```
> restart; EcuacNu := diff(lambda(r,t),t,t)+diff(lambda(r,t),t)^2
-diff(nu(r,t),t)*diff(lambda(r,t),t)=0;
```

$$EcuacNu := \frac{\partial^2}{\partial t^2}\lambda(r,t) + \left(\frac{\partial}{\partial t}\lambda(r,t)\right)^2 - \left(\frac{\partial}{\partial t}\nu(r,t)\right)\left(\frac{\partial}{\partial t}\lambda(r,t)\right) = 0 \quad (1.1)$$

```
> dsolve(EcuacNu,nu(r,t));
```

$$\nu(r,t) = \ln\left(\frac{\partial}{\partial t}\lambda(r,t)\right) + \lambda(r,t) + \_F1(r) \quad (1.2)$$

Esta ser'ia para el caso general

que para tu algoritmo de soluciones anis'otropas ser'ia

```
> EcuacZeta := diff(ln(diff(lambda(r, t), t))+lambda(r, t)+_F1
(r),r)= z(r,t) -1/r;
```

$$EcuacZeta := \frac{\frac{\partial^2}{\partial t\,\partial r}\lambda(r,t)}{\frac{\partial}{\partial t}\lambda(r,t)} + \frac{\partial}{\partial r}\lambda(r,t) + \frac{d}{dr}\_F1(r) = z(r,t) - \frac{1}{r} \quad (1.3)$$

Con lo cual queda por identificar la otra funci'on generadora

A continuaci'on presento lo que pudiera ser un modelo con la suposici'on de arriba 1.2

Colapso Lento

```
> restart:grtw():
```

GRTensorII Version 1.79 (R4)

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Developed by Peter Musgrave, Denis Pollney and Kayll Lake

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Latest version available from: http://grtensor.phy.queensu.ca/ (1.4)

```
> grOptionMetricPath :=
`/Users/luisnunez/Documents/MisDocumentos/maple/gravitacion/metr
icas/`;
```

grOptionMetricPath := (1.5)

/Users/luisnunez/Documents/MisDocumentos/maple/gravitacion/metricas/

```
> alias(lambda0 = lambda0(r,t), nu0 = nu0(r,t), K = K(t),omega=
omega(r,t), m=m(r,t), C=C(r), rhocB=rhocB(t), rhocFGM=rhocFGM
(t), mB=mB(r,t)):
> qload(esferica);
Calculated ds for esferica (0.002000 sec.)
```

Default spacetime = esferica

For the esferica spacetime:

Coordinates

$$x(up)$$

$$x^{a} = \left[\begin{array}{cccc} r & \theta & \phi & t \end{array}\right]$$

Line element

$$ds^2 = -e^{2\lambda 0}\, dr^2 - r^2\, d\theta^2 - r^2 \sin(\theta)^2\, d\phi^2 + e^{2\nu 0}\, dt^2 \qquad \textbf{(1.6)}$$

Ecuaciones de Einstein

```
> grcalcalter(G(dn, up), G(dn, dn), 1):
Created definition for G(dn,up)
Simplification will be applied during calculation.

Applying routine simplify to object g(dn,dn,pdn)
Calculated g(dn,dn,pdn) for esferica (0.022000 sec.)
Applying routine simplify to object Chr(dn,dn,dn)
Calculated Chr(dn,dn,dn) for esferica (0.007000 sec.)
Applying routine simplify to object g(up,up)
Calculated detg for esferica (0.003000 sec.)
Calculated g(up,up) for esferica (0.012000 sec.)
Applying routine simplify to object Chr(dn,dn,up)
Calculated Chr(dn,dn,up) for esferica (0.015000 sec.)
Applying routine simplify to object R(dn,dn)
Calculated R(dn,dn) for esferica (0.017000 sec.)
Applying routine simplify to object tRicciscalar
Calculated Ricciscalar for esferica (0.014000 sec.)
Applying routine simplify to object G(dn,dn)
Calculated G(dn,dn) for esferica (0.017000 sec.)
Applying routine simplify to object G(dn,up)
Calculated G(dn,up) for esferica (0.004000 sec.)
```

$$CPU\ Time = 0.108 \qquad \textbf{(2.1)}$$

```
> G00 := grcomponent(G(dn, up), [t, t]): G11 := grcomponent(G(dn,
  up), [r, r]):
> G22 := grcomponent(G(dn, up), [theta, theta]): G01 :=
  grcomponent(G(dn,dn),[t, r]):
> T00 := (rho+omega^2*P+2*omega*q)/(1-omega^2):
> T11 := -(P+omega^2*rho+2*omega*q)/(1-omega^2):
> T01 := -(omega*(rho+P)+q*(1+omega^2))*exp(nu0+lambda0)/(1-
  omega^2):
> T22 := -Pt:
```

Cambio a la definici'on de masa

```
> lambda0 := -ln(1 -2*m/r)/2;
```

$$\lambda 0 := -\frac{1}{2}\ln\left(1 - \frac{2\,m}{r}\right) \qquad \textbf{(2.2)}$$

Las ecuaciones de Einstein ser'an

```
> E00 := 8*Pi*simplify(T00) = simplify(G00);
```

$$E00 := -\frac{8\,\pi\left(\rho + \omega^2 P + 2\,\omega\, q\right)}{-1+\omega^2} = \frac{2\left(\frac{\partial}{\partial r} m\right)}{r^2} \qquad \textbf{(2.3)}$$

```
> E11 := 8*Pi*simplify(T11) = simplify(G11);
```

$$E11 := \frac{8\,\pi\,\left(P + \omega^2\,\rho + 2\,\omega\,q\right)}{-1 + \omega^2} = \frac{2\left(m - r^2\left(\frac{\partial}{\partial r}\,\nu 0\right) + 2\,r\left(\frac{\partial}{\partial r}\,\nu 0\right) m\right)}{r^3} \tag{2.4}$$

```
> E01 := 8*Pi*(T01) = simplify(G01);
```

$$E01 := -\frac{8\,\pi\left(\omega\left(\rho + P\right) + q\left(1 + \omega^2\right)\right) e^{-\frac{1}{2}\ln\left(1 - \frac{2\,m}{r}\right) + \nu 0}}{1 - \omega^2} = -\frac{2\left(\frac{\partial}{\partial t}\,m\right)}{r\left(-r + 2\,m\right)} \tag{2.5}$$

```
> E22 := 8*Pi*T22 = simplify(G22);
```

$$\begin{aligned} E22 := -8\,\pi\,Pt = {} & \frac{1}{r^3\left(-r + 2\,m\right)^2}\left(e^{-2\,\nu 0}\,r^4\left(\frac{\partial^2}{\partial t^2}\,m\right) - r^2\,m + 4\,r\,m^2 + \left(\frac{\partial}{\partial r}\,m\right) r^3\right. \\ & - r^4\left(\frac{\partial}{\partial r}\,\nu 0\right) - r^5\left(\frac{\partial}{\partial r}\,\nu 0\right)^2 - r^5\left(\frac{\partial^2}{\partial r^2}\,\nu 0\right) - e^{-2\,\nu 0}\left(\frac{\partial}{\partial t}\,m\right) r^4\left(\frac{\partial}{\partial t}\,\nu 0\right) \\ & - 4\,r^3\left(\frac{\partial}{\partial r}\,\nu 0\right)\left(\frac{\partial}{\partial r}\,m\right) m + 4\,r^2\left(\frac{\partial}{\partial r}\,\nu 0\right)\left(\frac{\partial}{\partial r}\,m\right) m^2 \\ & - 2\,e^{-2\,\nu 0}\,r^3\left(\frac{\partial^2}{\partial t^2}\,m\right) m - 4\left(\frac{\partial}{\partial r}\,m\right) r^2\,m + 3\,e^{-2\,\nu 0}\,r^3\left(\frac{\partial}{\partial t}\,m\right)^2 \\ & + 4\left(\frac{\partial}{\partial r}\,m\right) r\,m^2 + 5\,r^3\left(\frac{\partial}{\partial r}\,\nu 0\right) m - 8\,r^2\left(\frac{\partial}{\partial r}\,\nu 0\right) m^2 + 4\,r\left(\frac{\partial}{\partial r}\,\nu 0\right) m^3 \\ & + r^4\left(\frac{\partial}{\partial r}\,\nu 0\right)\left(\frac{\partial}{\partial r}\,m\right) + 6\,r^4\left(\frac{\partial}{\partial r}\,\nu 0\right)^2 m - 12\,r^3\left(\frac{\partial}{\partial r}\,\nu 0\right)^2 m^2 \\ & + 8\,r^2\left(\frac{\partial}{\partial r}\,\nu 0\right)^2 m^3 + 2\,e^{-2\,\nu 0}\left(\frac{\partial}{\partial t}\,m\right) r^3\left(\frac{\partial}{\partial t}\,\nu 0\right) m - 4\,m^3 \\ & \left. + 6\,r^4\left(\frac{\partial^2}{\partial r^2}\,\nu 0\right) m - 12\,r^3\left(\frac{\partial^2}{\partial r^2}\,\nu 0\right) m^2 + 8\,r^2\left(\frac{\partial^2}{\partial r^2}\,\nu 0\right) m^3\right) \end{aligned} \tag{2.6}$$

Es posible despejar las variables físicas: ρ, P, q y Pt de las ecuaciones anteriores:

```
> varfis:=solve({E00,E01,E11,E22},[rho,P,Pt,q]):
> assign(varfis);
```

y las organizo en 'ordenes de aproximaci'on de omega

```
> collect(simplify(P),omega);
```

$$\begin{aligned} & \frac{1}{4}\,\frac{1}{\left(\left(-r + 2\,m\right)\omega^2 + r - 2\,m\right)\pi\,r^3}\left(\left(\left(e^{\nu 0}\left(\frac{\partial}{\partial r}\,m\right) r^2 - 2\,r\,e^{\nu 0}\left(\frac{\partial}{\partial r}\,m\right) m\right)\omega^2\right.\right. \\ & + 2\,\omega\left(\frac{\partial}{\partial t}\,m\right) r^2\sqrt{-\frac{-r + 2\,m}{r}} + 4\,e^{\nu 0}\,r\left(\frac{\partial}{\partial r}\,\nu 0\right) m^2 + e^{\nu 0}\,r^3\left(\frac{\partial}{\partial r}\,\nu 0\right) \\ & \left.\left. - 4\,e^{\nu 0}\,r^2\left(\frac{\partial}{\partial r}\,\nu 0\right) m + 2\,e^{\nu 0}\,m^2 - e^{\nu 0}\,r\,m\right) e^{-\nu 0}\right) \end{aligned} \tag{2.7}$$

```
> collect(simplify(rho),omega);
```

$$\frac{1}{4}\,\frac{1}{\left(\left(-r+2m\right)\omega^{2}+r-2m\right)\pi r^{3}}\left(\left(\left(4\,e^{\nu 0}\,r\left(\frac{\partial}{\partial r}\,\nu 0\right)m^{2}+e^{\nu 0}\,r^{3}\left(\frac{\partial}{\partial r}\,\nu 0\right)\right.\right.\right.$$
$$\left.-4\,e^{\nu 0}\,r^{2}\left(\frac{\partial}{\partial r}\,\nu 0\right)m+2\,e^{\nu 0}\,m^{2}-e^{\nu 0}\,r\,m\right)\omega^{2}+2\,\omega\left(\frac{\partial}{\partial t}\,m\right)r^{2}\sqrt{-\frac{-r+2m}{r}}$$
$$\left.\left.+\,e^{\nu 0}\left(\frac{\partial}{\partial r}\,m\right)r^{2}-2\,r\,e^{\nu 0}\left(\frac{\partial}{\partial r}\,m\right)m\right)e^{-\nu 0}\right) \tag{2.8}$$

```
> collect(simplify(q),omega);
```

$$-\frac{1}{4}\,\frac{1}{\left(\left(-r+2m\right)\omega^{2}+r-2m\right)\pi r^{3}}\left(\left(\left(\left(\frac{\partial}{\partial t}\,m\right)r^{2}\omega^{2}\sqrt{-\frac{-r+2m}{r}}\right.\right.\right.$$
$$+\left(e^{\nu 0}\left(\frac{\partial}{\partial r}\,m\right)r^{2}+e^{\nu 0}\,r^{3}\left(\frac{\partial}{\partial r}\,\nu 0\right)-4\,e^{\nu 0}\,r^{2}\left(\frac{\partial}{\partial r}\,\nu 0\right)m-2\,r\,e^{\nu 0}\left(\frac{\partial}{\partial r}\,m\right)m\right.$$
$$\left.\left.-\,e^{\nu 0}\,r\,m+2\,e^{\nu 0}\,m^{2}+4\,e^{\nu 0}\,r\left(\frac{\partial}{\partial r}\,\nu 0\right)m^{2}\right)\omega+\left(\frac{\partial}{\partial t}\,m\right)r^{2}\sqrt{-\frac{-r+2m}{r}}\,\Bigg)$$
$$\left.e^{-\nu 0}\right) \tag{2.9}$$

```
> collect(simplify(Pt),omega);
```

$$-\frac{1}{8}\,\frac{1}{\pi r^{3}\left(r^{2}-4\,r\,m+4\,m^{2}\right)}\left(e^{-2\,\nu 0}\,r^{4}\left(\frac{\partial^{2}}{\partial t^{2}}\,m\right)-r^{2}\,m+4\,r\,m^{2}+\left(\frac{\partial}{\partial r}\,m\right)r^{3}\right.$$
$$-r^{4}\left(\frac{\partial}{\partial r}\,\nu 0\right)-r^{5}\left(\frac{\partial}{\partial r}\,\nu 0\right)^{2}-r^{5}\left(\frac{\partial^{2}}{\partial r^{2}}\,\nu 0\right)-e^{-2\,\nu 0}\left(\frac{\partial}{\partial t}\,m\right)r^{4}\left(\frac{\partial}{\partial t}\,\nu 0\right)$$
$$-4\,r^{3}\left(\frac{\partial}{\partial r}\,\nu 0\right)\left(\frac{\partial}{\partial r}\,m\right)m+4\,r^{2}\left(\frac{\partial}{\partial r}\,\nu 0\right)\left(\frac{\partial}{\partial r}\,m\right)m^{2}$$
$$-2\,e^{-2\,\nu 0}\,r^{3}\left(\frac{\partial^{2}}{\partial t^{2}}\,m\right)m-4\left(\frac{\partial}{\partial r}\,m\right)r^{2}\,m+3\,e^{-2\,\nu 0}\,r^{3}\left(\frac{\partial}{\partial t}\,m\right)^{2}$$
$$+4\left(\frac{\partial}{\partial r}\,m\right)r\,m^{2}+5\,r^{3}\left(\frac{\partial}{\partial r}\,\nu 0\right)m-8\,r^{2}\left(\frac{\partial}{\partial r}\,\nu 0\right)m^{2}+4\,r\left(\frac{\partial}{\partial r}\,\nu 0\right)m^{3}$$
$$+r^{4}\left(\frac{\partial}{\partial r}\,\nu 0\right)\left(\frac{\partial}{\partial r}\,m\right)+6\,r^{4}\left(\frac{\partial}{\partial r}\,\nu 0\right)^{2}m-12\,r^{3}\left(\frac{\partial}{\partial r}\,\nu 0\right)^{2}m^{2}$$
$$+8\,r^{2}\left(\frac{\partial}{\partial r}\,\nu 0\right)^{2}m^{3}+2\,e^{-2\,\nu 0}\left(\frac{\partial}{\partial t}\,m\right)r^{3}\left(\frac{\partial}{\partial t}\,\nu 0\right)m-4\,m^{3}$$
$$\left.+6\,r^{4}\left(\frac{\partial^{2}}{\partial r^{2}}\,\nu 0\right)m-12\,r^{3}\left(\frac{\partial^{2}}{\partial r^{2}}\,\nu 0\right)m^{2}+8\,r^{2}\left(\frac{\partial^{2}}{\partial r^{2}}\,\nu 0\right)m^{3}\right) \tag{2.10}$$

suponemos Colapso lento, esto es omega^2 = 0 y en las variables f'isicas ser'an

```
> rhoL:= collect(simplify(subs([omega^2=0,nu0=ln(diff(lambda0, t)
  )+lambda0+ C],rho)),omega);
```

(2.11)

$$rhoL := \frac{1}{4}\,\frac{e^{-C}\,(-4\,m+2\,r)\,\omega}{r^2\,\pi} + \frac{1}{4}\,\frac{e^{-C}\,e^{C}\left(\frac{\partial}{\partial r}\,m\right)}{r^2\,\pi} \tag{2.11}$$

```
> PL:= collect(simplify(subs([omega^2=0,nu0=ln(diff(lambda0, t))+
  lambda0+ C],P)),omega);
```

$$PL := \frac{1}{4}\,\frac{e^{-C}\left(-4\,m\left(\frac{\partial}{\partial t}\,m\right)r+2\left(\frac{\partial}{\partial t}\,m\right)r^2\right)\omega}{r^3\left(\frac{\partial}{\partial t}\,m\right)\pi} + \frac{1}{4}\,\frac{1}{r^3\left(\frac{\partial}{\partial t}\,m\right)\pi}\left(e^{-C}\left(\right.\right. \tag{2.12}$$

$$-2\,m\,e^{C}\,r\left(\frac{\partial^2}{\partial t\,\partial r}\,m\right) - 2\,m\left(\frac{\partial}{\partial t}\,m\right)e^{C} - 2\,m\,e^{C}\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{\partial}{\partial t}\,m\right)r$$

$$+ e^{C}\left(\frac{\partial^2}{\partial t\,\partial r}\,m\right)r^2 + 3\,e^{C}\,r\left(\frac{\partial}{\partial t}\,m\right)\left(\frac{\partial}{\partial r}\,m\right) + e^{C}\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{\partial}{\partial t}\,m\right)r^2$$

$$\left.\left. - r\left(\frac{\partial}{\partial t}\,m\right)e^{C}\right)\right)$$

```
> PtL:= collect(simplify(subs([omega^2=0,nu0=ln(diff(lambda0, t))
  +lambda0+ C],Pt)),omega);
```

$$PtL := -\frac{1}{8}\,\frac{1}{\left(\frac{\partial}{\partial t}\,m\right)r^3\,(-r+2\,m)\,\pi}\left(5\,r^3\left(\frac{\partial}{\partial r}\,m\right)\left(\frac{\partial^2}{\partial t\,\partial r}\,m\right)\right. \tag{2.13}$$

$$- r^3\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{\partial}{\partial t}\,m\right) + \left(\frac{\partial}{\partial t}\,m\right)r^2 + 5\,m\left(\frac{\partial}{\partial t}\,m\right)r$$

$$- 10\,m\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{\partial}{\partial t}\,m\right)r^2\left(\frac{\partial}{\partial r}\,m\right) - 2\left(\frac{\partial}{\partial t}\,m\right)m^2$$

$$+ 12\,r^2\left(\frac{\partial}{\partial r}\,m\right)^2\left(\frac{\partial}{\partial t}\,m\right) - 9\,r^2\left(\frac{\partial}{\partial t}\,m\right)\left(\frac{\partial}{\partial r}\,m\right) + 6\,m^2\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{\partial}{\partial t}\,m\right)r$$

$$- 6\,m\,r\left(\frac{\partial}{\partial t}\,m\right)\left(\frac{\partial}{\partial r}\,m\right) - m\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{\partial}{\partial t}\,m\right)r^2$$

$$+ 5\,r^3\left(\frac{\partial}{\partial r}\,m\right)\left(\frac{\partial}{\partial t}\,m\right)\left(\frac{\partial}{\partial r}\,C\right) + 4\,m^2\left(\frac{\partial}{\partial r}\,C\right)^2\left(\frac{\partial}{\partial t}\,m\right)r^2$$

$$- 4\,m\,r^3\left(\frac{\partial}{\partial r}\,C\right)^2\left(\frac{\partial}{\partial t}\,m\right) + r^4\left(\frac{\partial}{\partial r}\,C\right)^2\left(\frac{\partial}{\partial t}\,m\right) - r^3\left(\frac{\partial^2}{\partial t\,\partial r}\,m\right)$$

$$+ r^4\left(\frac{\partial^3}{\partial t\,\partial r^2}\,m\right) + 4\,m^2\left(\frac{\partial^2}{\partial r^2}\,C\right)\left(\frac{\partial}{\partial t}\,m\right)r^2 + 8\,m^2\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{\partial^2}{\partial t\,\partial r}\,m\right)r^2$$

$$- 6\,m\,r^2\left(\frac{\partial}{\partial t}\,m\right)\left(\frac{\partial^2}{\partial r^2}\,m\right) - 8\,m\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{\partial^2}{\partial t\,\partial r}\,m\right)r^3$$

$$- 10\,m\left(\frac{\partial^2}{\partial t\,\partial r}\,m\right)r^2\left(\frac{\partial}{\partial r}\,m\right) - 4\,m\left(\frac{\partial^2}{\partial r^2}\,C\right)\left(\frac{\partial}{\partial t}\,m\right)r^3 + 6\,m^2\,r\left(\frac{\partial^2}{\partial t\,\partial r}\,m\right)$$

$$- m \left(\frac{\partial^2}{\partial t\, \partial r} m \right) r^2 + 4 m^2 \left(\frac{\partial^3}{\partial t\, \partial r^2} m \right) r^2 + 3 \left(\frac{\partial}{\partial t} m \right) r^3 \left(\frac{\partial^2}{\partial r^2} m \right)$$

$$+ \left(\frac{\partial}{\partial t} m \right) r^4 \left(\frac{\partial^2}{\partial r^2} C \right) + 2 r^4 \left(\frac{\partial}{\partial r} C \right) \left(\frac{\partial^2}{\partial t\, \partial r} m \right) - 4 m \left(\frac{\partial^3}{\partial t\, \partial r^2} m \right) r^3 \Bigg)$$

```
> qL:= collect(simplify(subs([omega^2=0,nu0=ln(diff(lambda0, t))+
  lambda0+ C],q)),omega);
```

$$qL := -\frac{1}{4} \frac{1}{r^3 \left(\frac{\partial}{\partial t} m \right) \pi} \left(e^{-C} \left(-2 m \left(\frac{\partial}{\partial t} m \right) e^{C} - 2 m\, e^{C} r \left(\frac{\partial^2}{\partial t\, \partial r} m \right) \right. \right. \tag{2.14}$$

$$- 2 m\, e^{C} \left(\frac{\partial}{\partial r} C \right) \left(\frac{\partial}{\partial t} m \right) r + e^{C} \left(\frac{\partial}{\partial r} C \right) \left(\frac{\partial}{\partial t} m \right) r^2$$

$$\left. + 4\, e^{C} r \left(\frac{\partial}{\partial t} m \right) \left(\frac{\partial}{\partial r} m \right) + e^{C} \left(\frac{\partial^2}{\partial t\, \partial r} m \right) r^2 - r \left(\frac{\partial}{\partial t} m \right) e^{C} \right) \omega \Bigg)$$

$$- \frac{1}{4} \frac{e^{-C} \left(\left(\frac{\partial}{\partial t} m \right) r^2 - 2 m \left(\frac{\partial}{\partial t} m \right) r \right)}{r^3 \left(\frac{\partial}{\partial t} m \right) \pi}$$

Usamos los modelos anteriores B y FGM

Para el cso de B la funci'on masa ser'a

```
> mB := (12*Pi*rhocB*r^3)/(9 + 16*Pi*rhocB*r^2);
```

$$mB := \frac{12\, \pi\, rhocB\, r^3}{9 + 16\, \pi\, rhocB\, r^2} \tag{2.15}$$

```
> qLB := collect(simplify(subs(m=mB, qL )),omega);
```

$$qLB := \frac{1}{4} \left(e^{-C} \left(40\, e^{C} \pi\, rhocB\, r^2 \left(\frac{\partial}{\partial t} \left(\frac{12\, \pi\, rhocB\, r^3}{9 + 16\, \pi\, rhocB\, r^2} \right) \right) \right. \right. \tag{2.16}$$

$$+ 8\, e^{C} \pi\, rhocB\, r^3 \left(\frac{\partial^2}{\partial t\, \partial r} \left(\frac{12\, \pi\, rhocB\, r^3}{9 + 16\, \pi\, rhocB\, r^2} \right) \right)$$

$$+ 8\, e^{C} \pi\, rhocB\, r^3 \left(\frac{\partial}{\partial r} C \right) \left(\frac{\partial}{\partial t} \left(\frac{12\, \pi\, rhocB\, r^3}{9 + 16\, \pi\, rhocB\, r^2} \right) \right)$$

$$- 9\, e^{C} \left(\frac{\partial}{\partial r} C \right) \left(\frac{\partial}{\partial t} \left(\frac{12\, \pi\, rhocB\, r^3}{9 + 16\, \pi\, rhocB\, r^2} \right) \right) r$$

$$- 36\, e^{C} \left(\frac{\partial}{\partial t} \left(\frac{12\, \pi\, rhocB\, r^3}{9 + 16\, \pi\, rhocB\, r^2} \right) \right) \left(\frac{\partial}{\partial r} \left(\frac{12\, \pi\, rhocB\, r^3}{9 + 16\, \pi\, rhocB\, r^2} \right) \right)$$

$$- 64\, e^{C} \left(\frac{\partial}{\partial t} \left(\frac{12\, \pi\, rhocB\, r^3}{9 + 16\, \pi\, rhocB\, r^2} \right) \right) \left(\frac{\partial}{\partial r} \left(\frac{12\, \pi\, rhocB\, r^3}{9 + 16\, \pi\, rhocB\, r^2} \right) \right) \pi\, rhocB\, r^2$$

$$\left. - 9\, e^{C} \left(\frac{\partial^2}{\partial t\, \partial r} \left(\frac{12\, \pi\, rhocB\, r^3}{9 + 16\, \pi\, rhocB\, r^2} \right) \right) r + 9\, e^{C} \left(\frac{\partial}{\partial t} \left(\frac{12\, \pi\, rhocB\, r^3}{9 + 16\, \pi\, rhocB\, r^2} \right) \right) \right) \omega \Bigg)$$

$$\Big/\left(r^2\left(9+16\,\pi\,rhocB\,r^2\right)\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^3}{9+16\,\pi\,rhocB\,r^2}\right)\right)\pi\right)$$
$$+\frac{1}{4}\left(e^{-C}\left(-9\,r\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^3}{9+16\,\pi\,rhocB\,r^2}\right)\right)\right.\right.$$
$$\left.\left.+8\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^3}{9+16\,\pi\,rhocB\,r^2}\right)\right)r^3\,\pi\,rhocB\right)\right)\Big/\left(r^2\left(9\right.\right.$$
$$\left.\left.+16\,\pi\,rhocB\,r^2\right)\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^3}{9+16\,\pi\,rhocB\,r^2}\right)\right)\pi\right)$$

```
> qLB :=(1/4)*exp(-C)*(40*exp(C)*Pi*rhocB*r^2*(diff(12*Pi*rhocB*
r^3/(9+16*Pi*rhocB*r^2), t))+8*exp(C)*Pi*rhocB*r^3*(diff(12*Pi*
rhocB*r^3/(9+16*Pi*rhocB*r^2), t, r))+8*exp(C)*Pi*rhocB*r^3*
(diff(C, r))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))-9*
exp(C)*(diff(C, r))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2),
t))*r-36*exp(C)*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*
(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), r))-64*exp(C)*(diff
(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*(diff(12*Pi*rhocB*
r^3/(9+16*Pi*rhocB*r^2), r))*Pi*rhocB*r^2-9*exp(C)*(diff(12*Pi*
rhocB*r^3/(9+16*Pi*rhocB*r^2), t, r))*r+9*exp(C)*(diff(12*Pi*
rhocB*r^3/(9+16*Pi*rhocB*r^2), t)))*omega/(r^2*(9+16*Pi*rhocB*
r^2)*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*Pi)+(1/4)*
exp(-C)*(-9*r*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))+8*
(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*r^3*Pi*rhocB)/
(r^2*(9+16*Pi*rhocB*r^2)*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*
r^2), t))*Pi);
```

$$qLB := \frac{1}{4}\left(e^{-C}\left(40\,e^{C}\,\pi\,rhocB\,r^2\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.\right.\right. \tag{2.17}$$
$$\left.\left.-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)+8\,e^{C}\,\pi\,rhocB\,r^3\left(\frac{36\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^2}{9+16\,\pi\,rhocB\,r^2}\right.\right.$$
$$\left.\left.-\frac{1344\,\pi^2\,rhocB\,r^4\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}+\frac{12288\,\pi^3\,rhocB^2\,r^6\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^3}\right)\right.$$
$$+8\,e^{C}\,\pi\,rhocB\,r^3\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.$$
$$\left.-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)-9\,e^{C}\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.$$
$$\left.\left.-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)r-36\,e^{C}\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.\right.$$

$$
\left(-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\left(\frac{36\,\pi\,rhocB\,r^2}{9+16\,\pi\,rhocB\,r^2}-\frac{384\,\pi^2\,rhocB^2\,r^4}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)
$$

$$
-64\,e^{C}\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
\left.-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\left(\frac{36\,\pi\,rhocB\,r^2}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
\left.-\frac{384\,\pi^2\,rhocB^2\,r^4}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\pi\,rhocB\,r^2-9\,e^{C}\left(\frac{36\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^2}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
\left.-\frac{1344\,\pi^2\,rhocB\,r^4\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}+\frac{12288\,\pi^3\,rhocB^2\,r^6\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^3}\right)r
$$

$$
+9\,e^{C}\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\Bigg)\omega\Bigg)\Bigg/\Bigg(r^2\,(9
$$

$$
+16\,\pi\,rhocB\,r^2)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\pi\Bigg)
$$

$$
+\frac{1}{4}\left(e^{-C}\left(-9\,r\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\right.\right.
$$

$$
\left.\left.+8\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)r^3\,\pi\,rhocB\right)\right)\Bigg/
$$

$$
\left(r^2\left(9+16\,\pi\,rhocB\,r^2\right)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.\right.
$$

$$
\left.\left.-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\pi\right)
$$

```
> qLB :=collect(simplify(%),omega);
```

$qLB :=$ **(2.18)**

$$\frac{1}{4}\,\frac{\left(8\,e^{C}\,\pi\,rhocB\,r^{3}\left(\frac{\partial}{\partial r}\,C\right)-16\,e^{C}\,\pi\,rhocB\,r^{2}-9\,e^{C}\left(\frac{\partial}{\partial r}\,C\right)r-18\,e^{C}\right)e^{-C}\,\omega}{r^{2}\,\pi\,\left(9+16\,\pi\,rhocB\,r^{2}\right)}$$

$$+\,\frac{1}{4}\,\frac{\left(8\,\pi\,rhocB\,r^{3}-9\,r\right)e^{-C}}{r^{2}\,\pi\,\left(9+16\,\pi\,rhocB\,r^{2}\right)}$$

```
> PLB  := collect(simplify(subs(m=mB, PL )),omega);
```

$$PLB := -\frac{1}{4}\left(e^{-C}\left(-18\,r\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)\right.\right. \tag{2.19}$$

$$\left.+16\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)r^{3}\,\pi\,rhocB\right)\omega\Bigg)\Bigg/\left(r^{2}\,(9\right.$$

$$\left.+16\,\pi\,rhocB\,r^{2}\right)\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)\pi\Bigg)$$

$$-\frac{1}{4}\left(e^{-C}\left(8\,e^{C}\,\pi\,rhocB\,r^{3}\left(\frac{\partial^{2}}{\partial t\,\partial r}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)\right.\right.$$

$$+40\,e^{C}\,\pi\,rhocB\,r^{2}\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)$$

$$+8\,e^{C}\,\pi\,rhocB\,r^{3}\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)$$

$$-9\,e^{C}\left(\frac{\partial^{2}}{\partial t\,\partial r}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)r$$

$$-27\,e^{C}\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)\left(\frac{\partial}{\partial r}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)$$

$$-48\,e^{C}\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)\left(\frac{\partial}{\partial r}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)\pi\,rhocB\,r^{2}$$

$$-9\,e^{C}\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)r$$

$$\left.\left.+9\,e^{C}\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)\right)\right)\Bigg/\left(r^{2}\,(9\right.$$

$$\left.+16\,\pi\,rhocB\,r^{2}\right)\left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\,rhocB\,r^{3}}{9+16\,\pi\,rhocB\,r^{2}}\right)\right)\pi\Bigg)$$

```
> PLB := -(1/4)*exp(-C)*(-18*r*(diff(12*Pi*rhocB*r^3/(9+16*Pi*
rhocB*r^2), t))+16*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2),
t))*r^3*Pi*rhocB)*omega/(r^2*(9+16*Pi*rhocB*r^2)*(diff(12*Pi*
rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*Pi)-(1/4)*exp(-C)*(40*exp(C)
*Pi*rhocB*r^2*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))+8*
exp(C)*Pi*rhocB*r^3*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2),
t, r))+8*exp(C)*Pi*rhocB*r^3*(diff(C, r))*(diff(12*Pi*rhocB*
```

```
r^3/(9+16*Pi*rhocB*r^2), t))-9*exp(C)*(diff(12*Pi*rhocB*r^3/
(9+16*Pi*rhocB*r^2), t, r))*r-27*exp(C)*(diff(12*Pi*rhocB*r^3/
(9+16*Pi*rhocB*r^2), t))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*
r^2), r))-48*exp(C)*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2),
t))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), r))*Pi*rhocB*r^2
-9*exp(C)*(diff(C, r))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*
r^2), t))*r+9*exp(C)*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2),
t)))/(r^2*(9+16*Pi*rhocB*r^2)*(diff(12*Pi*rhocB*r^3/(9+16*Pi*
rhocB*r^2), t))*Pi);
```

$$PLB := -\frac{1}{4}\left(e^{-C}\left(-18\,r\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)+16\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)r^3\,\pi\,rhocB\right)\omega\right)\Bigg/\left(r^2\left(9+16\,\pi\,rhocB\,r^2\right)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\pi\right)-\frac{1}{4}\left(e^{-C}\left(40\,e^{C}\,\pi\,rhocB\,r^2\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)+8\,e^{C}\,\pi\,rhocB\,r^3\left(\frac{36\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^2}{9+16\,\pi\,rhocB\,r^2}-\frac{1344\,\pi^2\,rhocB\,r^4\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}+\frac{12288\,\pi^3\,rhocB^2\,r^6\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^3}\right)+8\,e^{C}\,\pi\,rhocB\,r^3\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)-9\,e^{C}\left(\frac{36\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^2}{9+16\,\pi\,rhocB\,r^2}-\frac{1344\,\pi^2\,rhocB\,r^4\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}+\frac{12288\,\pi^3\,rhocB^2\,r^6\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^3}\right)r \tag{2.20}$$

$$
\begin{aligned}
&- 27\, e^{C} \left(\frac{12\,\pi \left(\frac{\partial}{\partial t} rhocB \right) r^{3}}{9 + 16\,\pi\, rhocB\, r^{2}} \right. \\
&\left. - \frac{192\,\pi^{2}\, rhocB\, r^{5} \left(\frac{\partial}{\partial t} rhocB \right)}{\left(9 + 16\,\pi\, rhocB\, r^{2}\right)^{2}} \right) \left(\frac{36\,\pi\, rhocB\, r^{2}}{9 + 16\,\pi\, rhocB\, r^{2}} - \frac{384\,\pi^{2}\, rhocB^{2}\, r^{4}}{\left(9 + 16\,\pi\, rhocB\, r^{2}\right)^{2}} \right) \\
&- 48\, e^{C} \left(\frac{12\,\pi \left(\frac{\partial}{\partial t} rhocB \right) r^{3}}{9 + 16\,\pi\, rhocB\, r^{2}} \right. \\
&\left. - \frac{192\,\pi^{2}\, rhocB\, r^{5} \left(\frac{\partial}{\partial t} rhocB \right)}{\left(9 + 16\,\pi\, rhocB\, r^{2}\right)^{2}} \right) \left(\frac{36\,\pi\, rhocB\, r^{2}}{9 + 16\,\pi\, rhocB\, r^{2}} \right. \\
&\left. - \frac{384\,\pi^{2}\, rhocB^{2}\, r^{4}}{\left(9 + 16\,\pi\, rhocB\, r^{2}\right)^{2}} \right) \pi\, rhocB\, r^{2} - 9\, e^{C} \left(\frac{\partial}{\partial r} C \right) \left(\frac{12\,\pi \left(\frac{\partial}{\partial t} rhocB \right) r^{3}}{9 + 16\,\pi\, rhocB\, r^{2}} \right. \\
&\left. - \frac{192\,\pi^{2}\, rhocB\, r^{5} \left(\frac{\partial}{\partial t} rhocB \right)}{\left(9 + 16\,\pi\, rhocB\, r^{2}\right)^{2}} \right) r + 9\, e^{C} \left(\frac{12\,\pi \left(\frac{\partial}{\partial t} rhocB \right) r^{3}}{9 + 16\,\pi\, rhocB\, r^{2}} \right. \\
&\left. - \frac{192\,\pi^{2}\, rhocB\, r^{5} \left(\frac{\partial}{\partial t} rhocB \right)}{\left(9 + 16\,\pi\, rhocB\, r^{2}\right)^{2}} \right) \Bigg) \Bigg) \Bigg) \Bigg/ \left(r^{2} \left(9 \right. \right. \\
&\left. \left. + 16\,\pi\, rhocB\, r^{2} \right) \left(\frac{12\,\pi \left(\frac{\partial}{\partial t} rhocB \right) r^{3}}{9 + 16\,\pi\, rhocB\, r^{2}} - \frac{192\,\pi^{2}\, rhocB\, r^{5} \left(\frac{\partial}{\partial t} rhocB \right)}{\left(9 + 16\,\pi\, rhocB\, r^{2}\right)^{2}} \right) \pi \right)
\end{aligned}
$$

```
> PLB :=collect(simplify(%),omega);
```

$$
\begin{aligned}
PLB := &- \frac{1}{4} \frac{\left(-162\, r - 144\,\pi\, rhocB\, r^{3} + 256\,\pi^{2}\, rhocB^{2}\, r^{5}\right) e^{-C}\, \omega}{r^{2}\,\pi \left(9 + 16\,\pi\, rhocB\, r^{2}\right)^{2}} \\
&- \frac{1}{4} \frac{1}{r^{2}\,\pi \left(9 + 16\,\pi\, rhocB\, r^{2}\right)^{2}} \left(\left(-108\, e^{C}\,\pi\, rhocB\, r^{2} - 64\, e^{C}\,\pi^{2}\, rhocB^{2}\, r^{4} \right.\right. \\
&- 72\, e^{C}\,\pi\, rhocB\, r^{3} \left(\frac{\partial}{\partial r} C \right) + 128\, e^{C}\,\pi^{2}\, rhocB^{2}\, r^{5} \left(\frac{\partial}{\partial r} C \right) - 162\, e^{C} \\
&\left.\left. - 81\, e^{C} \left(\frac{\partial}{\partial r} C \right) r \right) e^{-C} \right)
\end{aligned}
\tag{2.21}
$$

```
> PtLB  := collect(simplify(subs(m=mB, PtL )),omega);
```

$$
PtLB := - \frac{1}{8} \left(405\, r \left(\frac{\partial}{\partial r} \left(\frac{12\,\pi\, rhocB\, r^{3}}{9 + 16\,\pi\, rhocB\, r^{2}} \right) \right) \left(\frac{\partial^{2}}{\partial t\, \partial r} \left(\frac{12\,\pi\, rhocB\, r^{3}}{9 + 16\,\pi\, rhocB\, r^{2}} \right) \right) \right. \tag{2.22}
$$

$$
\begin{aligned}
&+ 81 \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \\
&- 729 \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \left(\frac{\partial}{\partial r} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \\
&+ 972 \left(\frac{\partial}{\partial r} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right)^2 \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \\
&- 396 \pi \, rhocB \, r^3 \left(\frac{\partial}{\partial r} C \right) \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \\
&- 3240 \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \left(\frac{\partial}{\partial r} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \pi \, rhocB \, r^2 \\
&+ 416 \pi^2 \, rhocB^2 \, r^5 \left(\frac{\partial}{\partial r} C \right) \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \\
&+ 64 \pi^2 \, rhocB^2 \, r^6 \left(\frac{\partial}{\partial r} C \right)^2 \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \\
&- 144 \, r^4 \left(\frac{\partial}{\partial r} C \right)^2 \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \pi \, rhocB \\
&- 3456 \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \left(\frac{\partial}{\partial r} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \pi^2 \, rhocB^2 \, r^4 \\
&+ 3456 \left(\frac{\partial}{\partial r} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right)^2 \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \pi \, rhocB \, r^2 \\
&+ 3072 \left(\frac{\partial}{\partial r} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right)^2 \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \pi^2 \, rhocB^2 \, r^4 \\
&+ 828 \pi \, rhocB \, r^2 \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \\
&+ 405 \, r \left(\frac{\partial}{\partial r} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \left(\frac{\partial}{\partial r} C \right) \\
&+ 928 \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \pi^2 \, rhocB^2 \, r^4 \\
&+ 360 \pi \, rhocB \, r^3 \left(\frac{\partial}{\partial r} C \right) \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \left(\frac{\partial}{\partial r} \left(1 \Big/ \left(9 \right. \right. \right. \\
&\left. \left. \left. + 16 \pi \, rhocB \, r^2 \right) \left(12 \pi \, rhocB \, r^3 \right) \right) \right)
\end{aligned}
$$

$$
\begin{aligned}
&- 640\,\pi^2\, rhocB^2\, r^5 \left(\frac{\partial}{\partial r} C\right) \left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \left(\frac{\partial}{\partial r}\left(1/\left(9\right.\right.\right. \\
&\left.\left.\left.+16\,\pi\, rhocB\, r^2\right)\left(12\,\pi\, rhocB\, r^3\right)\right)\right) - 81 \left(\frac{\partial}{\partial r} C\right) \left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) r \\
&+ 81\, r^2 \left(\frac{\partial}{\partial r} C\right)^2 \left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \\
&- 396\,\pi\, rhocB\, r^3 \left(\frac{\partial^2}{\partial t\,\partial r}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \\
&- 144\,\pi\, rhocB\, r^4 \left(\frac{\partial^3}{\partial t\,\partial r^2}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \\
&+ 416\,\pi^2\, rhocB^2\, r^5 \left(\frac{\partial^2}{\partial t\,\partial r}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \\
&+ 64\,\pi^2\, rhocB^2\, r^6 \left(\frac{\partial^3}{\partial t\,\partial r^2}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \\
&- 81 \left(\frac{\partial^2}{\partial t\,\partial r}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) r + 81\, r^2 \left(\frac{\partial^3}{\partial t\,\partial r^2}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \\
&+ 64\,\pi^2\, rhocB^2\, r^6 \left(\frac{\partial^2}{\partial r^2} C\right) \left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \\
&+ 128\,\pi^2\, rhocB^2\, r^6 \left(\frac{\partial}{\partial r} C\right) \left(\frac{\partial^2}{\partial t\,\partial r}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \\
&+ 360\, r^3 \left(\frac{\partial}{\partial r}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \left(\frac{\partial^2}{\partial t\,\partial r}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \pi\, rhocB \\
&- 640\, r^5 \left(\frac{\partial}{\partial r}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \left(\frac{\partial^2}{\partial t\,\partial r}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \pi^2\, rhocB^2 \\
&- 144 \left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) r^4 \left(\frac{\partial^2}{\partial r^2} C\right) \pi\, rhocB \\
&+ 216 \left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) r^3 \left(\frac{\partial^2}{\partial r^2}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \pi\, rhocB \\
&- 384 \left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) r^5 \left(\frac{\partial^2}{\partial r^2}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \pi^2\, rhocB^2 \\
&- 288\, r^4 \left(\frac{\partial}{\partial r} C\right) \left(\frac{\partial^2}{\partial t\,\partial r}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) \pi\, rhocB \\
&+ 81 \left(\frac{\partial}{\partial t}\left(\frac{12\,\pi\, rhocB\, r^3}{9+16\,\pi\, rhocB\, r^2}\right)\right) r^2 \left(\frac{\partial^2}{\partial r^2} C\right)
\end{aligned}
$$

$$+ 243 \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) r \left(\frac{\partial^2}{\partial r^2} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right)$$

$$+ 162 \, r^2 \left(\frac{\partial}{\partial r} C \right) \left(\frac{\partial^2}{\partial t \, \partial r} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \Bigg) \Bigg/ \Bigg(r^2 \left(9 \right.$$

$$\left. + 16 \pi \, rhocB \, r^2 \right) \left(\frac{\partial}{\partial t} \left(\frac{12 \pi \, rhocB \, r^3}{9 + 16 \pi \, rhocB \, r^2} \right) \right) \left(-9 + 8 \pi \, rhocB \, r^2 \right) \pi \Bigg)$$

```
> PtLB := -(1/8)*(-81*r*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*
r^2), t, r))+81*r^2*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2),
t, r, r))-3240*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*
(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), r))*Pi*rhocB*
r^2+405*r*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), r))*(diff
(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*(diff(C, r))-640*r^5*
(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), r))*(diff(12*Pi*
rhocB*r^3/(9+16*Pi*rhocB*r^2), t, r))*Pi^2*rhocB^2+972*(diff
(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), r))^2*(diff(12*Pi*rhocB*
r^3/(9+16*Pi*rhocB*r^2), t))-144*r^4*(diff(C, r))^2*(diff(12*
Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*Pi*rhocB-396*r^3*(diff(C,
r))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*Pi*
rhocB+360*r^3*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), r))*
(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t, r))*Pi*rhocB+360*
Pi*rhocB*r^3*(diff(C, r))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*
r^2), t))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), r))+416*
Pi^2*rhocB^2*r^5*(diff(C, r))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*
rhocB*r^2), t))+64*Pi^2*rhocB^2*r^6*(diff(C, r))^2*(diff(12*Pi*
rhocB*r^3/(9+16*Pi*rhocB*r^2), t))+828*(diff(12*Pi*rhocB*r^3/
(9+16*Pi*rhocB*r^2), t))*Pi*rhocB*r^2+81*(diff(12*Pi*rhocB*r^3/
(9+16*Pi*rhocB*r^2), t))-396*r^3*(diff(12*Pi*rhocB*r^3/(9+16*
Pi*rhocB*r^2), t, r))*Pi*rhocB-144*r^4*(diff(12*Pi*rhocB*r^3/
(9+16*Pi*rhocB*r^2), t, r, r))*Pi*rhocB+128*Pi^2*rhocB^2*r^6*
(diff(C, r))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t, r))
+162*r^2*(diff(C, r))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*
r^2), t, r))+243*r*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2),
t))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), r, r))+81*r^2*
(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*(diff(C, r, r))
+3456*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), r))^2*(diff
(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*Pi*rhocB*r^2-288*r^4*
(diff(C, r))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t, r))*
Pi*rhocB-729*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*
(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), r))+416*Pi^2*
rhocB^2*r^5*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t, r))
+64*Pi^2*rhocB^2*r^6*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2),
t, r, r))+3072*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), r))
^2*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*Pi^2*rhocB^2*
r^4+928*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*Pi^2*
rhocB^2*r^4-144*r^4*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2),
t))*(diff(C, r, r))*Pi*rhocB-3456*(diff(12*Pi*rhocB*r^3/(9+16*
Pi*rhocB*r^2), t))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2),
r))*Pi^2*rhocB^2*r^4+64*Pi^2*rhocB^2*r^6*(diff(C, r, r))*(diff
(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))+216*r^3*(diff(12*Pi*
rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*(diff(12*Pi*rhocB*r^3/(9+16*
Pi*rhocB*r^2), r, r))*Pi*rhocB-384*r^5*(diff(12*Pi*rhocB*r^3/
(9+16*Pi*rhocB*r^2), t))*(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*
```

```
r^2), r, r))*Pi^2*rhocB^2-81*r*(diff(C, r))*(diff(12*Pi*rhocB*
r^3/(9+16*Pi*rhocB*r^2), t))-640*Pi^2*rhocB^2*r^5*(diff(C, r))*
(diff(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*(diff(12*Pi*
rhocB*r^3/(9+16*Pi*rhocB*r^2), r))+81*r^2*(diff(C, r))^2*(diff
(12*Pi*rhocB*r^3/(9+16*Pi*rhocB*r^2), t))+405*r*(diff(12*Pi*
rhocB*r^3/(9+16*Pi*rhocB*r^2), r))*(diff(12*Pi*rhocB*r^3/(9+16*
Pi*rhocB*r^2), t, r)))/(r^2*(9+16*Pi*rhocB*r^2)*(diff(12*Pi*
rhocB*r^3/(9+16*Pi*rhocB*r^2), t))*(-9+8*Pi*rhocB*r^2)*Pi);
```

$$
\begin{aligned}
PtLB := & -\frac{1}{8}\left(-81\left(\frac{\partial}{\partial r}C\right)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.\right. \\
& \left.\left.-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)r-\frac{15552\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right. \\
& +\frac{972\,\pi\left(\frac{\partial}{\partial t}rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}-729\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right. \\
& \left.\left.-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\left(\frac{36\,\pi\,rhocB\,r^2}{9+16\,\pi\,rhocB\,r^2}-\frac{384\,\pi^2\,rhocB^2\,r^4}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\right. \\
& -81\left(\frac{36\,\pi\left(\frac{\partial}{\partial t}rhocB\right)r^2}{9+16\,\pi\,rhocB\,r^2}-\frac{1344\,\pi^2\,rhocB\,r^4\left(\frac{\partial}{\partial t}rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right. \\
& \left.\left.+\frac{12288\,\pi^3\,rhocB^2\,r^6\left(\frac{\partial}{\partial t}rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^3}\right)r\right. \\
& -396\,\pi\,rhocB\,r^3\left(\frac{\partial}{\partial r}C\right)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right. \\
& \left.\left.-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)-3240\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right. \\
& \left.\left.-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\left(\frac{36\,\pi\,rhocB\,r^2}{9+16\,\pi\,rhocB\,r^2}\right.\right. \\
& \left.\left.-\frac{384\,\pi^2\,rhocB^2\,r^4}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\pi\,rhocB\,r^2+828\,\pi\,rhocB\,r^2\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.\right.
\end{aligned}
\tag{2.23}
$$

$$
\left(-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right) - 396\,\pi\,rhocB\,r^3 \left(\frac{36\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^2}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{1344\,\pi^2\,rhocB\,r^4\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} + \frac{12288\,\pi^3\,rhocB^2\,r^6\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^3} \right)
$$

$$
+ 360\,\pi\,rhocB\,r^3\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right)\left(\frac{36\,\pi\,rhocB\,r^2}{9+16\,\pi\,rhocB\,r^2} - \frac{384\,\pi^2\,rhocB^2\,r^4}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right)
$$

$$
- 640\,\pi^2\,rhocB^2\,r^5\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right)\left(\frac{36\,\pi\,rhocB\,r^2}{9+16\,\pi\,rhocB\,r^2} - \frac{384\,\pi^2\,rhocB^2\,r^4}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right)
$$

$$
+ 405\,r\left(\frac{36\,\pi\,rhocB\,r^2}{9+16\,\pi\,rhocB\,r^2} - \frac{384\,\pi^2\,rhocB^2\,r^4}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right)\left(\frac{36\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^2}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{1344\,\pi^2\,rhocB\,r^4\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} + \frac{12288\,\pi^3\,rhocB^2\,r^6\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^3} \right)
$$

$$
+ 81\,r^2\left(\frac{\partial}{\partial r}\,C\right)^2\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2} - \frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right)
$$

$$
+ 243\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right) r\left(\frac{72\,\pi\,rhocB\,r}{9+16\,\pi\,rhocB\,r^2} - \frac{2688\,\pi^2\,rhocB^2\,r^3}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right.
$$

$$
\left. + \frac{24576\,\pi^3\,rhocB^3\,r^5}{\left(9+16\,\pi\,rhocB\,r^2\right)^3} \right) + 162\,r^2\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{36\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^2}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
- \frac{1344\,\pi^2\,rhocB\,r^4\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}+\frac{12288\,\pi^3\,rhocB^2\,r^6\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^3}\Bigg)
$$

$$
+416\,\pi^2\,rhocB^2\,r^5\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
\left.-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)
$$

$$
+128\,\pi^2\,rhocB^2\,r^6\left(\frac{\partial}{\partial r}\,C\right)\left(\frac{36\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^2}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
- \frac{1344\,\pi^2\,rhocB\,r^4\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}+\frac{12288\,\pi^3\,rhocB^2\,r^6\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^3}\Bigg)
$$

$$
+64\,\pi^2\,rhocB^2\,r^6\left(\frac{\partial}{\partial r}\,C\right)^2\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
\left.-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)+360\,r^3\left(\frac{36\,\pi\,rhocB\,r^2}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
\left.-\frac{384\,\pi^2\,rhocB^2\,r^4}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\left(\frac{36\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^2}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
- \frac{1344\,\pi^2\,rhocB\,r^4\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}+\frac{12288\,\pi^3\,rhocB^2\,r^6\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^3}\Bigg)\pi\,rhocB
$$

$$
-640\,r^5\left(\frac{36\,\pi\,rhocB\,r^2}{9+16\,\pi\,rhocB\,r^2}-\frac{384\,\pi^2\,rhocB^2\,r^4}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\left(\frac{36\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^2}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
- \frac{1344\,\pi^2\,rhocB\,r^4\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}+\frac{12288\,\pi^3\,rhocB^2\,r^6\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^3}\Bigg)\pi^2\,rhocB^2
$$

$$
-144\,r^4\left(\frac{\partial}{\partial r}\,C\right)^2\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
\left(-\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right) \pi\,rhocB - 3456 \left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right) r^3}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right) \left(\frac{36\,\pi\,rhocB\,r^2}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{384\,\pi^2\,rhocB^2\,r^4}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right) \pi^2\,rhocB^2\,r^4 + 216 \left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right) r^3}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right) r^3 \left(\frac{72\,\pi\,rhocB\,r}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{2688\,\pi^2\,rhocB^2\,r^3}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} + \frac{24576\,\pi^3\,rhocB^3\,r^5}{\left(9+16\,\pi\,rhocB\,r^2\right)^3} \right) \pi\,rhocB
$$

$$
-384 \left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right) r^3}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right) r^5 \left(\frac{72\,\pi\,rhocB\,r}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{2688\,\pi^2\,rhocB^2\,r^3}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} + \frac{24576\,\pi^3\,rhocB^3\,r^5}{\left(9+16\,\pi\,rhocB\,r^2\right)^3} \right) \pi^2\,rhocB^2
$$

$$
+3456 \left(\frac{36\,\pi\,rhocB\,r^2}{9+16\,\pi\,rhocB\,r^2} - \frac{384\,\pi^2\,rhocB^2\,r^4}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right)^2 \left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right) r^3}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right) \pi\,rhocB\,r^2 + 3072 \left(\frac{36\,\pi\,rhocB\,r^2}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{384\,\pi^2\,rhocB^2\,r^4}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right)^2 \left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right) r^3}{9+16\,\pi\,rhocB\,r^2} \right.
$$

$$
\left. -\frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} \right) \pi^2\,rhocB^2\,r^4
$$

$$
\begin{aligned}
&- 288\, r^4 \left(\frac{\partial}{\partial r} C\right) \left(\frac{36\,\pi \left(\frac{\partial}{\partial t} rhocB\right) r^2}{9 + 16\,\pi\, rhocB\, r^2} - \frac{1344\,\pi^2\, rhocB\, r^4 \left(\frac{\partial}{\partial t} rhocB\right)}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^2} \right. \\
&\left. + \frac{12288\,\pi^3\, rhocB^2\, r^6 \left(\frac{\partial}{\partial t} rhocB\right)}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^3} \right) \pi\, rhocB + 405\, r \left(\frac{36\,\pi\, rhocB\, r^2}{9 + 16\,\pi\, rhocB\, r^2} \right. \\
&\left. - \frac{384\,\pi^2\, rhocB^2\, r^4}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^2} \right) \left(\frac{12\,\pi \left(\frac{\partial}{\partial t} rhocB\right) r^3}{9 + 16\,\pi\, rhocB\, r^2} \right. \\
&\left. - \frac{192\,\pi^2\, rhocB\, r^5 \left(\frac{\partial}{\partial t} rhocB\right)}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^2} \right) \left(\frac{\partial}{\partial r} C\right) \\
&- 144\,\pi\, rhocB\, r^4 \left(\frac{72\,\pi \left(\frac{\partial}{\partial t} rhocB\right) r}{9 + 16\,\pi\, rhocB\, r^2} - \frac{6528\,\pi^2\, rhocB\, r^3 \left(\frac{\partial}{\partial t} rhocB\right)}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^2} \right. \\
&\left. + \frac{159744\,\pi^3\, rhocB^2\, r^5 \left(\frac{\partial}{\partial t} rhocB\right)}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^3} - \frac{1179648\,\pi^4\, rhocB^3\, r^7 \left(\frac{\partial}{\partial t} rhocB\right)}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^4} \right) \\
&+ 416\,\pi^2\, rhocB^2\, r^5 \left(\frac{36\,\pi \left(\frac{\partial}{\partial t} rhocB\right) r^2}{9 + 16\,\pi\, rhocB\, r^2} - \frac{1344\,\pi^2\, rhocB\, r^4 \left(\frac{\partial}{\partial t} rhocB\right)}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^2} \right. \\
&\left. + \frac{12288\,\pi^3\, rhocB^2\, r^6 \left(\frac{\partial}{\partial t} rhocB\right)}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^3} \right) + 928 \left(\frac{12\,\pi \left(\frac{\partial}{\partial t} rhocB\right) r^3}{9 + 16\,\pi\, rhocB\, r^2} \right. \\
&\left. - \frac{192\,\pi^2\, rhocB\, r^5 \left(\frac{\partial}{\partial t} rhocB\right)}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^2} \right) \pi^2\, rhocB^2\, r^4 \\
&+ 64\,\pi^2\, rhocB^2\, r^6 \left(\frac{72\,\pi \left(\frac{\partial}{\partial t} rhocB\right) r}{9 + 16\,\pi\, rhocB\, r^2} - \frac{6528\,\pi^2\, rhocB\, r^3 \left(\frac{\partial}{\partial t} rhocB\right)}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^2} \right. \\
&\left. + \frac{159744\,\pi^3\, rhocB^2\, r^5 \left(\frac{\partial}{\partial t} rhocB\right)}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^3} - \frac{1179648\,\pi^4\, rhocB^3\, r^7 \left(\frac{\partial}{\partial t} rhocB\right)}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^4} \right) \\
&+ 972 \left(\frac{36\,\pi\, rhocB\, r^2}{9 + 16\,\pi\, rhocB\, r^2} - \frac{384\,\pi^2\, rhocB^2\, r^4}{\left(9 + 16\,\pi\, rhocB\, r^2\right)^2} \right)^2 \left(\frac{12\,\pi \left(\frac{\partial}{\partial t} rhocB\right) r^3}{9 + 16\,\pi\, rhocB\, r^2} \right.
\end{aligned}
$$

$$
\left. - \frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right) + 81\,r^2\left(\frac{72\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
- \frac{6528\,\pi^2\,rhocB\,r^3\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2} + \frac{159744\,\pi^3\,rhocB^2\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^3}
$$

$$
\left. - \frac{1179648\,\pi^4\,rhocB^3\,r^7\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^4}\right)
$$

$$
+ 64\,\pi^2\,rhocB^2\,r^6\left(\frac{\partial^2}{\partial r^2}\,C\right)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
\left. - \frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right) - 144\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.
$$

$$
\left. - \frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right) r^4\left(\frac{\partial^2}{\partial r^2}\,C\right)\pi\,rhocB
$$

$$
\left. + 81\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2} - \frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right) r^2\left(\frac{\partial^2}{\partial r^2}\,C\right)\right) \Big/
$$

$$
\left(r^2\left(9+16\,\pi\,rhocB\,r^2\right)\left(\frac{12\,\pi\left(\frac{\partial}{\partial t}\,rhocB\right)r^3}{9+16\,\pi\,rhocB\,r^2}\right.\right.
$$

$$
\left.\left. - \frac{192\,\pi^2\,rhocB\,r^5\left(\frac{\partial}{\partial t}\,rhocB\right)}{\left(9+16\,\pi\,rhocB\,r^2\right)^2}\right)\left(-9+8\,\pi\,rhocB\,r^2\right)\pi\right)
$$

```
> PtLB :=collect(simplify(%),omega);
```

$$
PtLB := -\frac{1}{8}\left(26244 + 29160\,\pi\,rhocB\,r^2 + 233280\,\pi^2\,rhocB^2\,r^4 + 32805\left(\frac{\partial}{\partial r}\,C\right)r \right. \tag{2.24}
$$

$$
+ 52488\,\pi\,rhocB\,r^3\left(\frac{\partial}{\partial r}\,C\right) - 9216\,\pi^3\,rhocB^3\,r^7\left(\frac{\partial}{\partial r}\,C\right) + 175104\,\pi^3\,rhocB^3\,r^6
$$

$$
+ 65536\,\pi^4\,rhocB^4\,r^8 + 6561\,r^2\left(\frac{\partial}{\partial r}\,C\right)^2 + 6561\,r^2\left(\frac{\partial^2}{\partial r^2}\,C\right)
$$

$$
\left. - 49152\,\pi^4\,rhocB^4\,r^9\left(\frac{\partial}{\partial r}\,C\right) + 11664\,r^4\left(\frac{\partial}{\partial r}\,C\right)^2\pi\,rhocB\right)
$$

$$- 15552\, r^6 \left(\frac{\partial}{\partial r} C\right)^2 \pi^2\, rhocB^2 - 18432\, r^8 \left(\frac{\partial}{\partial r} C\right)^2 \pi^3\, rhocB^3$$

$$+ 16384\, r^{10} \left(\frac{\partial}{\partial r} C\right)^2 \pi^4\, rhocB^4 + 11664\, r^4 \left(\frac{\partial^2}{\partial r^2} C\right) \pi\, rhocB$$

$$- 15552\, r^6 \left(\frac{\partial^2}{\partial r^2} C\right) \pi^2\, rhocB^2 - 18432\, r^8 \left(\frac{\partial^2}{\partial r^2} C\right) \pi^3\, rhocB^3$$

$$+ 16384\, r^{10} \left(\frac{\partial^2}{\partial r^2} C\right) \pi^4\, rhocB^4 \Big) \Big/ \left(\pi\, r^2 \left(-9 + 8\,\pi\, rhocB\, r^2\right) \left(9\right.\right.$$

$$\left.\left. + 16\,\pi\, rhocB\, r^2\right)^3\right)$$

```
> Ecua1:=simplify(PLB - PtLB);
```

$$Ecua1 := -\frac{1}{8}\left(-81920\,\pi^4\, rhocB^4\, r^7 - 193536\,\pi^3\, rhocB^3\, r^5 + 11664\,\pi\, rhocB\, r \right. \tag{2.25}$$

$$- 248832\,\pi^2\, rhocB^2\, r^3 - 29160\,\pi\, rhocB\, r^2 \left(\frac{\partial}{\partial r} C\right) - 19683 \left(\frac{\partial}{\partial r} C\right)$$

$$- 6561\, r \left(\frac{\partial}{\partial r} C\right)^2 - 6561\, r \left(\frac{\partial^2}{\partial r^2} C\right) - 27648\,\pi^3\, rhocB^3\, r^6 \left(\frac{\partial}{\partial r} C\right)$$

$$+ 81920\,\pi^4\, rhocB^4\, r^8 \left(\frac{\partial}{\partial r} C\right) - 11664\, r^3 \left(\frac{\partial}{\partial r} C\right)^2 \pi\, rhocB$$

$$+ 15552\, r^5 \left(\frac{\partial}{\partial r} C\right)^2 \pi^2\, rhocB^2 + 18432\, r^7 \left(\frac{\partial}{\partial r} C\right)^2 \pi^3\, rhocB^3$$

$$- 16384\, r^9 \left(\frac{\partial}{\partial r} C\right)^2 \pi^4\, rhocB^4 - 11664\, r^3 \left(\frac{\partial^2}{\partial r^2} C\right) \pi\, rhocB$$

$$+ 15552\, r^5 \left(\frac{\partial^2}{\partial r^2} C\right) \pi^2\, rhocB^2 + 18432\, r^7 \left(\frac{\partial^2}{\partial r^2} C\right) \pi^3\, rhocB^3$$

$$- 16384\, r^9 \left(\frac{\partial^2}{\partial r^2} C\right) \pi^4\, rhocB^4 - 31104\,\pi^2\, rhocB^2\, r^4 \left(\frac{\partial}{\partial r} C\right) + 26244\, e^{-C}\,\omega$$

$$+ 46656\, r^2\, e^{-C}\,\omega\,\pi\, rhocB - 62208\, r^4\, e^{-C}\,\omega\,\pi^2\, rhocB^2 - 73728\, r^6\, e^{-C}\,\omega\,\pi^3\, rhocB^3$$

$$\left. + 65536\, r^8\, e^{-C}\,\omega\,\pi^4\, rhocB^4\right) \Big/ \left(r\,\pi \left(-9 + 8\,\pi\, rhocB\, r^2\right) \left(9 + 16\,\pi\, rhocB\, r^2\right)^3\right)$$