Esta es la sustituci'on que se har'a en las expresiones para una m'etrica esf'erica

```
> restart:grtw();
```

GRTensorII Version 1.79 (R4)

6 February 2001

Developed by Peter Musgrave, Denis Pollney and Kayll Lake

Copyright 1994-2001 by the authors.

Latest version available from: http://grtensor.phy.queensu.ca/

/Applications/grii/metrics **(1.1)**

```
> grOptionMetricPath :=
  `/Users/luisnunez/Documents/MisDocumentos/maple/gravitacion/metr
  icas/`;
```

grOptionMetricPath := **(1.2)**

/Users/luisnunez/Documents/MisDocumentos/maple/gravitacion/metricas/

```
> qload(Lake1);
```

```
Calculated ds for Lake1 (0.000000 sec.)
```

Default spacetime = Lake1

For the Lake1 spacetime:

Coordinates

$x(up)$

$$x^a = \left[\begin{array}{cccc} r & \theta & \phi & t \end{array} \right]$$

Line element

$$ds^2 = \frac{dr^2}{1 - \frac{2\,m(r)}{r}} + r^2\, d\theta^2 + r^2 \sin(\theta)^2\, d\phi^2 - e^{2\,\Phi(r)}\, dt^2$$

Spherical LAKE all static (PhysRevD V67 104015) **(1.3)**

```
> grcalc(g(dn,dn)); grdisplay(_);
```

CPU Time = 0.001

For the Lake1 spacetime:

Covariant metric tensor

$g(dn, dn)$

(1.4)

$$g_{a\,b} = \begin{bmatrix} \dfrac{1}{1-\dfrac{2\,m(r)}{r}} & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2\sin(\theta)^2 & 0 \\ 0 & 0 & 0 & -e^{2\,\Phi(r)} \end{bmatrix} \tag{1.4}$$

```
> grcalc(G(dn,dn)); grdisplay(_);
Calculated g(dn,dn,pdn) for Lake1 (0.001000 sec.)
Calculated Chr(dn,dn,dn) for Lake1 (0.000000 sec.)
Calculated detg for Lake1 (0.000000 sec.)
Calculated g(up,up) for Lake1 (0.000000 sec.)
Calculated Chr(dn,dn,up) for Lake1 (0.001000 sec.)
Calculated R(dn,dn) for Lake1 (0.002000 sec.)
Calculated Ricciscalar for Lake1 (0.000000 sec.)
Calculated G(dn,dn) for Lake1 (0.002000 sec.)
```

CPU Time = 0.007

For the Lake1 spacetime:

Covariant Einstein

$G(dn, dn)$

$$G_{a\,b} = \left[\left[\frac{2\left(m(r)+2\left(\frac{d}{dr}\Phi(r)\right)r\,m(r)-\left(\frac{d}{dr}\Phi(r)\right)r^2\right)}{r^2\,(-r+2\,m(r))}, 0, 0, 0\right], \tag{1.5}$$

$$\left[0, -\frac{1}{r}\left(\left(\frac{d}{dr}m(r)\right)r - m(r) - \left(\frac{d}{dr}\Phi(r)\right)r^2 + \left(\frac{d}{dr}\Phi(r)\right)r\,m(r) - \left(\frac{d^2}{dr^2}\Phi(r)\right)r^3 + 2\left(\frac{d^2}{dr^2}\Phi(r)\right)r^2\,m(r) + \left(\frac{d}{dr}\Phi(r)\right)r^2\left(\frac{d}{dr}m(r)\right) - \left(\frac{d}{dr}\Phi(r)\right)^2 r^3 + 2\left(\frac{d}{dr}\Phi(r)\right)^2 r^2\,m(r)\right), 0, 0\right],$$

$$\left[0, 0, -\frac{1}{r}\left(\sin(\theta)^2\left(\left(\frac{d}{dr}m(r)\right)r - m(r) - \left(\frac{d}{dr}\Phi(r)\right)r^2 + \left(\frac{d}{dr}\Phi(r)\right)r\,m(r) - \left(\frac{d^2}{dr^2}\Phi(r)\right)r^3 + 2\left(\frac{d^2}{dr^2}\Phi(r)\right)r^2\,m(r) + \left(\frac{d}{dr}\Phi(r)\right)r^2\left(\frac{d}{dr}m(r)\right) - \left(\frac{d}{dr}\Phi(r)\right)^2 r^3 + 2\left(\frac{d}{dr}\Phi(r)\right)^2 r^2\,m(r)\right)\right), 0\right],$$

$$\left[0, 0, 0, \frac{2\, e^{2\,\Phi(r)} \left(\frac{d}{dr} m(r)\right)}{r^2}\right]\Bigg]$$

```
> grdef(`T{a b} := 1/(8*Pi)*G{a b} `);grcalc(T(up,up)); grcalc(T
  (dn,dn)); grdisplay(_);
Created definition for T(dn,dn)
Created definition for T(up,up)
Calculated T(dn,dn) for Lake1 (0.001000 sec.)
Calculated T(up,up) for Lake1 (0.001000 sec.)
```

$$CPU\ Time = 0.006$$

$$CPU\ Time = 0.$$

For the Lake1 spacetime:

T(dn,dn)

$$T(dn, dn)$$

$$T_{a\,b} = \left[\left[\frac{1}{4} \frac{m(r) + 2\left(\frac{d}{dr}\Phi(r)\right) r\, m(r) - \left(\frac{d}{dr}\Phi(r)\right) r^2}{\pi\, r^2\, (-r + 2\, m(r))}, 0, 0, 0\right], \tag{1.6}$$

$$\left[0, -\frac{1}{8} \frac{1}{\pi\, r}\left(\left(\frac{d}{dr} m(r)\right) r - m(r) - \left(\frac{d}{dr}\Phi(r)\right) r^2 + \left(\frac{d}{dr}\Phi(r)\right) r\, m(r)\right.\right.$$

$$- \left(\frac{d^2}{dr^2}\Phi(r)\right) r^3 + 2\left(\frac{d^2}{dr^2}\Phi(r)\right) r^2\, m(r) + \left(\frac{d}{dr}\Phi(r)\right) r^2 \left(\frac{d}{dr} m(r)\right)$$

$$\left.\left. - \left(\frac{d}{dr}\Phi(r)\right)^2 r^3 + 2\left(\frac{d}{dr}\Phi(r)\right)^2 r^2\, m(r)\right), 0, 0\right],$$

$$\left[0, 0, -\frac{1}{8} \frac{1}{\pi\, r}\left(\sin(\theta)^2 \left(\left(\frac{d}{dr} m(r)\right) r - m(r) - \left(\frac{d}{dr}\Phi(r)\right) r^2\right.\right.\right.$$

$$+ \left(\frac{d}{dr}\Phi(r)\right) r\, m(r) - \left(\frac{d^2}{dr^2}\Phi(r)\right) r^3 + 2\left(\frac{d^2}{dr^2}\Phi(r)\right) r^2\, m(r)$$

$$\left.\left.\left. + \left(\frac{d}{dr}\Phi(r)\right) r^2 \left(\frac{d}{dr} m(r)\right) - \left(\frac{d}{dr}\Phi(r)\right)^2 r^3 + 2\left(\frac{d}{dr}\Phi(r)\right)^2 r^2\, m(r)\right)\right), 0\right]$$

$$\left.\right],$$

$$\left[0, 0, 0, \frac{1}{4} \frac{e^{2\,\Phi(r)} \left(\frac{d}{dr} m(r)\right)}{\pi\, r^2}\right]\Bigg]$$

Construyo la cuadrivelocidad

```
> grdef(`u{^a}:= [0,0,0,u0] `); grcalc(u(up)); grdisplay(_);
Components assigned for metric: Lake1
Created definition for u(up)
```

$$CPU\ Time = 0.$$

For the Lake1 spacetime:

$$u(up)$$

$$u(up)$$

$$u^a = \begin{bmatrix} 0 & 0 & 0 & u0 \end{bmatrix} \tag{1.7}$$

```
> grdef( `normaU := g{a b}*u{^a}*u{^b}`  );
Created definition for normaU
> grcalc(normaU); grdisplay(_);
Calculated normaU for Lake1 (0.001000 sec.)
```

$$CPU\ Time = 0.001$$

For the Lake1 spacetime:

$$normaU$$

$$normaU = -e^{2\,\Phi(r)}\,u0^2 \tag{1.8}$$

```
> solU := solve( grcomponent(  normaU) = -1 , u0);
```

$$solU := \frac{1}{\sqrt{e^{2\,\Phi(r)}}}, -\frac{1}{\sqrt{e^{2\,\Phi(r)}}} \tag{1.9}$$

```
> sol1:= op( 1, [solU] ); sol2:= op( 2, [solU] );
```

$$sol1 := \frac{1}{\sqrt{e^{2\,\Phi(r)}}}$$

$$sol2 := -\frac{1}{\sqrt{e^{2\,\Phi(r)}}} \tag{1.10}$$

```
> subs( u0 = sol1 , grcomponent(normaU)    );
```

$$-1 \tag{1.11}$$

```
> subs( u0 = sol2 , grcomponent(normaU)    );
```

$$-1 \tag{1.12}$$

```
> u0:= sol1;
```

$$u0 := \frac{1}{\sqrt{e^{2\,\Phi(r)}}} \tag{1.13}$$

```
> grcalc(normaU); grdisplay(_);
```

$$CPU\ Time = 0.$$

For the Lake1 spacetime:

$$normaU$$

$$normaU = -1 \tag{1.14}$$

Las variables F'isicas quedan como

```
> grdef( ` density := T{a b}*u{^a}*u{^b}  ` ); grcalc(density);
  grdisplay(_);
Created definition for density
Calculated density for Lake1 (0.000000 sec.)
```

$$CPU\ Time = 0.$$

For the Lake1 spacetime:

density

$$density = \frac{1}{4}\,\frac{\frac{d}{dr}\,m(r)}{\pi\, r^2} \tag{1.15}$$

```
> rho:= grcomponent( density);
```

$$\rho := \frac{1}{4}\,\frac{\frac{d}{dr}\,m(r)}{\pi\, r^2} \tag{1.16}$$

```
> grdef(`n{^a}:= [n0,0,0,0] `); grcalc(n(up)); grdisplay(_);
Components assigned for metric: Lake1
Created definition for n(up)
```

CPU Time = 0.

For the Lake1 spacetime:

n(up)

$$n(up)$$

$$n^a = \left[\, n0 \;\; 0 \;\; 0 \;\; 0 \,\right] \tag{1.17}$$

```
> grdef( `normaN := g{a b}*n{^a}*n{^b}`  ); grcalc(normaN);
  grdisplay(_);
Created definition for normaN
Calculated normaN for Lake1 (0.000000 sec.)
```

CPU Time = 0.

For the Lake1 spacetime:

normaN

$$normaN = -\frac{r\, n0^2}{-r + 2\, m(r)} \tag{1.18}$$

```
> solN := solve( grcomponent(normaN) = -1 , n0);
```

$$solN := \frac{\sqrt{r\,(-r + 2\,m(r))}}{r}, -\frac{\sqrt{r\,(-r + 2\,m(r))}}{r} \tag{1.19}$$

```
> solN1:= op( 1, [solN] ); solN2:= op( 2, [solN] ); n0:= solN1;
```

$$solN1 := \frac{\sqrt{r\,(-r + 2\,m(r))}}{r}$$

$$solN2 := -\frac{\sqrt{r\,(-r + 2\,m(r))}}{r}$$

$$n0 := \frac{\sqrt{r\,(-r + 2\,m(r))}}{r} \tag{1.20}$$

```
> grdef(` ortogonal := n{^a}*u{a} = 0 `);grcalc(ortogonal)
  ;grdisplay(_);
> grdef( ` PresLake := T{a b}*n{^a}*n{^b}  ` ); grcalc(PresLake);
  grdisplay(_);
This object is already defined. The new definition has been
ignored.
```

CPU Time = 0.

For the Lake1 spacetime:

ortogonal

ortogonal = All components are zero

```
This object is already defined. The new definition has been
ignored.
Calculated PresLake for Lake1 (0.000000 sec.)
```

CPU Time = 0.

For the Lake1 spacetime:

PresLake

$$PresLake = \frac{1}{4}\,\frac{m(r) + 2\left(\frac{d}{dr}\Phi(r)\right) r\, m(r) - \left(\frac{d}{dr}\Phi(r)\right) r^2}{\pi\, r^3} \tag{1.21}$$

```
> PLake := grcomponent( PresLake);
```

$$PLake := \frac{1}{4}\,\frac{m(r) + 2\left(\frac{d}{dr}\Phi(r)\right) r\, m(r) - \left(\frac{d}{dr}\Phi(r)\right) r^2}{\pi\, r^3} \tag{1.22}$$

```
> grdef( ` pressure:= (T{a b}*g{^a ^b} + density)/3 ` );
Created definition for pressure
> grcalc(pressure); grdisplay(_);
Calculated pressure for Lake1 (0.000000 sec.)
```

CPU Time = 0.

For the Lake1 spacetime:

pressure

$$\begin{aligned} pressure = -\frac{1}{12}\,\frac{1}{r^2\,\pi}\Bigg(&3\left(\frac{d}{dr}\Phi(r)\right) m(r) - 2\left(\frac{d}{dr}\Phi(r)\right) r + \frac{d}{dr}\, m(r) \\ &- \left(\frac{d^2}{dr^2}\Phi(r)\right) r^2 + 2\left(\frac{d^2}{dr^2}\Phi(r)\right) r\, m(r) + \left(\frac{d}{dr}\Phi(r)\right) r\left(\frac{d}{dr}\, m(r)\right) \\ &- \left(\frac{d}{dr}\Phi(r)\right)^2 r^2 + 2\left(\frac{d}{dr}\Phi(r)\right)^2 r\, m(r)\Bigg) \end{aligned} \tag{1.23}$$

```
> P:= grcomponent( pressure);
```

$$\begin{aligned} P := -\frac{1}{12}\,\frac{1}{r^2\,\pi}\Bigg(&3\left(\frac{d}{dr}\Phi(r)\right) m(r) - 2\left(\frac{d}{dr}\Phi(r)\right) r + \frac{d}{dr}\, m(r) \\ &- \left(\frac{d^2}{dr^2}\Phi(r)\right) r^2 + 2\left(\frac{d^2}{dr^2}\Phi(r)\right) r\, m(r) + \left(\frac{d}{dr}\Phi(r)\right) r\left(\frac{d}{dr}\, m(r)\right) \\ &- \left(\frac{d}{dr}\Phi(r)\right)^2 r^2 + 2\left(\frac{d}{dr}\Phi(r)\right)^2 r\, m(r)\Bigg) \end{aligned} \tag{1.24}$$

```
> P;
```

$$-\frac{1}{12}\,\frac{1}{r^2\,\pi}\left(3\left(\frac{d}{dr}\Phi(r)\right) m(r) - 2\left(\frac{d}{dr}\Phi(r)\right) r + \frac{d}{dr}\, m(r) - \left(\frac{d^2}{dr^2}\Phi(r)\right) r^2\right) \tag{1.25}$$

$$+2\left(\frac{d^2}{dr^2}\Phi(r)\right) r\, m(r) + \left(\frac{d}{dr}\Phi(r)\right) r \left(\frac{d}{dr} m(r)\right) - \left(\frac{d}{dr}\Phi(r)\right)^2 r^2$$

$$+2\left(\frac{d}{dr}\Phi(r)\right)^2 r\, m(r)\Bigg)$$

```
> grdef( `TT{^a ^b} := (rho0 +P0)*u{^a}*u{^b} + P0*g{^a ^b}`  );
  grcalc(TT(up,dn));grdisplay(_);
Created definition for TT(up,up)
Created definition for TT(up,dn)
Calculated TT(up,up) for Lake1 (0.000000 sec.)
Calculated TT(up,dn) for Lake1 (0.001000 sec.)
```

$$CPU\ Time = 0.002$$

For the Lake1 spacetime:

TT(up,dn)

$$TT(up, dn)$$

$$TT^a{}_b = \begin{bmatrix} P0 & 0 & 0 & 0 \\ 0 & P0 & 0 & 0 \\ 0 & 0 & P0 & 0 \\ 0 & 0 & 0 & -\rho 0 \end{bmatrix} \tag{1.26}$$

```
> rho0 := rho; P0 := PLake;
```

$$\rho 0 := \frac{1}{4}\,\frac{\frac{d}{dr} m(r)}{\pi r^2}$$

$$P0 := \frac{1}{4}\,\frac{m(r) + 2\left(\frac{d}{dr}\Phi(r)\right) r\, m(r) - \left(\frac{d}{dr}\Phi(r)\right) r^2}{\pi r^3} \tag{1.27}$$

```
> grcalc(TT(up,dn));grdisplay(_);
```

$$CPU\ Time = 0.$$

For the Lake1 spacetime:

TT(up,dn)

$$TT(up, dn)$$

$$TT^a{}_b = \left[\left[\frac{1}{4}\,\frac{m(r) + 2\left(\frac{d}{dr}\Phi(r)\right) r\, m(r) - \left(\frac{d}{dr}\Phi(r)\right) r^2}{\pi r^3}, 0, 0, 0\right], \right.$$
$$\left[0, \frac{1}{4}\,\frac{m(r) + 2\left(\frac{d}{dr}\Phi(r)\right) r\, m(r) - \left(\frac{d}{dr}\Phi(r)\right) r^2}{\pi r^3}, 0, 0\right],$$
$$\left[0, 0, \frac{1}{4}\,\frac{m(r) + 2\left(\frac{d}{dr}\Phi(r)\right) r\, m(r) - \left(\frac{d}{dr}\Phi(r)\right) r^2}{\pi r^3}, 0\right], \tag{1.28}$$

$$\left[0, 0, 0, -\frac{1}{4}\,\frac{\frac{d}{dr}m(r)}{\pi r^2}\right]\Bigg]$$

```
> grdef(`Ec{b} := TT{^a b ; a}`); grcalc(Ec(dn)); grdisplay(_);
Created a definition for TT(up,dn,cdn)
Created definition for Ec(dn)
Calculated TT(up,dn,cdn) for Lake1 (0.001000 sec.)
Calculated Ec(dn) for Lake1 (0.001000 sec.)
```

$$CPU\ Time = 0.003$$

For the Lake1 spacetime:

Ec(dn)

$$Ec(dn)$$

$$Ec_a = \left[\frac{1}{4}\,\frac{1}{\pi r^4}\left(\left(\frac{d}{dr}m(r)\right)r + 2\left(\frac{d^2}{dr^2}\Phi(r)\right)r^2 m(r) - 3\left(\frac{d}{dr}\Phi(r)\right)r\,m(r)\right.\right. \tag{1.29}$$
$$+ 3\left(\frac{d}{dr}\Phi(r)\right)r^2\left(\frac{d}{dr}m(r)\right) - \left(\frac{d^2}{dr^2}\Phi(r)\right)r^3 + \left(\frac{d}{dr}\Phi(r)\right)r^2 - 3\,m(r)$$
$$\left.\left. + 2\left(\frac{d}{dr}\Phi(r)\right)^2 r^2 m(r) - \left(\frac{d}{dr}\Phi(r)\right)^2 r^3\right), 0, 0, 0\right]$$

```
> TOV := grcomponent(Ec(dn),[1]);
```

$$TOV := \frac{1}{4}\,\frac{1}{\pi r^4}\left(\left(\frac{d}{dr}m(r)\right)r + 2\left(\frac{d^2}{dr^2}\Phi(r)\right)r^2 m(r) - 3\left(\frac{d}{dr}\Phi(r)\right)r\,m(r)\right. \tag{1.30}$$
$$+ 3\left(\frac{d}{dr}\Phi(r)\right)r^2\left(\frac{d}{dr}m(r)\right) - \left(\frac{d^2}{dr^2}\Phi(r)\right)r^3 + \left(\frac{d}{dr}\Phi(r)\right)r^2 - 3\,m(r)$$
$$\left. + 2\left(\frac{d}{dr}\Phi(r)\right)^2 r^2 m(r) - \left(\frac{d}{dr}\Phi(r)\right)^2 r^3\right)$$

```
> collect(TOV, m(r));
```

$$\frac{1}{4}\,\frac{\left(-3\left(\frac{d}{dr}\Phi(r)\right)r + 2\left(\frac{d^2}{dr^2}\Phi(r)\right)r^2 - 3 + 2\left(\frac{d}{dr}\Phi(r)\right)^2 r^2\right)m(r)}{\pi r^4} \tag{1.31}$$
$$+ \frac{1}{4}\,\frac{1}{\pi r^4}\left(\left(\frac{d}{dr}m(r)\right)r + \left(\frac{d}{dr}\Phi(r)\right)r^2 + 3\left(\frac{d}{dr}\Phi(r)\right)r^2\left(\frac{d}{dr}m(r)\right)\right.$$
$$\left. - \left(\frac{d^2}{dr^2}\Phi(r)\right)r^3 - \left(\frac{d}{dr}\Phi(r)\right)^2 r^3\right)$$

```
> subs( diff(m(r),r) = m1(r), TOV); subs( diff(m1(r),r) = m11(r),
  %);
```

$$\frac{1}{4}\,\frac{1}{\pi r^4}\left(m1(r)\,r + 2\left(\frac{d^2}{dr^2}\Phi(r)\right)r^2 m(r) - 3\left(\frac{d}{dr}\Phi(r)\right)r\,m(r)\right.$$

$$+3\left(\frac{d}{dr}\Phi(r)\right)r^2\,m1(r)-\left(\frac{d^2}{dr^2}\Phi(r)\right)r^3+\left(\frac{d}{dr}\Phi(r)\right)r^2-3\,m(r)$$
$$+2\left(\frac{d}{dr}\Phi(r)\right)^2 r^2\,m(r)-\left(\frac{d}{dr}\Phi(r)\right)^2 r^3\Bigg)$$

$$\frac{1}{4}\,\frac{1}{\pi\,r^4}\left(m1(r)\,r+2\left(\frac{d^2}{dr^2}\Phi(r)\right)r^2\,m(r)-3\left(\frac{d}{dr}\Phi(r)\right)r\,m(r)\right. \tag{1.32}$$
$$+3\left(\frac{d}{dr}\Phi(r)\right)r^2\,m1(r)-\left(\frac{d^2}{dr^2}\Phi(r)\right)r^3+\left(\frac{d}{dr}\Phi(r)\right)r^2-3\,m(r)$$
$$\left.+2\left(\frac{d}{dr}\Phi(r)\right)^2 r^2\,m(r)-\left(\frac{d}{dr}\Phi(r)\right)^2 r^3\right)$$

```
> collect(collect(collect(%, m(r)),m1(r)),m11(r));
```

$$\frac{1}{4}\,\frac{\left(3\left(\frac{d}{dr}\Phi(r)\right)r^2+r\right)m1(r)}{\pi\,r^4} \tag{1.33}$$
$$+\frac{1}{4}\,\frac{\left(-3\left(\frac{d}{dr}\Phi(r)\right)r+2\left(\frac{d^2}{dr^2}\Phi(r)\right)r^2-3+2\left(\frac{d}{dr}\Phi(r)\right)^2 r^2\right)m(r)}{\pi\,r^4}$$
$$+\frac{1}{4}\,\frac{-\left(\frac{d}{dr}\Phi(r)\right)^2 r^3+\left(\frac{d}{dr}\Phi(r)\right)r^2-\left(\frac{d^2}{dr^2}\Phi(r)\right)r^3}{\pi\,r^4}$$

```
> EcuacDif := f1(x)*diff(y(x),x) +f0(x)*y(x) + g(x)=0;
```

$$EcuacDif := f1(x)\left(\frac{d}{dx}y(x)\right)+f0(x)\,y(x)+g(x)=0 \tag{1.34}$$

```
> dsolve(EcuacDif,y(x));
```

$$y(x)=\left(\int\left(-\frac{g(x)\,e^{\int\frac{f0(x)}{f1(x)}\,dx}}{f1(x)}\right)dx+\_C1\right)e^{\int\left(-\frac{f0(x)}{f1(x)}\right)dx} \tag{1.35}$$

```
> a := ((1/4)*(-3*(diff(Phi(r), r))*r+2*(diff(Phi(r), r, r))*r^2
-3+2*(diff(Phi(r), r))^2*r^2)/(Pi*r^4))/((1/4)*(3*(diff(Phi(r),
r))*r^2+r)/(Pi*r^4));
```

$$a := \frac{-3\left(\frac{d}{dr}\Phi(r)\right)r+2\left(\frac{d^2}{dr^2}\Phi(r)\right)r^2-3+2\left(\frac{d}{dr}\Phi(r)\right)^2 r^2}{3\left(\frac{d}{dr}\Phi(r)\right)r^2+r} \tag{1.36}$$

```
> b := ((1/4)*(-(diff(Phi(r), r))^2*r^3+(diff(Phi(r), r))*r^2-
(diff(Phi(r), r, r))*r^3)/(Pi*r^4))/((1/4)*(3*(diff(Phi(r), r))
*r^2+r)/(Pi*r^4));
```

$$b := \frac{-\left(\frac{d}{dr}\Phi(r)\right)^2 r^3 + \left(\frac{d}{dr}\Phi(r)\right) r^2 - \left(\frac{d^2}{dr^2}\Phi(r)\right) r^3}{3\left(\frac{d}{dr}\Phi(r)\right) r^2 + r} \tag{1.37}$$

```
> Phi(r) := (1/2)*N*ln(1-r^2/alpha);
```

$$\Phi(r) := \frac{1}{2} N \ln\left(1 - \frac{r^2}{\alpha}\right) \tag{1.38}$$

```
> simplify(a);
```

$$-\frac{N r^2 \alpha - 5 N r^4 - 3 \alpha^2 + 6 \alpha r^2 - 3 r^4 + 2 N^2 r^4}{\left(3 N r^2 - \alpha + r^2\right) r \left(\alpha - r^2\right)} \tag{1.39}$$

```
> simplify(b);
```

$$\frac{(N-2) r^4 N}{\left(3 N r^2 - \alpha + r^2\right)\left(\alpha - r^2\right)} \tag{1.40}$$

```
> exp(int(a,r));
```

$$e^{\frac{1}{3}\ln\left(-\alpha + r^2\right)N - \frac{2}{3}\ln\left(-\alpha + r^2\right) - 3\ln(r) + \frac{2}{3}\frac{\ln\left(3 N r^2 - \alpha + r^2\right)}{3N+1} + \frac{11}{3}\frac{\ln\left(3 N r^2 - \alpha + r^2\right)N}{3N+1}} \tag{1.41}$$

```
> integrando:= b*expand(%);
```

$$integrando := \frac{1}{\left(-\frac{3 N r^3}{\alpha\left(1 - \frac{r^2}{\alpha}\right)} + r\right)\left(-\alpha + r^2\right)^{2/3} r^3}\left(\left(-\frac{N^2 r^5}{\alpha^2\left(1 - \frac{r^2}{\alpha}\right)^2} - \frac{N r^3}{\alpha\left(1 - \frac{r^2}{\alpha}\right)} - \left(-\frac{N}{\alpha\left(1 - \frac{r^2}{\alpha}\right)} - \frac{2 N r^2}{\alpha^2\left(1 - \frac{r^2}{\alpha}\right)^2}\right) r^3\right)\left(-\alpha + r^2\right)^{\frac{1}{3}N}\left(3 N r^2 - \alpha + r^2\right)^{\frac{2}{3(3N+1)}}\left(3 N r^2 - \alpha + r^2\right)^{\frac{11}{3}\frac{N}{3N+1}}\right) \tag{1.42}$$

```
> int(integrando,r);
```

$$\int \frac{1}{\left(-\frac{3 N r^3}{\alpha\left(1 - \frac{r^2}{\alpha}\right)} + r\right)\left(-\alpha + r^2\right)^{2/3} r^3}\left(\left(-\frac{N^2 r^5}{\alpha^2\left(1 - \frac{r^2}{\alpha}\right)^2} - \frac{N r^3}{\alpha\left(1 - \frac{r^2}{\alpha}\right)} - \left(\right.\right. \tag{1.43}$$

$$\left(-\frac{N}{\alpha\left(1-\frac{r^2}{\alpha}\right)} - \frac{2Nr^2}{\alpha^2\left(1-\frac{r^2}{\alpha}\right)^2} \right) r^3 \Bigg) \left(-\alpha + r^2\right)^{\frac{1}{3}N} \left(3Nr^2 - \alpha + r^2\right)^{\frac{2}{3(3N+1)}} \left(3Nr^2 - \alpha + r^2\right)^{\frac{11}{3}\frac{N}{3N+1}} \Bigg) \mathrm{d}r$$