Abject mismatch tester gets us

Masterclass – session II

Find and solve a differential equation for the curve in the x-t plane that goes through $(c, 0)$ and is perpendicular to the solution curves of $\dot{x}(t) = -x(t)$.

A lake with constant volume V $[\mathrm{m}^3]$ is fed by a river with flow rate r $[\mathrm{m}^3/\mathrm{s}]$. A factory upstream begins releasing a pollutant into the river at a rate p $[\mathrm{m}^3/\mathrm{s}]$. What is the concentration $c(t)$ $[\mathrm{m}^3/\mathrm{m}^3]$ of the pollutant in the lake?

For the Fibonacci sequence

$$F_{n+2} = F_{n+1} + F_n, \qquad F_1 = 1, \quad F_0 = 0,$$

compute limit of F_{n+1}/F_n as $n \to \infty$.

What is the convergence rate?

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Solve:

- $\dot{x}(t) = t\,x(t)$
- $\dot{x}(t) = t/x(t)$
- $\dot{x}(t) = x^2(t)$
- $\dot{x}(t) = \sqrt{x(t)}$
- $\dot{x}(t) = (1+t^2)(1+x^2(t))$
- $\dot{x}(t) = x(t)\log x(t)$
- $\dot{x}(t) = g(t)h(x(t))$
- $\dot{x}(t) = a(t)x(t) + b(t)$
- $\dot{x}(t) = x(t) - x^{\alpha}(t)$

Tumor models:

- $\dot{V} = aV$ (Exponential)
- $\dot{V} = aV^b$ (Mendelsohn)
- $\dot{V} = aV(1 - \frac{1}{b}V)$ (Logistic)
- $\dot{V} = aV\,(b + V)^{-1}$ (Linear)
- $\dot{V} = aV\,(b + V)^{-1/3}$ (Surface)
- $\dot{V} = aV\log\frac{b}{c+V}$ (Gompertz)
- $\dot{V} = aV^{2/3} - bV$ (Bertalanffy)

What is

- the maximal V?
- the condition for growth?
- the doubling time?

Give the maximal interval of existence $(0, t^\star)$ for the initial value problem $\dot{x}(t) = x^2(t)$ with $x(0) := x_0 > 0$. Can you continue the solution past the blow up time?

Now consider $\dot{z}(t) = z^2(t)$, $t > 0$, for a complex-valued function z with the intial value $z(0) = x_0 + i\epsilon$, where $x_0 > 0$ and $\epsilon \neq 0$ are real. Where needed, assume $x_0 = 1$.

- What happens to the blow up time?
- Decomposing $z = x + iy$, write down the differential equations for x and y.
- Find the trajectory $\{(x(t), y(t)) : t > 0\} \subset \mathbb{R}^2$.
- Sketch the evolution in (t, x, y)-space for a small $\epsilon > 0$. What happens for $\epsilon \searrow 0$?

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- $\cos(y) - \frac{dy}{dx} x \sin(y) + \frac{dy}{dx} y^2 = 0$
- $\frac{dy}{dx} = \frac{x^2 - x + y^2}{e^y - 2xy}$
- $\frac{dy}{dx} + e^{3x} - 4y = 0$
- $x \frac{dy}{dx} + (x+1)y = 3 \quad (x < 0)$
- $\frac{dy}{dx} = 1 - \frac{y}{x}$
- $\frac{dy}{dx} = \frac{y-x}{y+x} \quad (x > 0)$
- $y'' - y' - 12y = 0$
- $y'' - 4y' + 9y = 0$
- $y'' - 6y' + 9y = 0$
- $y'' - 5y' + 6y = 3x + 3$
- $y'' - 5y' + 6y = e^{4x}$
- stability of equilibria of $\frac{dP}{dt} = P(P-5)(P-7)$

Problems by C. Tompkins

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