Abject mismatch tester gets us

Masterclass – session IV

Practice test by Ch. Rambo

- www.rambotutoring.com/GREpractice.pdf
- www.rambotutoring.com/GREpracticeanswers.pdf

1. B	23. A	45. A
2. A	24. D	46. E
3. A	25. D	47. E
4. E	26. C	48. A
5. D	27. C	49. D
6. C	28. E	50. A
7. C	29. C	51. E
8. B	30. D	52. D
9. D	31. D	53. E
10. D	32. D	54. C
11. A	33. E	55. A
12. B	34. A	56. C
13. B	35. A	57. C
14. B	36. C	58. B
15. A	37. E	59. B
16. B	38. A	60. D
17. E	39. E	61. C
18. D	40. C	62. A
19. B	41. B	63. E
20. C	42. D	64. A
21. C	43. A	65. C
22. A	44. D	66. B

ODE practice

Solve:

- $\dot{x}(t) = t\,x(t)$
- $\dot{x}(t) = t/x(t)$
- $\dot{x}(t) = x^2(t)$
- $\dot{x}(t) = \sqrt{x(t)}$
- $\dot{x}(t) = (1 + t^2)(1 + x^2(t))$
- $\dot{x}(t) = x(t) \log x(t)$
- $\dot{x}(t) = g(t)h(x(t))$
- $\dot{x}(t) = a(t)x(t) + b(t)$
- $\dot{x}(t) = x(t) - x^\alpha(t)$

Tumor models:

- $\dot{V} = aV$ (Exponential)
- $\dot{V} = aV^b$ (Mendelsohn)
- $\dot{V} = aV(1 - \frac{1}{b}V)$ (Logistic)
- $\dot{V} = aV\,(b + V)^{-1}$ (Linear)
- $\dot{V} = aV\,(b + V)^{-1/3}$ (Surface)
- $\dot{V} = aV \log \frac{b}{c+V}$ (Gompertz)
- $\dot{V} = aV^{2/3} - bV$ (Bertalanffy)

What is

- the maximal V?
- the condition for growth?
- the doubling time?

(Various sources)

- $\cos(y) - \frac{dy}{dx} x \sin(y) + \frac{dy}{dx} y^2 = 0$
- $\frac{dy}{dx} = \frac{x^2 - x + y^2}{e^y - 2xy}$
- $\frac{dy}{dx} + e^{3x} - 4y = 0$
- $x \frac{dy}{dx} + (x+1)y = 3 \quad (x < 0)$
- $\frac{dy}{dx} = 1 - \frac{y}{x}$
- $\frac{dy}{dx} = \frac{y - x}{y + x} \quad (x > 0)$
- $y'' - y' - 12y = 0$
- $y'' - 4y' + 9y = 0$
- $y'' - 6y' + 9y = 0$
- $y'' - 5y' + 6y = 3x + 3$
- $y'' - 5y' + 6y = e^{4x}$
- stability of equilibria of $\frac{dP}{dt} = P(P-5)(P-7)$

Problems by C. Tompkins

Information percolation model

Let $g(x, t)$ denote the cross-sectional density of posterior type x in the population at time t. The initial density $g(\cdot, t)$ of types is given. Then

$$\partial_t g(x, t) = -\lambda g(x, t) + \lambda \int_{-\infty}^{\infty} g(y, t) g(x - y, t) dy \qquad (1)$$

with the first term representing the rate of emigration from type x associated with meeting and leaving that type, and the second term representing the rate of immigration into type x due to "y" meeting "$x - y$", converting y to x.

- Show conservation of (probability) mass.
- Solve formally and interpret.

D. Duffie and G. Manzo, Information percolation in large markets, AER, 2007

Goodwin's cyclical macroeconomic model

Setup:

- (output per annum Y) = (capital stock K) / (accelerator ν)
- (employment L) = (output Y) / (productivity a)
- (employment rate λ) = (employment L) / (population N)
- $\frac{d}{dt}\log$(real wages w) = ($-c + d \times$ (employment rate λ))
- (profits Π) = (output Y) - (total wages wL)
- (investment I) = (profits Π)
- $\frac{d}{dt}$(capital K) = (investment I) - (depreciation γK)
- productivity growth: $\alpha \times 100\%$ per annum
- population growth: $\beta \times 100\%$ per annum

Find a conserved quantity.

S. Keen, A monetary Minsky model of the Great Moderation and Great Recession, JEBO, 2013

Proabability bits

The first passenger boarding a plane takes a random seat. Then each passenger takes their seat if available; otherwise a random seat. What is the probability that the last passenger gets the right seat?

There is a fair and an unfair die/dice. The unfair dice shows [•] with probability 1/2. You take one and throw three [•] in a row. How likely is the same to happen with the other dice?

You apply to *n* different grad schools but seal the envelopes before writing the addresses – randomly. What happens?

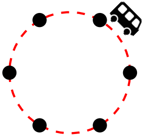
If the first time I get "heads" with a fair coin is on the N-th throw, you get N$. Would you play for 2$?

Interview questions, adapted

Busybus

The campus of a small university is served by one bus on a circular line. From each station ($N = 20$), the bus can go to an adjacent station or stay. Meanwhile, exactly one person arrives at a random station and waits for the bus to go to some other random station. Through an app, the destination of every person currently waiting anywhere along the route is known. The algorithm chooses who will board the bus at the current station respecting the bus capacity ($C = 10$). The passengers leave the bus as soon as it arrives at their destination.

Find an algorithm that is better than non-adaptive unidirectional driving.



See also:

- “Busybus” coding competition, http://bit.do/busybus, 2017
- A. Caicedo, Étude d’algorithmes pour le problème du “Busybus”, 2017

this slide is intentionally left blank