

```
> restart;
> TOV := (1/4)*((diff(m(r), r))*r+2*(diff(Phi(r), r, r))*r^2*m(r)
-3*(diff(Phi(r), r))*r*m(r)+3*(diff(Phi(r), r))*r^2*(diff(m(r),
r))-(diff(Phi(r), r, r))*r^3+(diff(Phi(r), r))*r^2-3*m(r)+2*(diff
(Phi(r), r))^2*r^2*m(r)-(diff(Phi(r), r))^2*r^3)/(r^4*Pi)= 0;
```

$$TOV := \frac{1}{4} \frac{1}{r^4 \pi} \left(\left(\frac{d}{dr} m(r) \right) r + 2 \left(\frac{d^2}{dr^2} \Phi(r) \right) r^2 m(r) - 3 \left(\frac{d}{dr} \Phi(r) \right) r m(r) \right. \\ \left. + 3 \left(\frac{d}{dr} \Phi(r) \right) r^2 \left(\frac{d}{dr} m(r) \right) - \left(\frac{d^2}{dr^2} \Phi(r) \right) r^3 + \left(\frac{d}{dr} \Phi(r) \right) r^2 - 3 m(r) \right. \\ \left. + 2 \left(\frac{d}{dr} \Phi(r) \right)^2 r^2 m(r) - \left(\frac{d}{dr} \Phi(r) \right)^2 r^3 \right) = 0 \quad (1)$$

```
> Phi := r -> (1/2)*N*ln(1 + alpha*r^2/R^2);
Phi := r -> \frac{1}{2} N \ln \left( 1 + \frac{\alpha r^2}{R^2} \right) \quad (2)
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```
> simplify(TOV);
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$$\frac{1}{4} \frac{1}{(R^2 + \alpha r^2)^2 r^4 \pi} \left(\left(\frac{d}{dr} m(r) \right) r R^4 + 2 \left(\frac{d}{dr} m(r) \right) r^3 R^2 \alpha + \left(\frac{d}{dr} m(r) \right) r^5 \alpha^2 \right. \\ \left. - N \alpha r^2 m(r) R^2 - 5 N \alpha^2 r^4 m(r) + 3 N \alpha r^3 \left(\frac{d}{dr} m(r) \right) R^2 \right. \\ \left. + 3 N \alpha^2 r^5 \left(\frac{d}{dr} m(r) \right) + 2 N \alpha^2 r^5 - 3 m(r) R^4 - 6 m(r) R^2 \alpha r^2 - 3 m(r) \alpha^2 r^4 \right. \\ \left. + 2 N^2 \alpha^2 r^4 m(r) - N^2 \alpha^2 r^5 \right) = 0 \quad (3)$$

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> with(DEtools, adjoint, firint, intfactor, redode, mutest,
odeadvisor):
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> #el factor integrador de esta ecuacion es
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```
> mu := intfactor(TOV,m(r));
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$$\mu := (R^2 + \alpha r^2 + 3 N \alpha r^2)^{-\frac{1}{3(1+3N)}} (R^2 + \alpha r^2 + 3 N \alpha r^2)^{\frac{2}{3} \frac{N}{1+3N}} (R^2 + \alpha r^2)^1 \\ /3 (R^2 + \alpha r^2)^{\frac{1}{3} N} \quad (4)$$

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> mutest(mu, TOV);
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$$0 \quad (5)$$

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> # y la solucion sera
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```
> firint(mu*TOV);
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$$\frac{1}{(R^2 + \alpha r^2)^{5/3} r^4} \left((R^2 + \alpha r^2)^{\frac{1}{3} N} (R^2 + \alpha r^2 + 3 N \alpha r^2)^{\frac{2}{3} \frac{N}{1+3N}} (R^2 + \alpha r^2 \right. \\ \left. + 3 N \alpha r^2)^{-\frac{1}{3(1+3N)}} (r R^4 + 2 r^3 R^2 \alpha + r^5 \alpha^2 + 3 N \alpha r^3 R^2 + 3 N \alpha^2 r^5) m(r) \right) \quad (6)$$

$$+ \int \left(-\frac{1}{(R^2 + \alpha r^2)^{8/3}} \left((R^2 + \alpha r^2 + 3 N \alpha r^2)^{-\frac{1}{3(1+3N)}} (R^2 + \alpha r^2 + 3 N \alpha r^2)^{\frac{2}{3} \frac{N}{1+3N}} (R^2 + \alpha r^2)^{\frac{1}{3} N} \alpha^2 N r (R^2 N + N \alpha r^2 - 2 R^2 - 2 \alpha r^2) \right) \right) dr$$

$$+ _CI = 0$$

> #el integrando se puede escribir

> Integrando := -(R^2+alpha*r^2+3*N*alpha*r^2)^(-1/(3*(1+3*N)))*
(R^2+alpha*r^2+3*N*alpha*r^2)^((2/3)*N/(1+3*N))*(R^2+alpha*r^2)^(
(1/3)*N)*alpha^2*N*r*(R^2*N+N*alpha*r^2-2*R^2-2*alpha*r^2)/(R^2+
alpha*r^2)^(8/3);

$$Integrando := -\frac{1}{(R^2 + \alpha r^2)^{8/3}} \left((R^2 + \alpha r^2 + 3 N \alpha r^2)^{-\frac{1}{3+9N}} (R^2 + \alpha r^2 + 3 N \alpha r^2)^{\frac{2}{3} \frac{N}{1+3N}} (R^2 + \alpha r^2)^{\frac{1}{3} N} \alpha^2 N r (R^2 N + N \alpha r^2 - 2 R^2 - 2 \alpha r^2) \right) \quad (7)$$

> simplify(Integrando);

$$-(N-2) (R^2 + \alpha r^2 + 3 N \alpha r^2)^{\frac{1}{3} \frac{-1+2N}{1+3N}} (R^2 + \alpha r^2)^{\frac{1}{3} N - \frac{5}{3}} \alpha^2 N r \quad (8)$$

> # esta integral se puede poner de la forma w^sigma*(w + K)^beta;

> #

> integrando2 := w^sigma*(w + K)^beta;

$$integrando2 := w^\sigma (w + K)^\beta \quad (9)$$

> assume(beta > 0, w > 0, sigma > -5/3, K < 0, N > 0);

> # y es integrable para

> Integral := int(integrando2, w);

$$Integral := \frac{1}{1 + \sigma} \left((-1)^{-\beta} K^{\beta} \left(-\text{signum} \left(1 + \frac{w}{K} \right) \right)^{\beta} \text{signum} \left(1 + \frac{w}{K} \right) \right. \\ \left. w^{1 + \sigma} \text{hypergeom} \left([-\beta, 1 + \sigma], [2 + \sigma], -\frac{w}{K} \right) \right) \quad (10)$$

> K := A - B; A := 1/(alpha*(3*N+1)); B := 1/alpha; simplify(K);

$$K := A - B$$

$$A := \frac{1}{\alpha (1 + 3 N)}$$

$$B := \frac{1}{\alpha}$$

$$- \frac{3 N}{\alpha (1 + 3 N)}$$

(11)

> w := u + B; u := (r/R)^2; simplify(w);

$$w := u + \frac{1}{\alpha}$$

$$u := \frac{r^2}{R^2}$$

$$\frac{R^2 + \alpha r^2}{R^2 \alpha} \quad (12)$$

> **beta := (2*N -1)/(9*N +3); sigma := (N-5)/3;**

$$\beta := \frac{-1 + 2 N}{3 + 9 N}$$

$$\sigma := \frac{1}{3} N - \frac{5}{3} \quad (13)$$

> **Integralfull:= simplify(Gammal*Integral/2);**

$$Integralfull := \frac{1}{2} \frac{1}{1 + \sigma} \left(\Gamma I (-1)^{-\beta} K^{\beta} \text{signum}(w + K) (-\text{signum}(w + K)) \right. \\ \left. - w^{1 + \sigma} \text{hypergeom} \left([-\beta, 1 + \sigma], [2 + \sigma], -\frac{w}{K} \right) \right) \quad (14)$$

> **Gammal:= -N*(N-2)*alpha^(2+sigma+beta)*R^(2+2*sigma+2*beta)*(3*N+1)^beta; simplify(%);**

$$\Gamma I := -N (N - 2) \alpha^{\frac{1}{3} + \frac{1}{3} N + \frac{-1 + 2 N}{3 + 9 N}} R^{-\frac{4}{3} + \frac{2}{3} N + \frac{2(-1 + 2 N)}{3 + 9 N}} (1 \\ + 3 N) \frac{-1 + 2 N}{3 + 9 N} \\ - N (N - 2) \alpha^{\frac{N(2 + N)}{1 + 3 N}} R^{\frac{2(-1 - N + N^2)}{1 + 3 N}} (1 + 3 N) \frac{1}{3} \frac{-1 + 2 N}{1 + 3 N} \quad (15)$$

> **simplify(Integralfull);**

$$\frac{1}{2} \frac{1}{1 + \sigma} \left((-1)^{1 - \beta} N (N - 2) \alpha^{\frac{N(2 + N)}{1 + 3 N}} R^{\frac{2(-1 - N + N^2)}{1 + 3 N}} (1 \right. \\ \left. + 3 N) \frac{1}{3} \frac{-1 + 2 N}{1 + 3 N} K^{\beta} \text{signum}(w + K) (-\text{signum}(w + K)) \right. \\ \left. - w^{1 + \sigma} \text{hypergeom} \left([-\beta, 1 + \sigma], [2 + \sigma], -\frac{w}{K} \right) \right) \quad (16)$$

> **masa := (((R^2+alpha*r^2)^(1/3)*N)*(R^2+alpha*r^2+3*N*alpha*r^2)^(2/3)*N/(1+3*N))*(R^2+alpha*r^2+3*N*alpha*r^2)^(-1/(3*(1+3*N)))*(r*R^4+2*r^3*R^2*alpha+r^5*alpha^2+3*N*alpha*r^3*R^2+3*N*alpha^2*r^5)/((R^2+alpha*r^2)^(5/3)*r^4)^(-1))*(Integralfull + C1);**

$$masa := \left((R^2 + \alpha r^2)^{5/3} r^4 \left(-\frac{1}{2} \frac{1}{1 + \sigma} \left(N (N \right. \right. \right. \quad (17)$$

$$\begin{aligned}
& -2) \alpha^{\frac{1}{3} + \frac{1}{3} N_{\sim} + \frac{-1+2 N_{\sim}}{3+9 N_{\sim}}} R^{-\frac{4}{3} + \frac{2}{3} N_{\sim} + \frac{2(-1+2 N_{\sim})}{3+9 N_{\sim}}} (1+3 N_{\sim})^{\frac{-1+2 N_{\sim}}{3+9 N_{\sim}}} (\\
& -1)^{-\beta_{\sim}} K_{\sim}^{\beta_{\sim}} \text{signum}(w_{\sim} + K_{\sim})^{\beta_{\sim}} (-\text{signum}(w_{\sim} + K_{\sim}))^{-\beta_{\sim}} w_{\sim}^{1+\sigma_{\sim}} \text{hypergeom}\left([-\beta_{\sim}, 1 \right. \\
& \left. + \sigma_{\sim}], [2 + \sigma_{\sim}], -\frac{w_{\sim}}{K_{\sim}}\right) + CI \Bigg) \Bigg/ \left((R^2 + \alpha r^2)^{\frac{1}{3} N_{\sim}} (R^2 + \alpha r^2 \right. \\
& \left. + 3 N_{\sim} \alpha r^2)^{\frac{2}{3} \frac{N_{\sim}}{1+3 N_{\sim}}} (R^2 + \alpha r^2 + 3 N_{\sim} \alpha r^2)^{-\frac{1}{3+9 N_{\sim}}} (r R^4 + 2 r^3 R^2 \alpha + r^5 \alpha^2 \right. \\
& \left. + 3 N_{\sim} \alpha r^3 R^2 + 3 N_{\sim} \alpha^2 r^5) \right)
\end{aligned}$$

> simplify(%);

$$\begin{aligned}
& \frac{1}{2} \frac{1}{1+\sigma_{\sim}} \left(r^3 (R^2 + \alpha r^2)^{2/3} (R^2 + \alpha r^2 + 3 N_{\sim} \alpha r^2)^{-\frac{1}{3} \frac{11 N_{\sim} + 3}{1+3 N_{\sim}}} (R^2 + \alpha r^2 \right. \\
& \left. + 3 N_{\sim} \alpha r^2)^{\frac{1}{3(1+3 N_{\sim})}} (R^2 + \alpha r^2)^{-\frac{1}{3} N_{\sim}} \left(\right. \right. \\
& \left. \left. - N_{\sim}^2 \alpha^{\frac{N_{\sim}(2+N_{\sim})}{1+3 N_{\sim}}} R^{\frac{2(-1-N_{\sim}+N_{\sim}^2)}{1+3 N_{\sim}}} (1+3 N_{\sim})^{\frac{1}{3} \frac{-1+2 N_{\sim}}{1+3 N_{\sim}}} (-1) \right. \right. \\
& \left. \left. - \beta_{\sim} K_{\sim}^{\beta_{\sim}} \text{signum}(w_{\sim} + K_{\sim})^{\beta_{\sim}} (-\text{signum}(w_{\sim} + K_{\sim}))^{-\beta_{\sim}} w_{\sim}^{1+\sigma_{\sim}} \text{hypergeom}\left([-\beta_{\sim}, 1 \right. \right. \right. \\
& \left. \left. + \sigma_{\sim}], [2 + \sigma_{\sim}], -\frac{w_{\sim}}{K_{\sim}}\right) + 2 N_{\sim} \alpha^{\frac{N_{\sim}(2+N_{\sim})}{1+3 N_{\sim}}} R^{\frac{2(-1-N_{\sim}+N_{\sim}^2)}{1+3 N_{\sim}}} (1 \right. \\
& \left. + 3 N_{\sim})^{\frac{1}{3} \frac{-1+2 N_{\sim}}{1+3 N_{\sim}}} (-1)^{-\beta_{\sim}} K_{\sim}^{\beta_{\sim}} \text{signum}(w_{\sim} + K_{\sim})^{\beta_{\sim}} (-\text{signum}(w_{\sim} + K_{\sim})) \right. \\
& \left. \left. - \beta_{\sim} w_{\sim}^{1+\sigma_{\sim}} \text{hypergeom}\left([-\beta_{\sim}, 1 + \sigma_{\sim}], [2 + \sigma_{\sim}], -\frac{w_{\sim}}{K_{\sim}}\right) + 2 CI + 2 CI \sigma_{\sim} \right) \right)
\end{aligned} \tag{18}$$