Differential equations: phase portraits ELTE, autumn 2019/20

R.A., November 15, 2019

3.1

Fix $m > 0$ and $g > 0$. Consider $E_{\mathrm{I}}(x, v) := \frac{1}{2}mv^2 + gx$ for $(x, v) \in \mathbb{R}^2$.

- In the x-v plane, sketch the level curves of E_{I}.
- If $E_{\mathrm{I}}(x(t), v(t))$ is constant and $\dot{x} = v$, what is $\dot{v}$?
- Sketch x and v as functions of t.
- Compute $F = m\ddot{x}$.

3.2

Same for $E_{\mathrm{II}}(x, v) := \frac{1}{2}mv^2 + \frac{1}{2}kx^2$, where $k > 0$.

3.3

Suppose $F = m\ddot{x} \overset{!}{=} -kx - \gamma v$, where $k > 0$ and $\gamma > 0$. Sketch the orbits in phase space.

3.4

Given the vectorfield $x \mapsto A\,x$ in $\mathbb{R}^2$, suggest a plausible matrix A.