HS-2020 PAI summary

$$\mathbf{RV} \ \boldsymbol{A} \mathcal{N}_{\boldsymbol{\mu},\boldsymbol{\Sigma}} = \mathcal{N}_{\boldsymbol{A}\boldsymbol{\mu},\boldsymbol{A}\boldsymbol{\Sigma}\boldsymbol{A}^{\mathsf{T}}} \qquad |2\pi\boldsymbol{\Sigma}|^{-\frac{1}{2}} e^{-\frac{1}{2}\|\boldsymbol{x}-\boldsymbol{\mu}\|_{\boldsymbol{\Sigma}^{-1}}^2} \qquad (\boldsymbol{w}|\boldsymbol{v}) \sim \mathcal{N}_{\bar{\boldsymbol{w}}+\boldsymbol{\Sigma}_{\times}\boldsymbol{\Sigma}_{\boldsymbol{v}}^{-1}(\boldsymbol{v}-\bar{\boldsymbol{v}}),\boldsymbol{\Sigma}_{\boldsymbol{w}}-\boldsymbol{\Sigma}_{\times}\boldsymbol{\Sigma}_{\boldsymbol{v}}^{-1}\boldsymbol{\Sigma}_{\times}^{\mathsf{T}}}$$

Proba 
$$P_{X,Y} = P_{X|Y} P_{Y}$$
  $P_{X=x} = \sum_{y} P_{X=x,Y=y}$   $P_{X=x,Y=y} = \sum_{y} P_{X=x,Y=y}$   $P_{X=x,Y=y$ 

$$\textbf{BL} \ \underline{p(\boldsymbol{y}|\boldsymbol{X},\theta) = \prod_{i} p(y_i|x_i,\theta)}_{\text{likelihood}} \qquad \underline{p(\theta)}_{\text{prior}} \qquad \underline{p(\theta|\boldsymbol{X},\boldsymbol{y}) \propto \text{likelihood} \times \text{prior}}_{\text{posterior}} \qquad \underline{p(y^*|x^*,\boldsymbol{X},\boldsymbol{y}) = \int p(y^*|x^*,\boldsymbol{X},\boldsymbol{y}) \times [\text{posterior}] \, d\theta}_{\text{predictions}}$$

$$\mathbf{Fact} \ \ \partial_{\theta}(\Sigma^{-1}) = -\Sigma^{-1}(\partial_{\theta}\Sigma)\Sigma^{-1} \qquad \qquad \partial_{\theta} \ln |\Sigma| = (\Sigma^{-\mathsf{T}} : \partial_{\theta}\Sigma) = \operatorname{tr}(\Sigma^{-1}\partial_{\theta}\Sigma) \qquad \qquad \underbrace{I(X,Y) = H[X] - H[X|Y]}_{\text{gain } \frac{1}{2}|I + \frac{1}{\sigma^2}\mathbb{V}[X]| \text{ for } Y = X + \varepsilon}$$

$$\mathbf{H} \ H[p,q] = -\mathbb{E}_p[\ln q] \qquad (p \parallel q) = \mathbb{E}_p[\ln \frac{p}{q}] \qquad H[\mathcal{N}] = \frac{1}{2}\ln((2\pi e)^d |\Sigma|) \qquad (\mathcal{N}_0 \parallel \mathcal{N}_1) = \frac{1}{2}(\operatorname{tr}(\frac{\Sigma_0}{\Sigma_1} - \mathbf{I}) + |\Delta \boldsymbol{\mu}|_{\boldsymbol{\Sigma}_1^{-1}}^2 - \ln \frac{|\boldsymbol{\Sigma}_0|}{|\boldsymbol{\Sigma}_1|})$$

$$\mathbf{BLR} \ \ y = X^\intercal w + \varepsilon \qquad \mathbb{E}[w|y] = \bar{w} + K(y - \bar{y}) \qquad \mathbb{V}[w|y] = \mathbf{\Sigma}_w - \underbrace{\mathbf{\Sigma}_w X (\mathbf{\Sigma}_\varepsilon + X^\intercal \mathbf{\Sigma}_w X)^{-1}}_{\text{Kalman gain } K, \ \mathcal{O}(|y|^2)} X^\intercal \mathbf{\Sigma}_w \overset{\text{\tiny smw}}{=} \underbrace{(\mathbf{\Sigma}_w^{-1} + X \mathbf{\Sigma}_\varepsilon^{-1} X^\intercal)^{-1}}_{\text{can be } \mathcal{O}(|w|^2 \times |y|)}$$

$$\mathbf{GP} \ \ \underline{w \overset{\mathrm{prior}}{\sim} \mathcal{N}(\bar{w}, \Sigma_w \equiv k(\cdot, \cdot))}_{X := (\delta_x)_{x \in A}} \quad \ \ \underline{\log p(x)}_{\mathrm{evidence}} = \underbrace{(q \parallel p(\cdot \mid x))}_{\geq 0} + \underbrace{H[q] - H[q, p(\cdot, x)]}_{\mathrm{ELBO}} \quad \ \underline{\int (t - y)_{+} p(y \mid x) dx}_{\mathrm{EI}(x) \ \mathrm{over \ target} \ t} \quad \ \underline{(Fw \mid y) + Bu' + \eta'}_{w_{\mathrm{new}} \ \mathrm{in \ Kalman \ filter}}$$

$$\mathbf{MC} \ \ \underline{\exists t: P^t > 0} \ \Rightarrow \ \underline{\exists ! \pi > 0}_{\mathrm{ergodic}} \ \Rightarrow \ \underline{\exists ! \pi > 0}_{+X_{\infty} \sim \pi} \quad \ \underbrace{P_{x' \leftarrow x} f_x = P_{x \leftarrow x'} f_{x'}}_{\mathrm{det \ bal \ for \ } f \ \Rightarrow \ P * f = f} \ \ \underbrace{u \leq \frac{R_{x \leftarrow x'} f_{x'}}{R_{x' \leftarrow x} f_x}}_{\mathrm{MH} \ r' \leftarrow r} \quad \underbrace{R_{x \leftarrow x}^{\mathrm{MALA}} = \mathcal{N}_{x - \tau} g_{x'}^{\prime}, 2\tau I}_{g \ = -\log f: \ \cup \ \Rightarrow \ \mathrm{convg}} \quad \underbrace{e^{-(\lambda |\theta|^2 + \frac{n}{m} \sum_{i}^{m} \ln(1 + e^{-y_i \theta \cdot x_i}))}}_{\log - \mathrm{reg} \ e^{-g} \ \mathrm{SGLD}: \ \rlap{$\psi$}, \ \tau \sim t^{-\frac{1}{3}}$}$$

$$\mathbf{MDP} \ \ Q_{x,a}^V := r_{x,a} + \gamma \sum_{x'} V_{x'} P_{x'|x,a} \quad \ \pi_x^Q := \arg Q_{x,\uparrow} \quad \underbrace{V_x^\pi \overset{!}{=} Q_{x,\pi(x)}^{V^\pi}}_{\text{solve for } V^\pi} \quad \underbrace{V_x^\star = Q_{x,\uparrow}^{V^\star}}_{\text{Bellman}} \quad \underbrace{V_x^\pi \leftrightarrow \pi := \pi^{(Q^V)}}_{\text{policy it. convg}} \quad \underbrace{V_x \leftarrow Q_{x,\uparrow}^V}_{\text{value it. $\varepsilon$-convg}}$$

$$\mathbf{QL} \ \ \underline{Q_{s_t,a_t} \leftarrow (1-\alpha_t) \, Q_{s_t,a_t} + \alpha_t \, (r_t + \gamma Q_{s_{t+1},\uparrow})}_{\text{(RM for } \alpha) \ \land \ (\text{all } (s,a) \text{ recurrent}) \Rightarrow \text{convg}} \ \ \underline{\min_{\theta} \sum_{xarx'} [r + \gamma Q_{x',\uparrow}^{\theta^{\text{old}}} - Q_{x,a}^{\theta}]^2}_{\text{DQN}} \ \ \underline{\sum_{t} \varepsilon_t^2 < \infty \leq \Sigma_t \varepsilon_t}_{\text{RM, $\epsilon$-greedy} \Rightarrow \text{convg}} \ \ \underline{r_{x,a} := R^{\text{max}}}_{P? \ x^*!, \text{ whp $\varepsilon$-cvg}}$$