{Imp}\beta\cdot\mathsf{NPC} \tag{8a}$$

$$\mathsf{Cargo}\cdot\mathsf{Imp}\beta_i + \mathsf{NPC}_{\mathsf{vacant}} \rightleftharpoons \mathsf{Cargo}\cdot\mathsf{Imp}\beta\cdot\mathsf{NPC} \tag{8b}$$

TODO(2): ref with $k_{\text{on}} = 10^{-3}\,\mu\text{M}^{-1}\,\text{s}^{-1}$ and $k_{\text{off}} = 10^{-4}\,\text{s}^{-1}$, as well as

$$\mathsf{Cargo}_i + \mathsf{Imp}\beta\cdot\mathsf{NPC} \rightleftharpoons \mathsf{Cargo}\cdot\mathsf{Imp}\beta\cdot\mathsf{NPC} \tag{9}$$

with $k_{\text{on}}^{\text{C}} = 0.11\,\mu\text{M}^{-1}\,\text{s}^{-1}$ and $k_{\text{off}}^{\text{C}} = 7.2 \times 10^{-4}\,\text{s}^{-1}$, where i can be "cytoplasmic" or "nuclear". This envelope compartment is in diffusive exchange with cytoplasm ($i = \text{cyt}$) and nucleus ($i = \text{nuc}$) with the permeability constant $D = 1\,\text{s}^{-1}$. In both, we also allow

$$\mathsf{Cargo} + \mathsf{Imp}\beta \rightleftharpoons \mathsf{Cargo}\cdot\mathsf{Imp}\beta \tag{10}$$

with the same $k_{\text{on}}^{\text{C}}/k_{\text{off}}^{\text{C}}$. For simplicity, we assume $[\mathsf{RanGTP}] = 3\,\mu\text{M}$ and $[\mathsf{RanGDP}] = 2\,\mu\text{M}$ are maintained at fixed concentrations and are only relevant at the envelope, where we have

$$\text{cargo knockoff:} \quad \mathsf{RanGTP}_{\mathsf{nuc}} + \mathsf{Cargo}\cdot\mathsf{Imp}\beta\cdot\mathsf{NPC} \longrightarrow \mathsf{Cargo} + \mathsf{RanGTP}\cdot\mathsf{Imp}\beta\cdot\mathsf{NPC} \tag{11}$$

$$\text{GTP hydrolysis:} \quad \mathsf{RanGTP}\cdot\mathsf{Imp}\beta\cdot\mathsf{NPC} \longrightarrow \mathsf{RanGDP}_{\mathsf{cyt}} + \mathsf{P} + \mathsf{Imp}\beta\cdot\mathsf{NPC}. \tag{12}$$

For hydrolysis we assume the reaction rate $v_{\text{GAP}}/(1+K_{\text{GAP}}/[\mathsf{RanGTP}\cdot\mathsf{Imp}\beta\cdot\mathsf{NPC}])$, similarly to (18a). We take $v_{\text{GAP}} = 0.07\,\mu\text{M}\,\text{s}^{-1}$ and $K_{\text{GAP}} = 0.1\,\mu\text{M}$. This reaction rate about $100\times$ smaller than in (18a) but in view of TODO(3): ref, this seems more realistic and has little effect on the course of the simulation.

Starting from $[\mathsf{Cargo}]_{\text{cyt}} = 1\,\mu\text{M}$ and $[\mathsf{Imp}\beta]_{\text{cyt}} = [\mathsf{Imp}\beta]_{\text{nuc}} = 0.5\,\mu\text{M}$, this model predicts a 6-fold accumulation of total cargo in the nucleus in steady-state. Meanwhile, the concentration of $\mathsf{Imp}\beta$ at the envelope is approximately $10^3\,\mu\text{M}$.

Code #4.

1.3 Role of RanBP1

According to [LM97, Fig. 4A], Impβ blocks hydrolysis of Ran · GTP by RanGAP but RanBP1 rescues it for most part. Similarly, [BG97] showed that RanBP1 transiently detaches Ran from the complex Kap · Ran · GTP (where Kap can be importin β, transportin or CAS), whereupon hydrolysis by RanGAP disassembles the complex; and that efficient disassembly of Impβ · Ran · GTP required RanBP1 *and* Impα [BG97, §3.2, cf. Fig. 4], [FBR97]. Importantly, Kaps and RanBP1 bind Ran at distinct sites [BG97, p.253].

Further, [See+03, Fig. 13] characterizes the kinetics of the formation of the complex between Ran · GTP, RanBP1 and RanGAP and its hydrolysis. In particular, the release of the γ-phosphate, which is the rate-limiting step of hydrolysis by RanGAP, is barely influenced by RanPB1, which instead stimulates the association of Ran with RanGAP.

In [RM05, p.1033], the initial rate of cargo import increased with co-addition of RanBP1, disagreeing with their simulation (in which RanBP1 acts catalytically [RM05, Fig. S1]).

Crystal structure: [Sar+07]

References

[Bis+95] F. R. Bischoff, H. Krebber, E. Smirnova, W. Dong, and H. Ponstingl. "Co-activation of RanGTPase and inhibition of GTP dissociation by Ran–GTP binding protein RanBP1". In: *The EMBO Journal* 14.4 (Feb. 1995), pp. 705–715. DOI: 10.1002/j.1460-2075.1995.tb07049.x (cit. on p. 7).

[Kle+95] C. Klebe, H. Prinz, A. Wittinghofer, and R. S. Goody. "The Kinetic Mechanism of Ran-Nucleotide Exchange Catalyzed by RCC1". In: *Biochemistry* 34.39 (Oct. 1995), pp. 12543–12552. DOI: 10.1021/bi00039a008 (cit. on pp. 7, 8).

[BG97] F. Bischoff and D. Görlich. "RanBP1 is crucial for the release of RanGTP from importin β-related nuclear transport factors". In: *FEBS Letters* 419.2-3 (Dec. 1997), pp. 249–254. DOI: 10.1016/s0014-5793(97)01467-1 (cit. on p. 5).

[FBR97] M. Floer, G. Blobel, and M. Rexach. "Disassembly of RanGTP-Karyopherin β Complex, an Intermediate in Nuclear Protein Import". In: *Journal of Biological Chemistry* 272.31 (Aug. 1997), pp. 19538–19546. DOI: 10.1074/jbc.272.31.19538 (cit. on p. 5).

[LM97] K. M. Lounsbury and I. G. Macara. "Ran-binding Protein 1 (RanBP1) Forms a Ternary Complex with Ran and Karyopherin β and Reduces Ran GTPase-activating Protein (RanGAP) Inhibition by Karyopherin β". In: *Journal of Biological Chemistry* 272.1 (Jan. 1997), pp. 551–555. DOI: 10.1074/jbc.272.1.551 (cit. on p. 5).

[Cat+01] B. Catimel, T. Teh, M. R. Fontes, I. G. Jennings, D. A. Jans, G. J. Howlett, E. C. Nice, and B. Kobe. "Biophysical Characterization of Interactions Involving Importin-α during Nuclear Import". In: *Journal of Biological Chemistry* 276.36 (Sept. 2001), pp. 34189–34198. DOI: 10.1074/jbc.m103531200 (cit. on pp. 2, 4).

[GSR03] D. Görlich, M. J. Seewald, and K. Ribbeck. "Characterization of Ran-driven cargo transport and the RanGTPase system by kinetic measurements and computer simulation". In: *The EMBO Journal* 22.5 (Mar. 2003), pp. 1088–1100. DOI: 10.1093/emboj/cdg113 (cit. on pp. 1, 4, 7, 8).

[See+03] M. J. Seewald, A. Kraemer, M. Farkasovsky, C. Körner, A. Wittinghofer, and I. R. Vetter. "Biochemical Characterization of the Ran–RanBP1–RanGAP System: Are RanBP Proteins and the Acidic Tail of RanGAP Required for the Ran-RanGAP GTPase Reaction?" In: *Molecular and Cellular Biology* 23.22 (Nov. 2003), pp. 8124–8136. DOI: 10.1128/mcb.23.22.8124-8136.2003 (cit. on p. 5).

[RM05] G. Riddick and I. G. Macara. "A systems analysis of importin-α–β mediated nuclear protein import". In: *Journal of Cell Biology* 168.7 (Mar. 2005), pp. 1027–1038. DOI: 10.1083/jcb.200409024 (cit. on pp. 4, 5).

[Sar+07] M. Sarić, X. Zhao, C. Körner, C. Nowak, J. Kuhlmann, and I. R. Vetter. "Structural and biochemical characterization of the Importin-β · Ran · GTP · RanBD1 complex". In: *FEBS Letters* 581.7 (Mar. 2007), pp. 1369–1376. DOI: 10.1016/j.febslet.2007.02.067 (cit. on p. 5).

2 Appendix

2.1 Minimal Ran gradient system

Here we recapitulate the minimal Ran gradient system from [GSR03, Fig. 2], cf. §1.1. The following account for the cytoplasmic species. Here, $[\dots]$ abbreviates the (cytoplasmic) concentration of the complex $\mathsf{RanBP1}\cdot\mathsf{Ran}\cdot\mathsf{GTP}$. Ex is an additional potentially useful flux of nuclear $\mathsf{Ran}\cdot\mathsf{GTP}$ to cytoplasmic $\mathsf{Ran}\cdot\mathsf{GDP}$, set by default to zero.

$$\tfrac{\mathrm{d}}{\mathrm{d}t}[\mathsf{Ran}\cdot\mathsf{GDP}]_{\mathrm{cyt}} = \mathsf{F}_{\mathsf{Ran}\cdot\mathsf{GDP}}\tfrac{V_{\mathrm{nuc}}}{V_{\mathrm{cyt}}} + \mathsf{GAP} + \mathsf{GAP}_{\mathsf{RanBP1}} + \mathsf{Ex}\tfrac{V_{\mathrm{nuc}}}{V_{\mathrm{cyt}}} \tag{13a}$$

$$\tfrac{\mathrm{d}}{\mathrm{d}t}[\mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{cyt}} = \mathsf{F}_{\mathsf{Ran}\cdot\mathsf{GTP}}\tfrac{V_{\mathrm{nuc}}}{V_{\mathrm{cyt}}} - \mathsf{GAP} - k_{\mathrm{on}}^{\mathrm{rbp}}[\mathsf{RanBP1}][\mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{cyt}} + k_{\mathrm{off}}^{\mathrm{rbp}}[\dots] \tag{13b}$$

$$\tfrac{\mathrm{d}}{\mathrm{d}t}[\mathsf{RanBP1}\cdot\mathsf{Ran}\cdot\mathsf{GTP}] = -\mathsf{GAP}_{\mathsf{RanBP1}} + k_{\mathrm{on}}^{\mathrm{rbp}}[\mathsf{RanBP1}][\mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{cyt}} - k_{\mathrm{off}}^{\mathrm{rbp}}[\dots] \tag{13c}$$

The following account for the nuclear species. As in [GSR03], E denotes free $\mathsf{RCC1}$.

$$\tfrac{\mathrm{d}}{\mathrm{d}t}[\mathsf{Ran}\cdot\mathsf{GDP}]_{\mathrm{nuc}} = -\mathsf{F}_{\mathsf{Ran}\cdot\mathsf{GDP}} + r_8[\mathsf{IntC}] - r_1[\mathsf{E}][\mathsf{Ran}\cdot\mathsf{GDP}]_{\mathrm{nuc}} \tag{14a}$$

$$\tfrac{\mathrm{d}}{\mathrm{d}t}[\mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{nuc}} = -\mathsf{F}_{\mathsf{Ran}\cdot\mathsf{GTP}} + r_4[\mathsf{IntA}] - r_5[\mathsf{E}][\mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{nuc}} - \mathsf{Ex} \tag{14b}$$

The nucleotide-exchange reaction $\mathsf{Ran}\cdot\mathsf{GDP} + \mathsf{GTP} \rightleftharpoons \mathsf{Ran}\cdot\mathsf{GTP} + \mathsf{GDP}$ is catalyzed by $\mathsf{RCC1}$. It is modeled as in [Kle+95, Fig. 6] / [GSR03, Fig. 1] with three intermediates. Note that it depends on the availability of GDP and GTP.

$$\tfrac{\mathrm{d}}{\mathrm{d}t}[\mathsf{IntA}] = -(r_4 + r_6)[\mathsf{IntA}] + r_5[\mathsf{E}][\mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{nuc}} + r_3[\mathsf{GTP}][\mathsf{IntB}] \tag{15a}$$

$$\tfrac{\mathrm{d}}{\mathrm{d}t}[\mathsf{IntB}] = r_6[\mathsf{IntA}] + r_2[\mathsf{IntC}] - (r_3[\mathsf{GTP}] + r_7[\mathsf{GDP}])[\mathsf{IntB}] \tag{15b}$$

$$\tfrac{\mathrm{d}}{\mathrm{d}t}[\mathsf{IntC}] = -(r_2 + r_8)[\mathsf{IntC}] + r_1[\mathsf{E}][\mathsf{Ran}\cdot\mathsf{GDP}]_{\mathrm{nuc}} + r_7[\mathsf{GDP}][\mathsf{IntB}] \tag{15c}$$

Constraints on the total concentration:

$$\text{Free } \mathsf{RCC1}: \qquad [\mathsf{E}] = \mathsf{RCC1}_{\mathsf{total}} - ([\mathsf{IntA}] + [\mathsf{IntB}] + [\mathsf{IntC}]) \tag{16a}$$

$$\text{Free } \mathsf{RanBP1}: \qquad [\mathsf{RanBP1}] = \mathsf{RanBP1}_{\mathsf{total}} - [\mathsf{RanBP1}\cdot\mathsf{Ran}\cdot\mathsf{GTP}] \tag{16b}$$

Gradient-driven fluxes from the nucleus to the cytoplasm:

$$\mathsf{F}_{\mathsf{Ran.GTP}} = D_{\mathsf{Ran}\cdot\mathsf{GTP}}\left([\mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{nuc}} - [\mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{cyt}}\right) \tag{17a}$$

$$\mathsf{F}_{\mathsf{Ran.GDP}} = D_{\mathsf{Ran}\cdot\mathsf{GDP}}\left([\mathsf{Ran}\cdot\mathsf{GDP}]_{\mathrm{nuc}} - [\mathsf{Ran}\cdot\mathsf{GDP}]_{\mathrm{cyt}}\right) \tag{17b}$$

RanGAP hydrolyzes the γ-phosphate of $\mathsf{Ran}\cdot\mathsf{GTP}$. This is more efficient when $\mathsf{Ran}\cdot\mathsf{GTP}$ is bound to $\mathsf{RanBP1}$ [Bis+95], reducing the IC50 seven-fold [GSR03, Table I, p. 1091].

$$\mathsf{GAP} = k_{\mathsf{GAP}}[\mathsf{RanGAP}]/(1 + K_{\mathsf{GAP}}/[\mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{cyt}}) \tag{18a}$$

$$\mathsf{GAP}_{\mathsf{RanBP1}} = k'_{\mathsf{GAP}}[\mathsf{RanGAP}]/(1 + K'_{\mathsf{GAP}}/[\mathsf{RanBP1}\cdot\mathsf{Ran}\cdot\mathsf{GTP}]) \tag{18b}$$

To determine the dynamic capacity Ex at steady-state we introduce the additional equation:

$$\tfrac{\mathrm{d}}{\mathrm{d}t}\mathsf{Ex} = k_{\mathsf{Ex}}\,[\mathsf{Ran}\cdot\mathsf{GTP}]_{\mathrm{nuc}}, \quad k_{\mathsf{Ex}} := 10\,\mathrm{s}^{-2}, \quad \text{initial} \quad \mathsf{Ex} := 0\,\mu\mathrm{M}\,\mathrm{s}^{-1}. \tag{19}$$

(13a)	$V_{\text{nuc}} = 1.2\,\text{pl}, \quad V_{\text{cyt}} = 1.8\,\text{pl}$	[GSR03, Table II]
(13a)	initial condition $[\mathsf{Ran}\cdot\mathsf{GDP}]_{\text{cyt}} = 5\,\mu\text{M}$	[GSR03, Table II]
(13b)–(13c)	$k_{\text{on}}^{\text{rbp}} = 0.3\,\mu\text{M}^{-1}\,\text{s}^{-1}, \quad k_{\text{off}}^{\text{rbp}} = 4\times10^{-4}\,\text{s}^{-1}$	[GSR03, Supp. Table A]
(14a)–(15c)	$r_1 = 74\,\mu\text{M}^{-1}\,\text{s}^{-1}, \quad r_8 = 55\,\text{s}^{-1}$ $r_7 = 11\,\mu\text{M}^{-1}\,\text{s}^{-1}, \quad r_2 = 21\,\text{s}^{-1}$ $r_3 = 0.6\,\mu\text{M}^{-1}\,\text{s}^{-1}, \quad r_6 = 19\,\text{s}^{-1}$ $r_5 = 100\,\mu\text{M}^{-1}\,\text{s}^{-1}, \quad r_4 = 55\,\text{s}^{-1}$	[GSR03, Supp. Table A] [Kle+95, Fig. 6]
(15a)–(15c)	$[\mathsf{GTP}] = 500\,\mu\text{M}, \quad [\mathsf{GDP}] = 1.6\,\mu\text{M}$	[GSR03, Table II]
(16a)	$\mathsf{RCC1}_{\mathsf{total}} = 0.7\,\mu\text{M}$	[GSR03, Supp. Table B]
(16b)	$\mathsf{RanBP1}_{\mathsf{total}} = 2\,\mu\text{M}$	[GSR03, Fig. 4]
(17a) (17b)	$D_{\mathsf{Ran}\cdot\mathsf{GTP}} = 0.03\,\text{s}^{-1}$ $D_{\mathsf{Ran}\cdot\mathsf{GDP}} = 0.12\,\text{s}^{-1}$	[GSR03, Table II]
(18a)	$k_{\mathsf{GAP}} = 10.6\,\text{s}^{-1}, \quad K_{\mathsf{GAP}} = 0.7\,\mu\text{M}$	[GSR03, Supp. Table A]
(18b)	$k'_{\mathsf{GAP}} = 10.8\,\text{s}^{-1}, \quad K'_{\mathsf{GAP}} = 0.1\,\mu\text{M}$	[GSR03, Table I]
(18a)–(18b)	cytoplasmic $[\mathsf{RanGAP}] = 0.7\,\mu\text{M}$	[GSR03, Table II / ST B]

Table 2: Constants for the "standard simulation condition" of §1.1 at 25 °C. Except for (13a), all species are initialized to zero at $t = 0$.

Condition	Affected parameters	Nuclear RanGTP, µM	Cytoplasmic RanGTP, nM	Dynamic capacity, µM/s
"Standard"	See Table 2	4.26 (4.3)	7.75 (7.7)	0.59 (0.60)
Omission of RanBP1	$\mathsf{RanBP1}_{\mathsf{total}} := 0$	4.27 (4.3)	8.13 (8.1)	0.59 (0.60)
200% RCC1	$\mathsf{RCC1}_{\mathsf{total}}$	3.95 (4.0)	7.17 (7.1)	0.59 (0.60)
50% RCC1	$\mathsf{RCC1}_{\mathsf{total}}$	4.31 (4.3)	7.82 (7.7)	0.58 (0.60)
10% RCC1	$\mathsf{RCC1}_{\mathsf{total}}$	3.59 (3.6)	6.50 (6.4)	0.46 (0.48)
1% RCC1	$\mathsf{RCC1}_{\mathsf{total}}$	1.40 (1.4)	2.52 (2.5)	0.075 (0.08)
GTP:GDP = 500:0	$[\mathsf{GDP}] := 0\,\mu\text{M}$	4.80 (4.8)	8.72 (8.6)	0.59 (0.60)
GTP:GDP = 500:50	$[\mathsf{GDP}] := \frac{1}{10}[\mathsf{GTP}]$	0.98 (0.8)	1.76 (1.5)	0.57 (0.58)
GTP:GDP = 500:500	$[\mathsf{GDP}] := [\mathsf{GTP}]$	0.12 (0.12)	0.22 (0.21)	0.34 (0.34)
Saturating NTF2	$D_{\mathsf{Ran}\cdot\mathsf{GDP}} := 0.48\,\text{s}^{-1}$	5.12 (5.1)	9.32 (9.2)	2.18 (2.2)
No NTF2	$D_{\mathsf{Ran}\cdot\mathsf{GDP}} := D_{\mathsf{Ran}\cdot\mathsf{GTP}}$	2.55 (2.5)	4.60 (4.5)	0.15 (0.16)
200% RanGAP	$[\mathsf{RanGAP}]$	4.27 (4.3)	3.95 (3.9)	0.59 (0.60)
50% RanGAP	$[\mathsf{RanGAP}]$	4.26 (4.3)	14.9 (14)	0.59 (0.60)
50% permeability	$D_{\mathsf{Ran}\cdot\mathsf{GTP}}$	4.91 (4.9)	4.44 (4.4)	0.59 (–)
200% permeability	$D_{\mathsf{Ran}\cdot\mathsf{GTP}}$	3.41 (3.4)	12.4 (12.3)	0.59 (–)
400% permeability	$D_{\mathsf{Ran}\cdot\mathsf{GTP}}$	2.46 (2.5)	18.0 (17.8)	0.59 (–)

Table 3: Steady-state concentrations for the simulation scenarios from [GSR03, Table II/III], with their results shown in brackets. Value for $D_{\mathsf{Ran}\cdot\mathsf{GDP}}$ is from [GSR03, Fig. 3].

2.2 List of codes

	page	https://github.com/numpde/nct1/tree/ ...
#1	p.1	main/code/20210225-GSR/v1
#2	p.2	main/code/20210407-Rearrangement
#3	p.2	main/code/20210225-GSR/v2
#4	p.4	main/code/20210403-StickyPore

3 TODO

TODOs:

1. p.4. ref
2. p.4. ref
3. p.4. ref

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