

Colapso lento..

El elemento de línea en coordenadas de Scharszchild es el siguiente:

$$ds^2 = e^{2\nu(t,r)} dt^2 - e^{2\lambda(t,r)} dr^2 - r^2 (d\theta^2 + \sin^2(\theta) d\phi^2)$$

> **restart:grtw():**

GRTensorII Version 1.79 (R4)

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Developed by Peter Musgrave, Denis Pollney and Kayll Lake

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(1.1)

> **grOptionMetricPath :=**

`/Users/luisnunez/Documents/MisDocumentos/maple/gravitacion/metricas/`;

grOptionMetricPath :=

(1.2)

/Users/luisnunez/Documents/MisDocumentos/maple/gravitacion/metricas/

> **alias(lambda0 = lambda0(r,t), nu0 = nu0(r,t), K = K(t), m=m(r,t)):**

> **gload(esferica);**

Calculated ds for esfera (0.001000 sec.)

Default spacetime = esfera

For the esfera spacetime:

Coordinates

x(up)

$$x^a = \begin{bmatrix} r & \theta & \phi & t \end{bmatrix}$$

Line element

$$ds^2 = -e^{2\lambda_0} dr^2 - r^2 d\theta^2 - r^2 \sin^2(\theta) d\phi^2 + e^{2\nu_0} dt^2$$

(1.3)

Ecuaciones de Einstein

> **grcalcalter(G(dn, up), G(dn, dn), 1):**

Created definition for G(dn,up)

Simplification will be applied during calculation.

Applying routine simplify to object g(dn,dn,pdn)

Calculated g(dn,dn,pdn) for esfera (0.015000 sec.)

Applying routine simplify to object Chr(dn,dn,dn)

Calculated Chr(dn,dn,dn) for esfera (0.008000 sec.)

Applying routine simplify to object g(up,up)

Calculated detg for esfera (0.002000 sec.)

Calculated g(up,up) for esfera (0.011000 sec.)

Applying routine simplify to object Chr(dn,dn,up)

Calculated Chr(dn,dn,up) for esfera (0.010000 sec.)

Applying routine simplify to object R(dn,dn)

Calculated R(dn,dn) for esfera (0.013000 sec.)

Applying routine simplify to object tRicciscalar

Calculated Ricciscalar for esfera (0.009000 sec.)

Applying routine simplify to object G(dn,dn)
 Calculated G(dn,dn) for esferica (0.018000 sec.)
 Applying routine simplify to object G(dn,up)
 Calculated G(dn,up) for esferica (0.006000 sec.)
 CPU Time = 0.091

(2.1)

```
> G00 := grcomponent(G(dn, up), [t, t]): G11 := grcomponent(G(dn,
up), [r, r]):
> G22 := grcomponent(G(dn, up), [theta, theta]): G01 :=
grcomponent(G(dn,dn),[t, r]):
> T00 := (rho+omega^2*P+2*omega*q)/(1-omega^2):
> T11 := -(P+omega^2*rho+2*omega*q)/(1-omega^2):
> T01 := -(omega*(rho+P)+q*(1+omega^2))*exp(nu0+lambda0)/(1-
omega^2):
> T22 := -Pt:
> grcalc(G(dn,dn,cdn));
Created a definition for G(dn,dn,cdn)
Calculated G(dn,dn,cdn) for esferica (0.007000 sec.)
CPU Time = 0.020
```

(2.2)

Cambio a la definici'on de masa

```
> #lambda0 := -ln(1 -2*m/r)/2;
```

Las ecuaciones de Einstein ser'an

```
> E00 := 8*Pi*simplify(T00) = simplify(G00);
```

$$E00 := - \frac{8 \pi (\rho + \omega^2 P + 2 \omega q)}{-1 + \omega^2} \quad (2.3)$$

$$= \frac{e^{-2 \nu 0} \left(2 e^{-2 \lambda 0 + 2 \nu 0} \left(\frac{\partial}{\partial r} \lambda 0 \right) r + e^{2 \nu 0} - e^{-2 \lambda 0 + 2 \nu 0} \right)}{r^2}$$

```
> E11 := 8*Pi*simplify(T11) = simplify(G11);
```

$$E11 := \frac{8 \pi (P + \omega^2 \rho + 2 \omega q)}{-1 + \omega^2} = \frac{e^{-2 \lambda 0} \left(e^{2 \lambda 0} - 1 - 2 r \left(\frac{\partial}{\partial r} \nu 0 \right) \right)}{r^2} \quad (2.4)$$

```
> E01 := 8*Pi*(T01) = simplify(G01);
```

$$E01 := - \frac{8 \pi (\omega (\rho + P) + q (1 + \omega^2)) e^{\lambda 0 + \nu 0}}{1 - \omega^2} = \frac{2 \left(\frac{\partial}{\partial t} \lambda 0 \right)}{r} \quad (2.5)$$

```
> E22 := 8*Pi*T22 = simplify(G22);
```

$$E22 := -8 \pi P t = \frac{1}{r} \left(\left(\frac{\partial}{\partial r} \lambda 0 \right) e^{-2 \lambda 0} - e^{-2 \lambda 0} \left(\frac{\partial}{\partial r} \nu 0 \right) + e^{-2 \nu 0} r \left(\frac{\partial^2}{\partial t^2} \lambda 0 \right) \right. \\ \left. - e^{-2 \nu 0} r \left(\frac{\partial}{\partial t} \lambda 0 \right) \left(\frac{\partial}{\partial t} \nu 0 \right) + e^{-2 \nu 0} r \left(\frac{\partial}{\partial t} \lambda 0 \right)^2 - e^{-2 \lambda 0} r \left(\frac{\partial^2}{\partial r^2} \nu 0 \right) \right. \\ \left. + e^{-2 \lambda 0} r \left(\frac{\partial}{\partial r} \lambda 0 \right) \left(\frac{\partial}{\partial r} \nu 0 \right) - e^{-2 \lambda 0} r \left(\frac{\partial}{\partial r} \nu 0 \right)^2 \right) \quad (2.6)$$

las ecuaciones de Einstein lentas

```
> E00Lenta := 8*Pi*convert(series(T00,omega,2),polynom)= simplify
```

(G00);

$$E00Lenta := 8 \pi (\rho + 2 \omega q)$$

$$= \frac{e^{-2 \nu 0} \left(2 e^{-2 \lambda 0 + 2 \nu 0} \left(\frac{\partial}{\partial r} \lambda 0 \right) r + e^{2 \nu 0} - e^{-2 \lambda 0 + 2 \nu 0} \right)}{r^2}$$

> E11Lenta := 8*Pi*convert(series(T11,omega,2),polynom) = simplify(G11);

$$E11Lenta := 8 \pi (-P - 2 \omega q) = \frac{e^{-2 \lambda 0} \left(e^{2 \lambda 0} - 1 - 2 r \left(\frac{\partial}{\partial r} \nu 0 \right) \right)}{r^2}$$

> E01Lenta := 8*Pi*convert(series(T01,omega,2),polynom) = simplify(G01);

$$E01Lenta := 8 \pi (-q e^{\lambda 0 + \nu 0} - (\rho + P) e^{\lambda 0 + \nu 0} \omega) = \frac{2 \left(\frac{\partial}{\partial t} \lambda 0 \right)}{r}$$

> E22Lenta := 8*Pi*convert(series(T22,omega,2),polynom)= simplify(G22);

$$E22Lenta := -8 \pi P t = \frac{1}{r} \left(\left(\frac{\partial}{\partial r} \lambda 0 \right) e^{-2 \lambda 0} - e^{-2 \lambda 0} \left(\frac{\partial}{\partial r} \nu 0 \right) + e^{-2 \nu 0} r \left(\frac{\partial^2}{\partial t^2} \lambda 0 \right) \right.$$

$$\left. - e^{-2 \nu 0} r \left(\frac{\partial}{\partial t} \lambda 0 \right) \left(\frac{\partial}{\partial t} \nu 0 \right) + e^{-2 \nu 0} r \left(\frac{\partial}{\partial t} \lambda 0 \right)^2 - e^{-2 \lambda 0} r \left(\frac{\partial^2}{\partial r^2} \nu 0 \right) \right.$$

$$\left. + e^{-2 \lambda 0} r \left(\frac{\partial}{\partial r} \lambda 0 \right) \left(\frac{\partial}{\partial r} \nu 0 \right) - e^{-2 \lambda 0} r \left(\frac{\partial}{\partial r} \nu 0 \right)^2 \right)$$

Usando unas variables efectivas

> P := Ptilde -2*omega*q; rho := rhotilde -2*omega*q;

$$P := Ptilde - 2 \omega q$$

$$\rho := rhotilde - 2 \omega q$$

> E00Lenta;

$$8 \pi rhotilde = \frac{e^{-2 \nu 0} \left(2 e^{-2 \lambda 0 + 2 \nu 0} \left(\frac{\partial}{\partial r} \lambda 0 \right) r + e^{2 \nu 0} - e^{-2 \lambda 0 + 2 \nu 0} \right)}{r^2}$$

> E11Lenta;

$$-8 \pi Ptilde = \frac{e^{-2 \lambda 0} \left(e^{2 \lambda 0} - 1 - 2 r \left(\frac{\partial}{\partial r} \nu 0 \right) \right)}{r^2}$$

> q:=solve(simplify(E01Lenta),q);

$$q := \frac{1}{4} \frac{4 \pi e^{\lambda 0 + \nu 0} r \omega rhotilde + 4 \pi e^{\lambda 0 + \nu 0} r \omega Ptilde + \frac{\partial}{\partial t} \lambda 0}{\pi e^{\lambda 0 + \nu 0} r (-1 + 4 \omega^2)}$$

> qLenta := subs(omega^2=0,simplify(q));

$$qLenta := -\frac{1}{4} \frac{\left(4 \pi e^{\lambda 0 + v 0} r \omega \text{rhotilde} + 4 \pi e^{\lambda 0 + v 0} r \omega \text{Ptilde} + \frac{\partial}{\partial t} \lambda 0 \right) e^{-\lambda 0 - v 0}}{\pi r} \quad (2.15)$$

Como siempre nos sobra una funcion. En este caso omega. Para encontrarla tenemos que suponer algo adicional

Para ello vamos a seguir suponiendo un colapso lento pero echamos mano de algunas funciones cinemáticas.

```
> alias(omega = omega(r,t));
```

$$\lambda 0, v 0, K, m, \omega \quad (2.16)$$

```
> grdef( `u{^a} := [ omega*exp(-lambda0)/sqrt(1-omega^2), 0, 0,
exp(-nu0)/sqrt(1-omega^2) ] );
Components assigned for metric: esferica
Created definition for u(up)
> grcalc(expsc[u]);
Created a definition for u(up,cdn)
Calculated u(up,cdn) for esferica (0.002000 sec.)
Calculated expsc[u] for esferica (0.001000 sec.)
CPU Time = 0.014 \quad (2.17)
```

```
> Expansion:=grcomponent(expsc[u]);
```

$$\text{Expansion} := \frac{1}{(1 - \omega^2)^{3/2} r} \left(r \left(\frac{\partial}{\partial r} \omega \right) e^{-\lambda 0} + r e^{-v 0} \left(\frac{\partial}{\partial t} \lambda 0 \right) - r e^{-v 0} \left(\frac{\partial}{\partial t} \lambda 0 \right) \omega^2 \right. \\ \left. + 2 \omega e^{-\lambda 0} - 2 \omega^3 e^{-\lambda 0} + \omega r e^{-v 0} \left(\frac{\partial}{\partial t} \omega \right) + \omega r e^{-\lambda 0} \left(\frac{\partial}{\partial r} v 0 \right) \right. \\ \left. - \omega^3 r e^{-\lambda 0} \left(\frac{\partial}{\partial r} v 0 \right) \right) \quad (2.18)$$

```
> subs(omega^3=0,Expansion):subs(omega^2=0,%):subs(diff(omega,t)=0,%):
> ExpansionLenta := collect(simplify(%),omega);collect(%,diff(omega,r));
```

$$\text{ExpansionLenta} := \frac{\left(2 e^{-\lambda 0} + r e^{-\lambda 0} \left(\frac{\partial}{\partial r} v 0 \right) \right) \omega}{r} \\ + \frac{r \left(\frac{\partial}{\partial r} \omega \right) e^{-\lambda 0} + r e^{-v 0} \left(\frac{\partial}{\partial t} \lambda 0 \right)}{r} \\ \left(\frac{\partial}{\partial r} \omega \right) e^{-\lambda 0} + \frac{\left(2 e^{-\lambda 0} + r e^{-\lambda 0} \left(\frac{\partial}{\partial r} v 0 \right) \right) \omega}{r} + e^{-v 0} \left(\frac{\partial}{\partial t} \lambda 0 \right) \quad (2.19)$$

Hacemos que la expansión lenta se anule y encontramos la forma del perfil de velocidades

```
> Ecuac := ExpansionLenta = 0;
```

$$\text{Ecuac} := \frac{\left(2 e^{-\lambda 0} + r e^{-\lambda 0} \left(\frac{\partial}{\partial r} v 0 \right) \right) \omega}{r} + \frac{r \left(\frac{\partial}{\partial r} \omega \right) e^{-\lambda 0} + r e^{-v 0} \left(\frac{\partial}{\partial t} \lambda 0 \right)}{r} = 0 \quad (2.20)$$

```
> dsolve(Ecuac,omega);
```

$$\omega = \frac{\left(\int \left(- \left(\frac{\partial}{\partial t} \lambda_0 \right) r^2 e^{\lambda_0} \right) dr + _F1(t) \right) e^{-v_0}}{r^2} \quad (2.21)$$

```
> restart:grtw():
```

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```
> grOptionMetricPath :=  
`/Users/luisnunez/Documents/MisDocumentos/maple/gravitacion/m  
etricas/`;
```

```
grOptionMetricPath := (2.1.2)
```

```
/Users/luisnunez/Documents/MisDocumentos/maple/gravitacion/metricas/
```

```
> alias(lambda0 = lambda0(r,t), nu0 = nu0(r,t), K = K(t), m=m  
(r,t));
```

```
> qload(esferica);
```

```
Default metric is now esferaica.
```

Ecuaciones de Einstein

```
> grcalcalter(G(dn, up), G(dn, dn), 1):
```

```
Created definition for G(dn,up)
```

```
Simplification will be applied during calculation.
```

```
Applying routine simplify to object g(dn,dn,pdn)
```

```
Calculated g(dn,dn,pdn) for esferaica (0.015000 sec.)
```

```
Applying routine simplify to object Chr(dn,dn,dn)
```

```
Calculated Chr(dn,dn,dn) for esferaica (0.006000 sec.)
```

```
Applying routine simplify to object g(up,up)
```

```
Calculated detg for esferaica (0.001000 sec.)
```

```
Calculated g(up,up) for esferaica (0.008000 sec.)
```

```
Applying routine simplify to object Chr(dn,dn,up)
```

```
Calculated Chr(dn,dn,up) for esferaica (0.010000 sec.)
```

```
Applying routine simplify to object R(dn,dn)
```

```
Calculated R(dn,dn) for esferaica (0.012000 sec.)
```

```
Applying routine simplify to object tRicciscalar
```

```
Calculated Ricciscalar for esferaica (0.008000 sec.)
```

```
Applying routine simplify to object G(dn,dn)
```

```
Calculated G(dn,dn) for esferaica (0.019000 sec.)
```

```
Applying routine simplify to object G(dn,up)
```

```
Calculated G(dn,up) for esferaica (0.004000 sec.)
```

```
CPU Time = 0.084
```

(2.2.1)

```
> G00 := grcomponent(G(dn, up), [t, t]): G11 := grcomponent(G  
(dn, up), [r, r]):
```

```
> G22 := grcomponent(G(dn, up), [theta, theta]): G01 :=  
grcomponent(G(dn,dn),[t, r]):
```

```
> T00 := (rho+omega^2*P+2*omega*q)/(1-omega^2):
```

```
> T11 := -(P+omega^2*rho+2*omega*q)/(1-omega^2):
```

```

> T01 := -(omega*(rho+P)+q*(1+omega^2))*exp(nu0+lambda0)/(1-
omega^2);
> T22 := -Pt;
> grcalc(G(dn,dn,cdn));
Created a definition for G(dn,dn,cdn)
Calculated G(dn,dn,cdn) for esferica (0.007000 sec.)
CPU Time = 0.020

```

(2.2.2)

Cambio a la definici'on de masa

```

> #lambda0 := -ln(1 -2*m/r)/2;

```

Las ecuaciones de Einstein ser'an

```

> E00 := 8*Pi*simplify(T00) = simplify(G00);

```

$$E00 := -\frac{8\pi(\rho + \omega^2 P + 2\omega q)}{-1 + \omega^2} \quad (2.2.3)$$

$$= \frac{e^{-2\nu_0} \left(2e^{-2\lambda_0 + 2\nu_0} \left(\frac{\partial}{\partial r} \lambda_0 \right) r + e^{2\nu_0} - e^{-2\lambda_0 + 2\nu_0} \right)}{r^2}$$

```

> E11 := 8*Pi*simplify(T11) = simplify(G11);

```

$$E11 := \frac{8\pi(P + \omega^2 \rho + 2\omega q)}{-1 + \omega^2} = \frac{e^{-2\lambda_0} \left(e^{2\lambda_0} - 1 - 2 \left(\frac{\partial}{\partial r} \nu_0 \right) r \right)}{r^2} \quad (2.2.4)$$

```

> E01 := 8*Pi*(T01) = simplify(G01);

```

$$E01 := -\frac{8\pi(\omega(\rho + P) + q(1 + \omega^2))e^{\lambda_0 + \nu_0}}{1 - \omega^2} = \frac{2 \left(\frac{\partial}{\partial t} \lambda_0 \right)}{r} \quad (2.2.5)$$

```

> E22 := 8*Pi*T22 = simplify(G22);

```

$$E22 := -8\pi Pt = \frac{1}{r} \left(\left(\frac{\partial}{\partial r} \lambda_0 \right) e^{-2\lambda_0} - e^{-2\lambda_0} \left(\frac{\partial}{\partial r} \nu_0 \right) + e^{-2\nu_0} r \left(\frac{\partial^2}{\partial t^2} \lambda_0 \right) \right. \\ \left. - e^{-2\nu_0} r \left(\frac{\partial}{\partial t} \lambda_0 \right) \left(\frac{\partial}{\partial t} \nu_0 \right) + e^{-2\nu_0} r \left(\frac{\partial}{\partial t} \lambda_0 \right)^2 - e^{-2\lambda_0} r \left(\frac{\partial^2}{\partial r^2} \nu_0 \right) \right. \\ \left. + e^{-2\lambda_0} r \left(\frac{\partial}{\partial r} \lambda_0 \right) \left(\frac{\partial}{\partial r} \nu_0 \right) - e^{-2\lambda_0} r \left(\frac{\partial}{\partial r} \nu_0 \right)^2 \right) \quad (2.2.6)$$

las ecuaciones de Einstein lentas

```

> E00Lenta := 8*Pi*convert(series(T00,omega,2),polynom)=
simplify(G00);

```

$$E00Lenta := 8\pi(\rho + 2\omega q) \quad (2.2.7)$$

$$= \frac{e^{-2\nu_0} \left(2e^{-2\lambda_0 + 2\nu_0} \left(\frac{\partial}{\partial r} \lambda_0 \right) r + e^{2\nu_0} - e^{-2\lambda_0 + 2\nu_0} \right)}{r^2}$$

```

> E11Lenta := 8*Pi*convert(series(T11,omega,2),polynom) =
simplify(G11);

```

(2.2.8)

$$E11Lenta := 8 \pi (-P - 2 \omega q) = \frac{e^{-2 \lambda 0} \left(e^{2 \lambda 0} - 1 - 2 \left(\frac{\partial}{\partial r} v 0 \right) r \right)}{r^2} \quad (2.2.8)$$

> **E01Lenta := 8*Pi*convert(series(T01,omega,2),polynom) = simplify(G01);**

$$E01Lenta := 8 \pi (-q e^{\lambda 0 + v 0} - (\rho + P) e^{\lambda 0 + v 0} \omega) = \frac{2 \left(\frac{\partial}{\partial t} \lambda 0 \right)}{r} \quad (2.2.9)$$

> **E22Lenta := 8*Pi*convert(series(T22,omega,2),polynom)=simplify(G22);**

$$E22Lenta := -8 \pi P t = \frac{1}{r} \left(\left(\frac{\partial}{\partial r} \lambda 0 \right) e^{-2 \lambda 0} - e^{-2 \lambda 0} \left(\frac{\partial}{\partial r} v 0 \right) + e^{-2 v 0} r \left(\frac{\partial^2}{\partial t^2} \lambda 0 \right) \right. \\ \left. - e^{-2 v 0} r \left(\frac{\partial}{\partial t} \lambda 0 \right) \left(\frac{\partial}{\partial t} v 0 \right) + e^{-2 v 0} r \left(\frac{\partial}{\partial t} \lambda 0 \right)^2 - e^{-2 \lambda 0} r \left(\frac{\partial^2}{\partial r^2} v 0 \right) \right. \\ \left. + e^{-2 \lambda 0} r \left(\frac{\partial}{\partial r} \lambda 0 \right) \left(\frac{\partial}{\partial r} v 0 \right) - e^{-2 \lambda 0} r \left(\frac{\partial}{\partial r} v 0 \right)^2 \right) \quad (2.2.10)$$

Usando unas variables efectivas

> **P := Ptilde -2*omega*q; rho := rhotilde -2*omega*q;**

$$P := Ptilde - 2 \omega q$$

$$\rho := rhotilde - 2 \omega q$$

(2.2.11)

> **E00Lenta;**

$$8 \pi rhotilde = \frac{e^{-2 v 0} \left(2 e^{-2 \lambda 0 + 2 v 0} \left(\frac{\partial}{\partial r} \lambda 0 \right) r + e^{2 v 0} - e^{-2 \lambda 0 + 2 v 0} \right)}{r^2} \quad (2.2.12)$$

> **E11Lenta;**

$$-8 \pi Ptilde = \frac{e^{-2 \lambda 0} \left(e^{2 \lambda 0} - 1 - 2 \left(\frac{\partial}{\partial r} v 0 \right) r \right)}{r^2} \quad (2.2.13)$$

> **q:=solve(simplify(E01Lenta),q);**

$$q := \frac{1}{4} \frac{4 \pi e^{\lambda 0 + v 0} r \omega rhotilde + 4 \pi e^{\lambda 0 + v 0} r \omega Ptilde + \frac{\partial}{\partial t} \lambda 0}{\pi e^{\lambda 0 + v 0} r (-1 + 4 \omega^2)} \quad (2.2.14)$$

> **qLenta := subs(omega^2=0,simplify(q));**

qLenta := (2.2.15)

$$-\frac{1}{4} \frac{\left(4 \pi e^{\lambda 0 + v 0} r \omega rhotilde + 4 \pi e^{\lambda 0 + v 0} r \omega Ptilde + \frac{\partial}{\partial t} \lambda 0 \right) e^{-\lambda 0 - v 0}}{\pi r}$$

> **#omega := (int((diff(m, t))*r^2/(sqrt((r-2*m)/r)*(-r+2*m)), r)+_F1(t))*exp(-nu0)/r^2;**

> **omega := (int(-(diff(lambda0, t))*r^2*exp(lambda0), r)+_F1(t))*exp(-nu0)/r^2;**

$$\omega := \frac{\left(\int \left(-\left(\frac{\partial}{\partial t} \lambda 0\right) r^2 e^{\lambda 0}\right) dr + {}_F1(t)\right) e^{-v 0}}{r^2} \quad (2.2.16)$$

> simplify(qLenta);

$$\begin{aligned} & -\frac{1}{4} \frac{1}{r^2 \pi} \left(\left(-4 \pi e^{\lambda 0} \text{rhotilde} \left(\int \left(\frac{\partial}{\partial t} \lambda 0 \right) r^2 e^{\lambda 0} dr \right) + 4 \pi e^{\lambda 0} \text{rhotilde} {}_F1(t) \right. \right. \\ & \quad \left. \left. - 4 \pi e^{\lambda 0} \text{Ptilde} \left(\int \left(\frac{\partial}{\partial t} \lambda 0 \right) r^2 e^{\lambda 0} dr \right) + 4 \pi e^{\lambda 0} \text{Ptilde} {}_F1(t) + r \left(\frac{\partial}{\partial t} \lambda 0 \right) \right) \right. \\ & \quad \left. e^{-\lambda 0 - v 0} \right) \end{aligned} \quad (2.2.17)$$