Colapso lento.. con m(r,t)

El elemento de línea en coordenadas de Scharszchild es el siguiente:

$$ds^2 = e^{2\nu(t,r)}dt^2 - e^{2\lambda(t,r)}dr^2 - r^2\left(d\theta^2 + sin^2(\theta)d\phi^2\right)$$

```
> restart:grtw():
```

GRTensorII Version 1.79 (R4)

6 February 2001

Developed by Peter Musgrave, Denis Pollney and Kayll Lake

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Latest version available from: http://grtensor.phy.queensu.ca/ **(1.1)**

```
> grOptionMetricPath := 
  `/Users/luisnunez/Documents/MisDocumentos/maple/gravitacion/metr
  icas/`;
```

grOptionMetricPath := **(1.2)**

/Users/luisnunez/Documents/MisDocumentos/maple/gravitacion/metricas/

```
> alias(lambda0 = lambda0(r,t), nu0 = nu0(r,t), K = K(t), m=m(r,
  t)):
> qload(esferica);
Calculated ds for esferica (0.001000 sec.)
```

Default spacetime = esferica

For the esferica spacetime:

Coordinates

$x(up)$

$$x^a = \begin{bmatrix} r & \theta & \phi & t \end{bmatrix}$$

Line element

$$ds^2 = -e^{2\,\lambda 0}\ d\,r^2 - r^2\ d\,\theta^2 - r^2\sin(\theta)^2\ d\,\phi^2 + e^{2\,\nu 0}\ d\,t^2 \qquad \textbf{(1.3)}$$

Ecuaciones de Einstein

```
> grcalcalter(G(dn, up), G(dn, dn), 1):
Created definition for G(dn,up)
Simplification will be applied during calculation.

Applying routine simplify to object g(dn,dn,pdn)
Calculated g(dn,dn,pdn) for esferica (0.023000 sec.)
Applying routine simplify to object Chr(dn,dn,dn)
Calculated Chr(dn,dn,dn) for esferica (0.005000 sec.)
Applying routine simplify to object g(up,up)
Calculated detg for esferica (0.002000 sec.)
Calculated g(up,up) for esferica (0.013000 sec.)
Applying routine simplify to object Chr(dn,dn,up)
Calculated Chr(dn,dn,up) for esferica (0.011000 sec.)
Applying routine simplify to object R(dn,dn)
Calculated R(dn,dn) for esferica (0.011000 sec.)
Applying routine simplify to object tRicciscalar
Calculated Ricciscalar for esferica (0.012000 sec.)
```

```
Applying routine simplify to object G(dn,dn)
Calculated G(dn,dn) for esferica (0.019000 sec.)
Applying routine simplify to object G(dn,up)
Calculated G(dn,up) for esferica (0.005000 sec.)
```

$$CPU\ Time = 0.100 \tag{2.1}$$

```
> G00 := grcomponent(G(dn, up), [t, t]): G11 := grcomponent(G(dn, up), [r, r]):
> G22 := grcomponent(G(dn, up), [theta, theta]): G01 := grcomponent(G(dn,dn),[t, r]):
> T00 := (rho+omega^2*P+2*omega*q)/(1-omega^2):
> T11 := -(P+omega^2*rho+2*omega*q)/(1-omega^2):
> T01 := -(omega*(rho+P)+q*(1+omega^2))*exp(nu0+lambda0)/(1-omega^2):
> T22 := -Pt:
> grcalc(G(dn,dn,cdn));
Created a definition for G(dn,dn,cdn)
Calculated G(dn,dn,cdn) for esferica (0.005000 sec.)
```

$$CPU\ Time = 0.018 \tag{2.2}$$

Cambio a la definici'on de masa

```
> lambda0 := -ln(1 -2*m/r)/2;
```

$$\lambda 0 := -\frac{1}{2}\ln\left(1-\frac{2\,m}{r}\right) \tag{2.3}$$

Las ecuaciones de Einstein ser'an

```
> E00 := 8*Pi*simplify(T00) = simplify(G00);
```

$$E00 := -\frac{8\,\pi\left(\rho+\omega^2 P+2\,\omega\,q\right)}{-1+\omega^2} = \frac{2\left(\frac{\partial}{\partial r}\,m\right)}{r^2} \tag{2.4}$$

```
> E11 := 8*Pi*simplify(T11) = simplify(G11);
```

$$E11 := \frac{8\,\pi\left(P+\omega^2\rho+2\,\omega\,q\right)}{-1+\omega^2} = \frac{2\left(m-\left(\frac{\partial}{\partial r}\,\nu 0\right)r^2+2\left(\frac{\partial}{\partial r}\,\nu 0\right)r\,m\right)}{r^3} \tag{2.5}$$

```
> E01 := 8*Pi*(T01) = simplify(G01);
```

$$E01 := -\frac{8\,\pi\left(\omega\left(\rho+P\right)+q\left(1+\omega^2\right)\right)\mathrm{e}^{-\frac{1}{2}\ln\left(1-\frac{2\,m}{r}\right)+\nu 0}}{1-\omega^2} = -\frac{2\left(\frac{\partial}{\partial t}\,m\right)}{r\,\left(-r+2\,m\right)} \tag{2.6}$$

```
> E22 := 8*Pi*T22 = simplify(G22);
```

$$\begin{aligned}E22 := -8\,\pi\,Pt = \frac{1}{r^3\left(-r+2\,m\right)^2}\Bigg(&-r^5\left(\frac{\partial^2}{\partial r^2}\,\nu 0\right)-r^5\left(\frac{\partial}{\partial r}\,\nu 0\right)^2-\left(\frac{\partial}{\partial r}\,\nu 0\right)r^4\\&+\left(\frac{\partial}{\partial r}\,m\right)r^3-r^2\,m+4\,r\,m^2+2\,\mathrm{e}^{-2\,\nu 0}\left(\frac{\partial}{\partial t}\,m\right)r^3\left(\frac{\partial}{\partial t}\,\nu 0\right)m\\&-\mathrm{e}^{-2\,\nu 0}\left(\frac{\partial}{\partial t}\,m\right)r^4\left(\frac{\partial}{\partial t}\,\nu 0\right)-4\left(\frac{\partial}{\partial r}\,\nu 0\right)r^3\left(\frac{\partial}{\partial r}\,m\right)m\\&+4\left(\frac{\partial}{\partial r}\,\nu 0\right)r^2\left(\frac{\partial}{\partial r}\,m\right)m^2+3\,\mathrm{e}^{-2\,\nu 0}\,r^3\left(\frac{\partial}{\partial t}\,m\right)^2+5\left(\frac{\partial}{\partial r}\,\nu 0\right)r^3\,m\end{aligned} \tag{2.7}$$

$$- 8 \left(\frac{\partial}{\partial r} \nu 0\right) r^2 m^2 + 4 \left(\frac{\partial}{\partial r} \nu 0\right) r\, m^3 + \left(\frac{\partial}{\partial r} \nu 0\right) r^4 \left(\frac{\partial}{\partial r} m\right)$$

$$+ 6 r^4 \left(\frac{\partial}{\partial r} \nu 0\right)^2 m - 12 r^3 \left(\frac{\partial}{\partial r} \nu 0\right)^2 m^2 + 8 r^2 \left(\frac{\partial}{\partial r} \nu 0\right)^2 m^3 - 4 \left(\frac{\partial}{\partial r} m\right) r^2 m$$

$$+ 4 \left(\frac{\partial}{\partial r} m\right) r\, m^2 - 2 e^{-2 \nu 0} r^3 \left(\frac{\partial^2}{\partial t^2} m\right) m - 4 m^3 + e^{-2 \nu 0} r^4 \left(\frac{\partial^2}{\partial t^2} m\right)$$

$$+ 6 r^4 \left(\frac{\partial^2}{\partial r^2} \nu 0\right) m - 12 r^3 \left(\frac{\partial^2}{\partial r^2} \nu 0\right) m^2 + 8 r^2 \left(\frac{\partial^2}{\partial r^2} \nu 0\right) m^3 \Bigg)$$

las ecuaciones de Einstein lentas

```
> E00Lenta := 8*Pi*convert(series(T00,omega,2),polynom)= simplify
  (G00);
```

$$E00Lenta := 8 \pi \left(\rho + 2 \omega q\right) = \frac{2 \left(\frac{\partial}{\partial r} m\right)}{r^2} \tag{2.8}$$

```
> E11Lenta := 8*Pi*convert(series(T11,omega,2),polynom) =
  simplify(G11);
```

$$E11Lenta := 8 \pi \left(-P - 2 \omega q\right) = \frac{2 \left(m - \left(\frac{\partial}{\partial r} \nu 0\right) r^2 + 2 \left(\frac{\partial}{\partial r} \nu 0\right) r\, m\right)}{r^3} \tag{2.9}$$

```
> E01Lenta := 8*Pi*convert(series(T01,omega,2),polynom) =
  simplify(G01);
```

$$E01Lenta := 8 \pi \left(-q\, e^{-\frac{1}{2} \ln\left(1 - \frac{2 m}{r}\right) + \nu 0} - \left(\rho + P\right) e^{-\frac{1}{2} \ln\left(1 - \frac{2 m}{r}\right) + \nu 0} \omega\right) = -\frac{2 \left(\frac{\partial}{\partial t} m\right)}{r \left(-r + 2 m\right)} \tag{2.10}$$

```
> E22Lenta := 8*Pi*convert(series(T22,omega,2),polynom)= simplify
  (G22);
```

$$E22Lenta := -8 \pi Pt = \frac{1}{r^3 \left(-r + 2 m\right)^2} \left(-r^5 \left(\frac{\partial^2}{\partial r^2} \nu 0\right) - r^5 \left(\frac{\partial}{\partial r} \nu 0\right)^2 - \left(\frac{\partial}{\partial r} \nu 0\right) r^4\right. \tag{2.11}$$

$$+ \left(\frac{\partial}{\partial r} m\right) r^3 - r^2 m + 4 r\, m^2 + 2 e^{-2 \nu 0} \left(\frac{\partial}{\partial t} m\right) r^3 \left(\frac{\partial}{\partial t} \nu 0\right) m$$

$$- e^{-2 \nu 0} \left(\frac{\partial}{\partial t} m\right) r^4 \left(\frac{\partial}{\partial t} \nu 0\right) - 4 \left(\frac{\partial}{\partial r} \nu 0\right) r^3 \left(\frac{\partial}{\partial r} m\right) m$$

$$+ 4 \left(\frac{\partial}{\partial r} \nu 0\right) r^2 \left(\frac{\partial}{\partial r} m\right) m^2 + 3 e^{-2 \nu 0} r^3 \left(\frac{\partial}{\partial t} m\right)^2 + 5 \left(\frac{\partial}{\partial r} \nu 0\right) r^3 m$$

$$- 8 \left(\frac{\partial}{\partial r} \nu 0\right) r^2 m^2 + 4 \left(\frac{\partial}{\partial r} \nu 0\right) r\, m^3 + \left(\frac{\partial}{\partial r} \nu 0\right) r^4 \left(\frac{\partial}{\partial r} m\right)$$

$$+ 6 r^4 \left(\frac{\partial}{\partial r} \nu 0\right)^2 m - 12 r^3 \left(\frac{\partial}{\partial r} \nu 0\right)^2 m^2 + 8 r^2 \left(\frac{\partial}{\partial r} \nu 0\right)^2 m^3 - 4 \left(\frac{\partial}{\partial r} m\right) r^2 m$$

$$+ 4 \left(\frac{\partial}{\partial r} m\right) r m^2 - 2 e^{-2 \nu 0} r^3 \left(\frac{\partial^2}{\partial t^2} m\right) m - 4 m^3 + e^{-2 \nu 0} r^4 \left(\frac{\partial^2}{\partial t^2} m\right)$$

$$+ 6 r^4 \left(\frac{\partial^2}{\partial r^2} \nu 0\right) m - 12 r^3 \left(\frac{\partial^2}{\partial r^2} \nu 0\right) m^2 + 8 r^2 \left(\frac{\partial^2}{\partial r^2} \nu 0\right) m^3\Bigg)$$

Usando unas variables efectivas

```
> P := Ptilde -2*omega*q; rho := rhotilde -2*omega*q;
```

$$P := Ptilde - 2 \omega q$$

$$\rho := rhotilde - 2 \omega q \qquad (2.12)$$

```
> E00Lenta;
```

$$8 \pi\, rhotilde = \frac{2 \left(\frac{\partial}{\partial r} m\right)}{r^2} \qquad (2.13)$$

```
> E11Lenta;
```

$$-8 \pi\, Ptilde = \frac{2 \left(m - \left(\frac{\partial}{\partial r} \nu 0\right) r^2 + 2 \left(\frac{\partial}{\partial r} \nu 0\right) r m\right)}{r^3} \qquad (2.14)$$

```
> q:=solve(simplify(E01Lenta),q);
```

$$q := \frac{1}{4} \frac{1}{\pi e^{\nu 0} r \left(r - 2 m - 4 r \omega^2 + 8 \omega^2 m\right)} \Bigg(-4 \pi e^{\nu 0} r^2 \omega\, rhotilde \qquad (2.15)$$

$$+ 8 \pi e^{\nu 0} r \omega\, rhotilde\, m - 4 \pi e^{\nu 0} r^2 \omega\, Ptilde + 8 \pi e^{\nu 0} r \omega\, Ptilde\, m$$

$$- \left(\frac{\partial}{\partial t} m\right) \sqrt{-\frac{-r + 2 m}{r}}\Bigg)$$

```
> qLenta := subs(omega^2=0,simplify(q));
```

$$qLenta := \frac{1}{4} \frac{1}{\pi r (r - 2 m)} \Bigg(\Bigg(-4 \pi e^{\nu 0} r^2 \omega\, rhotilde + 8 \pi e^{\nu 0} r \omega\, rhotilde\, m \qquad (2.16)$$

$$- 4 \pi e^{\nu 0} r^2 \omega\, Ptilde + 8 \pi e^{\nu 0} r \omega\, Ptilde\, m - \left(\frac{\partial}{\partial t} m\right) \sqrt{-\frac{-r + 2 m}{r}}\Bigg) e^{-\nu 0}\Bigg)$$

Como siempre nos sobra una funcion. En este caso omega. Para encontrarla tenemos que suponer algo adicional

Para ello vamos a seguir suponiento un colapso lento pero echamos mano de algunas funciones cinem'aticas.

```
> alias(omega = omega(r,t));
```

$$\lambda 0, \nu 0, K, m, \omega \qquad (2.17)$$

```
> grdef( `u{^a} := [ omega*exp(-lambda0)/sqrt(1-omega^2), 0, 0,
  exp(-nu0)/sqrt(1-omega^2) ]`);
Components assigned for metric: esferica
Created definition for u(up)
> grcalc(expsc[u]);
Created a definition for u(up,cdn)
Calculated u(up,cdn) for esferica (0.003000 sec.)
Calculated expsc[u] for esferica (0.002000 sec.)
```

$$CPU\ Time\ = 0.015 \tag{2.18}$$

```
> Expansion:=grcomponent(expsc[u]);
```

$$\begin{aligned} Expansion := {} & \frac{1}{\left(1-\omega^2\right)^{3/2} r\left(-r+2m\right)} \left(-\left(\frac{\partial}{\partial r}\omega\right)\sqrt{-\frac{-r+2m}{r}}\, r^2 \right. \\ & + 2r\left(\frac{\partial}{\partial r}\omega\right)\sqrt{-\frac{-r+2m}{r}}\, m - e^{-\nu 0}\left(\frac{\partial}{\partial t}m\right) r + r\, e^{-\nu 0}\left(\frac{\partial}{\partial t}m\right)\omega^2 \\ & - 2\omega\sqrt{-\frac{-r+2m}{r}}\, r + 2\omega^3\sqrt{-\frac{-r+2m}{r}}\, r + 4\omega\sqrt{-\frac{-r+2m}{r}}\, m \\ & - 4\omega^3\sqrt{-\frac{-r+2m}{r}}\, m - \omega r^2 e^{-\nu 0}\left(\frac{\partial}{\partial t}\omega\right) + 2\omega r\, e^{-\nu 0}\left(\frac{\partial}{\partial t}\omega\right) m \\ & - \omega r^2\sqrt{-\frac{-r+2m}{r}}\left(\frac{\partial}{\partial r}\nu 0\right) + 2\omega r\sqrt{-\frac{-r+2m}{r}}\left(\frac{\partial}{\partial r}\nu 0\right) m \\ & \left. + \omega^3 r^2\sqrt{-\frac{-r+2m}{r}}\left(\frac{\partial}{\partial r}\nu 0\right) - 2\omega^3 r\sqrt{-\frac{-r+2m}{r}}\left(\frac{\partial}{\partial r}\nu 0\right) m \right) \end{aligned} \tag{2.19}$$

```
> subs(omega^3=0,Expansion):subs(omega^2=0,%):subs(diff(omega,t)=
  0,%):
> ExpansionLenta := collect(simplify(%),omega);collect(%,diff
  (omega,r));
```

$$\begin{aligned} ExpansionLenta := {} & \frac{1}{r\left(-r+2m\right)} \left(\left(-2\sqrt{-\frac{-r+2m}{r}}\, r + 4\sqrt{-\frac{-r+2m}{r}}\, m \right.\right. \\ & \left.\left. - r^2\sqrt{-\frac{-r+2m}{r}}\left(\frac{\partial}{\partial r}\nu 0\right) + 2r\sqrt{-\frac{-r+2m}{r}}\left(\frac{\partial}{\partial r}\nu 0\right) m \right)\omega \right) \\ & + \frac{1}{r\left(-r+2m\right)} \left(-\left(\frac{\partial}{\partial r}\omega\right)\sqrt{-\frac{-r+2m}{r}}\, r^2 \right. \\ & \left. + 2r\left(\frac{\partial}{\partial r}\omega\right)\sqrt{-\frac{-r+2m}{r}}\, m - e^{-\nu 0}\left(\frac{\partial}{\partial t}m\right) r \right) \end{aligned}$$

$$\begin{aligned} & \frac{\left(-r^2\sqrt{-\frac{-r+2m}{r}} + 2r\sqrt{-\frac{-r+2m}{r}}\, m\right)\left(\frac{\partial}{\partial r}\omega\right)}{r\left(-r+2m\right)} + \frac{1}{r\left(-r+2m\right)} \left(\left(\right.\right. \\ & -2\sqrt{-\frac{-r+2m}{r}}\, r + 4\sqrt{-\frac{-r+2m}{r}}\, m - r^2\sqrt{-\frac{-r+2m}{r}}\left(\frac{\partial}{\partial r}\nu 0\right) \end{aligned} \tag{2.20}$$

$$+2\,r\,\sqrt{-\frac{-r+2\,m}{r}}\,\left(\frac{\partial}{\partial r}\,\nu 0\right)\,m\Bigg)\,\omega\Bigg)-\frac{e^{-\nu 0}\left(\frac{\partial}{\partial t}\,m\right)}{-r+2\,m}$$

Hacemos que la expansi'on lenta se anule y encontramos la forma del perfil de velocidades

```
> Ecuac := ExpansionLenta = 0;
```

$$Ecuac := \frac{1}{r\,(-r+2\,m)}\Bigg(\Bigg(-2\,\sqrt{-\frac{-r+2\,m}{r}}\,r+4\,\sqrt{-\frac{-r+2\,m}{r}}\,m - r^2\,\sqrt{-\frac{-r+2\,m}{r}}\,\left(\frac{\partial}{\partial r}\,\nu 0\right)+2\,r\,\sqrt{-\frac{-r+2\,m}{r}}\,\left(\frac{\partial}{\partial r}\,\nu 0\right)\,m\Bigg)\,\omega\Bigg) + \frac{1}{r\,(-r+2\,m)}\Bigg(-\left(\frac{\partial}{\partial r}\,\omega\right)\sqrt{-\frac{-r+2\,m}{r}}\,r^2 + 2\,r\,\left(\frac{\partial}{\partial r}\,\omega\right)\sqrt{-\frac{-r+2\,m}{r}}\,m - e^{-\nu 0}\left(\frac{\partial}{\partial t}\,m\right)\,r\Bigg) = 0 \quad \textbf{(2.21)}$$

```
> dsolve(Ecuac,omega);
```

$$\omega = \frac{\left(\int \frac{\left(\frac{\partial}{\partial t}\,m\right)\,r^2}{\sqrt{\frac{r-2\,m}{r}}\,(-r+2\,m)}\,dr + \_F1(t)\right)\,e^{-\nu 0}}{r^2} \quad \textbf{(2.22)}$$

```
> restart:grtw():
```

GRTensorII Version 1.79 (R4)

6 February 2001

Developed by Peter Musgrave, Denis Pollney and Kayll Lake

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Latest version available from: http://grtensor.phy.queensu.ca/ **(2.1.1)**

```
> grOptionMetricPath :=
`/Users/luisnunez/Documents/MisDocumentos/maple/gravitacion/m
etricas/`;
```

grOptionMetricPath := **(2.1.2)**

/Users/luisnunez/Documents/MisDocumentos/maple/gravitacion/metricas/

```
> alias(lambda0 = lambda0(r,t), nu0 = nu0(r,t), K = K(t), m=m
(r,t)):
> qload(esferica);
Calculated ds for esferica (0.000000 sec.)
```

Default spacetime = esferica

For the esferica spacetime:

Coordinates

x(up)

$$x^a = [\; r \;\; \theta \;\; \phi \;\; t \;]$$

Line element

$$ds^2 = -e^{2\lambda 0}\, dr^2 - r^2\, d\theta^2 - r^2 \sin(\theta)^2\, d\phi^2 + e^{2\nu 0}\, dt^2 \tag{2.1.3}$$

Ecuaciones de Einstein

```
> grcalcalter(G(dn, up), G(dn, dn), 1):
Created definition for G(dn,up)
Simplification will be applied during calculation.

Applying routine simplify to object g(dn,dn,pdn)
Calculated g(dn,dn,pdn) for esferica (0.014000 sec.)
Applying routine simplify to object Chr(dn,dn,dn)
Calculated Chr(dn,dn,dn) for esferica (0.005000 sec.)
Applying routine simplify to object g(up,up)
Calculated detg for esferica (0.001000 sec.)
Calculated g(up,up) for esferica (0.008000 sec.)
Applying routine simplify to object Chr(dn,dn,up)
Calculated Chr(dn,dn,up) for esferica (0.008000 sec.)
Applying routine simplify to object R(dn,dn)
Calculated R(dn,dn) for esferica (0.011000 sec.)
Applying routine simplify to object tRicciscalar
Calculated Ricciscalar for esferica (0.009000 sec.)
Applying routine simplify to object G(dn,dn)
Calculated G(dn,dn) for esferica (0.017000 sec.)
Applying routine simplify to object G(dn,up)
Calculated G(dn,up) for esferica (0.007000 sec.)
```

$$CPU\ Time = 0.079 \tag{2.2.1}$$

```
> G00 := grcomponent(G(dn, up), [t, t]): G11 := grcomponent(G
  (dn, up), [r, r]):
> G22 := grcomponent(G(dn, up), [theta, theta]): G01 :=
  grcomponent(G(dn,dn),[t, r]):
> T00 := (rho+omega^2*P+2*omega*q)/(1-omega^2):
> T11 := -(P+omega^2*rho+2*omega*q)/(1-omega^2):
> T01 := -(omega*(rho+P)+q*(1+omega^2))*exp(nu0+lambda0)/(1-
  omega^2):
> T22 := -Pt:
> grcalc(G(dn,dn,cdn));
Created a definition for G(dn,dn,cdn)
Calculated G(dn,dn,cdn) for esferica (0.006000 sec.)
```

$$CPU\ Time = 0.020 \tag{2.2.2}$$

Cambio a la definici'on de masa

```
> lambda0 := -ln(1 -2*m/r)/2;
```

$$\lambda 0 := -\frac{1}{2}\ln\left(1 - \frac{2\,m}{r}\right) \tag{2.2.3}$$

Las ecuaciones de Einstein ser'an

```
> E00 := 8*Pi*simplify(T00) = simplify(G00);
```

$$E00 := -\frac{8\,\pi\left(\rho + \omega^2 P + 2\,\omega\, q\right)}{-1 + \omega^2} = \frac{2\left(\frac{\partial}{\partial r} m\right)}{r^2} \tag{2.2.4}$$

```
> E11 := 8*Pi*simplify(T11) = simplify(G11);
```

(2.2.5)

$$E11 := \frac{8\pi\left(P+\omega^2\rho+2\omega q\right)}{-1+\omega^2} = \frac{2\left(m-r^2\left(\frac{\partial}{\partial r}\nu0\right)+2r\left(\frac{\partial}{\partial r}\nu0\right)m\right)}{r^3} \tag{2.2.5}$$

```
> E01 := 8*Pi*(T01) = simplify(G01);
```

$$E01 := -\frac{8\pi\left(\omega(\rho+P)+q\left(1+\omega^2\right)\right)e^{-\frac{1}{2}\ln\left(1-\frac{2m}{r}\right)+\nu0}}{1-\omega^2} = -\frac{2\left(\frac{\partial}{\partial t}m\right)}{r\,(-r+2m)} \tag{2.2.6}$$

```
> E22 := 8*Pi*T22 = simplify(G22);
```

$$\begin{aligned} E22 := -8\pi Pt = \frac{1}{r^3(-r+2m)^2}\Bigg(&4r^2\left(\frac{\partial}{\partial r}\nu0\right)\left(\frac{\partial}{\partial r}m\right)m^2 \\ &- e^{-2\nu0}\left(\frac{\partial}{\partial t}m\right)r^4\left(\frac{\partial}{\partial t}\nu0\right)+2e^{-2\nu0}\left(\frac{\partial}{\partial t}m\right)r^3\left(\frac{\partial}{\partial t}\nu0\right)m \\ &- 2e^{-2\nu0}r^3\left(\frac{\partial^2}{\partial t^2}m\right)m-4r^3\left(\frac{\partial}{\partial r}\nu0\right)\left(\frac{\partial}{\partial r}m\right)m+r^4\left(\frac{\partial}{\partial r}\nu0\right)\left(\frac{\partial}{\partial r}m\right) \\ &+ 6r^4\left(\frac{\partial}{\partial r}\nu0\right)^2 m-12r^3\left(\frac{\partial}{\partial r}\nu0\right)^2 m^2+8r^2\left(\frac{\partial}{\partial r}\nu0\right)^2 m^3-4m^3 \\ &- r^4\left(\frac{\partial}{\partial r}\nu0\right)-r^2m+4rm^2+\left(\frac{\partial}{\partial r}m\right)r^3-r^5\left(\frac{\partial}{\partial r}\nu0\right)^2-r^5\left(\frac{\partial^2}{\partial r^2}\nu0\right) \\ &+ 3e^{-2\nu0}r^3\left(\frac{\partial}{\partial t}m\right)^2-4\left(\frac{\partial}{\partial r}m\right)r^2m+4\left(\frac{\partial}{\partial r}m\right)rm^2+5r^3\left(\frac{\partial}{\partial r}\nu0\right)m \\ &- 8r^2\left(\frac{\partial}{\partial r}\nu0\right)m^2+4r\left(\frac{\partial}{\partial r}\nu0\right)m^3+6r^4\left(\frac{\partial^2}{\partial r^2}\nu0\right)m \\ &- 12r^3\left(\frac{\partial^2}{\partial r^2}\nu0\right)m^2+8r^2\left(\frac{\partial^2}{\partial r^2}\nu0\right)m^3+e^{-2\nu0}r^4\left(\frac{\partial^2}{\partial t^2}m\right)\Bigg) \end{aligned} \tag{2.2.7}$$

las ecuaciones de Einstein lentas

```
> E00Lenta := 8*Pi*convert(series(T00,omega,2),polynom)=
  simplify(G00);
```

$$E00Lenta := 8\pi\,(\rho+2\omega q) = \frac{2\left(\frac{\partial}{\partial r}m\right)}{r^2} \tag{2.2.8}$$

```
> E11Lenta := 8*Pi*convert(series(T11,omega,2),polynom) =
  simplify(G11);
```

$$E11Lenta := 8\pi\,(-P-2\omega q) = \frac{2\left(m-r^2\left(\frac{\partial}{\partial r}\nu0\right)+2r\left(\frac{\partial}{\partial r}\nu0\right)m\right)}{r^3} \tag{2.2.9}$$

```
> E01Lenta := 8*Pi*convert(series(T01,omega,2),polynom) =
  simplify(G01);
```

$$E01Lenta := 8\pi\left(-q\,e^{-\frac{1}{2}\ln\left(1-\frac{2m}{r}\right)+\nu0}-(\rho+P)\,e^{-\frac{1}{2}\ln\left(1-\frac{2m}{r}\right)+\nu0}\omega\right) = \tag{2.2.10}$$

$$-\frac{2\left(\frac{\partial}{\partial t} m\right)}{r\,(-r+2\,m)}$$

```
> E22Lenta := 8*Pi*convert(series(T22,omega,2),polynom)=
  simplify(G22);
```

$$E22Lenta := -8\,\pi\,Pt = \frac{1}{r^3\,(-r+2\,m)^2}\Bigg(4\,r^2\left(\frac{\partial}{\partial r}\nu 0\right)\left(\frac{\partial}{\partial r} m\right) m^2 - e^{-2\,\nu 0}\left(\frac{\partial}{\partial t} m\right) r^4\left(\frac{\partial}{\partial t}\nu 0\right) + 2\,e^{-2\,\nu 0}\left(\frac{\partial}{\partial t} m\right) r^3\left(\frac{\partial}{\partial t}\nu 0\right) m - 2\,e^{-2\,\nu 0}\,r^3\left(\frac{\partial^2}{\partial t^2} m\right) m - 4\,r^3\left(\frac{\partial}{\partial r}\nu 0\right)\left(\frac{\partial}{\partial r} m\right) m + r^4\left(\frac{\partial}{\partial r}\nu 0\right)\left(\frac{\partial}{\partial r} m\right) + 6\,r^4\left(\frac{\partial}{\partial r}\nu 0\right)^2 m - 12\,r^3\left(\frac{\partial}{\partial r}\nu 0\right)^2 m^2 + 8\,r^2\left(\frac{\partial}{\partial r}\nu 0\right)^2 m^3 - 4\,m^3 - r^4\left(\frac{\partial}{\partial r}\nu 0\right) - r^2\,m + 4\,r\,m^2 + \left(\frac{\partial}{\partial r} m\right) r^3 - r^5\left(\frac{\partial}{\partial r}\nu 0\right)^2 - r^5\left(\frac{\partial^2}{\partial r^2}\nu 0\right) + 3\,e^{-2\,\nu 0}\,r^3\left(\frac{\partial}{\partial t} m\right)^2 - 4\left(\frac{\partial}{\partial r} m\right) r^2\,m + 4\left(\frac{\partial}{\partial r} m\right) r\,m^2 + 5\,r^3\left(\frac{\partial}{\partial r}\nu 0\right) m - 8\,r^2\left(\frac{\partial}{\partial r}\nu 0\right) m^2 + 4\,r\left(\frac{\partial}{\partial r}\nu 0\right) m^3 + 6\,r^4\left(\frac{\partial^2}{\partial r^2}\nu 0\right) m - 12\,r^3\left(\frac{\partial^2}{\partial r^2}\nu 0\right) m^2 + 8\,r^2\left(\frac{\partial^2}{\partial r^2}\nu 0\right) m^3 + e^{-2\,\nu 0}\,r^4\left(\frac{\partial^2}{\partial t^2} m\right)\Bigg) \tag{2.2.11}$$

Usando unas variables efectivas

```
> P := Ptilde -2*omega*q; rho := rhotilde -2*omega*q;
```

$$P := Ptilde - 2\,\omega\,q$$

$$\rho := rhotilde - 2\,\omega\,q \tag{2.2.12}$$

```
> E00Lenta;
```

$$8\,\pi\,rhotilde = \frac{2\left(\frac{\partial}{\partial r} m\right)}{r^2} \tag{2.2.13}$$

```
> E11Lenta;
```

$$-8\,\pi\,Ptilde = \frac{2\left(m - r^2\left(\frac{\partial}{\partial r}\nu 0\right) + 2\,r\left(\frac{\partial}{\partial r}\nu 0\right) m\right)}{r^3} \tag{2.2.14}$$

```
> q:=solve(simplify(E01Lenta),q);
```

$$q := \frac{1}{4}\,\frac{1}{\pi\,e^{\nu 0}\,r\left(r - 2\,m - 4\,r\,\omega^2 + 8\,\omega^2\,m\right)}\Big(-4\,\pi\,e^{\nu 0}\,r^2\,\omega\,rhotilde \tag{2.2.15}$$

$$+ 8\,\pi\,e^{\nu 0}\,r\,\omega\,rhotilde\,m - 4\,\pi\,e^{\nu 0}\,r^2\,\omega\,Ptilde + 8\,\pi\,e^{\nu 0}\,r\,\omega\,Ptilde\,m - \left(\frac{\partial}{\partial t}\,m\right)\sqrt{-\frac{-r+2\,m}{r}}\Bigg)$$

```
> qLenta := subs(omega^2=0,simplify(q));
```

$$qLenta := \frac{1}{4}\,\frac{1}{\pi\,r\,(r-2\,m)}\Bigg(\bigg(-4\,\pi\,e^{\nu 0}\,r^2\,\omega\,rhotilde + 8\,\pi\,e^{\nu 0}\,r\,\omega\,rhotilde\,m - 4\,\pi\,e^{\nu 0}\,r^2\,\omega\,Ptilde + 8\,\pi\,e^{\nu 0}\,r\,\omega\,Ptilde\,m - \left(\frac{\partial}{\partial t}\,m\right)\sqrt{-\frac{-r+2\,m}{r}}\bigg)\,e^{-\nu 0}\Bigg) \tag{2.2.16}$$

```
> #omega := (int((diff(m, t))*r^2/(sqrt((r-2*m)/r)*(-r+2*m)),
  r)+_F1(t))*exp(-nu0)/r^2;
> omega := (int(-(diff(lambda0, t))*r^2*exp(lambda0), r)+_F1(t)
  )*exp(-nu0)/r^2;
```

$$\omega := \frac{\left(\int\left(-\frac{\left(\frac{\partial}{\partial t}\,m\right)r}{\left(1-\frac{2\,m}{r}\right)^{3/2}}\right)dr + \_F1(t)\right)e^{-\nu 0}}{r^2} \tag{2.2.17}$$

```
> simplify(qLenta);
```

$$-\frac{1}{4}\,\frac{1}{r^2\,\pi\,(-r+2\,m)}\Bigg(\bigg(-4\,\pi\,rhotilde\,r\left(\int\frac{\left(\frac{\partial}{\partial t}\,m\right)r^2}{(-r+2\,m)\sqrt{-\frac{-r+2\,m}{r}}}\,dr\right) - 4\,\pi\,rhotilde\,r\,\_F1(t) + 8\,\pi\,rhotilde\,m\left(\int\frac{\left(\frac{\partial}{\partial t}\,m\right)r^2}{(-r+2\,m)\sqrt{-\frac{-r+2\,m}{r}}}\,dr\right) + 8\,\pi\,rhotilde\,m\,\_F1(t) - 4\,\pi\,Ptilde\,r\left(\int\frac{\left(\frac{\partial}{\partial t}\,m\right)r^2}{(-r+2\,m)\sqrt{-\frac{-r+2\,m}{r}}}\,dr\right) - 4\,\pi\,Ptilde\,r\,\_F1(t) + 8\,\pi\,Ptilde\,m\left(\int\frac{\left(\frac{\partial}{\partial t}\,m\right)r^2}{(-r+2\,m)\sqrt{-\frac{-r+2\,m}{r}}}\,dr\right) + 8\,\pi\,Ptilde\,m\,\_F1(t) - \left(\frac{\partial}{\partial t}\,m\right)\sqrt{-\frac{-r+2\,m}{r}}\,r\bigg)\,e^{-\nu 0}\Bigg) \tag{2.2.18}$$