Abject mismatch tester gets us

Masterclass – session V

A large class is to be divided into teams and each student must be a member of exactly one team. However, each student dislikes three of their classmates. How many teams must be created so that no student is the teammate of someone they dislike?
Dislike need not be mutual. Teams need not be equally sized.

[2019-Rambo, #14]

What is the radius of convergence of

$$\sum_{n \geq 0} \frac{(nx)^n}{2 \times 4 \times 6 \times \cdots \times 2n}$$

[2019-Rambo, #40]

Let X_i be binary iid rv with $p = \frac{1}{2}$, and set $X = X_1 + \ldots + X_{100}$. Which is largest?

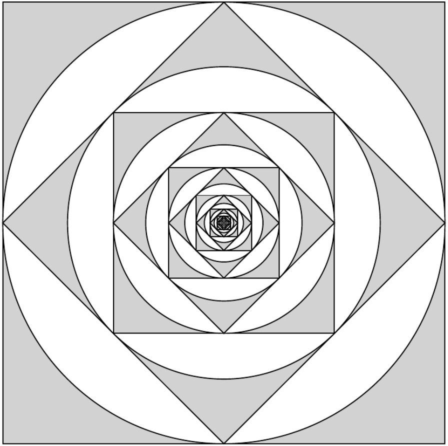
- $\mathrm{Var}(X)$
- $100 \times \mathbb{P}(|X - 50| > 25)$
- $100 \times \mathbb{P}(X \geq 60)$
- $\sum_{k \geq 0} \frac{k}{2^k} \binom{100}{k}$
- 30

[2019-Rambo, #56]

Compute

$$\lim_{n\to\infty} \frac{1}{n} + \frac{1}{2+n} + \frac{1}{4+n} + \ldots + \frac{1}{3n}$$

[2019-Rambo, #57]



[2019-Rambo, #65]

Give the maximal interval of existence $(0, t^\star)$ for the initial value problem $\dot{x}(t) = x^2(t)$ with $x(0) := x_0 > 0$. Can you continue the solution past the blow up time?

Now consider $\dot{z}(t) = z^2(t)$, $t > 0$, for a complex-valued function z with the intial value $z(0) = x_0 + i\epsilon$, where $x_0 > 0$ and $\epsilon \neq 0$ are real. Where needed, assume $x_0 = 1$.

- What happens to the blow up time?
- Decomposing $z = x + iy$, write down the differential equations for x and y.
- Find the trajectory $\{(x(t), y(t)) : t > 0\} \subset \mathbb{R}^2$.
- Sketch the evolution in (t, x, y)-space for a small $\epsilon > 0$. What happens for $\epsilon \searrow 0$?

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- $\cos(y) - \frac{dy}{dx} x \sin(y) + \frac{dy}{dx} y^2 = 0$
- $\frac{dy}{dx} = \frac{x^2 - x + y^2}{e^y - 2xy}$
- $\frac{dy}{dx} + e^{3x} - 4y = 0$
- $x \frac{dy}{dx} + (x+1)y = 3 \quad (x < 0)$
- $\frac{dy}{dx} = 1 - \frac{y}{x}$
- $\frac{dy}{dx} = \frac{y - x}{y + x} \quad (x > 0)$
- $y'' - y' - 12y = 0$
- $y'' - 4y' + 9y = 0$
- $y'' - 6y' + 9y = 0$
- $y'' - 5y' + 6y = 3x + 3$
- $y'' - 5y' + 6y = e^{4x}$
- stability of equilibria of $\frac{dP}{dt} = P(P-5)(P-7)$

[C. Tompkins]

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