Eötvös Loránd University Faculty of Science Institute of Mathematics

MSc in applied mathematics

Description of the program



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I. Course requirements

Students enrolled in the program must obtain at least **120 credits** in the following distribution:

- at least 18 **credits** from so called **basic courses** (B)
- at least 24 **credits** from so called **core courses** (C)
- at least 40 **credits** from so called **differentiated courses** (D)

In the framework of Modelling activities the Modelling project works (6 credits) are compulsory, and the Modelling week can be chosen freely. On top of these, 6 credits can be chosen freely from the list of all subjects offered to MSc students in mathematics and applied mathematics. Furthermore, a thesis (worth 20 credits) must be written at the end of the studies.

Under special circumstances it is possible to get a waiver from taking basic courses. In this case the missing credits can be obtained by taking more free courses.

It is expected – although not enforced – that the students should finish in two years (i.e. four semesters).

Subject

Stochastics

Multivariate statistical methods

Statistical computing and modelling 1

II. List of subjects

Contact hours

4 h/w (lecture)

2 h/w (practice)

exam

term mark

6+0

3+0

Coordinator	(hours/week)	Credits	Evaluation
. Basic courses (min. 18 cr.)			
Real analysis	2 h/w (lecture)	2.2	exam
Árpád Tóth		3+3	term mark
Functional analysis	2 h/w (lecture)	2.2	exam
János Karátson	2 h/w (practice)	3+3	term mark
Linear and abstract algebra	2 h/w (lecture)	3+3	exam
Péter Frenkel	2 h/w (practice)	3+3	term mark
Operations research	2 h/w (lecture)	3+3	exam
Tamás Király	1 /	575	term mark
Discrete mathematics	2 h/w (lecture)	3+3	exam
Péter Sziklai	2 h/w (practice)	575	term mark
Probability and statistics	3 h/w (lecture)	4+3	exam
Tamás Móri	1 /	- +⊤J	term mark
Non-Euclidean geometries	2 h/w (lecture)	3+3	exam
Gábor Moussong	2 h/w (practice)	3+3	term mark
Core courses (min. 24 cr.)			
Applied analysis			
Numerical modeling and numerical methods for ODEs	2 h/w (lecture)	3+3	exam
István Faragó	2 h/w (practice)	313	term mark
Theory of algorithms			-
Algorithms	2 h/w (lecture)	3+3	exam
Zoltán Király	2 h/w (practice)	313	term mark
Stochastics			
Stochastic processes	3 h/w (lecture)	4.0	exam
Vilmos Prokaj		4+3	term mark
Discrete mathematics	1 1	<u> </u>	<u> </u>
Applied discrete mathematics	2 h/w (lecture)		exam
Péter Csikvári		3+3	term mark
Operations research	(1-30000)	<u>-</u>	
Discrete and continuous optimization 1	2 h/w (lecture)	1	exam
Tamás Király	2 h/w (recture) 2 h/w (practice)	3+3	term mark
Tumus Kiruty	2 II/w (practice)		term mark
. Differentiated courses (min. 40 cr.)			
Applied analysis			
Differential equations	2 h/w (lecture)	2+2	exam
Péter Simon	2 h/w (practice)	3+3	term mark
Partial differential equations	2 h/w (lecture)	2+2	exam
Ádám Besenyei		3+3	term mark
Numerical solution of elliptic partial differential equations	2 h/w (lecture)	2.2	exam
János Karátson		3+3	term mark
Numerical solution of time-dependent partial differential	2 h/w (lecture)	2.2	exam
equations Ferenc Izsák	2 h/w (practice)	3+3	term mark
Dynamical systems and differential equations	2 h/w (lecture)	2.2	exam
Péter Simon	2 h/w (practice	3+3	term mark

György Michaletzky

András Zempléni

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2 h/w (practice)	3+0	term mark		
2 h/w (lecture)	3+3	exam term mark		
2 h/w (lecture)	3+0	exam		
2 h/w (lecture)	3+0	exam		
2 h/w (lecture)	3+0	exam		
2 h/w (lecture)	3+0	exam		
2 h/w (lecture)	3+0	exam		
2 h/w (lecture)	3+0	exam		
2 h/w (practice)	0+3	term mark		
2 h/w (lecture) 2 h/w (practice)	3+3	exam term mark		
Computer geometry				
2 h/w (lecture)	3+0	exam		
2 h/w (lecture) 2 h/w (practice)	3+3	exam term mark		
2 h/w (lecture)	3+0	exam		
2 h/w (lecture)	3+0	exam		
	0+3	term mark		
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	0+4	term mark		
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III. List of lecturers

Name	Affiliation	Research areas
Miklós Arató	PTS	statistics, random fields, actuarial mathematics
Ádám Besenyei	AAC	mahematical analysis, differential equations
Balázs Csikós	GEO	differential geometry, Riemannian geometry, Lie groups
Péter Csikvári	CSC	extremal and algebraic combinatorics
Villő Csiszár	PTS	statistics, random permutations, random graphs
István Faragó	AAC	numerical analysis, numerical linear algebra, mathematical modelling
Péter Frenkel	ANT	combinatorial algebra
Katalin Gyarmati	ANT	pseudorandomness, combinatorial number theory, exponential and character sums
Ferenc Izsák	AAC	partial differential equations, finite element method, numerical modeling
Alpár Jüttner	OPR	combinatorial optimization
János Karátson	AAC	numerical function analysis, partial differential equations
Tamás Király	OPR	submodular functions, combinatorial optimization
Zoltán Király	CSC	algorithms, data structures, graph theory, combinatorial optimization, complexity theory
János Kiss Hubert	ECO	financial markets, bank systems, game theory
László Márkus	PTS	financial mathematics, environmental applications of statistics
György Michaletzky	PTS	stochastic processes, realization theory for stationary processes
Tamás Móri	PTS	probability theory, random graphs and networks, martingales
Gábor Moussong	GEO	geometric topology, geometric group theory, hyperbolic geometry
Vilmos Prokaj	PTS	probability theory, stochastic processes
Péter Simon	AAC	dynamical systems, differential equations, network processes
Péter Sziklai	CSC	structures over finite fields, finite geometries, polynomials over finite fields, posets of words over a finite alphabet, cryptography
Tamás Szőnyi	CSC	finite geometry, block designs, coding theory, combinatorics
Árpád Tóth	ANA	analysis
László Verhóczki	GEO	differential geometry, Riemannian geometry
András Zempléni	PTS	statistics, extreme value modeling, multivariate models

Department codes

AAC	Applied Analysis and Computational Mathematics
ANA	Analysis
ANT	Algebra and Number Theory
CSC	Computer Science
ECO	Economics
GEO	Geometry
OPR	Operations Research
PTS	Probability Theory and Statistics
TEA	Teaching and Education Centre

IV. Description of the courses (in alphabetical order)

Title of the course: Algorithms

Number of contact hours per week: 2+2 Credit value: 3+3

Course coordinator(s): Zoltán Király

Department(s): Department of Computer Science Evaluation: oral exam and tutorial mark

Prerequisites: none

A short description of the course:

Sorting and selection. Applications of dynamic programming (maximal interval-sum, knapsack, order of multiplication of matrices, optimal binary search tree, optimization problems in trees). Graph algorithms: BFS, DFS, applications (shortest paths, 2-colorability, strongly connected orientation, 2-connected blocks, strongly connected components). Dijkstra's algorithm and applications (widest path, safest path, PERT method, Jhonson's algorithm). Applications of network flows. Stable matching. Algorithm of Hopcroft and Karp. Concept of approximation algorithms, examples (Ibarra-Kim, metric TSP, Steiner tree, bin packing). Search trees. Amortization time. Fibonacci heap and its applications. Data compression. Counting with large numbers, algorithm of Euclid, RSA. Fast Fourier transformation and its applications. Strassen's method for matrix multiplication.

Textbook: -

Further reading: T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein: *Introduction to Algorithms*, McGrawHill, 2002.

Title of the course: Algorithms

Number of contact hours per week: 2+2 Credit value: 2+3

Course coordinator(s): Zoltán Király

Department(s): Department of Computer Science Evaluation: oral examination and tutorial mark

Prerequisites: none

A short description of the course:

Sorting and selection. Applications of dynamic programming (maximal interval-sum, knapsack, order of multiplication of matrices, optimal binary search tree, optimization problems in trees).

Graph algorithms: BFS, DFS, applications (shortest paths, 2-colorability, strongly connected orientation, 2-connected blocks, strongly connected components). Dijkstra's algorithm and applications (widest path, safest path, PERT method, Jhonson's algorithm). Applications of network flows. Stable matching. Algorithm of Hopcroft and Karp.

Concept of approximation algorithms, examples (Ibarra-Kim, metric TSP, Steiner tree, bin packing). Search trees. Amortization time. Fibonacci heap and its applications.

Data compression. Counting with large numbers, algorithm of Euclid, RSA. Fast Fourier transformation and its applications. Strassen's method for matrix multiplication.

Textbook:

Further reading:

T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein: Introduction to Algorithms, McGraw-Hill, 2002

Title of the course: Applied discrete mathematics

Number of contact hours per week: 2+2 Credit value: 3+3

Course coordinator(s): Péter Csikvári

Department(s): Department of Computer Science Evaluation: oral or written exam and tutorial grade

Prerequisites: none

A short description of the course:

Spectral graph theory, pseudorandom graphs and expander graphs. Expander graphs and derandomization. Page rank algorithm. Shannon-capacity and Lovász-theta number. Clustering and k-means algorithms. Generating functions and recurrences. Inversion formulas for partially ordered sets. Mechanical summation. Fields, polynomials, combinatorial Nullstellensatz. Randomized methods: expectation and second moment method. Random graphs, threshold functions. Colorings of graphs and hypergraphs, perfect graphs. Matching theory.

Textbook: -

Further reading:

- S. Jukna, Extremal Combinatorics with Applications in Computer Science
- J. H. van Lint, R.J. Wilson, A course in combinatorics, Cambridge Univ. Press, 1992; 2001.
- L. Lovász: Combinatorial Problems and Exercises, AMS, Providence, RI, 2007
- R. L. Graham, D. E. Knuth, O. Patashnik, Concrete Mathematics.

Title of the course: Codes and symmetric structures

Number of contact hours per week: 2+0 Credit value: 3+0

Course coordinator(s): Tamás Szőnyi

Department(s): Department of Computer Science Evaluation: oral or written examination

Prerequisites:

A short description of the course:

Error-correcting codes; important examples: Hamming, BCH (Bose, Ray-Chaudhuri, Hocquenheim) codes. Bounds for the parameters of the code: Hamming bound and perfect codes, Singleton bound and MDS codes. Reed-Solomon, Reed-Muller codes. The Gilbert-Varshamov bound. Random codes, explicit asymptotically good codes (Forney's concatenated codes, Justesen codes). Block designs t-designs and their links with perfect codes. Binary and ternary Golay codes and Witt designs. Fisher's inequality and its variants. Symmetric designs, the Bruck-Chowla-Ryser condition. Constructions (both recursive and direct) of block designs.

Textbook: none

Further reading:

- P.J. Cameron, J.H. van Lint: Designs, graphs, codes and their links Cambridge Univ. Press, 1991.
- J. H. van Lint: Introduction to Coding theory, Springer, 1992.
- J. H. van Lint, R.J. Wilson, A course in combinatorics, Cambridge Univ. Press, 1992; 2001

Title of the course: Computational geometry

Number of contact hours per week: 2+0 Credit value: 3+0

Course coordinator(s): Márton Naszódi

Department(s): Department of Geometry Evaluation: oral or written examination

Prerequisites: none

A short description of the course:

Basic notions: affine subspaces in \mathbb{R}^n , convex hull, polytopes, norms.

Algorithmic problems: Triangulations, Dirichlet-Voronoi diagrams. Delaunay triangulations. Divide-and-conquer methods. Range searching. Ray tracing. Art gallery problem. Epsilon nets.

Topological methods in computational geometry. Estimating the volume of a convex body.

Textbook:

- S. Devadoss, J. O'Rourke: *Discrete and Computational Geometry*. 2011 Princeton University Press.
- J. Matoušek: Lectures on Discrete Geometry. 2002 Springer Graduate Text in Mathematics.

Title of the course: Computational methods in operation research

Number of contact hours per week: 0+2 Credit value: 0+3

Course coordinator(s): Alpár Jüttner

Department(s): Department of Operations Research

Evaluation: tutorial mark

Prerequisites: none

A short description of the course:

Implementation questions of mathematical programming methods.

Formulation of mathematical programming problems, and interpretation of solutions: progress from standard input/output formats to modeling tools.

The LINDO and LINGO packages for linear, nonlinear, and integer programming. The CPLEX package for linear, quadratic, and integer programming.

Modeling tools: XPRESS, GAMS, AMPL.

Textbook: -

Further reading:

Maros, I.: *Computational Techniques of the Simplex Method*, Kluwer Academic Publishers, Boston, 2003.

Title of the course: Computational number theory

Number of contact hours per week: 2+0 Credit value: 3+0

Course coordinator(s): Katalin Gyarmati

Department(s): Department of Algebra and Number Theory

Evaluation: oral or written examination

Prerequisites: basic number theory (see the content of the BSc course

Number theory 1); classical algebra (polynomials), linear

algebra, finite fields

A short description of the course:

Time complexity of elementary operations and basic arithmetic problems. For n=pq the determination of p and q is polynomially equivalent with that of $\varphi(n)$. Modular computations of powers. Factorization with algebraic identities. Foundations of cryptography. RSA, discrete logarithm and the Diffie–Hellman key exchange system. Primality testing, pseudoprimes. Fermat-factorization, the factor-base algorithm, the quadratic sieve. Elliptic curves, the analog of the Diffie–Hellman key exchange system. Pseudorandom sequences and their applications in connection with Monte Carlo methods and in cryptography.

Textbook:

Neal Koblitz, A Course in Number Theory and Cryptography

A. Das: Computational Number Theory, CRC Press

Title of the course: Differential equations

Number of contact hours per week: 2+2 Credit value: 3+3

Course coordinator(s): Péter Simon

Department(s): Department of Applied Analysis and Computational

Mathematics

Evaluation: oral exam and home exercise

Prerequisites: none

A short description of the course:

Differential equations in physics, chemistry, biology and economics.

Existence and uniqueness of solutions.

Methods for solving differential equations.

Linear systems. Higher order linear equations.

Autonomous differential equations, dynamical systems. Stability, Liapunov's method.

Poincaré-Bendixson theory. Stability of periodic orbits, the Poincaré map.

Discrete time dynamical systems.

Boundary value problems. Elements of the calculus of variation.

Textbook:

M. Braun, Differential equations and their applications, Springer 1993.

Further reading:

L. Perko, Differential Equations and Dynamical systems, Springer.

Title of the course:	Differential geometry	v of curves and surfaces
Title of the course:	Differential geometr	v of curves and surfaces

Course coordinator(s): László Verhóczki

Department(s): Department of Geometry

Evaluation: oral or written examination and tutorial mark

Prerequisites: Multivariable calculus, Linear algebra

A short description of the course:

Smooth parameterized curves in the 3-dimensional Euclidean space \mathbb{R}^3 . Arc length parameterization. Distinguished Frenet frame. Curvature, torsion. Frenet formulas. Fundamental theorem of the theory of curves. Signed curvature of a plane curve. Four vertex theorem. Theorems on total curvatures of closed curves.

Smooth surfaces in \mathbb{R}^3 . Parameterizations. Tangent space at a point. First fundamental form. Normal curvature, Meusnier's theorem. Weingarten mapping, principal curvatures and directions. Christoffel symbols. Compatibility equations. Theorema Egregium. Fundamental theorem of the local theory of surfaces. Geodesic curves.

Vector fields on \mathbb{R}^3 . Gradient vector field of a smooth function, curl of vector fields on \mathbb{R}^2 and \mathbb{R}^3 , divergence of a vector field. Integral formulae (divergence theorem, Stokes' theorem, Green's theorem).

Textbook: B. Csikós: Differential Geometry. Typotex, Budapest, 2014.

Further reading:

M. P. do Carmo: Differential geometry of curves and surfaces. Prentice Hall, Englewood

Cliffs, 1976.

Course coordinator(s): Péter Sziklai

Department(s): Computer Science

Evaluation: exam and term mark

Prerequisites: none

A short description of the course:

Basic problems in combinatorics. Further enumeration problems, principle of inclusion-exclusion. Pigeon hole principle and its applications. Binomial theorem, combinatorial identities. Fibonacci numbers, linear recurrences. Catalan numbers.

Graphs. Realization of degree sequences. Paths, cycles, Euler-trails, Hamilton-cycles. Connectivity, trees, Prüfer code, forests. Colourings of graphs. Planar graphs, Euler's formula. Number of edges and colourings of planar graphs.

Matchings in bipartite graphs, Hall's theorem. Graph parameters, Gallai's theorem. Double and k-fold connectivity. Ramsey's theorem for graphs. The theorem of Erdős-Szekeres. Extremal problems, Turán type theorems.

(Algorithmic questions for graphs.) Linear algebra methods in combinatorics

Brief introduction to complexity of algorithms. The classes P, NP, co-NP. NP-completeness.

Textbook: Lovász, L., Pelikán, J, Vesztergombi, K., Discrete Mathematics (Elementary and Beyond), Springer

Further reading: va Lint, J.H., Wilson, *A Course in Combinatorics*, Cambridge University Press

Title of the course: Discrete and continuous optimization 1 (Discrete

optimization)

Number of contact hours per week: 2+2 Credit value: 3+3

Course coordinator(s): Tamás Király

Department(s): Department of Operations Research

Evaluation: oral exam and home exercise

Prerequisites: none

A short description of the course:

Fundamental notions in graph theory and matroid theory (matchings, flows and circulations, spanning trees, greedy algorithm). Polyhedral combinatorics: totally unimodular matrices and their applications. Combinatorial graph algorithms: dynamic programming, maximum flows, Hungarian method. Basics of integer linear programming (Lagrangian relaxation, branch-and-bound).

Textbook:

András Frank: *Connections in combinatorial optimization*. Oxford Lecture Series in Mathematics and Its Applications 38 (2011)

Further reading:

- J. Matousek, B. Gärtner: *Understanding and Using Linear Programming*. Springer (2007)
- B. Guenin, J. Könemann, L. Tunçel: *A Gentle Introduction to Optimization*. Cambridge University Press (2014)

Title of the course: Discrete and continuous optimization 2 (Continuous optimization)

Number of contact hours per week: 2+2 Credit value: 3+3

Course coordinator(s): Tamás Király

Department(s): Department of Operations Research oral exam and home exercise

Prerequisites: none

A short description of the course:

Linear programming duality, interior point methods.

Matrix games: Nash equilibrium, Neumann's theorem on equilibrium in zero-sum games. Convex optimization: duality, convex Farkas theorem, Karush-Kuhn-Tucker theorem.

Textbook:

András Frank: *Connections in combinatorial optimization*. Oxford Lecture Series in Mathematics and Its Applications 38 (2011)

Further reading:

- J. Matousek, B. Gärtner: *Understanding and Using Linear Programming*. Springer (2007)
- B. Guenin, J. Könemann, L. Tunçel: *A Gentle Introduction to Optimization*. Cambridge University Press (2014)

Course coordinator(s): Péter Sziklai

Department(s): Computer Science

Evaluation: exam and term mark

Prerequisites: none

A short description of the course:

Basic problems in combinatorics. Further enumeration problems, principle of inclusion-exclusion. Pigeon hole principle and its applications. Binomial theorem, combinatorial identities. Fibonacci numbers, linear recurrences. Catalan numbers.

Graphs. Realization of degree sequences. Paths, cycles, Euler-trails, Hamilton-cycles. Connectivity, trees, Prüfer code, forests. Colourings of graphs. Planar graphs, Euler's formula. Number of edges and colourings of planar graphs.

Matchings in bipartite graphs, Hall's theorem. Graph parameters, Gallai's theorem. Double and k-fold connectivity. Ramsey's theorem for graphs. The theorem of Erdős-Szekeres. Extremal problems, Turán type theorems.

(Algorithmic questions for graphs.) Linear algebra methods in combinatorics

Brief introduction to complexity of algorithms. The classes P, NP, co-NP. NP-completeness.

Textbook: Lovász, L., Pelikán, J, Vesztergombi, K., Discrete Mathematics (Elementary and Beyond), Springer

Further reading: va Lint, J.H., Wilson, *A Course in Combinatorics*, Cambridge University Press

Title of the course: Dynamical systems and differential equations

Number of contact hours per week: 2+2 Credit value: 3+3

Course coordinator(s): Péter Simon

Department(s): Dept. of Appl. Analysis and Computational Math.

Evaluation: oral or written exam and tutorial mark

Prerequisites: Differential equations

A short description of the course:

Topological equivalence, classification of linear systems. Stable, unstable, centre manifolds theorems, Hartman - Grobman theorem. Periodic solutions and their stability. Index of two-dimensional vector fields, behaviour of trajectories at infinity. Applications to models in biology and chemistry. Chaos in the Lorenz equation. Bifurcations in dynamical systems, basic examples. Definitions of local and global bifurcations. Saddle-node bifurcation, Andronov-Hopf bifurcation. Discrete dynamical systems. Classification according to topological equivalence. 1D maps, the tent map and the logistic map. Symbolic dynamics. Chaotic systems.

Textbook: -

Further reading:

M. Braun, Differential equations and their applications, Springer, 1993.

L. Perko, Differential Equations and Dynamical systems, Springer, 2001.

Title of the course:	Finance and economics
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Course coordinator(s): János Kiss Hubert

Department(s): Department of Economics (Faculty of Social Sciences)

Evaluation: written exam

Prerequisites: none

A short description of the course:

The financial system: flow of funds, financial markets and institutions

The time value of money: interest rate (rate of interest), simple interest, compound interest, accumulation function, future value, current value, present value, net present value, discount factor, discount rate (rate of discount), nominal rate, effective rate, inflation and real rate of interest, force of interest, equation of value, the term structure of interest rates, spot rates, forward rates.

Principles of Market Valuation:

Valuation of known cash flows: level payment annuities, annuity-immediate, annuity due, perpetuity, payable m-thly or payable continuously, arithmetic increasing/decreasing annuity, geometric increasing/decreasing annuity, term of annuity

Loans: principal, interest, term of loan, outstanding balance, final payment (drop payment, balloon payment), amortization, sinking fund

Bond valuation: price, book value, amortization of premium, accumulation of discount, redemption value, par value/face value, yield rate, coupon, coupon rate, term of bond, callable/non-callable

Stock valuation: the dividend discount model, common and preferred stocks, stock price, stock dividend, mutual funds, stock indexes and exchange traded funds, capital asset pricing model

Interest rate sensitivity: duration, convexity, immunization

Principles of risk management:

Hedging, forwards and futures, swaps: hedging, arbitrage, diversifiable risk, non-diversifiable risk, forward contract, futures contract, cost of carry, swap, swap term, prepaid swap, notional amount, swap spread, deferred swap, simple commodity swap, interest rate swap

Insuring and options: derivative, underlying asset, over the counter market, short selling, short position, long position, ask price, bid price, bid-ask spread, spot price, credit risk, dividends, margin, maintenance margin, margin call, mark to market, no-arbitrage, risk-averse, call option, put option, expiration, expiration date, strike price/exercise price, European option, American option, option writer, in-the-money, at-the-money, out-of-the-money, covered call, put-call parity

Diversification and Portfolio choice

Textbooks:

Bodie, Z., Merton, R., Cleeton, D., *Financial Economics* (Second Edition), 2012, Pearson Learning Solutions

Broverman, S.A., *Mathematics of Investment and Credit* (Fifth Edition), 2010, ACTEX Publications

Daniel, J.W., and Vaaler, L.J.F., *Mathematical Interest Theory* (Second Edition), 2009, The Mathematical Association of America

Title of the course:	Functional analysis	
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Course coordinator(s): János Karátson

Department(s): Department of Applied Analysis and Computational

Mathematics

Evaluation: oral exam and home exercise

Prerequisites: none

A short description of the course:

Banach spaces, function spaces. A review of Riemann-Stieltjes and Lebesgue integral.

Sobolev spaces in 1D.

Hilbert spaces, orthonormal systems, Fourier series. Orthogonal polynomials.

Bounded linear functionals in Banach space, Hahn-Banach theorem.

Banach-Steinhaus theorem. Uniform convergence of trigonometric series.

Banach homeomorphism theorem.

Bounded linear functionals in Hilbert space, Riesz representation theorem.

Bounded linear operators in Hilbert space. Adjoint, projectors; selfadjoint, isometric, unitary operators.

Operator equations in Hilbert space. Bilinear forms, Lax-Milgram lemma.

Compact selfadjoint operators in Hilbert space.

Textbook: J. B. Conway: A Course in Functional Analysis, 1997

Further reading: Rudin, W., Functional Analysis, McGraw-Hill, 1991.

Title of the course: Geometric modelling

Number of contact hours per week: 2+0 Credit value: 3+0

Course coordinator(s): László Verhóczki

Department(s): Department of Geometry
Evaluation: oral or written examination
Prerequisites: Multivariable calculus

A short description of the course:

Affine combination of points, barycentric coordinates. Affine maps. Parameterized curves in R³. Linear interpolation, the de Casteljau algorithm, Bézier curves. Bernstein polynomials. Blossomings. Degree elevation on a Bézier curve. Subdivision of a Bézier curve. Cubic Hermite interpolation. Piecewise cubic interpolations to a sequence of points. B-spline curves, the Cox–de Boor algorithm, knot insertions. Homogeneous coordinates. Rational Bézier curves defined by control points and weights. Conics as rational quadratics. Rational B-spline curves. Composite curves, continuity conditions.

Parameterized surfaces in R³. Constructing polynomial patches. Bilinearly and bicubically blended Coons patches. Bicubic Hermite patches. Bilinear interpolation. Tensor product patches. Bézier triangles, the de Casteljau algorithm. B-spline surfaces. Rational Bézier and B-spline surfaces. Composite surfaces, continuity conditions.

Textbook:

G. Farin: Curves and surfaces for CAGD. *A practical guide*. 5th Edition. Academic Press, Boston, 2002.

Further reading:

J. Gallier: *Curves and surfaces in geometric modeling: theory and algorithms*. Morgan Kaufmann Publishers, San Francisco, 2000.

Title of the course: Introduction to information theory

Number of contact hours per week: 2+0 Credit value: 3+0

Course coordinator(s): Villő Csiszár

Department(s): Department of Probability Theory and Statistics

Evaluation: oral or written exam
Prerequisites: Probability and Statistics

A short description of the course:

Coding by codes of different length and blocks. Entropy and its properties. I-divergence and its properties. The notion of the noisy channel, channel coding. The capacity of a channel and its calculations. Source and channel coding using linear codes. Systems with multiple users: individual codes for correlated sources. A channel with additive Gauss-noise.

Textbook: -

Further reading:

Csiszár – Körner: Information Theory: Coding Theorems for Discrete Memoryless Systems.

Akadémiai Kiadó, 1981.

Cover – Thomas: *Elements of Information Theory*. Wiley, 1991.

Title of the course: Integral geometry

Number of contact hours per week: 2+0 Credit value: 3+0

Course coordinator(s): Balázs Csikós

Department(s): Department of Geometry
Evaluation: oral or written examination
Prerequisites: Multivariable calculus

A short description of the course:

Crofton type formulae: Formula for the length of a curve in the plane, in space, or on the sphere, formula for the surface area of a surface. Poincaré's formula.

Radon transform: Definition. The support theorem. Inversion formula. Plancherel formula.

Funk transform: Definition. Spherical harmonics. The kernel of the Funk transform.

Inversion formulae. Applications to convex geometry and tomography.

Textbook:

B. Csikós: Differential Geometry. Typotex, Budapest, 2014.

S. Helgason: *The Radon Transform*. Progress in Mathematics Vol. 5. Birkhäuser, 1999.

Further reading:

R. Schneider: The use of spherical harmonics in convex geometry

(http://home.mathematik.uni-freiburg.de/rschnei/Vortr%E4ge.Kent.pdf)

Title of the course: Linear and abstract algebra

Number of contact hours per week: 2+2 Credit value: 3+3

Course coordinator(s): Péter Frenkel

Department(s): Department of Algebra and Number Theory Evaluation: two midterm tests, home exercises, oral exam

Prerequisites: none

A short description of the course:

Vector spaces, linear maps, eigenvalues, minimal polynomial, diagonalizable matrices, Cayley-Hamilton theorem. Matrices of polynomials, Smith normal form. Jordan canonical form.

Unitary spaces, unitary and self-adjoint transformations, Spectral theorem. Quadratic forms, Sylvester's law of inertia.

Groups, Lagrange theorem, normal subgroup and quotient group, permutation groups. The structure of finite Abelian groups.

Unique factorization in rings. The basics of number theory, quadratic congruences, continued fractions.

Field extensions, finite fields.

Groebner bases.

Textbooks:

P.R. Halmos, Finite dimensional vector spaces

S. Lang, *Algebra*

Title of the course: Mathematical finance 1

Number of contact hours per week: 2+0 Credit value: 3+0

Course coordinator(s): Miklós Arató

Department(s): Probability Theory and Statistics

Evaluation: Oral exam

Prerequisites: Probability and statistics

A short description of the course:

Discrete time, finite horizon, one-bond - one-stock markets. Derivatives: options, forwards, warrants and swaps. Self-financing strategies. Options and contingent claims. Arbitrage. Hedge. The binomial model. Martingale measure. "Integral" representation of discrete time martingales in the binomial model. Pricing of European options - the sufficiency lemma. The Cox-Ross-Rubinstein formula. Snell envelope and optimal stopping. Doob-Meyer decomposition of sub/supermartingales. Pricing of American options. Fundamental theorems of asset pricing: absence of arbitrage and the existence of martingale measures. Market completeness and the uniqueness of martingale measures. Non-complete markets - atomic martingale measures. The seller's and the buyer's prices of an option in non-complete markets. Transaction costs. Black-Scholes (B-SCH) formula as a limit of CRR. The role of the Numeraire. Market price of risk. Completeness of the Black-Scholes market.

Textbook: -

Recommended reading:

R. J. Elliott - E. P. Kopp: *Mathematics of financial markets*, 1999. Springer Finance.

M. Musiela - M. Rutkowski: Martingale Methods in Financial Modelling

Title of the course:	Mathematical finance 2	
Title of the course:	Mathematical finance 2	

Course coordinator(s): Miklós Arató

Department(s): Probability Theory and Statistics

Evaluation: Oral exam

Prerequisites: Stochastic processes of mathematical finance I

A short description of the course:

Overwiev of some aspects of the stochastic integral by Wiener process, Ito's calculus and stochastic differential equations. Integral representation of martingales. Bond-Stock market, self financing strategies and equivalent martingale measures in continuous time. Pricing of European options: the B-SCH formula and PDE. Implied volatility, smile. Local volatility models, Breeden-Litzenbereger formula and Dupire equation. Stochastic volatility models, the Heston model in particular.

Ito diffusions. Doob-Meyer decomposition. Continuous Markov processes and the infinitesimal operator. Dynkin-Kinney condition, the locality of the infinitesimal operator. Infinitesimal operator for Ito diffusions. Feynman-Kac (F-K) formula. The B-SCH PDE from the F-K formula. Optimal stopping by level crossing and pricing of American options. Snell envelope and the American options.

Overwiev of Lévy-processes (LP). Compound Poisson processes. Lévy measure. Jump measure. Lévy-Ito and Lévy-Khinchin representation. Jump diffusions. LPs of bounded variation, subordinators. Markov LPs and the infinitesimal generator. Martingale LPs. Modelling asset prices by exponential Lévy-processes. Merton's, Kou's and Bates' models. Infinite activity models. Variance gamma and NIG models. Ito and F-K formula for LPs. PIDEs and the value of options. Esscher transform.

Textbook: -

Recommended reading:

Robert J. Elliott - P. Ekkehard Kopp: *Mathematics of financial markets*, 1999. Springer Finance.

Ioannis Karatzas, Steven E. Shreve: *Brownian Motion and Stochastic Calculus*, 2nd edition 1991. Springer

M. Musiela, M. Rutkowski: *Martingale methods in financial modelling*, 1997, 2nd ed. Springer 2005.

A. N. Shiryaev: Essentials of Stochastic Mathematical Finance. World Scientific, Singapore, 1999.

Rama Cont and Peter Tankov: *Financial Modelling with Jump Processes*, Chapman and Hall, 2004.

Title of the course: Modelling project work 1

Number of contact hours per week: 0+2 Credit value: 0+3

Course coordinator(s): Alpár Jüttner, András Zempléni

Department(s): all

Evaluation: term mark Prerequisites: none

A short description of the course:

At the beginning of the term students can choose from a list of topics for possible research. The first term is spent by understanding of the problem, the overview of the relevant literature, studying the possible approaches and to start working independently on the relevant methods under the guidance of a supervisor. At the end of the semester the students have to write a summary (3-4 pages) and give a short presentation about the work they have done. The note is determined by the quality of the summary and the presentation, and it is given in accordance to the opinion of the supervisor.

Textbook: -	
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Title of the course: Modelling project work 2

Number of contact hours per week: 0+2 Credit value: 0+3

Course coordinator(s): Alpár Jüttner, András Zempléni

Department(s): all

Evaluation: term mark

Prerequisites: Modelling project work 1

A short description of the course:

The students can continue their work from Project work 1 or can choose (at the beginning of the term) a new topic from a list of topics for possible research. During this term the task of the student is to start implementing the previously found methods for the particular problem and evaluate the obtained results, under the guidance of a supervisor. At the end of the term a 5-6 page long report and a 10-12 minute long presentation is expected from the student. The term grade is determined by the quality of the summary and the presentation, and it is given in accordance to the opinion of the supervisor.

Textbook: -

Title of the course: Modelling project work 3

Number of contact hours per week: 0+3 Credit value: 0+4

Course coordinator(s): Alpár Jüttner, András Zempléni

Department(s): all

Evaluation: term mark

Prerequisites: Modelling project work 2

A short description of the course:

The students can continue their work from Project work 2 or in exceptional cases can choose (at the beginning of the term) a new topic from a list of topics for possible research. During this term the task of the student is to complete the implementation of the previously found methods for the particular problem, investigating its applicability to similar/more general problems and sketch possible continuations of the research, under the guidance of a supervisor. At the end of the term a 8-10 page long report and a 15 minute long presentation is expected from the student. The term grade is determined by the quality of the summary and the presentation, and it is given in accordance to the opinion of the supervisor.

Textbook: -

Title of the course:	Modelling week	
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Number of contact hours per week:

Credit value: 0+3

Course coordinator(s): Alpár Jüttner, András Zempléni

Department(s): all

Evaluation: term mark Prerequisites: none

A short description of the course:

The students work in groups and are introduced to the problem – usually formulated in non-mathematical terms – on the first day and then an instructor helps to guide the students to a solution during the week. The students present their results to the other groups on the last day and then write up their work as a report (see below). The main aims of the Modelling Week are to train students in Mathematical Modelling and stimulate their collaboration and communication skills, in a multinational environment. The term grade depends on the report and presentation.

Textbook: -		

Title of the course: Multivariate statistical methods

Number of contact hours per week: 4+0 Credit value: 6+0

Course coordinator(s): György Michaletzky

Department(s): Department of Probability Theory and Statistics

Evaluation: oral or written exam

Prerequisites: Probability Theory and Statistics

A short description of the course:

Estimation of the parameters of multidimensional normal distribution. Matrix valued distributions. Wishart distribution: density function, determinant, expected value of its inverse.

Hypothesis testing for the parameters of multivariate normal distribution. Independence, goodness-of-fit test for normality. Linear regression.

Correlation, maximal correlation, partial correlation, canonical correlation.

Principal component analysis, factor analysis, analysis of variances.

Contingency tables, maximum likelihood estimation in loglinear models. Kullback–Leibler divergence. Linear and exponential families of distributions. Numerical method for determining the L-projection (Csiszár's method, Darroch–Ratcliff method)

Textbook: -

Further reading:

- J. D. Jobson, Applied Multivariate Data Analysis, Vol. I-II. Springer Verlag, 1991, 1992.
- C. R. Rao: Linear statistical inference and its applications, Wiley and Sons, 1968.

Title of the course:	Non-Euclidean geometries	
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Course coordinator(s): Gábor Moussong

Department(s): Department of Geometry

Evaluation: oral or written examination and tutorial mark

Prerequisites: linear algebra, fundamental concepts of group theory,

high school level Euclidean geometry

A short description of the course:

Affine geometry: affine transformations of the plane, affine invariants, linear algebra and affine geometry. Orthogonal matrices and Euclidean geometry.

Spherical geometry: spherical triangles and trigonometry, structure of orthogonal groups.

Inversive geometry: inversions, Möbius transformations, and their invariants, Poincaré extension.

Projective geometry: the projective plane, homogeneous coordinates, projective transformations, cross ratio, conics, polarity.

Hyperbolic geometry: Projective, conformal, and quadratic form models of hyperbolic plane. Transformations, distance, angle, area. Some formulas of hyperbolic trigonometry.

Textbook: G. Moussong, Notes on Non-Euclidean Geometries.

Further reading: E. Rees, *Notes on Geometry*. Universitext, Springer, 1983.

Title of the course:	Numerical modeling and numerical methods for
	ODEs

Course coordinator(s): István Faragó

Department(s): Department of Applied Analysis and Computational

Mathematics

Evaluation: project work + exam Prerequisites: Differential equations

A short description of the course:

Basics of the numerical analysis: interpolation and solution of systems of linear algebraic equations.

Several topics of the theory of ordinary differential equtions are considered which are important in real-life applications and play a special role in the numerical theory (stability, semidiscretization of time-dependent partial differential equations and their qualitative analysis, stiff problems, etc.)

The simplest one-step numerical methods for first order Cauchy problems. Stability notions and their different criteria, consistency, convergence. Runge-Kutta methods, their A-stability and convergence. Criteria of absolute stability. Stiff systems.

Linear multistep methods, Adams-type methods, consistency. Boundary value problems for second order ODEs. Some basic theorems for the existence and uniqueness. Shooting methods and finite difference methods. Stability and convergence of the methods. Special problems of the computer realizations are also considered.

Textbook:

Faragó István: Numerical Methods for Ordinary Differential Equations, TypTech, 2014

Further reading:

D. Kincaid, W. Cheney. Numerical analysis, AMS, 2009, 787 p.

Title of the course: Numerical solution of elliptic partial differential equations

Course coordinator(s): János Karátson

Department(s): Department of Applied Analysis and Computational

Mathematics

Evaluation: oral exam and home exercise

Prerequisites: Numerical modelling and numerical solution of ordinary

differential equations

A short description of the course:

Linear elliptic boundary value problems. Theoretical basis, bilinear forms, weak solutions. Some basic approximation schemes in numerical analysis.

Finite difference method. Stability and convergence.

Finite element method. Variational formulation, basis functions, convergence.

Iterative solution of the discretized problems. Conjugate gradients, preconditioning.

Two-grid and multigrid methods.

Textbook: -

Further reading:

Evans, G., Blackledge, J., Yardley, P., Numerical methods for partial differential equations. Springer, 2000.

Faragó I., Karátson J.: *Numerical solution of nonlinear elliptic problems via preconditioning operators: theory and applications.* Nova Science, 2002.

Title of the course: Numerical solution of time-dependent partial differential equations

Number of contact hours per week: 2+2 Credit value: 3+3

Course coordinator(s): Ferenc Izsák

Department(s): Department of Applied Analysis and Computational

Mathematics

Evaluation: tests, programming tasks, exam

Prerequisites: programing (Matlab)

A short description of the course:

Discretization and semidiscretization. (Conditional) stability, Lax's theorem. Analysis of stability: Fourier transform, Neumann condition. Neumann condition for (transfer) matrices. Numerical solution of linear parabolic problems. Explicit, implicit schemes, Crank–Nicolson scheme, CFL condition.

Textbook:

Thomas: Numerical PDEs: Finite-Difference Methods, Springer, New York, 1995.

Further reading:

Richtmyer, R.D., Morton, K.W. *Differnce methods for initial-value problems*, Interscience Pub., New York, 1967.

Title of the course: Operations research

Number of contact hours per week: 2+2 Credit value: 3+3

Course coordinator(s): Tamás Király

Department(s): Department of Operations Research Evaluation: oral or written exam and tutorial mark

Prerequisites: none

A short description of the course:

Shortest paths, conservative weightings (algorithms of Dijkstra and Bellman-Ford), Critical path method. Assignment and transportation problems, Kuhn's Hungarian method. Algorithms for maximum flows and feasible circulations. Linear inequality systems, basic and strong basic solutions. Polyhedra and polytopes. Farkas lemma, optimality criteria, duality theorem. The simplex method. Totally unimodular matrices and their applications.

Textbook:

András Frank: Connections in combinatorial optimization. Oxford Lectire Series in Mathematics and Its Applications 38 (2011)

- J. Matousek, B. Gärtner: Understanding and Using Linear Programming. Springer (2007)
- B. Guenin, J. Könemann, L. Tunçel: A Gentle Introduction to Optimization. Cambridge University Press (2014)

Title of the course: Partial differential equations

Number of contact hours per week: 2+2 Credit value: 3+3

Course coordinator(s): Ádám Besenyei

Department(s): Applied Analysis and Computational Mathematics

Evaluation: oral exam

Prerequisites:

A short description of the course:

Notion and types of partial differential equations, examples in physics, heat equation.

Classification of second order linear partial differential equations, canonical forms.

Cauchy problems for parabolic and hyperbolic equations. Duhamel principle.

Elliptic problems, Green's identities, Green's functions.

Sobolev spaces, embeddings, trace.

Weak solution of elliptic boundary value problems.

Eigenvalues, eigenfunctions and applications.

Initial-boundary value problems for parabolic and hyperbolic equations, Fourier's method.

Textbook:

L.C. Evans: *Partial Differential Equations*, Second Edition, Graduate Studies in Mathematics, 19, AMS: Providence, RI, 2010.

Haim Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2010.

Title of the course: Probability and statistics

Number of contact hours per week: 3+2 Credit value: 4+3

Course coordinator(s): Tamás F. Móri

Department(s): Department of Probability Theory and Statistics

Evaluation: oral or written exam and tutorial mark

Prerequisites: none

A short description of the course:

Measurable space, measurable mappings, measure, signed measure. Complete measure space. Integral and its properties. Interchangeability of limit and integral. Extension of measures, Lebesgue measure, Lebesgue-Stieltjes measure. Signed measure and its total variation. Hahn and Jordan decompositions. Absolutely continuous and singular measures. Lebesgue decomposition, Radon–Nikodym derivative. Product of measure spaces, Fubini's theorem. Probability space, random variables, distribution function, density function, expectation, variance, covariance, independence. Types of convergence: a.s., in probability, in Lp, weak. Uniform integrability. Characteristic function, central limit theorems. Conditional expectation, conditional probability, regular version of conditional distribution, conditional density function. Martingales, submartingales, limit theorem, regular martingales. Strong law of large numbers, series of independent random variables, the 3 series theorem. Statistical field, sufficiency, completeness. Fisher information. Informational inequality. Blackwell-Rao theorem. Point estimation: method of moments, maximum likelihood, Bayes estimators. Hypothesis testing, the likelihood ratio test, asymptotic properties. The multivariate normal distribution, ML estimation of the parameters. Linear model, least squares estimator. Testing linear hypotheses in Gaussian linear models.

Textbook: -

Further reading:

- J. Galambos: Advanced Probability Theory. Marcel Dekker, New York, 1995.
- E. L. Lehmann: Theory of Point Estimation. Wiley, New York, 1983.
- E. L. Lehmann: Testing Statistical Hypotheses, 2nd Ed., Wiley, New York, 1986.

Title of the course: Real analysis

Number of contact hours per week: 2+2 Credit value: 3+3

Course coordinator(s): Árpád Tóth Department(s): Analysis

Evaluation: exam and term mark

Prerequisites: none

A short description of the course:

Basic concepts and theorems of continuity, differentiability and integration in one and several variables. Riemann-Stieltjes integral, line integrals. Generalizations of the fundamental theorem of calculus to higher dimensions. The inverse and implicit function theorems, conditional extrema, Lagrange multipliers.

Measure theore and Lebesgue integral. The Lebesque dominated convergence theorem. Fubini's theorem.

The Fourier transform and its applications.

Textbook:

Marsden, J.E., Hoffman, M. J.: Elementary classical analysis

Rudin, W.:Principles of mathematical analysis

Further reading: -

Title of the course: Statistical computing and modelling 1

Number of contact hours per week: 0+2 Credit value: 0+3

Course coordinator(s): András Zempléni

Department(s): Department of Probability and Statistics Evaluation: weekly homework or final practical exam

Prerequisites: Probability and statistics

A short description of the course:

Statistical hypothesis testing and parameter estimation: algorithmic aspects and technical instruments. Numerical-graphical methods of descriptive statistics. Estimation of the location and scale parameters. Testing statistical hypotheses. Probability distributions.

Representation of distribution functions, random variate generation, estimation and fitting probability distributions. The analysis of dependence. Analysis of variance. Linear regression models. A short introduction to statistical programs of different category: instruments for demonstration and education, office environments, limited tools of several problems, closed programs, expert systems for users and specialists.

Work in computer lab (EXCEL, Statistica, Matlab, SPSS, R-project).

Textbook:

Further reading:

http://www.statsoft.com/Textbook

http://www.spss-tutorials.com/basics/

http://www.r-project.org/doc/bib/R-books.html

Pröhle, T.-Zempléni, A.: Statistical Problem Solving in R (2016)

http://www.mathworks.com/access/helpdesk/help/pdf_doc/stats/stats.pdf

Title of the course: Statistical computing and modelling 2

Number of contact hours per week: 0+2 Credit value: 0+3

Course coordinator(s): András Zempléni

Department(s): Department of Probability and Statistics Evaluation: weekly homework or final practical exam

Prerequisites: Multivariate statistical methods

A short description of the course:

Multivariate statistics: review of the methods and introduction of the used software packages. Dimension reduction. Principal components, factor analysis, canonical correlation. Multivariate analysis of categorical data. Modelling binary data, linear-logistic model.

Principle of multidimensional scaling. Correspondence analysis. Grouping. Cluster analysis and classification. Statistical methods for survival data analysis.

Probit, logit and nonlinear regression. Life tables, Cox-regression.

Simulation techniques: jackknife, bootstrap

Work in computer lab (EXCEL, Statistica, SPSS, SAS, R-project).

Textbook: -

Further reading:

http://www.statsoft.com/Textbook

http://www.spss-tutorials.com/more/

http://www.r-project.org/doc/bib/R-books.html

Pröhle, T.-Zempléni, A.: Statistical Problem Solving in R (2016)

http://www.mathworks.com/access/helpdesk/help/pdf_doc/stats/stats.pdf

Title of the course: Stochastic processes

Number of contact hours per week: 3+2 Credit value: 4+3

Course coordinator(s): Vilmos Prokaj

Department(s): Department of Probability Theory and Statistics

Evaluation: oral or written exam and tutorial mark

Prerequisites: Probability and Statistics

A short description of the course:

Markov chains in discrete and continuous time. Introduction to the Brownian motion. Donsker theorem. Integral with square integrable integrands. Ito lemma. Stochastic differential equations. Diffusions.

Textbook: -

Further reading:

Privault, N. *Notes on Markov chains* (available on-line http://www.ntu.edu.sg/home/nprivault/indext.html)

Kuo, H.-H. Introduction to stochastic integration, Springer, 2006

Klebaner, F. C. Introduction to stochastic calculus with applications, Imperial College Press, 2005

Title of the course: Time series analysis I

Number of contact hours per week: 2+2 Credit value: 3+3

Course coordinator(s): László Márkus

Department(s): Probability Theory and Statistics Evaluation: oral exam and tutorial mark

Prerequisites: Stationary Processes

A short description of the course:

Stationary processes, autocovariance, autocorrelation, partial autocorrelation functions and their properties. Spectral representation, Herglotz's theorem.

Introduction and basic properties of specific time series models:

Linear models: AR(1), AR(2) AR(p), Yule-Walker equations, MA(q), ARMA(p,q), ARIMA(p,d,q) conditions for the existence of stationary solutions and invertibility, the spectral density function.

Nonlinear models: ARCH(1), ARCH(p), GARCH(p,q), Bilinear(p,q,P,Q), TAR, SETAR. Stochastic recursion equations, stability, the Ljapunov-exponent and conditions for the existence of stationary solutions, Kesten-Vervaat-Goldie theorem on stationary solutions with regularly varying distributions.

Estimation of the mean for stationary processes. Properties of the sample mean, with respect to the spectral measure. Estimation of the autocovariance function: bias, variance and covariance of the estimator. Limit theorems for the estimated autocovariance.

Estimation of the discrete spectrum, the periodogram. Properties of periodogram values at Fourier frequencies. Expectation, variance, covariance and distribution of the periodogram at arbitrary frequencies. Linear processes, linear filter, impulse-response and transfer functions, spectral density and periodogram transformation by the linear filter. The periodogram as useless estimation of the spectral density function. Windowed periodogram as spectral density estimation. Window types. Bias and variance of the windowed estimation. Tailoring the windows.

Textbook:

Further reading: Priestley, M.B.: *Spectral Analysis and Time Series*, Academic Press 1981. Brockwell, P. J., Davis, R. A.: *Time Series: Theory and Methods*. Springer, N.Y. 1987. Tong, H.: *Non-linear time series: a dynamical systems approach*, Oxford University Press, 1991.

Hamilton, J. D.: *Time series analysis*, Princeton University Press, Princeton, N. J. 1994 Brockwell, P. J., Davis, R. A.: *Introduction to time series and forecasting*, Springer. 1996. Pena, D., Tiao and Tsay, R.: *A Course in Time Series Analysis*, Wiley 2001.