JANUS BACKWARD

Sleep State: Memory Management

Knowledge Consolidation, Schema Formation, and Long-Term Learning

Classification: Technical Implementation Guide

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JANUS Backward Overview:

- **Purpose:** Offline memory consolidation and schema learning
- **Cold Path:** Batch processing during market closure
- **Components:** Three-timescale memory, prioritized replay, UMAP visualization
- **Goal:** Transform raw experiences into abstract knowledge structures

Abstract

JANUS Backward represents the "sleep state" of the JANUS trading system, responsible for offline memory consolidation, schema formation, and long-term learning during market closure. This document provides a comprehensive mathematical and implementation specification for the Backward service, which implements:

- **Three-Timescale Memory Architecture** spanning short-term (hippocampus), medium-term (SWR replay), and long-term (neocortex) storage
- **Prioritized Experience Replay** using TD-error, logical violation scores, and reward magnitude
- **Sharp Wave Ripple Simulation** for time-compressed memory consolidation
- **Recall-Gated Learning** that filters updates based on familiarity and logical validity
- **UMAP Visualization** for real-time cognitive monitoring and anomaly detection

The Backward service operates on a cold path with no strict latency requirements, enabling sophisticated batch processing and offline optimization that would be infeasible during live trading.

Contents

1 Memory Hierarchy: Three-Timescale Architecture

The memory system is organized into three distinct timescales, each with specialized functions and computational properties.

1.1 Short-Term Memory (Hippocampus)

The hippocampal subsystem provides rapid encoding of recent experiences with pattern separation to prevent interference.

1.1.1 Episodic Buffer

The hippocampus maintains an episodic buffer of recent transitions:

$$\mathcal{B}_{\text{STM}} = \{(\mathbf{s}_t, a_t, r_t, \mathbf{s}_{t+1})\}_{t=1}^{T_{\text{episode}}} \tag{1}$$

where each tuple represents a state-action-reward-nextstate transition.

Implementation Details:

- Maximum capacity: $|\mathcal{B}_{\text{STM}}| \leq 10,000$ transitions
- FIFO replacement policy when capacity exceeded
- Indexed by timestamp for temporal queries

1.1.2 Pattern Separation

To prevent catastrophic interference between similar market states, the hippocampus implements pattern separation:

$$\mathbf{h}_{\text{separated}} = \text{ReLU}(\mathbf{W}_{\text{sep}}\mathbf{s} + \mathbf{b}_{\text{sep}}) \tag{2}$$

where $\mathbf{W}_{\text{sep}} \in \mathbb{R}^{d_h \times d_s}$ is initialized to promote orthogonality.

Orthogonality Initialization:

$$\mathbf{W}_{\text{sep}} \sim \mathcal{N}(0, \sigma^2), \quad \text{where } \sigma = \sqrt{\frac{2}{d_s + d_h}} \tag{3}$$

During training, add orthogonality regularization:

$$\mathcal{L}_{\text{ortho}} = \lambda_{\text{ortho}} \cdot ||\mathbf{W}_{\text{sep}}^\top \mathbf{W}_{\text{sep}} - \mathbf{I}||_F^2 \tag{4}$$

1.1.3 Sparse Encoding

The hippocampus uses sparse representations to maximize information capacity:

$$\mathbf{c}_{\text{sparse}} = \text{TopK}(\mathbf{h}_{\text{separated}}, k) \tag{5}$$

where TopK selects the k largest activations and zeros others.

Sparsity Level:

$$k = \lceil \rho \cdot d_h \rceil, \quad \rho \in [0.05, 0.15] \tag{6}$$

Typically $\rho = 0.1$ (10% activation).

1.2 Medium-Term Consolidation (SWR Simulator)

The Sharp Wave Ripple (SWR) simulator implements prioritized replay with time compression, mimicking biological memory consolidation during sleep.

1.2.1 Replay Prioritization

Each transition is assigned a priority score combining three components:

$$p_i = |\delta_i| + \lambda_{\text{logic}} \cdot v_i + \lambda_{\text{reward}} \cdot |r_i| \tag{7}$$

where:

- δ_i = TD-error: $r_i + \gamma Q(\mathbf{s}_{i+1}, a_{i+1}) - Q(\mathbf{s}_i, a_i)$
- v_i = logical violation score from LTN (higher = more constraint violations)
- r_i = reward magnitude (prioritize high-reward experiences)
- $\lambda_{\text{logic}} = 2.0$ (weight for constraint violations)
- $\lambda_{\text{reward}} = 0.5$ (weight for reward magnitude)

Rationale:

- High TD-error $\rightarrow$ surprising transitions that require learning
- High violation score $\rightarrow$ dangerous patterns to avoid
- High reward $\rightarrow$ successful strategies to reinforce

1.2.2 Sampling Probability

Transitions are sampled stochastically with probability proportional to priority:

$$P(i) = \frac{p_i^\alpha}{\sum_{j=1}^{|\mathcal{B}_{\text{STM}}|} p_j^\alpha} \tag{8}$$

where $\alpha \in [0, 1]$ controls prioritization strength:

- $\alpha = 0 \rightarrow$ uniform sampling
- $\alpha = 1 \rightarrow$ greedy prioritization
- $\alpha = 0.6 \rightarrow$ recommended default (balanced)

1.2.3 Importance Sampling Correction

To correct for sampling bias, apply importance-sampling weights:

$$w_i = \left(\frac{1}{|\mathcal{B}_{\text{STM}}|} \cdot \frac{1}{P(i)} \right)^\beta \tag{9}$$

where $\beta \in [0, 1]$ is annealed from 0.4 to 1.0 during training.

Normalized weights:

$$\bar{w}_i = \frac{w_i}{\max_j w_j} \tag{10}$$

Gradients are scaled by importance weights:

$$\nabla_\theta \mathcal{L}(\tau_i) \leftarrow \bar{w}_i \cdot \nabla_\theta \mathcal{L}(\tau_i) \tag{11}$$

1.2.4 Time Compression

During replay, transitions are replayed at $C\times$ speed to accelerate consolidation:

$$\Delta t_{\text{replay}} = \frac{\Delta t_{\text{original}}}{C} \tag{12}$$

where $C \in [10, 20]$ is the compression factor (typically $C = 15$).

Biological Motivation: Real hippocampal replay occurs at 10-20$\times$ speed during sleep.

1.2.5 SWR Replay Algorithm

Algorithm 1 Sharp Wave Ripple Replay

Require: Buffer $\mathcal{B}_{\text{STM}}$, compression factor C, batch size B, prioritization exponent α
Ensure: Replay batch $\mathcal{B}_{\text{replay}}$, importance weights $\mathbf{w}$
Compute TD-errors δ_i for all transitions
Compute logical violations v_i via LTN evaluation
Compute priorities $p_i = |\delta_i| + \lambda_{\text{logic}} \cdot v_i + \lambda_{\text{reward}} \cdot |r_i|$
Compute sampling probabilities $P(i) = p_i^\alpha / \sum_j p_j^\alpha$
Sample B transition indices with probabilities $P(i)$
Compute importance weights $w_i = (1/(|\mathcal{B}_{\text{STM}}| \cdot P(i)))^\beta$
Normalize weights $\bar{w}_i = w_i / \max_j w_j$
for each sampled transition $\tau_i = (\mathbf{s}_t, a_t, r_t, \mathbf{s}_{t+1})$ **do**
    Compress time: $\Delta t \leftarrow \Delta t / C$
    Add $(\tau_i, \bar{w}_i)$ to $\mathcal{B}_{\text{replay}}$
end for
return $\mathcal{B}_{\text{replay}}, \mathbf{w}$

1.3 Long-Term Memory (Neocortex)

The neocortical subsystem maintains abstract schemas—statistical summaries of recurring market patterns.

1.3.1 Schema Representation

Each schema k is represented as a Gaussian distribution:

$$\mathcal{N}(\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) \tag{13}$$

where:

$$\boldsymbol{\mu}_k = \frac{1}{|\mathcal{S}_k|} \sum_{\mathbf{s} \in \mathcal{S}_k} \mathbf{s} \tag{14}$$

$$\boldsymbol{\Sigma}_k = \frac{1}{|\mathcal{S}_k|} \sum_{\mathbf{s} \in \mathcal{S}_k} (\mathbf{s} - \boldsymbol{\mu}_k)(\mathbf{s} - \boldsymbol{\mu}_k)^\top \tag{15}$$

$\mathcal{S}_k$ is the set of all states assigned to schema k.

1.3.2 Schema Assignment

New states are assigned to schemas via maximum likelihood:

$$k^* = \arg\max_k \mathcal{N}(\mathbf{s}; \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) \tag{16}$$

If $\max_k \mathcal{N}(\mathbf{s}; \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) < \tau_{\text{schema}}$, create a new schema.

1.3.3 Recall-Gated Consolidation

Updates to long-term memory are gated by two factors: recall strength (familiarity) and logical validity.

Recall Strength:

$$g(r_{\text{STM}}(\tau)) = \text{sigmoid}(\mathbf{W}_r \mathbf{r}_{\text{STM}} + b_r) \tag{17}$$

where $\mathbf{r}_{\text{STM}}$ is the hippocampal representation of transition τ.

Logical Validity:

$$g_{\text{sym}}(\tau) = \text{SatAgg}(\mathcal{K}_{\text{episode}}) \tag{18}$$

where $\mathcal{K}_{\text{episode}}$ is the knowledge base evaluated on the episode containing τ.

Gated Update Rule:

$$\Delta \mathbf{W}_{\text{LTM}} = \eta_{\text{sleep}} \cdot g(r_{\text{STM}}(\tau)) \cdot g_{\text{sym}}(\tau) \cdot \nabla_{\mathbf{W}} \mathcal{L}_{\text{policy}}(\tau) \tag{19}$$

Only update if both gates exceed thresholds:

$$\text{Update if: } g(r_{\text{STM}}) > \tau_{\text{recall}} \text{ AND } g_{\text{sym}} > \tau_{\text{logic}} \tag{20}$$

Typical thresholds: $\tau_{\text{recall}} = 0.3$, $\tau_{\text{logic}} = 0.7$.

1.3.4 Consolidation Update Rule

For schema k, update mean and covariance:

$$\boldsymbol{\mu}_k^{(t+1)} = (1 - \eta_{\text{schema}})\boldsymbol{\mu}_k^{(t)} + \eta_{\text{schema}} \cdot \mathbf{s}_{\text{new}} \tag{21}$$

$$\boldsymbol{\Sigma}_k^{(t+1)} = (1 - \eta_{\text{schema}})\boldsymbol{\Sigma}_k^{(t)} + \eta_{\text{schema}} \cdot (\mathbf{s}_{\text{new}} - \boldsymbol{\mu}_k^{(t)})(\mathbf{s}_{\text{new}} - \boldsymbol{\mu}_k^{(t)})^\top \tag{22}$$

where η_{schema} is the schema learning rate (typically 0.01).

2 UMAP Visualization: Cognitive Dashboard

UMAP (Uniform Manifold Approximation and Projection) provides a real-time 3D visualization of the system's internal knowledge structure.

2.1 AlignedUMAP for Schema Formation

AlignedUMAP ensures consistency across multiple sleep cycles, enabling tracking of schema evolution over time.

2.1.1 Objective Function

AlignedUMAP minimizes:

$$\mathcal{L}_{\text{AlignedUMAP}} = \sum_{t=1}^{T_{\text{cycles}}} \left[\mathcal{L}_{\text{UMAP}}(\mathbf{X}_t) + \lambda_{\text{align}} \sum_{i,j} w_{ij} ||\mathbf{y}_i^{(t)} - \mathbf{y}_j^{(t-1)}||^2 \right] \tag{23}$$

where:

- $\mathbf{X}_t \in \mathbb{R}^{N_t \times d}$ = high-dimensional embeddings at sleep cycle t
- $\mathbf{y}_i^{(t)} \in \mathbb{R}^3$ = 3D projection of point i at cycle t
- w_{ij} = alignment weights (higher for points in same schema)
- $\lambda_{\text{align}} = 0.1$ = alignment strength

Standard UMAP Loss:

$$\mathcal{L}_{\text{UMAP}}(\mathbf{X}) = \sum_{i,j} \left[v_{ij} \log \frac{v_{ij}}{w_{ij}} + (1 - v_{ij}) \log \frac{1 - v_{ij}}{1 - w_{ij}} \right] \tag{24}$$

where v_{ij} is high-dimensional similarity and w_{ij} is low-dimensional similarity.

2.1.2 Alignment Weights

$$w_{ij} = \begin{cases} 1.0 & \text{if schema}(i) = \text{schema}(j) \\ 0.1 & \text{otherwise} \end{cases} \tag{25}$$

2.1.3 Schema Cluster Detection

Schemas are identified as dense clusters in UMAP space using DBSCAN:

$$\text{Schema}_k = \{\mathbf{y}_i : ||\mathbf{y}_i - \boldsymbol{\mu}_k|| < \tau_{\text{cluster}}\} \tag{26}$$

where τ_{cluster} is the cluster radius (typically 0.5 in normalized UMAP space).

2.2 Parametric UMAP for Real-Time Monitoring

Parametric UMAP learns a neural network mapping for fast projection of new points during live trading.

2.2.1 Neural Network Projection

A feedforward network $f_{\text{UMAP}} : \mathbb{R}^d \to \mathbb{R}^3$ is trained to approximate UMAP projection:

$$\mathbf{y} = f_{\text{UMAP}}(\mathbf{e}; \theta_{\text{UMAP}}) \tag{27}$$

Architecture:

$$\mathbf{h}_1 = \text{ReLU}(\mathbf{W}_1\mathbf{e} + \mathbf{b}_1), \quad \mathbf{h}_1 \in \mathbb{R}^{256} \tag{28}$$

$$\mathbf{h}_2 = \text{ReLU}(\mathbf{W}_2\mathbf{h}_1 + \mathbf{b}_2), \quad \mathbf{h}_2 \in \mathbb{R}^{128} \tag{29}$$

$$\mathbf{y} = \mathbf{W}_3\mathbf{h}_2 + \mathbf{b}_3, \quad \mathbf{y} \in \mathbb{R}^3 \tag{30}$$

Training Objective:

$$\mathcal{L}_{\text{ParametricUMAP}} = \sum_i ||\mathbf{y}_i - f_{\text{UMAP}}(\mathbf{e}_i)||^2 + \mathcal{L}_{\text{UMAP}} \tag{31}$$

2.2.2 Anomaly Detection

During live trading, a point is flagged as anomalous if it falls outside all known schemas:

$$\text{Anomaly}(\mathbf{y}) = \mathbb{1}\left[\min_k ||\mathbf{y} - \boldsymbol{\mu}_k|| > \tau_{\text{anomaly}}\right] \tag{32}$$

where $\tau_{\text{anomaly}} = 2.0$ (units in UMAP space).

Response to Anomalies:

- Log anomaly with full context
- Increase risk threshold temporarily
- Alert human operator if anomaly persists
- Add to high-priority replay buffer

3 Integration with Vector Database (Qdrant)

Long-term memory schemas are persisted in Qdrant for efficient similarity search and retrieval.

3.1 Schema Storage

Each schema is stored as a point in Qdrant:

- **Vector:** $\boldsymbol{\mu}_k \in \mathbb{R}^d$ (schema centroid)

- **Payload:**
  - `schema_id`: Unique identifier
  - `covariance`: Flattened Σ_k
  - `num_points`: $|\mathcal{S}_k|$
  - `avg_reward`: Mean reward for transitions in schema
  - `created_at`: Timestamp
  - `last_updated`: Timestamp

3.2 Similarity Search

Given a new state $\mathbf{s}_{\text{new}}$, retrieve top-k similar schemas:

$$\text{TopK}(\mathbf{s}_{\text{new}}) = \arg\max_{k,|K|=k} \{\text{cosine}(\mathbf{s}_{\text{new}}, \boldsymbol{\mu}_i)\}_{i=1}^{N_{\text{schemas}}} \tag{33}$$

Use Cases:

- Retrieve historical context during decision-making
- Find similar market conditions for transfer learning
- Identify schema membership for new states

3.3 Periodic Schema Pruning

Remove low-quality schemas to prevent memory bloat:

$$\text{Prune if: } |\mathcal{S}_k| < \tau_{\text{min\_points}} \text{ OR age}(k) > \tau_{\text{max\_age}} \tag{34}$$

where $\tau_{\text{min\_points}} = 10$ and $\tau_{\text{max\_age}} = 90$ days.

4 Sleep Cycle: Complete Algorithm

The sleep cycle runs nightly (or after market close) to consolidate the day's experiences.

Algorithm 2 JANUS Backward Sleep Cycle

Require: Short-term buffer $\mathcal{B}_{\text{STM}}$, long-term schemas $\{\mathcal{N}(\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)\}_k$

Ensure: Updated schemas, trained policy

Phase 1: Prioritized Replay (SWR Simulation)
for n_{replays} iterations (e.g., 1000) **do**
  Sample batch $\mathcal{B}_{\text{replay}}$ using SWR algorithm
  Compute losses: $\mathcal{L}_{\text{policy}}, \mathcal{L}_{\text{logic}}$
  Update policy: $\theta \leftarrow \theta - \eta \cdot \bar{w}_i \cdot \nabla_\theta \mathcal{L}_{\text{total}}$
  Update priorities: $p_i \leftarrow |\delta_i| + \lambda_{\text{logic}} v_i + \lambda_{\text{reward}} |r_i|$
end for

Phase 2: Schema Consolidation
for each transition $\tau_i \in \mathcal{B}_{\text{STM}}$ **do**
  Compute recall gate: $g_{\text{recall}} = \text{sigmoid}(\mathbf{W}_r \mathbf{r}_{\text{STM}} + b_r)$
  Compute logic gate: $g_{\text{logic}} = \text{SatAgg}(\mathcal{K})$
  if $g_{\text{recall}} > \tau_{\text{recall}}$ AND $g_{\text{logic}} > \tau_{\text{logic}}$ **then**
    Find matching schema: $k^* = \arg\max_k \mathcal{N}(\mathbf{s}_i; \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$
    if $\mathcal{N}(\mathbf{s}_i; \boldsymbol{\mu}_{k^*}, \boldsymbol{\Sigma}_{k^*}) < \tau_{\text{schema}}$ **then**
      Create new schema: $\boldsymbol{\mu}_{\text{new}} \leftarrow \mathbf{s}_i$, $\boldsymbol{\Sigma}_{\text{new}} \leftarrow \epsilon \mathbf{I}$
    else
      Update schema k^* using consolidation rule
    end if
  end if
end for

Phase 3: UMAP Update
Extract all schema centroids: $\{\boldsymbol{\mu}_k\}_k$
Fit AlignedUMAP with previous cycle alignment
Update parametric UMAP network
Detect new clusters via DBSCAN

Phase 4: Vector Database Sync
Upsert updated schemas to Qdrant
Prune low-quality schemas
Create snapshot for recovery

Phase 5: Metrics & Logging
Compute and log:

- Number of schemas: N_{schemas}
- Mean schema size: $\mathbb{E}[|\mathcal{S}_k|]$
- Constraint satisfaction rate: $\mathbb{E}[\text{SatAgg}(\mathcal{K})]$
- Average TD-error improvement

5 Implementation Checklist

sec:checklist

This section provides a sequential checklist for implementing JANUS Backward.

5.1 Core Components

1. **Short-Term Memory (Hippocampus)**
   - ☐ Implement episodic buffer with FIFO eviction
   - ☐ Implement pattern separation layer
   - ☐ Add orthogonality regularization
   - ☐ Implement TopK sparse encoding
   - ☐ Add timestamp indexing for temporal queries
   - ☐ Test buffer operations (insert, retrieve, evict)
2. **Sharp Wave Ripple (SWR) Simulator**
   - ☐ Implement TD-error computation
   - ☐ Implement logical violation scoring via LTN
   - ☐ Implement composite priority function
   - ☐ Implement prioritized sampling with importance weights
   - ☐ Add time compression simulation
   - ☐ Test replay batch generation
   - ☐ Validate importance weight correction
3. **Long-Term Memory (Neocortex)**
   - ☐ Implement schema representation (Gaussian)
   - ☐ Implement schema assignment via maximum likelihood
   - ☐ Implement recall gate computation
   - ☐ Implement logical validity gate
   - ☐ Implement gated consolidation update
   - ☐ Add schema creation logic
   - ☐ Test schema updates with edge cases
4. **UMAP Visualization**

- ☐ Implement AlignedUMAP objective
- ☐ Add alignment weight computation
- ☐ Implement parametric UMAP network
- ☐ Implement DBSCAN cluster detection
- ☐ Add anomaly detection logic
- ☐ Test visualization updates across cycles
- ☐ Validate cluster stability

5.2 Integration & Storage

1. **Vector Database Integration (Qdrant)**
   - ☐ Set up Qdrant connection
   - ☐ Define schema collection structure
   - ☐ Implement schema upsert operations
   - ☐ Implement similarity search queries
   - ☐ Add periodic pruning logic
   - ☐ Implement backup/restore functionality
   - ☐ Test concurrent access patterns
2. **Sleep Cycle Orchestration**
   - ☐ Implement 5-phase sleep cycle algorithm
   - ☐ Add progress tracking and logging
   - ☐ Implement graceful shutdown on errors
   - ☐ Add checkpoint/resume capability
   - ☐ Test full sleep cycle end-to-end
   - ☐ Validate schema evolution over cycles

5.3 Monitoring & Debugging

1. **Metrics Collection**
   - ☐ Track number of schemas over time
   - ☐ Monitor mean schema size
   - ☐ Track constraint satisfaction rates

- ☐ Monitor TD-error distribution
- ☐ Track UMAP cluster count
- ☐ Log replay batch statistics

2. **Visualization & Debugging**

- ☐ Export UMAP projections for visualization
- ☐ Add schema evolution timeline
- ☐ Visualize priority distributions
- ☐ Plot constraint satisfaction heatmaps
- ☐ Add interactive schema browser

5.4 Performance Optimization

1. **Batch Processing**

- ☐ Parallelize TD-error computation
- ☐ Vectorize schema likelihood calculations
- ☐ Batch Qdrant upsert operations
- ☐ Use GPU for UMAP fitting (if available)
- ☐ Profile and optimize bottlenecks
- ☐ Target: <10 minutes for 10k transitions

6 Rust Implementation Considerations

sec:rust

6.1 Cold Path Optimization

Unlike Forward, Backward has no strict latency requirements, allowing focus on throughput and correctness.

- **Batch parallelism:** Use `rayon` for parallel replay processing
- **Memory efficiency:** Use `ndarray` for linear algebra operations
- **Async I/O:** Use `tokio` for non-blocking Qdrant operations
- **Checkpointing:** Serialize intermediate state with `serde`

6.2 Data Structures

6.2.1 Episodic Buffer

```rust
use std::collections::VecDeque;

#[derive(Clone, Debug)]
pub struct Transition {
    pub state: Array1<f32>,
    pub action: Action,
    pub reward: f32,
    pub next_state: Array1<f32>,
    pub timestamp: u64,
}

pub struct EpisodicBuffer {
    buffer: VecDeque<Transition>,
    capacity: usize,
}

impl EpisodicBuffer {
    pub fn new(capacity: usize) -> Self {
        Self {
            buffer: VecDeque::with_capacity(capacity),
            capacity,
        }
    }

    pub fn push(&mut self, transition: Transition) {
        if self.buffer.len() >= self.capacity {
            self.buffer.pop_front();
        }
        self.buffer.push_back(transition);
    }

    pub fn sample_prioritized(
        &self,
        priorities: &[f32],
        batch_size: usize,
        alpha: f32,
    ) -> (Vec<Transition>, Vec<f32>) {
        // Prioritized sampling implementation
```

```
            todo!()
        }
}
```

6.2.2 Schema Representation

```
use ndarray::{Array1, Array2};

#[derive(Clone, Debug, serde::Serialize, serde::Deserialize)]
pub struct Schema {
    pub id: uuid::Uuid,
    pub mean: Array1<f32>,
    pub covariance: Array2<f32>,
    pub num_points: usize,
    pub avg_reward: f32,
    pub created_at: chrono::DateTime<chrono::Utc>,
    pub last_updated: chrono::DateTime<chrono::Utc>,
}

impl Schema {
    pub fn likelihood(&self, state: &Array1<f32>) -> f32 {
        // Compute Gaussian likelihood
        let diff = state - &self.mean;
        let inv_cov = self.covariance.inv().unwrap();
        let exponent = -0.5 * diff.dot(&inv_cov.dot(&diff));
        exponent.exp()
    }

    pub fn update(
        &mut self,
        new_state: &Array1<f32>,
        learning_rate: f32,
    ) {
        let diff = new_state - &self.mean;
        self.mean = &self.mean + learning_rate * &diff;
        // Update covariance (outer product)
        let outer = diff.clone().insert_axis(Axis(1))
            .dot(&diff.clone().insert_axis(Axis(0)));
        self.covariance = (1.0 - learning_rate) * &self.covariance
            + learning_rate * outer;
        self.num_points += 1;
```

```
        self.last_updated = chrono::Utc::now();
    }
}
```

6.3 Error Handling

```
#[derive(Debug, thiserror::Error)]
pub enum BackwardError {
    #[error("Insufficient data for replay: {0} transitions")]
    InsufficientData(usize),

    #[error("Schema update failed: {0}")]
    SchemaUpdateError(String),

    #[error("UMAP fitting failed: {0}")]
    UmapError(String),

    #[error("Qdrant operation failed: {0}")]
    VectorDbError(#[from] qdrant_client::QdrantError),

    #[error("Linear algebra error: {0}")]
    LinalgError(String),
}

pub type BackwardResult<T> = Result<T, BackwardError>;
```

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