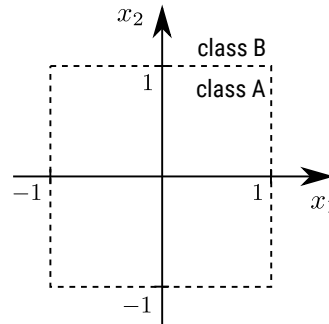


## Exercise Sheet 8

### Exercise 1: Designing a Neural Network (25 P)

We would like to implement a neural network that classifies data points in  $\mathbb{R}^2$  according to decision boundary given in the figure below.



We consider as an elementary computation the *threshold neuron* whose relation between inputs  $(a_i)_i$  and output  $a_j$  is given by

$$z_j = \sum_i a_i w_{ij} + b_j \quad a_j = 1_{z_j > 0}.$$

- (a) *Design* at hand a neural network that takes  $x_1$  and  $x_2$  as input and produces the output “1” if the input belongs to class A, and “0” if the input belongs to class B. *Draw* the neural network model and *write down* the weights  $w_{ij}$  and bias  $b_j$  of each neuron.

### Exercise 2: Backward Propagation (5 + 20 P)

We consider a neural network that takes two inputs  $x_1$  and  $x_2$  and produces an output  $y$  based on the following set of computations:

$$\begin{aligned} z_3 &= x_1 \cdot w_{13} + x_2 \cdot w_{23} & z_5 &= a_3 \cdot w_{35} + a_4 \cdot w_{45} & y &= a_5 + a_6 \\ a_3 &= \tanh(z_3) & a_5 &= \tanh(z_5) \\ z_4 &= x_1 \cdot w_{14} + x_2 \cdot w_{24} & z_6 &= a_3 \cdot w_{36} + a_4 \cdot w_{46} \\ a_4 &= \tanh(z_4) & a_6 &= \tanh(z_6) \end{aligned}$$

- (a) *Draw* the neural network graph associated to this set of computations.
- (b) *Write* the set of backward computations that leads to the evaluation of the partial derivative  $\partial y / \partial w_{13}$ . Your answer should avoid redundant computations. Hint:  $\tanh'(t) = 1 - (\tanh(t))^2$ .

### Exercise 3: Programming (50 P)

Download the programming files on ISIS and follow the instructions.